Group			
Schönflies	Abstract		
K_h K $D_{\infty h}$ D_{∞} $C_{\infty v}$ $C_{\infty h}$ C_{∞}	$\begin{array}{c} {\rm O}(3) \\ {\rm SO}(3) \\ {\rm O}(2) \times {\rm Z}_2 \\ {\rm O}(2) \\ {\rm O}(2) \\ {\rm SO}(2) \times {\rm Z}_2 \\ {\rm SO}(2) \end{array}$		

Continu	ous group	Finite	group
Schönflies	Abstract	Schönflies	Abstract
K_h	O(3)		
		I_h	$A_5 \times Z_2$
		$O_h \ T_h$	$S_4 \times Z_2$ $A_4 \times Z_2$
		T_d	S_4
K	SO(3)		
		I	A_5
		$O \ T$	$egin{array}{c} \mathrm{S}_4 \ \mathrm{A}_4 \end{array}$
$D_{\infty h}$	$O(2) \times Z_2$	D	Dil 7
		$D_{nh} \\ D_{nd}$	$Dih_n \times Z_2$ Dih_{2n}
		D _{nd}	D1112n
D_{∞}	O(2)	D	Dil
		D_n	Dih_n
$C_{\infty v}$	O(2)		
		C_{nv}	Dih_n
$C_{\infty h}$	$SO(2) \times Z_2$		
		C_{nh}	$\mathbf{Z}_n \times \mathbf{Z}_2$
		S_{2n}	\mathbf{Z}_{2n}
C_{∞}	SO(2)		
		C_n	Z_n

Group				
Schönflies	Abstract			
$K_h \\ I_h \\ O_h \\ T_h$	$\begin{array}{c} O(3) \\ A_5 \times Z_2 \\ S_4 \times Z_2 \\ A_4 \times Z_2 \end{array}$			
K I O T_d T	$SO(3)$ A_5 S_4 S_4 A_4			
$D_{\infty h} \\ D_{nh} \\ D_{nd}$	$O(2) \times Z_2$ $Dih_n \times Z_2$ Dih_{2n}			
$D_{\infty} \\ D_{n}$	$O(2)$ Dih_n			
$C_{\infty v} \\ C_{nv}$	$O(2)$ Dih_n			
$C_{\infty h} \\ C_{nh} \\ S_{2n}$	$SO(2) \times Z_2$ $Z_n \times Z_2$ Z_{2n}			
$C_{\infty} \\ C_{n}$	$SO(2)$ Z_n			

Number of atoms	Possible groups (not including subgroups)
1 2 3 4	$K_h \\ D_{\infty h}, C_{\infty v} \\ D_{\infty h}, C_{\infty v}, D_{3h}, C_{2v}, C_s \\ D_{\infty h}, C_{\infty v}, D_{3h}, C_{2v}, C_s, T_d, D_{4h}, D_{2h}, D_{2d}, D_2, C_{3v}, C_{2h}, C_2, C_1$

Number of atoms	Possible groups (not including subgroups)	
1 2 3 4	$K_h \\ D_{\infty h}, C_{\infty v} \\ D_{\infty h}, C_{\infty v}, D_{3h}, C_{2v}, C_s \\ D_{\infty h}, C_{\infty v}, D_{3h}, C_{2v}, C_s, T_d, D_{4h}, D_{2h}, D_{2d}, D_2, C_{3v}, C_{2h}, C_2, C_1$	

N Possible groups (not including subgroups)

```
1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  K_h
 2
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      D_{\infty h} C_{\infty v}
3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    D_{3h}
                                 C_s
                                                                                                                                                                                                                                                                                C_{2v}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      D_{\infty h} C_{\infty v}
                                                                                                                                                                                                       C_{2h}
  4
                                 C_s
                                                                                                                                                                                                                                                                               C_{2v} C_{3v}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        D_2
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          D_{2h} D_{3h} D_{4h}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    D_{\infty h} C_{\infty v}
                                                                                                       C_1 C_2
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          D_{2d}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 T_d
5
                                                                                                                                                                                                                                                                               C_{2v} C_{3v} C_{4v}
                                                                                                                                                                                                       C_{2h}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      D_2
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          D_{2d}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           D_{2h} D_{3h} D_{4h} D_{5h}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 T_d
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       D_{\infty h} C_{\infty v}
                                                                                                     C_1 C_2
                               C_s C_i C_1 C_2 C_3 C_{2h} C_{3h} C_{2v} C_{3v} C_{4v} C_{5v}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   D_2 D_3 D_{2d} D_{3d} D_{2h} D_{3h} D_{4h} D_{5h} D_{6h}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               T_d O_h D_{\infty h} C_{\infty v}
                                 C_s \quad C_i \quad C_1 \quad C_2 \quad C_3 \quad C_{2h} \quad C_{3h} \quad C_{2v} \quad C_{3v} \quad C_{4v} \quad C_{5v} \quad C_{6v} \quad D_2 \quad D_3 \quad D_{2d} \quad D_{3d} \quad D_{2h} \quad D_{3h} \quad D_{4h} \quad D_{5h} \quad D_{6h} \quad D_{7h} \quad T_d \quad O_h \quad D_{\infty h} \quad C_{\infty v} \quad D_{5h} \quad
```

N Possible groups (not including subgroups)

