Problem Chosen	2024	Team Control Number
B	MCM/ICM Summary Sheet	2427171

Submarine Evolution: Breaking deep sea location and rescue bottlenecks

Summary

With the tourism industry, undersea exploration in search of shipwrecks is becoming popular, and how can we prevent the submarine from having problems and losing contact with the main ship during the exploration? Therefore, this paper models the analysis and develops relevant safety procedures.

For locate, this paper collects the three-dimensional model of the Ionian Sea, ocean current information, and other relevant sea parameters, establishes the submarine dynamics equations according to the force situation, combined with **Kalman filtering** to build the **Model I: Dynamic Position Prediction Model**, and after the establishment of the relevant simulation program for simulation, analyzes the uncertainty factors associated with these predictions. Finally, it is suggested that the submarine's position and current environmental parameters can be sent to the main ship on a regular basis, and relevant equipment can be carried to reduce these uncertainties, as well as can reduce the uncertainty of these related types of equipment.

For prepare, this paper collects the commonly used underwater search equipments in the market, such as Klein3000 side-scan sonar, MEPUS-AUV3000L (AUV), and Falcon-DR (ROV). This paper describes the relevance of these search types of equipments, as well as the search efficiency, builds **Model II: Cost-Benefit Ratio Model**, which compares the cost-benefit of different equipments and concludes that the equipment that should carry the main ship for deployment, in addition, this paper discusses the additional equipments that may be required to be needed to equip the SAR vessel.

For searching, this paper first analyzes the commonly used search modes, compares their efficiencies, and then recommends the best search mode, after which the search area is divided into n grids, establishes **model III: a model based on Bayesian Network Theory**, updates the probability distribution according to the new observations searched by the search device using Bayes' Theorem, and uses the grid with the highest probability i i.e., the optimal deployment point, and finally gives the probability of finding the submersible as a function of the time and the cumulative search results.

For the extrapolation, this paper collects the three-dimensional model of the Caribbean Sea, ocean current information and other related sea parameters, discusses the changes needed to extend the model to the Caribbean Sea area, and establishes the related simulation program for simulation in conjunction with the model I. For the case of the existence of multiple submarines, this paper uses the **Deep-Q-Network** algorithm to carry out tracking and prediction of the targets of multiple submarines, and finally verifies the validity of the model through simulation.

Finally, the sensitivity analysis of index weights shows that the model is not sensitive to the change in index weights. The advantages and disadvantages of the model are discussed, and an improvement scheme is proposed.

Keywords: Dynamics; Cost-benefit ratio; Bayesian search; Deep-Q-Network;

Contents

1 Introduction

1.1 Background

The shipwrecks at the bottom of the Ionian Sea are full of mystery. To lead humans on a journey of undersea exploration, a Greece-based company, Maritime Cruises Mini-Submarines (MCMS), has dedicated itself to building submersibles capable of diving to the deepest depths of the oceans for this journey. To avoid life-threatening situations for tourists if the submersible loses contact with the main vessel or the submarine loses power in the deep ocean, it makes sense to develop a model that predicts the position of the undersea submersible over time. With these capabilities, accurate positions can help rescue lost submersibles, detect and repair mechanical problems in time, and evaluate and improve submarine design, thus enhancing the development and application of submarine technology.

1.2 Restatement of the Problem

We need to construct a model for predicting how a submarine's position in the water evolves, taking into account that a submersible may be located on the seafloor or in the middle layer of the water column and that its movement is affected by factors such as currents, the density of seawater, and the topography of the seafloor.

For problem 1, We need to consider what information a submarine needs to report regularly to the main ship and what equipment it needs to be equipped with to reduce forecasting uncertainty.

For problem 2, We should consider what additional search equipment the company should equip the main ship with to respond to emergencies, factoring in the cost, maintenance, readiness, and utilization needs of such equipment.

For problem 3, We need to create a model that will reduce the time required to locate a lost submarine by analyzing the submarine location information to guide the selection of the initial deployment point and search strategy. We will then calculate how the likelihood of successfully locating the submarine changes as time passes and search results accumulate.

For problem 4, We want to clarify how the model can apply to other tourist locations, such as the Caribbean Sea. Also, discuss what adjustments need to be made to the model to make it capable of handling the movement of multiple submarines in the same area.

1.3 Literature Review

Submersible positioning technology is one of the critical technologies in deep-sea exploration, which is of great significance for deep-sea resource exploration, seabed geological surveys, and marine biology research, among others. The current research has made significant progress in the submersible positioning technology.

The technology of submersible positioning mainly relies on acoustic positioning systems [1], such as long baseline (LBL), ultra-short baseline (USBL), and short baseline (SBL) methods, which combine with inertial navigation systems, satellite communication technology, and other means to improve the positioning accuracy and reliability. These techniques determine the position of the submersible by transmitting and receiving acoustic waves with high accuracy, but the acoustic signals are easily interfered with in deep-sea environments, limiting their application range. In 2009, the WHOI Research Institute successfully developed the hybrid remotely operated vehicle (HROV) "Nereus". However, it was accidentally lost during the operation at about 9990m in the Kermadec Trench of New Zealand on May 10, 2014[2][3].

Therefore, it is significant to improve the submarine localization function further. In this, we provide a model that can accurately predict the position of a submarine in the water, taking into account the influence of various factors such as the external environment and the cost of equipment, and construct a model based on the Bayesian search theory to search for a lost submarine.

1.4 Our Work

To avoid complicated descriptions, and intuitively reflect our work process, the flow chart is shown as the following figure:

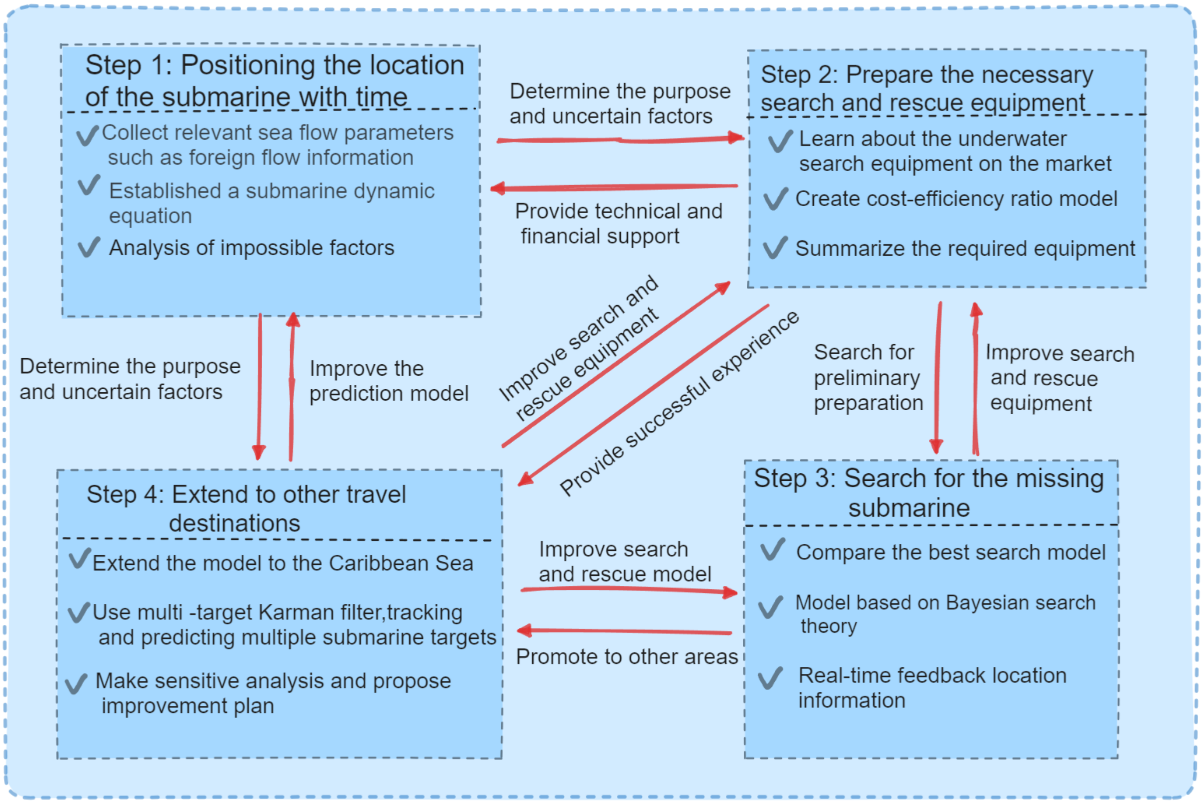


Figure 1. Our Work

2 Assumptions and Justifications

To simplify the problem, we make the following basic assumptions, each of which is properly justified.

- **Assumption 1: The preprocessed data is reliable.**
  Justification: this assumption is made to ensure the accuracy of the model solution.
- **Assumption 2: There are no other emergencies, such as extreme weather.**
  Justification: To simplify the model, this paper ignores the interference of extreme weather to the information. Extreme weather can have a big impact on the model, but the probability is small enough to ignore for now.
- **Assumption 3: The density of seawater in the Ionian and Caribbean Seas is 1 g/cm³.**
  Justification: Due to the large amount of data involved in the model, the density of seawater is very close to 1g/cm³, so the final error is negligible.
- **Assumption 4: Submarine malfunctioned with no communication capabilities with power system unavailable.**
  Justification: Simplified modeling for ease of calculation.

3 Notations

Table 1: Notations

Symbol	Description	Unit
K	Kalman gain	
$\vec{r}$	Position vector of the submersible	
Cd	Drag coefficient	
t	The time of submersible movement	s
$C_{\text{equipment}}$	Equipment cost	$
C_{training}	Personnel training cost	$
C_{storage}	Storage and maintenance costs	$
ρ	Sea water density	g/cm^3
v	The speed of the submersible relative to the water	m/s
vi	Time step Speed of moment i	m/s

4 Locate

4.1 Problem Analysis

In this question, we need to establish specific measures to predict the positional situation of a submarine after a failure. We plan to build a dynamic position prediction model to forecast the submarine's position and establish a set of constraints to achieve this goal.

4.2 Preparation of the Model

- The submersible is treated as a mass, disregarding the effects of its size and shape
- Effects of bottom topography on submersible motion can be modeled by adjusting buoyancy and drag parameters
- The velocity of the current is constant and there is no rotation, according to the figure below, the current velocity of 0.11m/s in the Ionian Sea at latitude 36°36'N and longitude 21°78'E is pointing to the southeast.

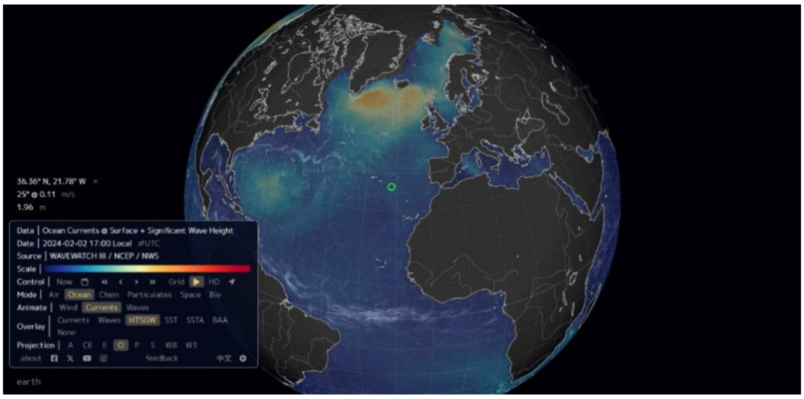


Figure 2. Ionian ocean current

- ✧ Obtained seafloor topographic data from the GEBCO website, used Global Mapper pro v23.1 to analyze local features and some parameters of the seafloor topography, and then used Python to visualize the data.

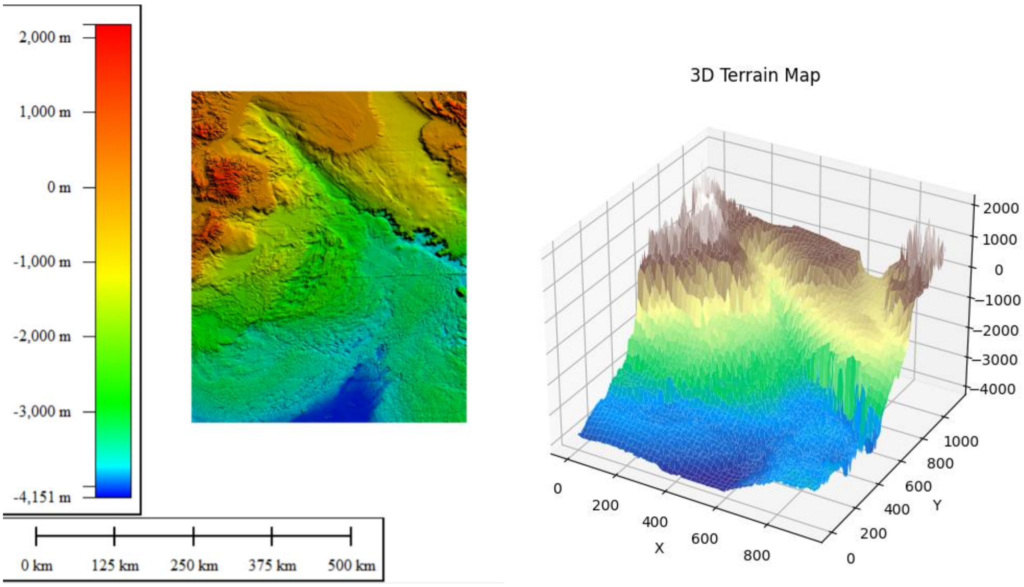


Figure 3. Ionian Sea terrain

Note： Analysis shows that the Ionian Sea is the deepest sub-basin of the Mediterranean Sea and consists of several trenches and plateaus with depths of more than 4,000 m. The 3D Terrian Map shows that the Ionian Sea seafloor spans between 1,000-2,000, and explorers need to take care of their safety in case they lose communication with the main ship.

