Embedding quadratization gadgets on Chimera and Pegasus graphs

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We group all known quadratizations of cubic and quartic binary optimization problems into five and seven unique graphs respectively. We then perform a minor embedding of these graphs onto the well-known Chimera graph, and the brand new *Pegasus* graph. In cases where two or more graphs have a minor embedding with the same overhead in terms of auxiliary variables, we make recommendations for which gadgets are best to use for certain problems.

Discrete optimization problems are often naturally formulated in terms of minimizing some polynomial of degree > 2, which are then 'quadratized' into a quadratic function which can be solved using standard algorithms for universal classical computers [1], using special-purpose classical annealers [2], or using quantum annealers [3]. With dozens of quadratization methods available [4], one should choose the best quadratization for a given problem, and for a given method for solving the problem.

When quadratizing a high-degree term, the resulting quadratic function (called the 'gadget') will have some connectivity between the binary variables (or bits, or qubits, herein referred to for convenience only as qubits) which can be represented by a graph in which vertices represent qubits and edges indicate when two different qubits appear together in a quadratic term. Since this graph incorporates no information about the linear terms, constant term, or the coefficient of the quadratic term, many different gadgets have the same graph, therefore in this paper we will classify all known quadratization gadgets into categories according to their corresponding graph (herein called their 'gadget graph').

Gadget graphs tell us a lot about how costly the quadratic optimization problem will be, and those with larger connectivity tend to yield more difficult functions to optimize. Furthermore, some optimization methods only work if their corresponding graph has a certain connectivity, two example's being D-Wave's well-known Chimera graph [5], and their very recently presented *Pegasus* graph, both shown in Fig. 1.

Any graph, can be mapped onto the Chimera or Pegasus graphs by minor-embedding [6].

Figure 1.