1					
2					$D_{\infty h}$ $C_{\infty e}$
3	C_s		C_{2v}		D_{3h} $D_{\infty h}$ $C_{\infty h}$
4	C_s	C_1 C_2 C_{2h}	C_{2v} C_{3v}	D_2	D_{2d} D_{2h} D_{3h} D_{4h} T_d $D_{\infty h}$ $C_{\infty h}$
5	C_s	C_1 C_2 C_{2h}	C_{2v} C_{3v} C_{4v}	D_2	D_{2d} D_{2h} D_{3h} D_{4h} D_{5h} T_d $D_{\infty h}$ $C_{\infty h}$
6	C_s C_i	C_1 C_2 C_3 C_{2h} C_{3h}	C_{2v} C_{3v} C_{4v} C_{5v}	D_2 D_3	D_3 D_{2d} D_{3d} D_{2h} D_{3h} D_{4h} D_{5h} D_{6h} D_{6h} T_d O_h $D_{\infty h}$ $C_{\infty h}$
7	C_s C_i	C_1 C_2 C_3 C_{2h} C_{3h}	$C_{2v} \ C_{3v} \ C_{4v} \ C_{5v} \ C_{6v}$	D_2 D_3	D_{3} D_{2d} D_{3d} D_{2h} D_{3h} D_{4h} D_{5h} D_{6h} D_{7h} T_{d} O_{h} $D_{\infty h}$ $C_{\infty h}$
8	C_s C_i	C_1 C_2 C_3 C_4 C_{2h} C_{3h}	$C_{4h} \ C_{2v} \ C_{3v} \ C_{4v} \ C_{5v} \ C_{6v} \ C_{6v}$	$_{7v}$ D_2	D_{2d} D_{3d} D_{2h} D_{3h} D_{4h} D_{5h} D_{6h} D_{7h} T_d O_h $D_{\infty h}$ $C_{\infty h}$

Point groups	Possible number of a	toms
K_h	1	1
$D_{\infty h}$	$2,3,4,\dots$	$n \ge 2$
$C_{\infty v}$	$2,3,4,\ldots$	$n \ge 2$
D_{nd}	$2n, 2n+1, 2n+2, \dots, n \ge 2$	$2n, n \ge 2$
D_{nh}	$n, n+1, n+2, \dots, n \ge 3$	$n, n \ge 3$
D_{2h}	$4,5,6,\ldots$	$n \ge 4$
D_n	$2n, 2n+1, 2n+2, \dots, n \ge 2$	$2n, n \ge 2$
S_{2n}	$4n, 4n + 1, 4n + 2, n \ge 2$	$4n, n \ge 2$
C_{nv}	$n+1, n+2, n+3\dots$	$n \ge 2$
C_{nh}	$2n, 2n+1, 2n+2, \dots$	$n \ge 2$
C_s	$3,4,5,\ldots$	$n \ge 3$
C_{i}	$6,7,8,\ldots$	$n \ge 6$
C_n	$2n, 2n+1, 2n+2m \dots$	$2n, n \ge 2$
C_1	$4,5,6,\ldots$	$n \ge 4$
T_d	$4, 5, 8, 9, 12, 13, \dots$	4n, 4n + 1
T_h	$12, 13, 24, 25, 36, 37 \dots$	12n, 12n + 1
T	12, 13, 24, 25, 36, 37	12n, 12n + 1
O_h	$6, 7, 12, 13, 18, 19 \dots$	6n, 6n + 1
O	24, 25, 48, 49, 72, 73	24n, 24n + 1
I_h	12, 13, 24, 25, 36, 37	12n, 12n + 1
I	$60, 61, 120, 121, 180, 181, \dots$	60n, 60n + 1

Point groups	Minim	um number of atoms
K_h	1	
$D_{\infty h}$	2	
$C_{\infty v}$	2	
D_{nd}	2n	$n \ge 2$
D_{nh}	n	$n \ge 3$
D_{2h}	4	
D_n	2n	$n \ge 2$
S_{2n}	4n	$n \ge 2$
C_{nv}	n+1	$n \ge 2$
C_{nh}	2n	$n \ge 2$
C_s	3	
C_{i}	6	
C_n	2n	$n \ge 2$
C_1	4	
T_d	4	
T_h	12	
T	12	
O_h	6	
Ö	24	
I_h	12	
I	60	

Point groups			Num	ber c	of ato	ms				
		Minimum	1	2	3	4	5	6	7	8
K_h		1	1	X	X	X	X	X	X	X
$D_{\infty h}$		2	X	/	/	/	/	/	/	/
$C_{\infty v}$		2	X	/	/	/	/	/	/	/
	$n \ge 3$	n	X	X	/	/	/	/	/	/
D_{nh}	n = 2	4	X	X	X	1	1	1	1	1
D_{nd}	$n \ge 2$	2n	X	X	X	1	1	1	1	1
D_n	$n \ge 2$	2n	X	X	X	1	1	1	1	/
S_{2n}	$n \ge 2$	4n	X	X	X	X	X	X	X	/
C_{nv}	$n \ge 2$	n+1	X	X	/	1	/	1	/	/
C_{nh}	$n \ge 2$	2n	X	X	X	1	/	1	/	/
C_n	$n \ge 2$	2n	X	X	X	1	/	1	/	/
C_s		3	X	X	1	/	1	/	1	/
C_i		6	X	X	X	X	X	/	1	/
C_1		4	X	X	X	/	/	/	/	✓

Possible point groups for 3-atom (triatomic) molecules

Point group	Chemical formula type		
	A_3	A_2B	ABC
$ \begin{array}{c} D_{\infty h} \\ C_{\infty v} \\ D_{3h} \\ C_{2v} \\ C_{s} \end{array} $	C_3 Unlikely H_3^+ O_3 Unlikely	CO ₂ N ₂ O X H ₂ O HO ₂	HOS X HNO