4.3 Establishment of the Model

4.3.1 Establishment of a coordinate system

In the modeling of submersibles, the choice of coordinate system generally includes two coordinate systems, i.e., the fixed coordinate system and the kinematic coordinate system, and the establishment of the coordinate system is generally by the right-hand rule. When studying the dynamics of a submersible, the earth is generally chosen as the reference system. The fixed coordinate system and the body coordinate system of a manned submersible are selected as shown in the figure below. The origin E of the fixed coordinate system $E-\xi\eta\zeta$ is taken as a point on the sea surface according to the actual situation, the $E\zeta$ axis points to the center of the earth, the $E\xi$ axis points to the geographic north, and the $E\eta$ axis points to the east, which constitutes a right-handed right-angle coordinate system.

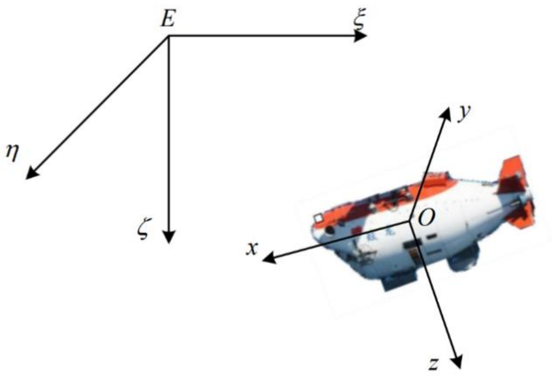


Figure 4. Submersible coordinate system

The origin O of the submarine hull coordinate system $O-xyz$ is taken as the midpoint of the main axis of symmetry of the submarine, the Ox-axis coincides with the main axis of symmetry of the submarine, and points to the bow of the submarine, the Oy-axis coincides with the auxiliary axis of symmetry and points to the right side of the submarine in the forward direction, and the Oz-axis is perpendicular to the waterline surface, and points to the bottom direction of the submarine. $O-xyz$ also forms a right-handed

4.3.2 kinetic equation

Considering the dynamics of a submarine, the equations for the dynamics of a submarine can be expressed using Newton's second law:

$$m\frac{d^2\vec{r}}{dt^2}=\overrightarrow{F_b}+\overrightarrow{F_g}+\overrightarrow{F_c}-\overrightarrow{F_d} \tag{1}$$

where $\vec{r}$ is the position vector of the submarine and t is the time of the movement of the submarine.

m is the mass of the submarine.

Fg=mg is the gravitational force of the submersible.

Fc is the force of the current on the submersible.

The buoyancy force Fb can be calculated from the displacement of the submersible and the density of the water.

The drag force Fd can be estimated from the shape of the submersible, surface roughness, and speed of motion, ignoring forces in other directions and taking the vertical direction:

$$F_d=\frac{1}{2}\rho v^2 C_d A \tag{2}$$

where ρ is the density of the water, v is the velocity of the submersible relative to the water, Cd is the drag coefficient, and A.the area of the submersible facing the water.

Establish the following system of differential equations：

$$\begin{cases} m\frac{d^2\vec{r}}{dt^2}=\overrightarrow{F_b}+\overrightarrow{F_g}+\overrightarrow{F_c}-\vec{F}_d \\ F_g=m\cdot g \\ F_d=\frac{1}{2}\rho v^2 C_d A \end{cases} \tag{3}$$

4.3.3 nonlinear Kalman filter

Although we have predicted the position of the submersible with the kinetic equations, in the real physical world, the submersible will be affected by a series of external disturbances in the ocean, and this problem becomes complicated at once. Suppose there is a GPS on the submersible that can tell us its position at moment t. Doesn't that solve the problem? But we can't trust GPS completely, because it also has "accuracy errors". The first error is based on the time, the mass of the submersible, the current force on the submersible, the buoyancy, the resistance, and the position, which is called "process error"; the second error is generated by the observation of the sensor, which is called "observation error". Therefore, we hope to use the Kalman filtering model for the calculation, that is, in the real world where there are both "process error" and "observation error", to find an optimal estimation that is closer to the real value.

For Equation (1) we can divide the time interval Δt into tiny steps:

$$m\frac{v_{i+1}-v_i}{\Delta t}=F_{b_i}+F_{g_i}+F_{c_i}-F_{d_i} \tag{4}$$

v_i is the speed of the time step moment. F_{b_i} is the buoyancy force at the time step.
F_{g_i} is the force of gravity at the time step moment.
F_{c_i} is the force of the current on the submersible at the time step.
F_{d_i} is the drag force at the time step moment.

We can obtain the nonlinear Kalman filter equation of state:

$$x_{i+1}=Ax_i+Bu_i \tag{5}$$

$$x_i=\begin{bmatrix} r_i \\ v_i \end{bmatrix}, u_i=\begin{bmatrix} F_{b_i} \\ F_{g_i} \\ F_{c_i} \\ F_{d_i} \end{bmatrix}, A=\begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}, B=\frac{\Delta t}{m}\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \tag{6}$$

A: State transfer matrix to describe the state vector x

B: Input control matrix to describe the effect of external control inputs on the system state

Eventually, a position equation that can be used to describe the change in position of the submarine at different points in time is obtained:

$$m\frac{\vec{r}(t+\Delta t)-2\vec{r}(t)+\vec{r}(t-\Delta t)}{\Delta t^2}=(\rho v^2 C_d A-F_c)\vec{e}_z\Delta t \tag{7}$$

4.4 Solution of the model

We now give the simulation parameters used to model the submersible position and velocity over time:

Table 2. Analog parameter

symbol	dt	num steps	m	Fb	Fg	Fc
value	1	1000	1000	2000	9800	500

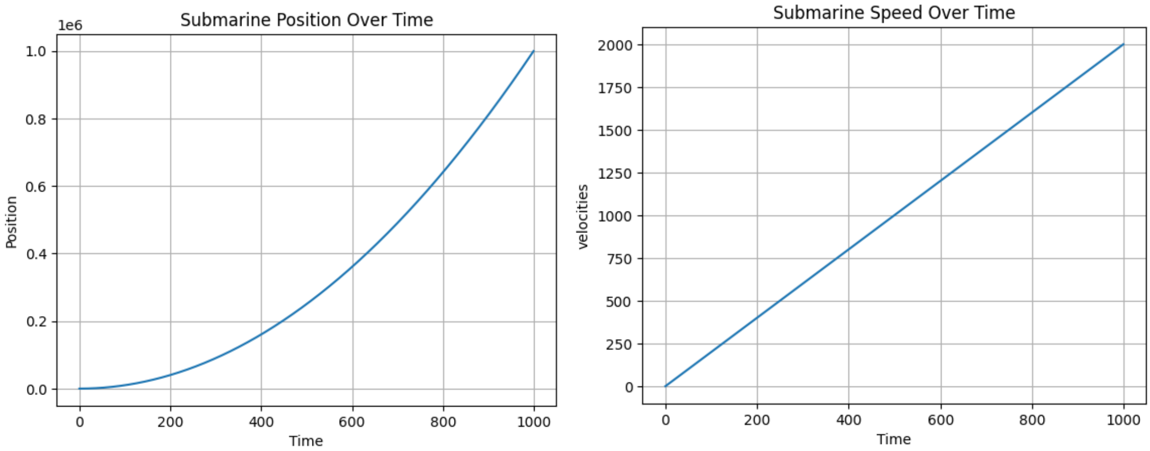


Figure 5. Position and velocity over time

4.5 Error analysis of the model

The positioning model we adopt is nonlinear Kalman model, which includes EKF and UKF, among which EKF is widely used in various fields. The basic principle is to use the system observations to correct the system state until the system error is infinitely close to the Cramer-Rao lower bound. UKF is a method to obtain the mean and variance of higher order by means of multiparticle approximation to the probability density distribution of function for strongly nonlinear systems. Our positioning model is suitable for EKF.

4.5.1 Relational derivation

The derivation process of the relationship between the best estimated value, predicted value and observed value is shown in the figure below：

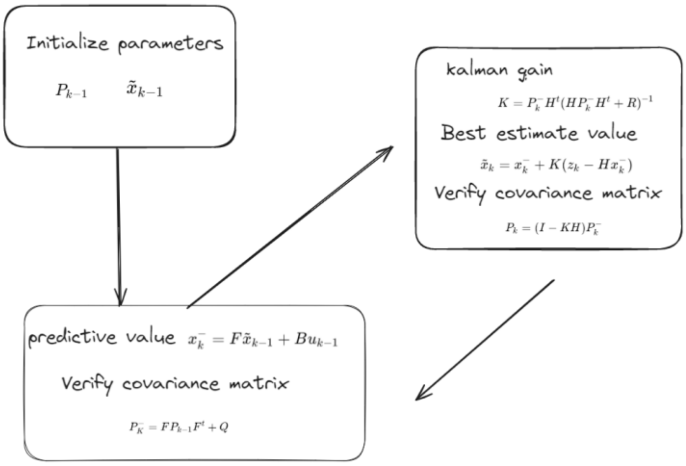


Figure 6. Kalman filter processing steps

4.5.2 Reinforcement model

In the first step, the stern position information $Xt = (xt, yt, zt)$ was calculated with the main ship GPS information $Xs = (xs, ys, zs)$ and the bow direction h; In the second step, the position of the transducer array $Xw = (xw, yw, zw)$ is estimated by combining cable length l and stern speed information vt. The third step is to estimate the position of the submersible $Xh = (xh, yh, zh)$ by using the ranging results r and depth data z. First, the position of the submersible is initialized by using pair information interaction, and then the position of the submersible is tracked by EKF. It is necessary to consider the sparsity of the ranging results, and the motion state of the submersible is mainly considered to float.

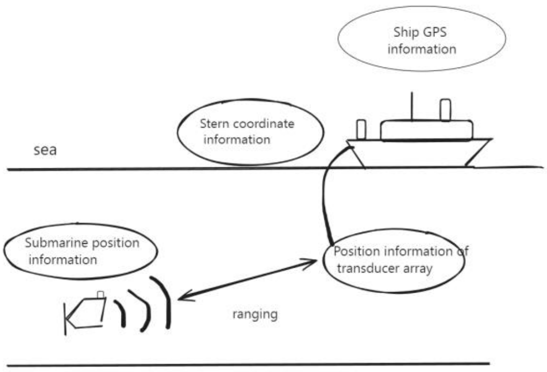


Figure 7. Manned submersible positioning

In order to better predict the positioning of the submersible, we continue to use the EKF model for enhancement, we give:

$$X_k = F_{k-1}X_{k-1} + w_{k-1} \tag{8}$$

$$z_k = Hx_k + v_k \tag{9}$$

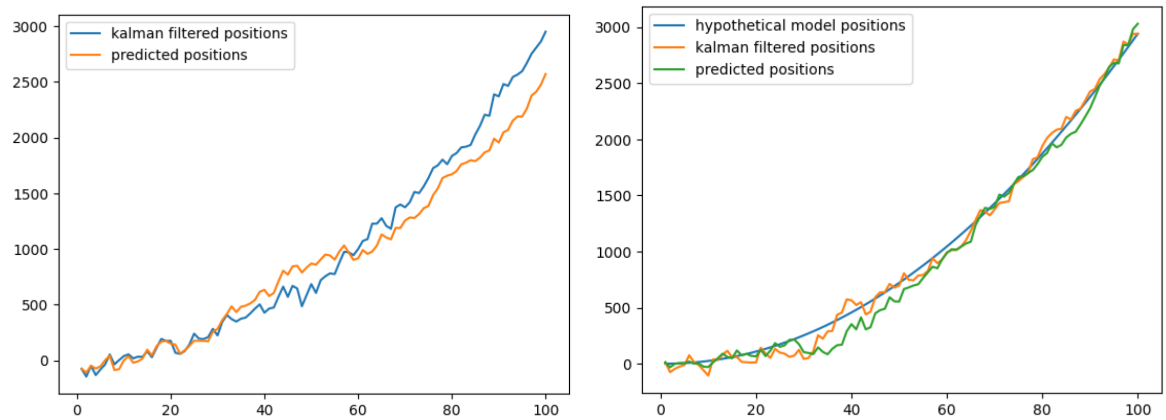
$$X_k = \left[x_k, v_{x_k}, y_k, v_{y_k}\right]^T \tag{10}$$

$$F = \begin{bmatrix} 1 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta t \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{11}$$

$$r_k = h(X_k) = \sqrt{\left(x_k - x_{b_k}\right)^2 + \left(y_k - y_{b_k}\right)^2} + v_k \tag{12}$$

$$H_k = \left.\frac{\partial h(X_k)}{\partial X}\right|_{X=X_k} = \left[\frac{x_k - x_{b_k}}{r_k}, 0, \frac{y_k - y_{b_k}}{r_k}, 0\right]^T \tag{13}$$

