# A hierarchy of classes for Hamiltonian transformations

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We define a hierarchy of classes for Hamiltonian transformations and categorize all known quadratization techniques according to these classes, and give examples of the types of problems which would require gadgets of each individual class.

Discrete optimization problems are often naturally formulated in terms of minimizing some polynomial of degree $> 2$, which is then 'quadratized' into a quadratic function which can be solved using standard algorithms for universal classical computers [1], using special-purpose classical annealers [2], or using quantum annealers [3].

When quadratizing a function, depending on what we need to learn from the optimization, sometimes it is only necessary to preserve the value of the minimum (the ground state energy), and sometimes it is necessary to preserve the value of the minimum and an input that gives this minimum value (a ground state). Sometimes we even need to know not just any ground state but all of them (all degeneracies of the ground state). Sometimes we need not just the ground state but the full spectrum, sometimes the full spectrum and its degeneracies. Sometimes we may be content with a quadratization that preserves the ground state most of the time. All of this does not only apply to quadratizations, but to any Hamiltonian transformation in general.

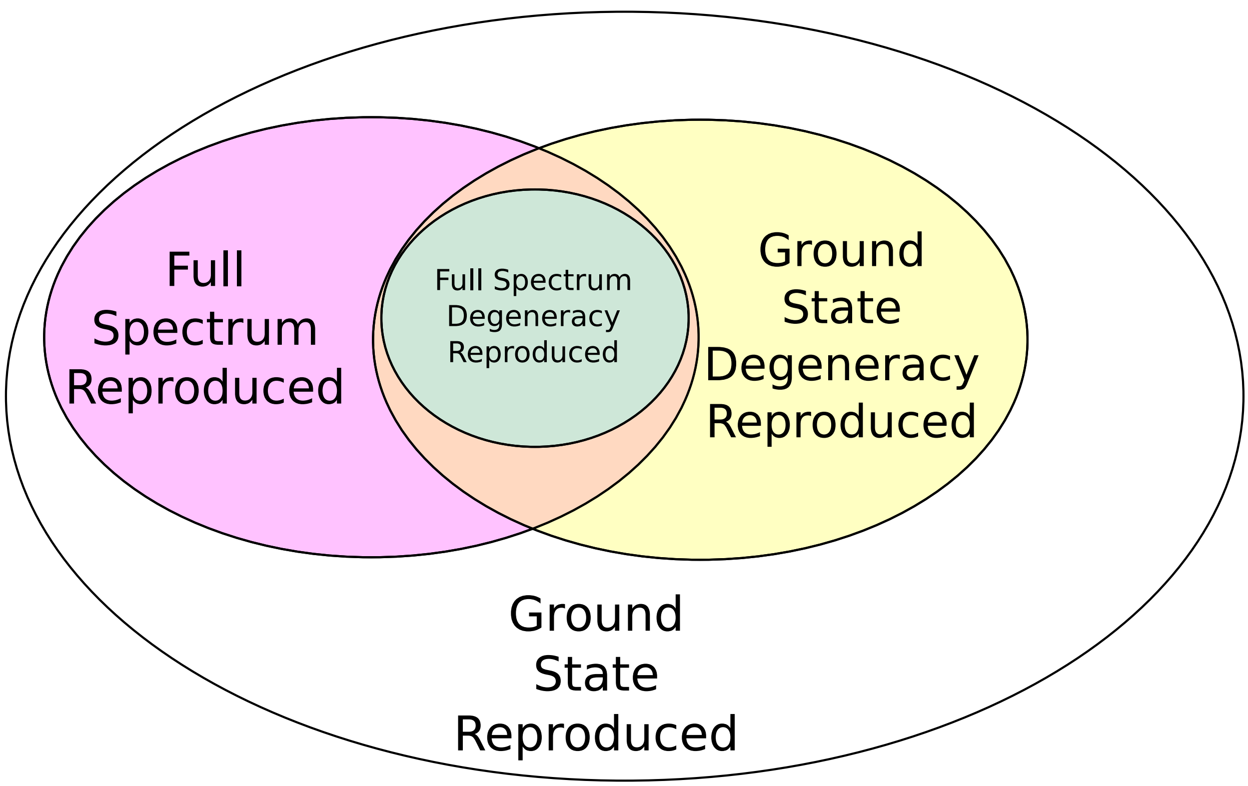


Figure 1. Hierarchy of the senses in which a problem term can be mapped. Outside of the outer circle is the space of all Hamiltonians which do not map the problem in any sense. The loosest sense in which a problem can be mapped is for the ground state(s) to be repoduced, but not necessality in a way which preserves degeneracy, or any higher excited states. A stronger way in which a term can be mapped is either to correctly reporduce the degeraracy of the ground state or to reproduce the whole spectrum. Finally, the strictest requirement is to reproduce the entire spectrum and its degeneracy.