Possible point groups for 4-atom (tetraatomic) molecules

Point group	Che	emical form	ula type		
	${ m A}_4$	A_3B	A_2B_2	A_2BC	ABCD
T_d	P_4	X	X	×	X
$D_{\infty h}$		×	C_2H_2	X	×
$C_{\infty v}$	Unlikely			H_2BN	
D_{4h}	S_4^{2+}	X	×	X	×
D_{3h}		H_3B	X	X	X
D_{2h}		Unlikely	$\mathrm{Br_2Na_2}$	X	X
D_{2d}	Possible but unknown	X	X	X	×
D_2	Possible but unknown	X	X	X	×
C_{3v}		$_{ m H_3N}$	X	X	X
C_{2v}			H_2Si_2	X	X
C_{2h}		Unlikely	H_2N_2	X	X
C_2		Unlikely	H_2O_2	X	X
C_s		v		Cl_2OS	
C_1				H_2OS	

Impossible point groups for 3-atom (triatomic) molecules

Reason for elimination	Point groups
Absence of σ planes	$C_n, S_{2n}D_n, T, O, I$
Presence of inversion center Presence of a C_{n+3} axis with $n \ge 1$ Presence of S_4 axis Presence of σ_d plane Not enough atoms	$C_{i}, C_{(2n)h}, S_{4n+2}, D_{(2n)h}, D_{(2n)d+1}, \\ C_{n+3}, C_{(n+3)v}, C_{(n+3)h}, D_{(n+3)}, D_{(n+3)h}, D_{(n+3)d}, S_{2n+6}, O, O_{h} \\ S_{4} \\ D_{(2n)d} \\ C_{i}, C_{n+3}, C_{(n+3)v}, C_{(n+3)h}, D_{(n+3)}, D_{(n+3)h}, D_{(n+3)d}, S_{2n+6}, T, O, T_{h}, T_{d}, O_{h}, I, I_{h}$

 C_n

 S_{2n}

 D_n

Any three points form a plane, so any triatomic molecule will have a mirror-plane formed by the three atoms. Since all these point groups do not contain any mirror-plane elements, it is not possible for any of these point groups to be the "highest" point group for a triatomic molecule. For example, the existence of a mirror-plane will promote the molecule from C_1 to the higher point group C_s .

 C_i

 $C_{2h}, C_{4h}, C_{6h}, \dots$

 $D_{2h}, D_{4h}, D_{6h}, \dots$

 $D_{3d}, D_{5d}, D_{7d}, \dots$

If we choose the origin (0,0,0) to be the center of inversion, and one atom is at the position (x,y,z), then an identical atom would need to be at the position (-x,-y,-z) and the third atom would need to be at the origin. We therefore have two identical atoms on a line going through the origin, and a third atom in the center of that line, which would promote the molecule to $D_{\infty h}$ (the "highest" possible symmetry group for a system with more than one atom). Furthermore, any point group that contains an inversion element, will for the same reason either not be possible for a triatomic molecule, or will be promoted to $D_{\infty h}$ from which no further promotion is possible for a system with more than one atom. For each point group that contians an inversion element, we will refer back to the proof in this section.

 D_{2d}

 $C_{3h}, C_{5h}, C_{7h}, \dots$

 $C_{3v}, C_{4v}, C_{5v}, \dots$

For a molecule to have a C_n axis with $n \geq 3$, the molecule must contain a subset of atoms that form a regular n-sided polygon (e.g. an equilateral triangle, a square, a regular pentagon, etc.). For a triatomic molecule, this would not be possible for n > 3, which makes $C_{5h}, C_{7h}, C_{9h}, \ldots$ and $C_{4v}, C_{5v}, C_{6v} \ldots$ impossible; and for n = 3 a C_n axis would be possible (only) for an equilateral triangle, but an equilateral triangle would have a C_3 axis going through the center and perpendicular to the plane formed by the molecule, and also three more C_3 axes in the plane of the molecule and each passing through one of the atoms/vertices, hence promoting the molecule's point group to at least D_3 (in fact D_{3h} due to the horizontal mirror-plane formed by the three atoms) and making it impossible for a molecule's highest point group to be C_{3h} or C_{3v} . The S_{4+4n} point groups all have a C_4 element.

 S_4

 $C_{nv}, n \geq 3$

A minimum of n+1 atoms is needed, so for n=3 we would need 4 atoms, and for n>3 we would need even more.

 D_{2h}

Proof still needed

 $D_{nh}, n \geq 4$

A minimum of n atoms is needed, so for n = 4 we would need 4 atoms, and for n > 4 we would need even more.

C_1

Any three points form a plane, so any triatomic molecule will have a mirror-plane formed by the three atoms. The existence of a mirror-plane will promote the molecule to C_s .

C_1

Any three points form a plane, so any triatomic molecule will have a mirror-plane formed by the three atoms. The existence of a mirror-plane will promote the molecule to C_s .

C_1

Any three points form a plane, so any triatomic molecule will have a mirror-plane formed by the three atoms. The existence of a mirror-plane will promote the molecule to C_s .

C_1

Any three points form a plane, so any triatomic molecule will have a mirror-plane formed by the three atoms. The existence of a mirror-plane will promote the molecule to C_s .

C_1

Any three points form a plane, so any triatomic molecule will have a mirror-plane formed by the three atoms. The existence of a mirror-plane will promote the molecule to C_s .

