An Eight-Parameter Function for Simulating Model Rocket Engine Thrust Curves

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An Eight-Parameter Function for Simulating Model Rocket Engine Thrust Curves

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The toy model rocket is used extensively as an example of a realistic physical system. Teachers from grade school to the university level use them. Many teachers and students write computer programs to investigate rocket physics since the problem involves nonlinear functions related to air resistance and mass loss. This paper describes a nonlinear eight-parameter function that correctly models the thrust profile for model rocket engines. Examples are given for commonly used Estes rocket engines.

A good starting point on model rocket physics is the classic *TPT* article written by Nelson and Wilson.[1] Keeports[2] has a more current paper on numerical calculations, and Widmark[3] has another fairly recent paper on rocket physics. I also suggest Gregorek's[4] technical report for a good discussion of air resistance.

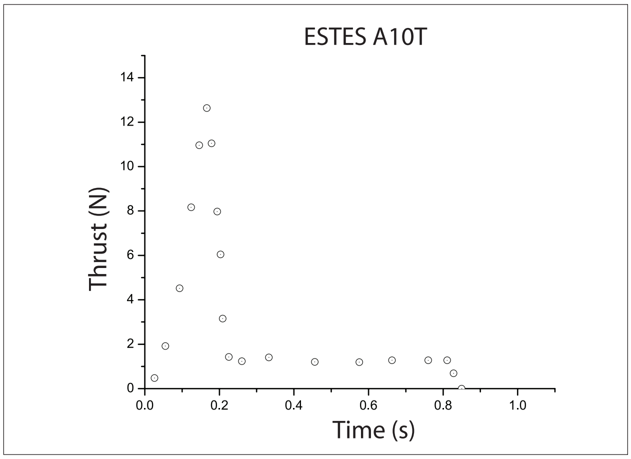

Fig 1. Thrust (in Newtons) versus time (in seconds) for an Estes A10T engine.

One problem all programmers face is the need to interpolate a set of thrust data over time. To help simplify the incorporation of the thrust data, I developed an eight-parameter function, which is used to give a continuous function of thrust versus time for most model rocket engines.

The thrust function has the following goals:
It should

- give the correct total impulse
- follow the general curve of the thrust profile
- go to zero naturally just as the engine thrust dies when the fuel is expended
- reasonably model an Estes engine (most commonly used in classrooms)

This function has been mainly designed for Estes Model Rocket[5] engines and data for all engines were obtained from the website http://www.thrustcurve.org.[6] A typical thrust curve is shown for an Estes A10T engine (Fig. 1).

The typical curve shows a big initial peak followed by a long plateau. Nelson[1] points out that this is to give the model rocket a high initial velocity so that the fins can stabilize the motion of the rocket as it comes off the launching pad. This is typical of a so-called "Series I" engine.[7] The solid propellant is packed into the tube with an indentation at the ignition point. This exposes more surface area and initially burns more fuel. After this initial burn, the surface flattens out and exposes less area. This gives the engine a longer, steady burn that results in a plateau following the initial peak.

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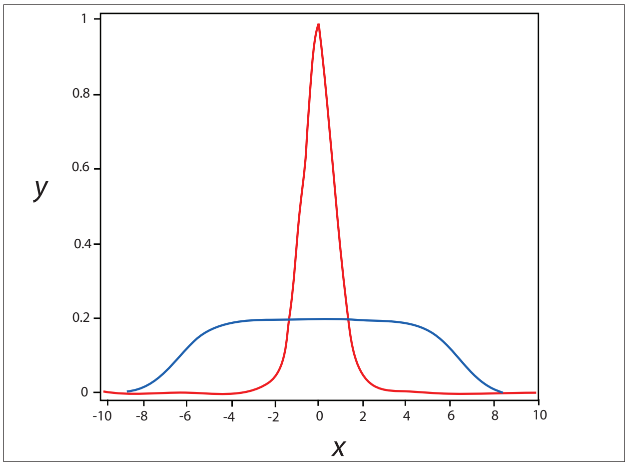

Fig. 2. Two curves using Eq. (1): Red line, *a*=1.0, *b*=0.0, *m*=0.8, *k*=1.0, Blue line, *a*=0.2, *b*=0.0, *m*=3.0, *k*=0.00001.