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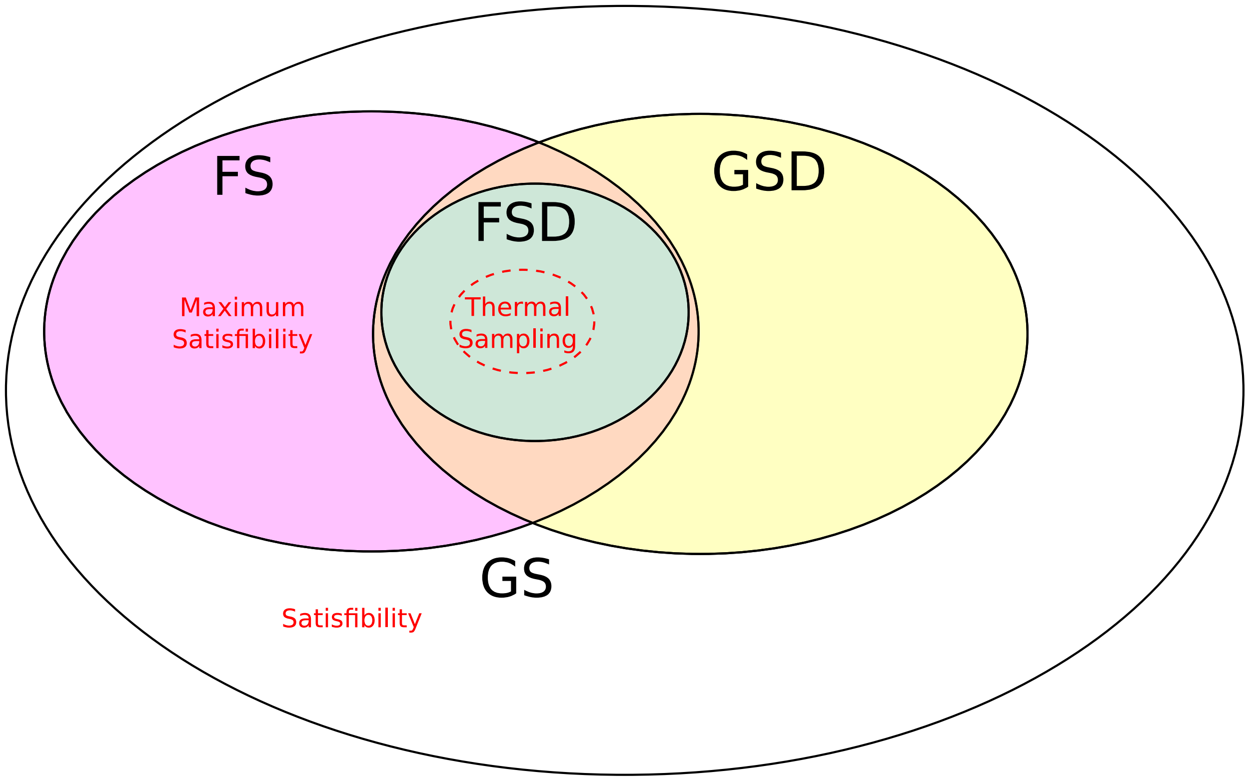


Figure 2. Version of the diagram in Fig. 1 including some of the applications that different classes would be useful for. For example, only the ground state needs to be reproduced for satisfiability problems because the answer is only relevant if each term can simultaineously be in its ground state. On the other hand, maximum statisfiability needs the higher parts of the spectrum to be reproduced so that the cost of an unsatisfied clause is the same for all clauses. In neither of these cases does the degerneracy need to be reproduced. Finally, reproducing the full spectrum and the degerearacy is a necissary, but not sufficient condition for thermal sampling, the reason it is not sufficient is that thermal sampling may still not be practically feasible if there are 'spurious' states close in energy to the highest energy state of the mapped term.

| | **Energy** | | | **State** | | | | | | | | | **Degeneracy** | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Ground | Partial Spec. | Full Spec. | Ground | | | Partial Spec. | | | Full Spec. | | | Ground | | | Partial Spec. | | | Full Spec. | | |
| % accuracy | 100 | 100 | 100 | > 0 | > 50 | 100 | > 0 | > 50 | 100 | > 0 | > 50 | 100 | > 0 | > 50 | 100 | > 0 | > 50 | 100 | > 0 | > 50 | 100 |
| GS | ✓ | ✓ | ✓ | | | ✓ | | | | | | | | | | | | | | | |
| FS | ✓ | ✓ | ✓ | | | ✓ | | | ✓ | | | ✓ | | | | | | | | | |
| FSGSD | ✓ | ✓ | ✓ | | | ✓ | | | ✓ | | | ✓ | | | ✓ | | | | | | |
| GSD | ✓ | ✓ | ✓ | | | ✓ | | | | | | | | | ✓ | | | | | | |
| FSD | ✓ | ✓ | ✓ | | | ✓ | | | ✓ | | | ✓ | | | ✓ | | | | | | ✓ |
| QC | ✓ | | | | | | | | | | | | | | | | | | | | |
| QC | ✓ | ✓ | | | | | | | | | | | | | | | | | | | |
| QC | ✓ | ✓ | ✓ | | | | | | | | | | | | | | | | | | |
| QC | ✓ | | | | | ✓ | | | | | | | | | | | | | | | |
| QC | ✓ | ✓ | | | | ✓ | | | ✓ | | | | | | | | | | | | |
| QC+P | ✓ | | | | | ✓ | | | | | | | | | ✓ | | | | | | |
| QC+P | ✓ | ✓ | | | | ✓ | | | ✓ | | | | | | ✓ | | | ✓ | | | |