Structure	Point group	Graph	Formula	Dimensionality	Example	Dihedrals	Diagram
1	T_d	K_4	${ m A}_4$	Pyramidal	P_4	$\cos^{-1}\left(\frac{1}{3}\right)$	
2	C_{3v}	S_3	A_3B	Pyramidal	${ m H_3N}$	$\cos^{-1}\left(\frac{\cos\theta(1-\cos\theta)}{\sin^2\theta}\right)$	
3	C_s	S_3	A_2BC	Pyramidal	$\mathrm{Cl_2OS}$	$\cos^{-1}\left(\frac{\cos\theta - \cos^2\varphi}{\sin^2\varphi}\right)$	
4	D_{2h}	$K_4 - e$	A_2B_2	Planar	$\mathrm{Cl}_2\mathrm{Cu}_2$	0 or 180	Cu Ci Cu
5	C_{2v}	$K_4 - e$	A_2B_2	Pyramidal	$\mathrm{H_2Si_2}$	$\cos^{-1}\left(\frac{\cos\theta-\cos^2\varphi}{\sin^2\varphi}\right)$	
6	C_2	P_4	A_2B_2	Pyramidal	$\mathrm{H_2O_2}$	$\cos^{-1}\left(\frac{\cos\theta - \cos^2\varphi}{\sin^2\varphi}\right)$	
7	C_1	P_4	A_2BC	Pyramidal	${ m H_2OS}$	$\cos^{-1}\left(\frac{\cos\theta - \cos\varphi\cos\phi}{\sin\varphi\sin\phi}\right)$	\$ — ®
8	D_{3h}	S_3	A_3B	Planar	BH_3	0 or 180	51 B S2
9	C_s	P_4	A_2BC	Planar	HNSi_2	0 or 180	61 62
10	D_{2h}	C_4	A_2B_2	Planar	$\mathrm{Br_2Na_2}$	0 or 180	•
11	C_{2v}	P_4	A_2B_2	Planar	O_2S_2	0 or 180	
12	C_{2v}	S_3	A_2BC	Planar	CFO_2	0 or 180	
13	$D_{\infty h}$	P_4	A_2B_2	Linear	C_2H_2	0 or 180	
14	$C_{\infty v}$	P_4	A_2BC	Linear	C_2AuH	0 or 180	Au - C - C - C
15	C_s	$T_{3,1}$	A_2B_2	Planar	$\mathrm{H_{2}Si_{2}}$	0 or 180	H(1) S(1) S(2)
16	C_s	S_3	ABCD	Planar	CBrFO	0 or 180	P C Br
17	C_{2v}	$K_4 - e$	A_3B	Planar	C_3Si	0 or 180	S C(2)
18	C_{2v}	$K_4 - e$	A_3B	Planar	C_3Si	0 or 180	G(1)

Structure	Point group	Graph	Formula	Dimensionality	Example	Dihedrals	Diagram
19	C_{3v}	K_4	A_3B	Pyramidal	AsP_3		
20	C_{2h}	P_4	A_2B_2	Planar	$\mathrm{H_2N_2}$		
21	C_s	S_3	A_2B_2	Pyramidal	$\mathrm{S}_2\mathrm{F}_2$	$\cos^{-1}\left(\frac{\cos\theta - \cos^2\varphi}{\sin^2\varphi}\right)$	
22	C_{2v}	$T_{3,1}$	A_3B	Planar	C_3H	0 or 180	Å
23	C_{2v}	C_4	ABC_2	Planar	$CsNO_2$	0 or 180	•

Shape 1: P_4

P-P bond length

Information missing

 $\begin{array}{ll} \theta & \text{P-P-P bond angle} \\ \varphi & \text{P-P-P-P dihedral angle} \end{array}$

Since all faces of a regular tetrahedron are equilateral triangles, $\theta = 60^{\circ}$. The dihedral angles φ in a regular tetrahedron are given by $\cos^{-1}\left(\frac{1}{3}\right)$. We therefore have the following z-matrix:

 $\begin{array}{|c|c|c|c|c|c|c|} \hline P & & & & & \\ P & 1 & r & & & \\ P & 1 & r & 2 & \theta_3 & & \\ P & 1 & r & 2 & \theta_3 & 3 & \varphi \\ \hline \end{array}$

Shape 2: NH₃

Information provided

 r_1 N-H bond length

 θ_1 H-N-H bond angle

Information missing

 r_2 H-H distance

 θ_2 H-H-H bond angle

 θ_3 H-H-N bond angle

 φ_i Dihedral angles between various pairs of planes

Since the three H atoms form an equilateral triangle, $\theta_2 = 60^{\circ}$. Also, (at least some of) the dihedral angles φ are given by the following simplified form for the dihedral law of cosines:

$$\varphi = \cos^{-1} \left(\frac{\cos \theta \left(1 - \cos \theta \right)}{\sin^2 \theta} \right). \tag{1}$$

We therefore have the following z-matrix:

 $\begin{array}{|c|c|c|c|c|c|} \hline \textbf{N} & & & & & \\ \textbf{H} & 1 & r_1 & & & \\ \textbf{H} & 1 & r_1 & 2 & \theta_1 & & \\ \textbf{H} & 1 & r_1 & 2 & \theta_1 & 3 & \varphi \\ \hline \end{array}$

Shape 3: Cl₂OS

	Information provided						
r_1	S-O bond length						
r_2	S-Cl bond length						
$ heta_1$	Cl-S-O bond angle						
θ_2	Cl-S-Cl bond angle						
φ	Cl-S-O-Cl dihedral angle						
Information missing							
r_3	Cl-O distance						
θ_3							
θ_4							
θ_5							
φ_i	Dihedral angles between various other pairs of planes						

They gave the dihedral angle but didn't need to, because it's exactly what we would get from the dihedral law of cosines. The missing geometric information is provided below:

$$r_3 = \sqrt{r_1^2 + r_2^2 - 2\cos\theta_1} \tag{2}$$

Shape 4: Cl₂Cu₂

We need to first undersated why one pair of atoms has a dashed line between them and the other pair does not.

