E.1:

We have four categories of vectors:

$$\text{Category I}: \left(\begin{bmatrix} -2 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \end{bmatrix} \right), \qquad \text{Category II:} \left(\begin{bmatrix} -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right)$$

$$\text{Category III}: \left(\begin{bmatrix} 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix} \right), \qquad \text{Category IV:} \left(\begin{bmatrix} -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right)$$

i. Design a two-neuron perceptron network (single layer) to recognize the four categories of vectors. Select the ”BEST” decision boundaries, and explain what ”BEST” means. Sketch the decision boundaries, and find the weights and biass. SHOW ALL WORK. Use the following targets:

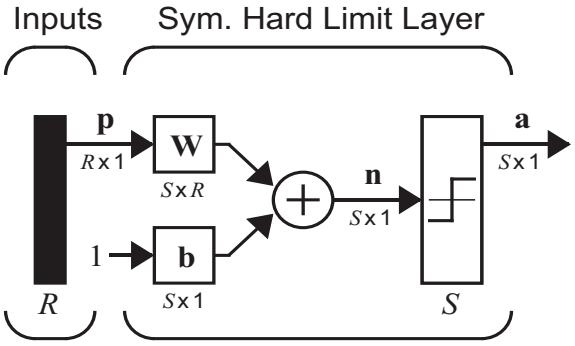
$$\text{Category I}: \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \text{Category II:} \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \text{Category III:} \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \text{Category IV:} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

ii. Draw the network diagram. Use the exact abbreviated notation that is used in class and textbook.

iii. Suppose the following vector is added to category IV. Perform one iteration of the perceptron rule with this vector. (Start with the weights you determined in part i.) $\begin{bmatrix} -3 \\ 1 \end{bmatrix}$

E.2:

Consider the following perceptron network.



$$\mathbf{W} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

i. How many different classes can this network classify?

ii. Draw a diagram illustrating the regions corresponding to each class. Label each region with the corresponding network output.

iii. Calculate the network output for the following input. $p = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

iv. Plot the input from part iii in your diagram from part ii, and verify that it falls in the correctly labeled region.

E.3:

Consider the following function:

$F(x) = [1 + (x_1 + x_2 - 5)^2][1 + (3x_1 - 2x_2)^2]$

i. Perform one iteration of Newtons method, starting from the initial guess $\begin{bmatrix} 10 & 10 \end{bmatrix}^T$

ii. Repeat part (i), starting from the initial guess $\begin{bmatrix} 2 & 2 \end{bmatrix}^T$

iii. Find the minimum of the function, and compare with your results from the previous two parts.

E.4:

For the network shown below the initial weights and biases are chosen to be

$$w^1(0) = 1, b^1(0) = -2, w^2(0) = 1, b^2(0) = 1$$

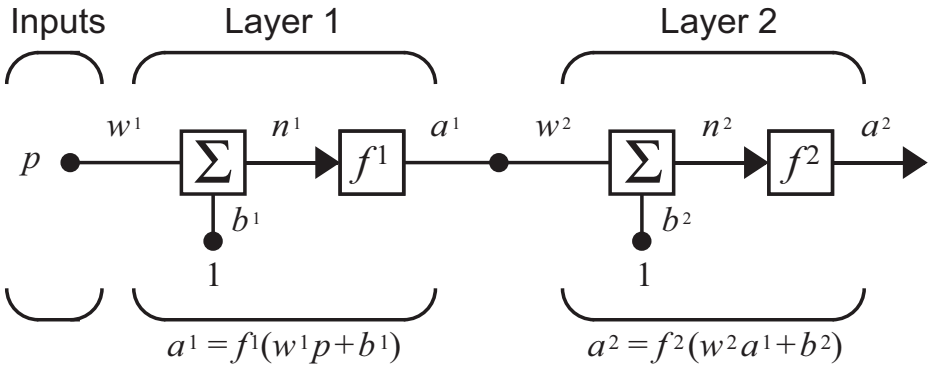
The network transfer functions are

$$f^1(n) = (n)^2, f^2(n) = \frac{1}{n}$$

and an input/target pair is given to be

$$\{p = 1, t = 1\}$$

Perform one iteration of backpropagation with $\alpha = 1$



Note 1: Write a Python Script to solve E.5. This problem is to implement the Perceprton learning rule. Please read the the summary page chapter 4 and just follow the update equations.

E.5: Python Exercise

The vectors in the ordered set defined below were obtained by measuring the weight and ear lengths of toy rabbits and bears in the Fuzzy Wuzzy Animal Factory. The target values indicate whether the respective input vector was taken from a rabbit (0) or a bear (1). The first element of the input vector is the weight of the toy, and the second element is the ear length.

$$p_1 = \begin{bmatrix}1\\4\end{bmatrix}, t_1 = 0; \quad p_2 = \begin{bmatrix}1\\5\end{bmatrix}, t_2 = 0; \quad p_3 = \begin{bmatrix}2\\4\end{bmatrix}, t_3 = 0; \quad p_4 = \begin{bmatrix}2\\5\end{bmatrix}, t_4 = 0;$$

$$p_5 = \begin{bmatrix}3\\1\end{bmatrix}, t_5 = 1; \quad p_6 = \begin{bmatrix}3\\2\end{bmatrix}, t_6 = 1; \quad p_7 = \begin{bmatrix}4\\1\end{bmatrix}, t_7 = 1; \quad p_8 = \begin{bmatrix}4\\2\end{bmatrix}, t_8 = 1;$$

i. i. Use Python to initialize and train a network to solve this ”practical” problem.

ii. Use Python test the resulting weight and bias values against the input vectors.

iii. Please plot the inputs and check your trained weight vector and validate your results by plotting the trained weight and bias.