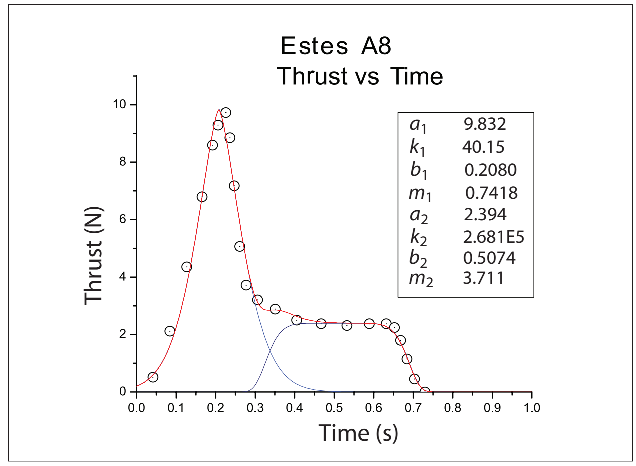

Fig. 3. The fitted eight-parameter function to an Estes A8 engine. The two separate peaks are shown. A more pointed peak to the left and a broader, shorter plateau to the right. The red line shows the sum of the two functions.

The total impulse (change in momentum) delivered by the engine equals the area under the thrust-versus-time curve. This area is numerically integrated to find the total impulse in Newton-seconds (N·s). Estes engines can deliver impulses in a range from 0.625 N·s to 90 N·s depending on the engine.

A fitting function must then be able to duplicate both the big initial peak and the long plateau region. The function

$$y = a \cdot \exp\left\{-k\left[\left(x-b\right)^2\right]^m\right\} \tag{1}$$

can fit the peak region or the plateau region. This function is a modified Gaussian and is a variation on the Sigmoidal Weibull function type two.[8] The parameter a controls the amplitude while b controls the centroid of the curve. The parameter k determines the width while m determines how sharply the function can drop off. If m is equal to one, then the function will go as the square of x, which is a normal Gaussian curve. A small value of k will cause the peak to broaden, while a large value of k will cause a more narrow peak.

As a result, values of m close to one with a large k will give sharp peaks, while large values of m with small values of k will cause a flat broad plateau (see Fig. 2).

Fitting two of these functions to the data can duplicate the entire trust curve. The function will require two sets of four parameters for an eight-parameter function of time:

$$\begin{aligned}\text{Thrust} &= a_1 \cdot \exp\left\{-k_1\left[\left(t-b_1\right)^2\right]^{m_1}\right\} \\ &+ a_2 \cdot \exp\left\{-k_2\left[\left(t-b_2\right)^2\right]^{m_2}\right\} .\end{aligned} \tag{2}$$

Equation (2) expresses the engine thrust as a function of time. This function was fitted to the Estes A8 thrust curve as a function of time (see Fig. 3). All fitting was done using Origin 6.1.[9]

Figure 3 shows that the function meets the desired goals. This is a continuous function that gives the correct general features and the correct total impulse. Since the function is smooth and continuous, there is no need for special programming statements that interpolate the thrust between fixed data points. The function also goes to zero in a natural manner at the correct time.

In Fig. 4, there is a fit to an Estes C5 engine. It has a longer plateau region with a sharper cutoff. Again the fit gives very good results.

The function was devised to duplicate the initial peak followed by a broad plateau. However, it has turned out to be a surprisingly flexible function and can handle very different curves. Figure 5 shows a radically different curve for the more powerful Estes NCR G70 engine.