	Information provided
$r_1 \\ \theta_1$	
	Information missing
r_2 θ_2 φ_i	Dihedral angles between various pairs of planes

We therefore have the following z-matrix:

$$\begin{array}{|c|c|c|c|c|c|c|} \hline \mathbb{N} & & & & & \\ \mathbb{H} & 1 & r_2 & & & \\ \mathbb{H} & 1 & r_2 & 2 & \theta_1 & \\ \mathbb{H} & 1 & r_2 & 2 & \theta_1 & 3 & \varphi \\ \hline \end{array}$$

Shape 5: H_2Si_2

T f 1 :	
Information	provided

- r_1 Si-Si bond length
- r_2 Si-H bond length
- φ Dihedral angle between two Si-Si-H planes

Information missing

- r_3 H-H distance
- θ_1 H-Si-H bond angle
- θ_2 Si-H-Si bond angle
- θ_3 Si-Si-H bond angle
- θ_4 H-H-Si angle
- φ_i Dihedral angles between various other pairs of planes

For a z-matrix, in addition to the information provided we would need at least one planar angle, despite none being provided from the experimental paper. The missing geometric information can be provided based on the information provided from the experimental paper though:

$$r_3 = \sin\frac{\varphi}{2}\sqrt{4r_2^2 - r_1^2},\tag{3}$$

$$\theta_1 = \cos^{-1} \left(\frac{\cos \varphi \left(4r_1^2 - r_2^2 \right) + r_2^2}{4r_1^2} \right),\tag{4}$$

$$\theta_2 = \cos^{-1}\left(1 - \frac{1}{2}\left(\frac{r_1}{r_2}\right)^2\right),\tag{5}$$

$$\theta_3 = \cos^{-1}\left(\frac{r_1}{2r_2}\right),\tag{6}$$

$$\theta_4 = \frac{\pi - \theta_1}{2}.\tag{7}$$

An alternative formula for θ_1 is:

$$\theta_1 = \sin^{-1}\left(\frac{\sin\left(\frac{\varphi}{2}\right)\sqrt{4r_2^2 - r_1^2}}{r_2}\right). \tag{8}$$

For the first column of the z-matrix, we have 6 possibilities which are listed below along with the possible planar angles that could be used for each of these possibilities:

This means that if we know θ_3 or θ_4 then we have enough to complete the planar angles column of the z-matrix, but if we only know θ_1 or θ_2 , we would need to determine two of the angles rather than one. Since θ_3 is a "bond angle" in the original reference and θ_4 is not, we will present a formula for θ_3 :

We can now write a z-matrix. Since the first option in the above table will lead to usage of r_1 and r_2 in lexicographical order (these bond angles are presented as they were in Landolt-Bornstein), we will use that option:

Shape 6: H_2O_2

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ed	provid	mation	Infor
\mathbf{e}	DIOVIG	шалюп	ппп

- r_1 H-O bond length
- r_2 O-O bond length
- θ_1 H-O-O bond angle
- φ H-O-O-H dihedral angle

Information missing

- r_3 H...H distance
- r_4 O...H distance
- θ_2 H-H-B bond angle
- θ_3 H-H-H angle
- φ_i Dihedral angles between various pairs of planes

We therefore have the following z-matrix:

Shape 7: H_2OS

Peilin's molecule

Information provided

- r_1 H-O bond length
- r_2 O-S bond length
- r_3 S-H bond length
- θ_1 H-O-S bond angle
- θ_2 H-S-O bond angle

Information missing

- r_4 H...H distance
- r_5 H...O distance
- r_6 H...S distance
- θ_3 H...H-O angle
- θ_4 H...H...O angle
- θ_5 H-O...H angle
- θ_6 $\,$ H... O-S angle
- θ_7 H-S...H angle
- θ_8 H...S-O angle
- φ_i Dihedral angles between various pairs of planes

We therefore have the following z-matrix:

$$\begin{bmatrix} \mathsf{B} & & & \\ \mathsf{H} & 1 & r_1 & & \\ \mathsf{H} & 1 & r_1 & 2 & \theta_1 & \\ \mathsf{H} & 1 & r_1 & 2 & \theta_1 & 3 & \varphi \\ \end{bmatrix}$$

Shape 8: BH₃

Hemanth's molecule

	Information provided
r_1	B-H bond length
	Information missing
r_2 θ_1 θ_2 θ_3	H-H distance H-B-H bond angle H-H-B bond angle H-H-H angle
$arphi_i$	Dihedral angles between various pairs of planes

Since the three H atoms form an equilateral triangle, $\theta_3=60^\circ$. We therefore have the following z-matrix:

Shape 9: HNSi₂

Mia finished this but still needs to type it in LaTeX.

	Information provided							
r_1	H-Si bond length							
r_2	Si-N bond length							
r_3	N-Si bond length							
$ heta_1$	Si-N-Si bond angle							
θ_2	H-Si-N bond angle							
	Information missing							
r_4	NH distance							
r_5	SiSi distance							
r_6	SiH distance							
θ_3	N=SiSi angle							
$ heta_4$	HSiSi angle							
θ_5	H-SiSi angle							
θ_6	HSi-N angle							
φ_i	Dihedral angles between various other pairs of planes							

We have the following expressions for the missing geometric information (waiting for Mia): But we can make the z-matrix with only information given to us originally:

$$\begin{array}{|c|c|c|c|c|c|c|c|c|} \hline \textbf{Si} & & & & & & \\ \textbf{N} & 1 & r_1 & & & & \\ \textbf{Si} & 2 & r_2 & 1 & \theta_1 & & & \\ \textbf{H} & 3 & r_3 & 2 & \theta_2 & 1 & \varphi \\ \hline \end{array}$$

Shape 10: Br_2Na_2

Abdul's molecule

	Information provided
'1	Br-Na bond length
) ₁	Br-Na-Br bond angle

Information missing

- r_2 Br...Br distance
- r_3 Na...Na distance
- θ_1 Br...Br-Na angle
- θ_2 Br-Na...Na angle
- θ_3 Na-Br-Na angle
- φ_i Dihedral angles between various other pairs of planes