After simulation, we can get:



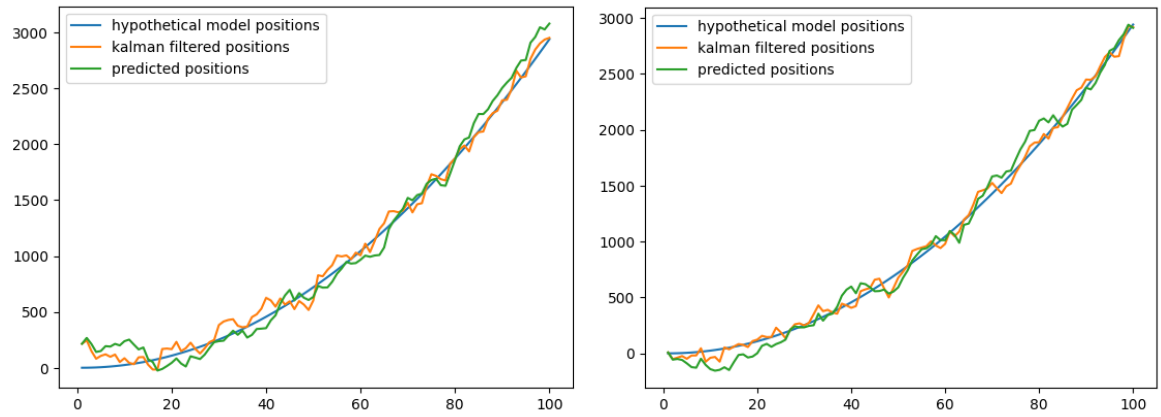


Figure 8. Goodness of fit analysis

4.5.3 Model Analysis

It can be seen that the simulated positioning model of the submersible enhanced by Kalman filtering becomes more accurate, which provides more guarantees for the communication between the main ship and the submersible. We can approximately replace the predicted positions with kalman filtered positions. However, this model still has many problems and is affected by many uncertainties. Below we will try our best to list solutions to reduce the influence of uncertainties.

4.6 uncertainties

In the process of modeling, we have to consider a lot of uncertainties, such as the geological environment, our attitude, etc., and we list them as much as possible:

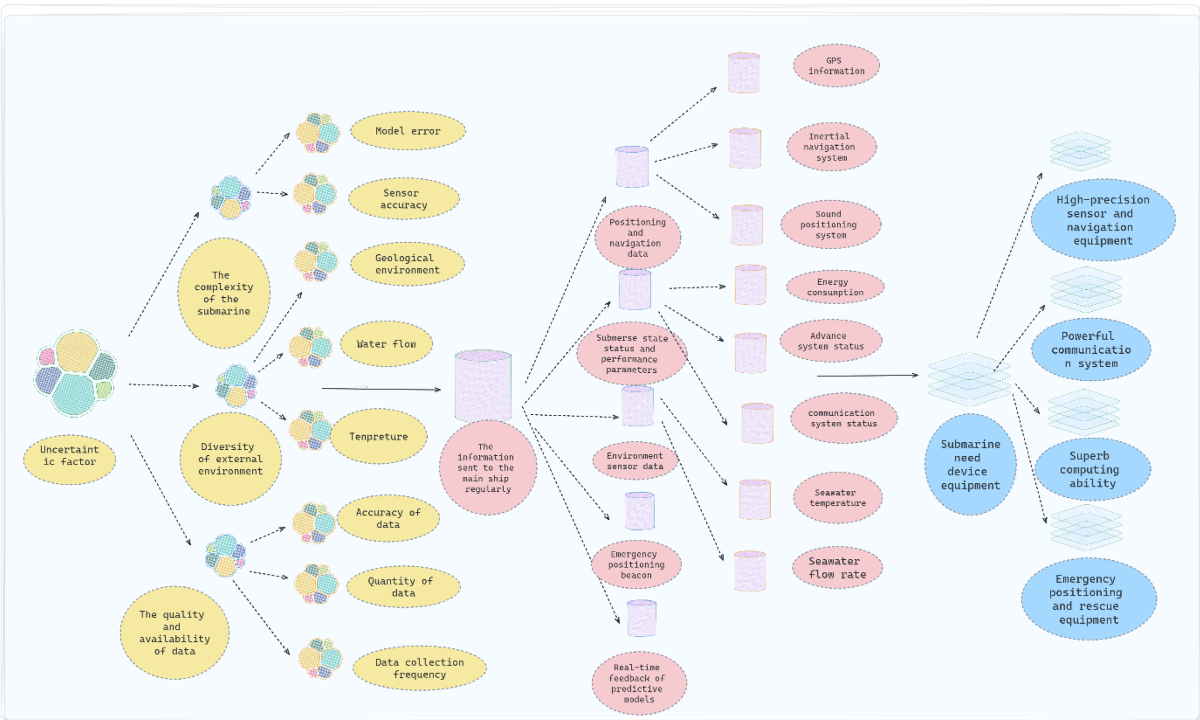


Figure 9. Uncertainties, information, and equipment

5 Prepare

5.1 Problem Analysis

In this case, we need to develop a comprehensive benefit analysis model to determine which equipment is best, and we need to analyze which equipment has the lowest overall cost and the best results.

5.2 Preparation of the Model

5.2.1 Search Equipment

- ✧ **Klein 3000 side scan sonar:** Side-scan sonar is used to scan underwater terrain and targets, usually mounted on a ship, and can be used for seabed surveys and underwater searches.
- ✧ **MEPUS-AUV3000L(AUV):** Auvs are typically equipped with high-definition cameras, sonar systems, robotic arms, etc., and are able to navigate autonomously underwater, collect data, perform tasks, and transmit acquired information to ground control centers.
- ✧ **Falcon-DR(ROV):** Rovs can carry a variety of sensors and tools, such as cameras, maneuver arms, sampling devices, to perform different tasks and are typically controlled by the operator via a remote control device.

