A hierarchy of classes for Hamiltonian transformations

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We define a hierarchy of classes for Hamiltonian transformations and categorize all known quadratization techniques according to these classes, and give examples of the types of problems which would require gadgets of each individual class.

Discrete optimization problems are often naturally formulated in terms of minimizing some polynomial of degree > 2, which is then 'quadratized' into a quadratic function which can be solved using standard algorithms for universal classical computers [1], using special-purpose classical annealers [2], or using quantum annealers [3].

When quadratizing a function, depending on what we need to learn from the optimization, sometimes it is only necessary to preserve the value of the minimum (the ground state energy), and sometimes it is necessary to preserve the value of the minimum and an input that gives this minimum value (a ground state). Sometimes we even need to know not just any ground state but all of them (all degeneracies of the ground state). Sometimes we need not just the ground state but the full spectrum, sometmies the full spectrum and its degeneracies. Sometimes we may be content with a quadratization that preserves the ground state most of the time. All of this does not only apply to quadratizations, but to any Hamiltonian transformation in general.

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