Fitting with this function is a bit tricky. It usually requires repeated attempts at inputting good initial values for the parameters. After some practice, it can

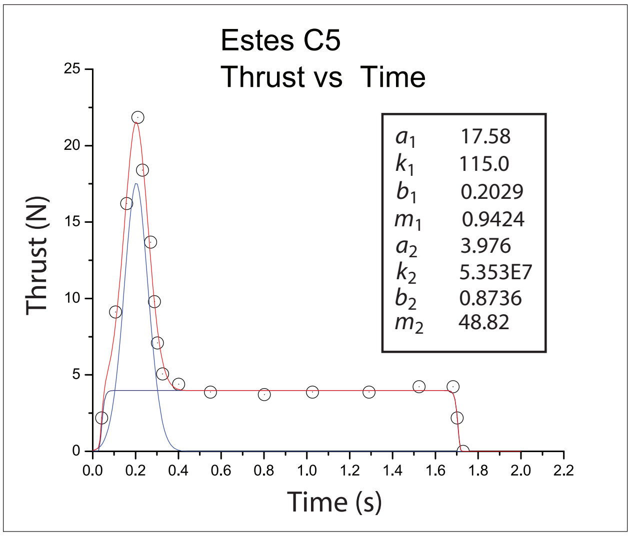

Fig. 4. The eight-parameter fit to an Estes C5 engine.

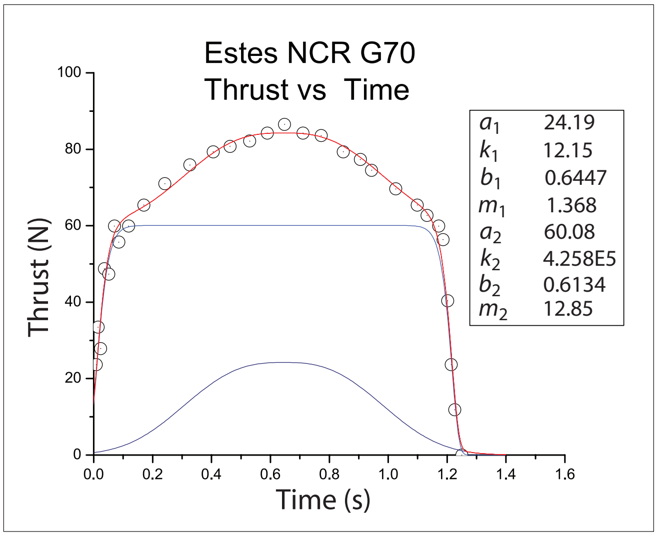

Fig. 5. The eight-parameter fit to an Estes NCR G70 engine. This is a high impulse engine, *I* = 90 N·s, and shows that the eight-parameter function can fit a wide variety of thrust curves.

be done quickly. It helps to make a good estimate of the centroid and amplitude of each curve and hold them fixed while the other constants are varied. A table of fitted parameters, to four significant figures, for some common engines is given in Table. I.

An Excel file with these parameters and the thrust functions they generate can be found at the website http://www.uncp.edu/home/dooling/engines/engine-curves.htm. In it is a link to the original Excel file. For each set of parameters there is a plot. The plot will automatically change if the user alters the parameters in the original Excel file.

I have used this function extensively in my own simulations and with students who use it to learn physics through modeling. It is especially useful with students since they do not have to generate a more complex conditional code to carry out interpolation. They can put all the parameters into an array and easily select which engine to use in the simulation. Also, the student can easily experiment with the parameters to invent new thrust profiles to see how different profiles with similar impulses can affect the rocket altitude.

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Table I. Fitted parameters for some common Estes engines to four significant figures.

Estes	Impulse (N·s)	a_1	k_1	b_1	m_1	a_2	k_2	b_2	m_2
A10T	2.0	12.00	200.0	0.1574	0.8592	1.206	6.915E+8	0.4268	11.45
A8	2.32	9.851	39.02	0.2082	0.7357	2.392	2.929E+5	0.5078	3.742
B4	4.29	12.58	203.7	0.1430	1.012	3.584	5.371E+6	0.6204	8.347
B6	4.33	9.534	87.45	0.1525	0.8735	4.622	2.029E+9	0.4892	9.738
C5	9.1	17.58	114.8	0.2029	0.9421	3.978	5.353E+7	0.8736	48.82
C6	8.82	11.50	122.2	0.1776	0.7806	4.485	1.576E+2	0.9423	25.00
NCR G70	87.88	24.22	11.97	0.6447	1.360	60.07	4.258E+5	0.6134	12.85

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