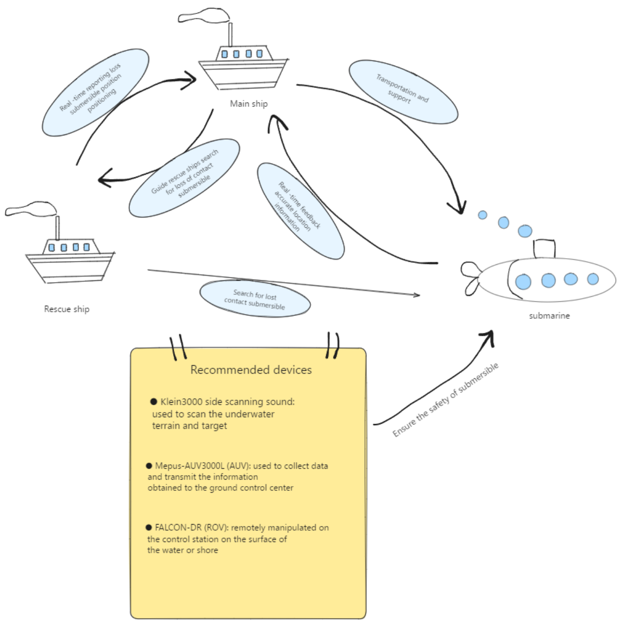


Figure 10. Diagram of main ship, submarine and rescue ship

5.2.2 Costs

If they are to be used, the following costs need to be considered when preparing and using these search devices:

Table 3. Equipment cost

Equipment Name	Equipment cost	Personnel training	Storage cost
Klein3000	$13903.4	$500	$500
MEPUS-AUV3000L	$20855.1	$1500	$1000
Falcon-DR	$10427.6	$1200	$1000

5.2.3 Search Efficiency

Considering the cost, it is also essential to use the results, of the following search efficiency of the search equipment:

Table 4. Search Efficiency

Equipment Name	Search Depth	Search Scope	Search speed
Klein3000	$2000m$	$600m$	$170m/s$
MEPUS-AUV3000L	$3000m$	$1852m$	$1m/s$
Falcon-DR	$1100m$	$1100m$	$2m/s$

5.3 Establishment of the Model

Assuming that the total number of equipment allowed to be carried by the main vessel and the rescue vessel is less than or equal to 5 and based on the cost of the equipment collected and the parameters of the equipment search and rescue, we need to consider the availability of the equipment and at the same time save as much as possible the related cost of expenditure.

We start by modeling the cost-benefit ratio:

$$CER(e) = E(e)/C(e) \tag{14}$$

Where e denotes different search devices, $E(e)$ denotes the benefit value of the device.

$$E(e) = a \cdot E_{\text{profundity}}(e) + b \cdot E_{\text{radius}}(e) + c \cdot E_{\text{speed}}(e) \tag{15}$$

Where $E_{\text{profundity}}$ is the search depth, E_{radius} is the search range, E_{speed} is the search speed . a, b, c are the weights of these factors.

$C(e)$ denotes the total cost of the equipment, which can be further refined as:

$$C(e) = C_{\text{equipment}}(e) + C_{\text{training}}(e) + C_{\text{storage}}(e) \tag{16}$$

Among these $C_{\text{equipment}}$ are the cost of equipment, C_{training} are the cost of personnel training, and C_{storage} are the cost of storage and maintenance.

5.4 Solution of the model

According to the data we can list:

$$s.t.\begin{cases} C(e) = 14903.4X_1 + 23355.1X_2 + 12627.6X_3 \\ X_1 + X_2 + X_3 \le 5 \\ X_1, X_2, X_3 > 0 \end{cases} \tag{17}$$

$$s.t.\begin{cases} E(e) = (2000a + 600b + 170c)X_1 + (3000a + 1852b + c)X_2 + (1100a + 1100b + c)X_3 \\ X_1 + X_2 + X_3 \le 5 \\ X_1, X_2, X_3 > 0 \end{cases} \tag{18}$$

$$CER(e) = E(e)/C(e) = \frac{(2000\mathrm{a} + 600\mathrm{b} + 170\mathrm{c})\mathrm{X}_1 + (3000\mathrm{a} + 1852\mathrm{b} + \mathrm{c})\mathrm{X}_2 + (1100\mathrm{a} + 1100\mathrm{b} + \mathrm{c})\mathrm{X}_3}{14903.4\mathrm{X}_1 + 23355.1\mathrm{X}_2 + 12627.6\mathrm{X}_3} \tag{19}$$

For the linear integer programming problem, we can use Monte Carlo method for estimation, we assume that a, b, c are 0.4, 0.3, 0.3 , respectively, and then we proceed with the analysis:

$$s.t.\begin{cases} E(e) = 1031X_1 + 1755.9X_2 + 770.3X_3 \\ X_1 + X_2 + X_3 \leq 5 \\ X_1, X_2, X_3 > 0 \end{cases} \tag{20}$$

$$CER(e) = E(e)/C(e) = \frac{1031X_1 + 1755.9X_2 + 770.3X_3}{14903.4X_1 + 23355.1X_2 + 12627.6X_3} \tag{21}$$

Af s implanting $C(e)$ and $E(e)$ using Monte Carlo algorithm for 1000 times respectively, the following figure is obtained, and it is easy to see that $C(e)$ is mostly distributed in the range of 60000-80000 interval, and $E(e)$ is mostly distributed in the range of 3000-6000 interval.

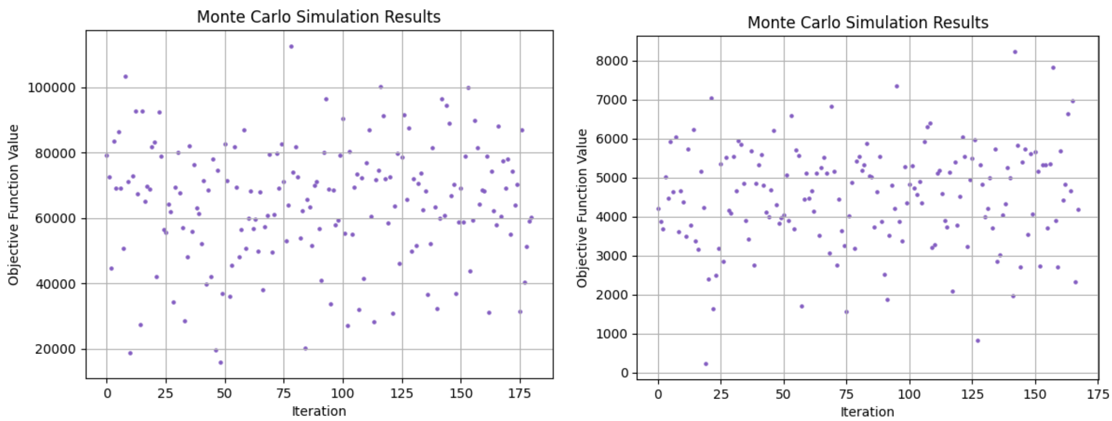


Figure 11. Monte Carlo simulation results for C(e) and E(e)

For the cost-effectiveness ratio $CER(e)$, we first determine the probability distribution of each variable $X1$、$X2$、 $X3$, generate a large number of random samples based on the probability distribution, and then calculate the corresponding objective function values, collect all the values of the objective function, carry out statistical analysis, calculate the mean and variance, and obtain them after processing:

Mean value of CER(e): 0.07217944998576989

Standard deviation of CER(e): 0.0036991256889698423

5.5 Additional equipment

- **Emergency Breathing Appliances:** for emergency treatment of the injured to save their lives.
- **Deep-sea Camera:** Using cameras that can withstand high water pressure, images can be captured in the deep sea to help locate submersibles.
- **Inertial Navigation System:** Inertial navigation systems utilize accelerometers and gyroscopes to measure and calculate dynamic parameters such as the position, velocity, and direction of the submersible; it does not rely on external signals, and is therefore very suitable for use in underwater environments.
- **Global Positioning System:** Although GPS signals cannot be used directly underwater, submersibles can use GPS to pinpoint their position when surfacing, and the data can be used to calibrate and verify the accuracy of other navigation systems.