We have the following expressions for the missing geometric information (waiting for Abdul): But we can make the z-matrix with only information given to us originally:

Si						
N	1	r_1				
Si	2	r_2	1	θ_1		
N Si H	3	r_3	2	θ_2	1	φ

Shape 11: O_2S_2

Abdul's molecule

Information provided

- r_1 O-S bond length
- r_2 S-S bond length
- θ_1 O-S-S bond angle

Information missing

- r_3 O...O distance
- r_4 O...S distance
- θ_2 O...O-S angle
- θ_3 O...O...S angle
- θ_4 O-S...O angle
- θ_5 O...S-S angle
- φ_i Dihedral angles between various other pairs of planes

We have the following expressions for the missing geometric information (waiting for Abdul): But we can make the z-matrix with only information given to us originally:

Shape 12: $CBrO_2$

Hemanth's molecule

Information provided

- r_1 C-Cl bond length
- r_2 C=O bond length
- θ_1 Cl-C-Cl bond angle

Information missing

- r_3 Cl...Cl distance
- r_4 Cl...O distance
- θ_2 C-Cl...Cl angle
- θ_3 C-Cl...O angle
- θ_4 C-O...Cl angle
- θ_5 Cl-C...O angle
- θ_6 Cl...Cl...O angle
- θ_7 Cl...O-C angle
- 67 Cl...O-C angle
- θ_8 Cl...O...Cl angle
- φ_i Dihedral angles between various other pairs of planes

We have the following expressions for the missing geometric information:

$$r_3 = \sqrt{2r_1^2 - 2r_1^2 \cos \theta_1},\tag{9}$$

$$r_4 = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos\theta_2},\tag{10}$$

$$\theta_2 = \tag{11}$$

But we can make the z-matrix with only information given to us originally:

$$\begin{array}{|c|c|c|c|c|c|c|c|c|} \hline \text{Si} & & & & & \\ \text{N} & 1 & r_1 & & & \\ \text{Si} & 2 & r_2 & 1 & \theta_1 & & \\ \text{H} & 3 & r_3 & 2 & \theta_2 & 1 & \varphi \\ \hline \end{array}$$

Shape 13: C_2H_2

Aimun's molecule

Information provided

- r_1 H-Si bond length
- r_2 Si-N bond length
- r_3 N-Si bond length
- θ_1 Si-N-Si bond angle
- θ_2 H-Si-N bond angle

Information missing

- r_4 N...H distance
- r_5 Si...Si distance
- r_6 Si...H distance
- θ_3 N=Si...Si angle
- θ_4 H...Si...Si angle
- θ_5 H-Si...Si angle
- θ_6 H...Si-N angle
- φ_i Dihedral angles between various other pairs of planes

We have the following expressions for the missing geometric information (waiting for Mia): But we can make the z-matrix with only information given to us originally:

Si						
N	1	r_1				
Si	2	r_2	1	$ heta_1$		
Н	3	r_3	2	θ_2	1	φ

Shape 15: H_2Si_2

Justin's molecule

Information provided

- r_1 Si-Si bond length
- r_2 Si-N bond length
- r_3 N-Si bond length
- θ_1 Si-N-Si bond angle
- θ_2 H-Si-N bond angle

Information missing

- r_4 N...H distance
- r_5 Si...Si distance
- r_6 Si...H distance
- θ_3 N=Si...Si angle
- θ_4 H...Si...Si angle
- θ_5 H-Si...Si angle
- θ_6 H...Si-N angle
- φ_i Dihedral angles between various other pairs of planes

We have the following expressions for the missing geometric information (waiting for Mia): But we can make the z-matrix with only information given to us originally:

Shape 16: CBrFO

Hemanth's molecule

Information provided

- r_1 C-F bond length
- r_2 C=O bond length
- θ_1 Si-N-Si bond angle
- θ_2 H-Si-N bond angle

Information missing

- r_4 N...H distance
- r_5 Si...Si distance
- r_6 Si...H distance
- θ_3 N=Si...Si angle
- θ_4 H...Si...Si angle
- θ_5 H-Si...Si angle
- θ_6 H...Si-N angle
- φ_i Dihedral angles between various other pairs of planes

We have the following expressions for the missing geometric information (waiting for Mia): But we can make the z-matrix with only information given to us originally:

Shape 17: C_3Si

Sam's molecule

Information provided

 r_1 C(1)-C(2) bond length r_2 C(2)-C(2)' bond length r_3 C(2)-Si bond length

Information missing

 r_4 C(1)...Si distance

 θ_1 C(2)-C(1)-C(2)' angle

 θ_2 C(1)-C(2)-C(2)' angle

 θ_3 Si-C(2)-C(2)' angle

 θ_4 C(2)-Si-C(2)' angle

 θ_5 Si-C(2)-C(1) angle

 θ_6 Si...C(1)-C(2) angle

 θ_7 C(1)...Si-C(2) angle

 φ_i Dihedral angles between various other pairs of planes

We have the following expressions for the missing geometric information:

$$r_4 = r_3 \sqrt{1 - \frac{r_2^2}{4r_3^2}} + r_1 \sqrt{1 - \frac{r_2^2}{4r_1^2}} \tag{12}$$

$$\theta_1 = \arccos\left(1 - \frac{r_2^2}{2r_1^2}\right) \tag{13}$$

$$\theta_2 = \arccos\left(\frac{r_2}{2r_1}\right) \tag{14}$$

$$\theta_3 = \arccos\left(\frac{r_2}{2r_3}\right) \tag{15}$$

$$\theta_4 = \arccos\left(1 - \frac{r_2^2}{2r_3^2}\right) \tag{16}$$

$$\theta_5 = \arccos\left(\frac{r_2}{2r_1}\right) + \arccos\left(\frac{r_2}{2r_3}\right) \tag{17}$$

$$\theta_6 = \frac{1}{2}\arccos\left(1 - \frac{r_2^2}{2r_1^2}\right) \tag{18}$$

$$\theta_7 = \frac{1}{2}\arccos\left(1 - \frac{r_2^2}{2r_3^2}\right) \tag{19}$$

$$\begin{array}{|c|c|c|c|c|c|c|} \hline C(1) & & & & \\ C(2) & 1 & r_1 & & \\ C(2)' & 2 & r_2 & 1 & \theta_2 \\ Si & 3 & r_3 & 2 & \theta_3 & 1 & \varphi \\ \hline \end{array}$$