6 Search

6.1 Problem Analysis

We need to establish a suitable model to recommend the initial deployment points and search patterns of the equipment, in order to minimize the positioning time of the lost submersible as much as possible.

6.2 Preparation of the Model

6.2.1 Model assumption

- ✧ Assume that the search area is around the predicted path.
- ✧ assume that no message is received from the submarine for 10min and start searching it.
- ✧ Considering that submarine localization is a dynamic process, the model may need to be updated periodically to adapt to changing environments and situations.

6.2.2 Processing Steps

✧ **Data collection and preprocessing.**

We save the randomly generated coordinate positions of the submersible and the main ship as a dataset, and the samples contain x, y, z coordinate values.

✧ **Extract the eigenvalues.**

According to the randomly generated coordinate values, we simulate the sailing route between the main ship and the submersible, first assume that the sailing route between the main ship and the submersible is a short distance, and then increase the number of coordinate points one by one, observe the simulated route, and take the distance and time between the main ship and the submersible as the eigenvalues.

✧ **Model selection and training.**

We will simulate the dataset and use Bayesian network classifiers for training. Then, the main ship and submersible paths, as well as the decision boundaries of the classifier, were visualized and displayed.

✧ **Model evaluation and optimization.**

A suitable balance needs to be found by considering the impact of indicators such as the search efficiency and positioning time of the submersible as well as changing environmental factors on the model's accuracy.

6.2.3 The search method

- ✧ **Circular Search:** This is a relatively simple and straightforward method, usually a series of circular searches around a fixed reference point, gradually increasing the diameter of the circle.
- ✧ **Spiral Search:** This is a method of traversing the elements of a two-dimensional matrix by following a spiral path; in a spiral search, we start at the top left corner of the matrix and access the matrix elements in a clockwise direction until we have traversed all the elements.
- ✧ **Grid Search:** This is an exhaustive search method that finds the optimal hyperparameter by traversing all possible combinations of hyperparameters.

6.3 Establishment of the Model

Bayesian network (BN) is known as causal probability network, based on the Bayesian formula; the conditional probability of each node variable of the Bayesian network is developed on the basis of the Bayesian formula and statistics.

The Bayesian formula is as follows：

$$P(x_i \mid Y) = \frac{P(Y \mid x_i)P(x_i)}{\sum_{i=1}^{n} P(x_i)P(Y \mid x_i)} \tag{22}$$

We divide the search region into n grids, and the a priori probability of the presence of a diver in each grid is Pi , Xi denotes that the diver is located in grid i, Y denotes the observation information, and $P(x_i \mid Y)$ is the a posteriori probability that the diver is located in grid i given the observation information Y.

A Bayesian network treats each variable as a node and represents the conditional probability dependencies between them with directed edges, as in Figure:

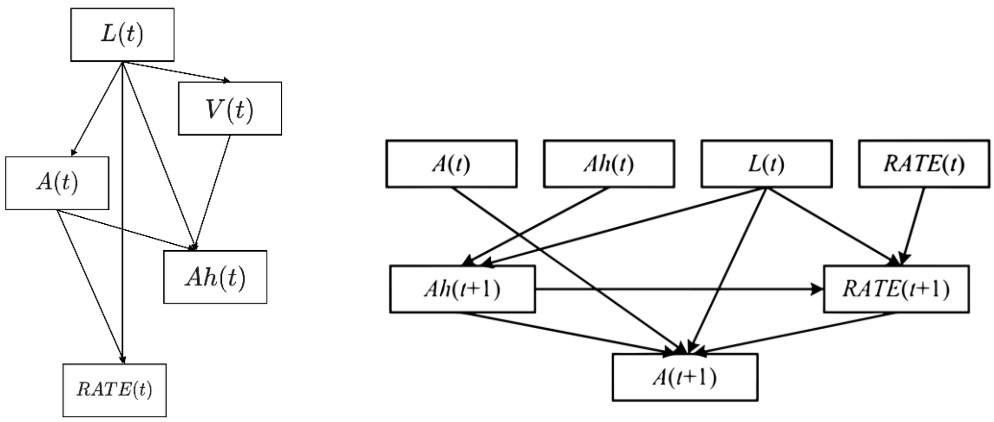


Figure 12. Bayesian network trajectory

As can be seen from the above figure, given the values of four variables, namely, depth $L(t)$, acceleration $A(t)$, lift rate $Ah(t)$, and turn rate $RATE(t)$ at the current moment, according to the dynamic Bayesian network and the conditional probability matrices of each variable, the trajectory prediction and generation for the next moment can be realized.

The velocity is obtained by integrating the known velocity components:

$$V(t) = \sqrt{V_{\mathrm{E}}^{1}(t) + V_{\mathrm{N}}^{2}(t) + V_{\mathrm{G}}^{3}(t)} \tag{23}$$

Differencing the two moments before and after gives the value of acceleration:

$$A(t) = V(t+1) - V(t) \tag{24}$$

The lift rate variable can be obtained by making a difference between the depth values of the 2 moments before and after:

$$Ah(t) = L(t+1) - L(t) \tag{25}$$

Ratio the eastward velocity to the northward velocity to get the heading reference data, set north as the axis positive direction and east as the axis positive direction, and convert the heading reference data to the heading angle in the coordinate system：

$$\theta(t) = \frac{\arctan \left|\left(\frac{V_E(t)}{V_N(t)}\right)\right| \times 180^\circ}{\pi} \tag{26}$$