Shape 18: C₃Si

Sam's molecule

Information provided

 r_1 C(1)-C(2) bond length

 r_2 Si-C(2) bond length

 r_3 Si-C(1) bond length

Information missing

 r_4 C(1)...C(1)' distance

 θ_1 C(1)...C(1)'-C(2) angle

 θ_2 C(1)-C(2)-C(1)' angle

 θ_3 C(1)-C(2)-Si angle

 θ_4 Si-C(1)-C(2) angle

 θ_5 C(1)-Si-C(2) angle

 θ_6 C(1)-Si-C(1)' angle

 θ_7 C(1)...C(1)'-Si angle

 φ_i Dihedral angles between various other pairs of planes

We have the following expressions for the missing geometric information:

$$r_4 = 2r_1 \cos\left(90 - \arccos\left(\frac{r_1^2 + r_2^2 - r_3^2}{2r_1 r_2}\right)\right) \tag{20}$$

$$\theta_1 = 90 - \arccos\left(\frac{r_1^2 + r_2^2 - r_3^2}{2r_1r_2}\right) \tag{21}$$

$$\theta_2 = 2\arccos\left(\frac{r_1^2 + r_2^2 - r_3^2}{2r_1r_2}\right) \tag{22}$$

$$\theta_3 = \arccos\left(\frac{r_1^2 + r_2^2 - r_3^2}{2r_1r_2}\right) \tag{23}$$

$$\theta_4 = \arccos\left(\frac{r_1^2 + r_3^2 - r_2^2}{2r_1r_3}\right) \tag{24}$$

$$\theta_5 = \arccos\left(\frac{r_2^2 + r_3^2 - r_1^2}{2r_2r_3}\right) \tag{25}$$

$$\theta_6 = 2\arccos\left(\frac{r_2^2 + r_3^2 - r_1^2}{2r_2r_3}\right) \tag{26}$$

$$\theta_7 = 90 - \arccos\left(\frac{r_2^2 + r_3^2 - r_1^2}{2r_2r_3}\right) \tag{27}$$

$$\begin{array}{|c|c|c|c|c|c|} \hline \textbf{C(1)} & & & & \\ \textbf{C(2)} & 1 & r_1 & & \\ \textbf{C(1)'} & 2 & r_1 & 1 & \theta_2 \\ \textbf{Si} & 3 & r_3 & 2 & \theta_4 & 1 & \varphi \\ \hline \end{array}$$

Shape 19: AsP₃

Sichao's molecule

Information provided

 r_1 As-P bond length

 r_2 P-P bond length

Information missing

 θ_1 P-As-P angles

 θ_2 As-P-P angle

 θ_3 P-P-P angle

 φ_1 – Dihedral angle between AsP_2 plane and AsP_2 plane

 φ_2 Dihedral angle between AsP_2 plane and P_3 plane

We have the following expressions for the missing geometric information:

$$\theta_1 = \cos^{-1}\left(\frac{2r_1^2 - r_2^2}{2r_1^2}\right)$$

$$\theta_2 = \frac{180 - \theta_1}{2}$$

$$\theta_3 = 60$$

$$\varphi_1 = \cos^{-1}\left(\frac{\cos\theta_1(1 - \cos\theta_1)}{\sin^2\theta_1}\right)$$

$$\varphi_2 = \cos^{-1}\left(\frac{\cot\theta_2}{\sqrt{3}}\right)$$

We are able to construct the z-matrix using just the information on r_1, θ_1 and φ_1 :

$$\begin{array}{|c|c|c|c|c|c|c|} \hline \textbf{As} & & & & & \\ \textbf{P} & 1 & r_1 & & & \\ \textbf{P} & 1 & r_1 & 2 & \theta_1 & \\ \textbf{P} & 1 & r_1 & 2 & \theta_1 & 3 & \varphi_1 \\ \hline \end{array}$$

Shape 20: H_2N_2

Justin's molecule

Information provided

 r_1 H-Si bond length

 r_2 Si-N bond length

 r_3 N-Si bond length

 θ_1 Si-N-Si bond angle

 θ_2 H-Si-N bond angle

Information missing

 r_4 N...H distance

 r_5 Si...Si distance

 r_6 Si...H distance

 θ_3 N=Si...Si angle

 θ_4 H...Si...Si angle

 θ_5 H-Si...Si angle

 θ_6 H...Si-N angle

 φ_i Dihedral angles between various other pairs of planes

We have the following expressions for the missing geometric information (waiting for Mia): But we can make the z-matrix with only information given to us originally:

Shape 21: S_2F_2

Sam's molecule

Information p	provided
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 r_1 F-S bond length

 r_2 S=S bond length

 θ_1 F-S-F bond angle

 θ_2 F-S=S bond angle

Information missing

 r_3 F...F distance

 r_4 F...S distance

 θ_3 S-F...F angle

 θ_4 S...F...F angle

 θ_5 S...F-S angle

 θ_6 F...S=S angle

 φ_1 Dihedral angle between S_2F plane and S_2F plane

 φ_2 Dihedral angle between SF_2 plane and S_2F plane

We have the following expressions for the missing geometric information:

$$r_3 = r_1 \sqrt{2 \left(1 - \cos \theta_1\right)} \tag{28}$$

$$r_4 = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos\theta_2} \tag{29}$$

$$\theta_3 = \frac{180 - \theta_1}{2} \tag{30}$$

$$\theta_4 = \arccos\left(r_1 \sqrt{\frac{1 - \cos\theta_1}{2(r_1^2 + r_2^2 - 2r_1 r_2 \cos\theta_2)}}\right) \tag{31}$$

$$\theta_5 = \arccos\left(\frac{r_1 - r_2 \cos \theta_2}{\sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos \theta_2}}\right) \tag{32}$$

$$\theta_6 = \arccos\left(\frac{r_2 - r_1 \cos \theta_2}{\sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos \theta_2}}\right) \tag{33}$$

$$\varphi_1 = \arccos\left(\frac{\cos\theta_1 - \cos^2\theta_2}{\sin^2\theta_2}\right) \tag{34}$$

$$\varphi_2 = \arccos\left(\frac{\cos\theta_2 \left(1 - \cos\theta_1\right)}{\sin\theta_1 \sin\theta_2}\right) \tag{35}$$

$$\begin{array}{|c|c|c|c|c|c|c|} \hline F & & & & & \\ S & 1 & r_1 & & & \\ S & 2 & r_2 & 1 & \theta_2 & & \\ F & 2 & r_1 & 3 & \theta_2 & 1 & \varphi_1 \\ \hline \end{array}$$

Shape 22: C_3H

Sam's molecule

Information provided

 r_1 C \equiv C bond length

 r_2 C-C bond length

 r_3 C-H bond length

Information missing

 r_4 C...H distance

 θ_1 C \equiv C-C bond angle

 θ_2 C-C-C bond angle

 θ_3 C-C-H bond angle

 θ_4 C \equiv C...H angle

 θ_5 C-C...H angle

 θ_6 C-H...C angle

 φ_i Dihedral angles between various other pairs of planes

We have the following expressions for the missing geometric information:

$$r_4 = \sqrt{r_2^2 + r_3^2 - 2r_2r_3\cos\left(90 + \arccos\left(\frac{r_1}{2r_2}\right)\right)}$$
 (36)

$$\theta_1 = \arccos\left(\frac{r_1}{2r_2}\right) \tag{37}$$

$$\theta_2 = 180 - 2\arccos\left(\frac{r_1}{2r_2}\right) \tag{38}$$

$$\theta_3 = 90 + \arccos\left(\frac{r_1}{2r_2}\right) \tag{39}$$

$$\theta_4 = \arccos\left(\frac{r_1}{2\sqrt{r_2^2 + r_3^2 - 2r_2r_3\cos\left(90 + \arccos\left(\frac{r_1}{2r_2}\right)\right)}}\right)$$
 (40)

$$\theta_5 = \arccos\left(\frac{r_1}{2\sqrt{r_2^2 + r_3^2 - 2r_2r_3\cos\left(90 + \arccos\left(\frac{r_1}{2r_2}\right)\right)}}\right) - \arccos\left(\frac{r_1}{2r_2}\right)$$
(41)

$$\theta_6 = \arcsin\left(\frac{r_1}{2\sqrt{r_2^2 + r_3^2 - 2r_2r_3\cos\left(90 + \arccos\left(\frac{r_1}{2r_2}\right)\right)}}\right)$$

$$\tag{42}$$

$$\begin{array}{|c|c|c|c|c|c|c|c|} \hline \textbf{C} & & & & & \\ \textbf{C} & 1 & r_1 & & & \\ \textbf{C} & 2 & r_2 & 1 & \theta_1 & & \\ \textbf{H} & 3 & r_3 & 2 & \theta_3 & 1 & \varphi \\ \hline \end{array}$$

Shape 23: $CsNO_2$

Sam's molecule

Information provided

 r_1 Cs-O bond length

 r_2 O-N bond length

 θ_1 O-N-O bond angle

Information missing

 r_3 Cs...N distance

 r_4 O...NO distance

 θ_2 Cs-O-N bond angle

 θ_3 O-Cs-O bond angle

 θ_4 Cs...N-O angle

 θ_5 Cs-O...O angle

 θ_6 N...Cs-O angle

 θ_7 O...O-N angle

 φ_i Dihedral angles between various other pairs of planes

We have the following expressions for the missing geometric information:

$$r_3 = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos\left(\frac{180 - \theta_1}{2} + \arccos\left(\frac{r_2}{2r_1}\sqrt{2(1 - \cos\theta_1)}\right)\right)}$$
(43)

$$r_4 = r_2 \sqrt{2 \left(1 - \cos \theta_1\right)} \tag{44}$$

$$\theta_2 = \frac{180 - \theta_1}{2} + \arccos\left(\frac{r_2}{2r_1}\sqrt{2\left(1 - \cos\theta_1\right)}\right) \tag{45}$$

$$\theta_3 = 2\arcsin\left(\frac{r_2}{2r_1}\sqrt{2\left(1-\cos\theta_1\right)}\right) \tag{46}$$

$$\theta_4 = \frac{\theta_1}{2} \tag{47}$$

$$\theta_5 = \arccos\left(\frac{r_2}{2r_1}\sqrt{2\left(1-\cos\theta_1\right)}\right) \tag{48}$$

$$\theta_6 = \arcsin\left(\frac{r_2}{2r_1}\sqrt{2\left(1-\cos\theta_1\right)}\right) \tag{49}$$

$$\theta_7 = \frac{180 - \theta_1}{2} \tag{50}$$