$\theta(t)$ is the heading angle of the submersible's coordinate system, In turn, calculate the turn rate:

$$RATE(t) = \theta(t+1) - \theta(t) \tag{27}$$

- Similar conditional probability matrices can be created for submersible lift rate, turn rate, and acceleration. During subsequent iterations, the conditional probability matrices of the variables are invoked to generate predictions based on the probability distribution of the values of the child nodes.
- The predicted submersible trajectory data are visualized and a three-dimensional coordinate system is established. The Z-axis is positive if the ground is vertically upward, the X-axis is positive if the ground is eastward, and the Y-axis is positive if the ground is northward. In the X-Y plane, the initial value of heading is set, and the predicted changes in the turn rate are continuously superimposed to iteratively generate a new value of heading. Based on the kinematic equations, the real-time positions of the UAV in the X-axis, Y-axis, and Z-axis are generated:

$$\begin{aligned} x(t+1) &= x(t) + V(t)\cos\,\theta(t) \\ y(t+1) &= y(t) + V(t)\sin\,\theta(t) \end{aligned} \tag{28}$$

$$\theta(t) = \sum_{t=1}^{t} RATE(t) + \theta_0 \tag{29}$$

$$z(t) = L(t) \tag{30}$$

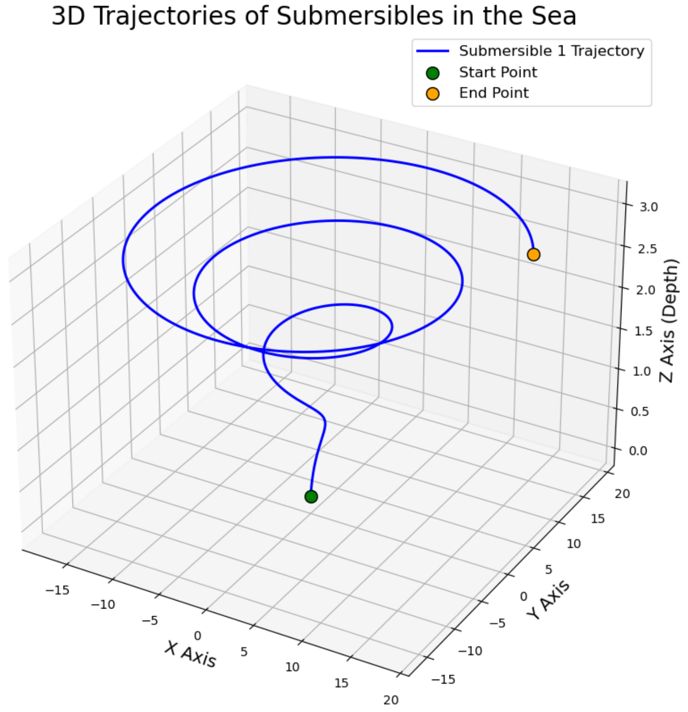


Figure 13. Submarine 3D Tracks

We generate simple simulation data to observe the paths of the main ship and the submersible, then we construct features and labels, 0 for the main ship and 1 for the submersible, and then we use Bayesian network classification for training, and we construct a grid to draw the decision boundary to get the path maps of the main ship and the submersible, this is just our simple simulation

ship_path： [[1, 2], [2, 3], [3, 4], [4, 5], [5, 6]]
submarine_path： [[1, 1], [2, 2], [3, 3], [4, 4], [5, 5]]

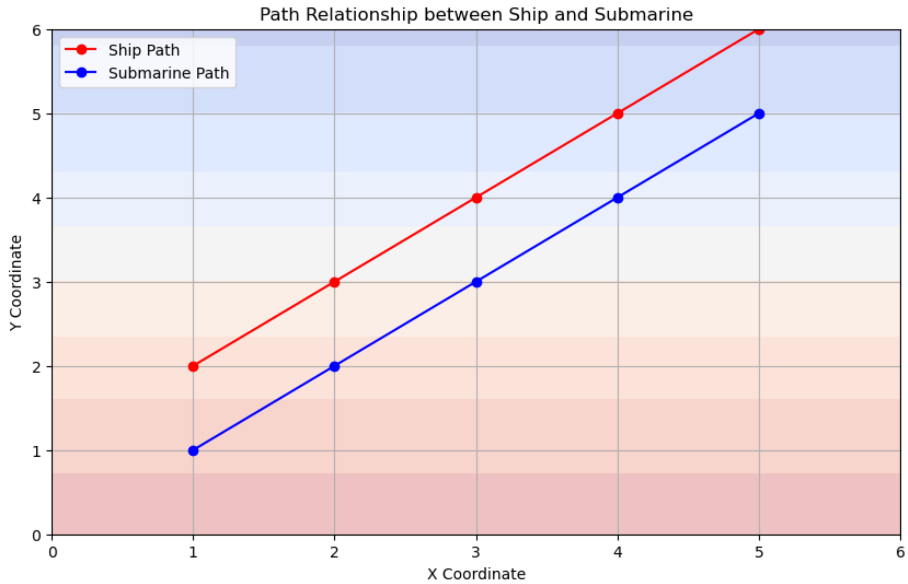


Figure 14. Ship-Submarine Relationship

Next we use random generation which will generate 100 random pairs of points to simulate the coordinates of the main ship and dive, because it is simulated data, so there will be a large deviation from the actual, as follows

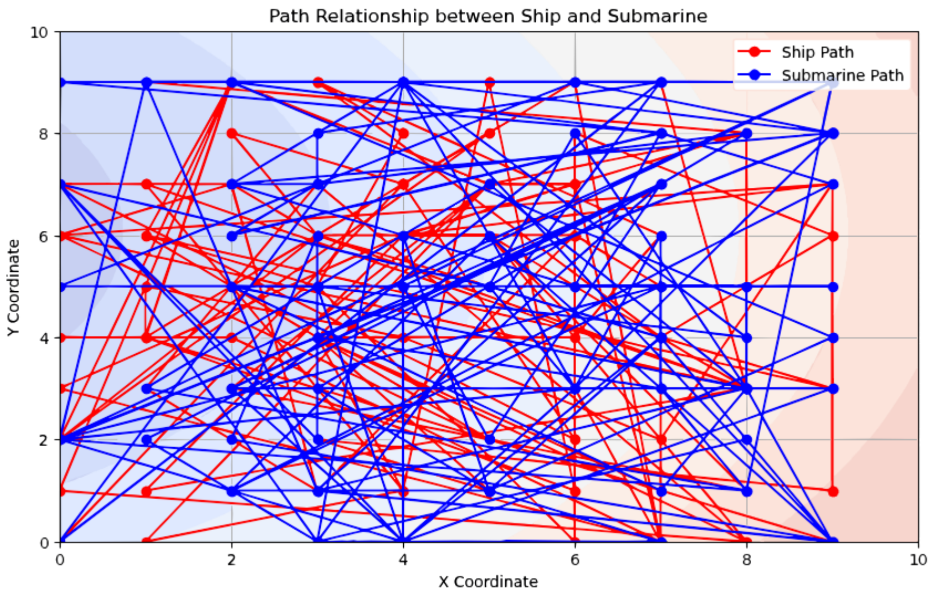


Figure 15. Ship-Submarine Relationship

Next we use random generation which will generate 100 random pairs of points to simulate the coordinates of the main ship and dive, because it is simulated data, so there will be a large deviation from the actual, as follows

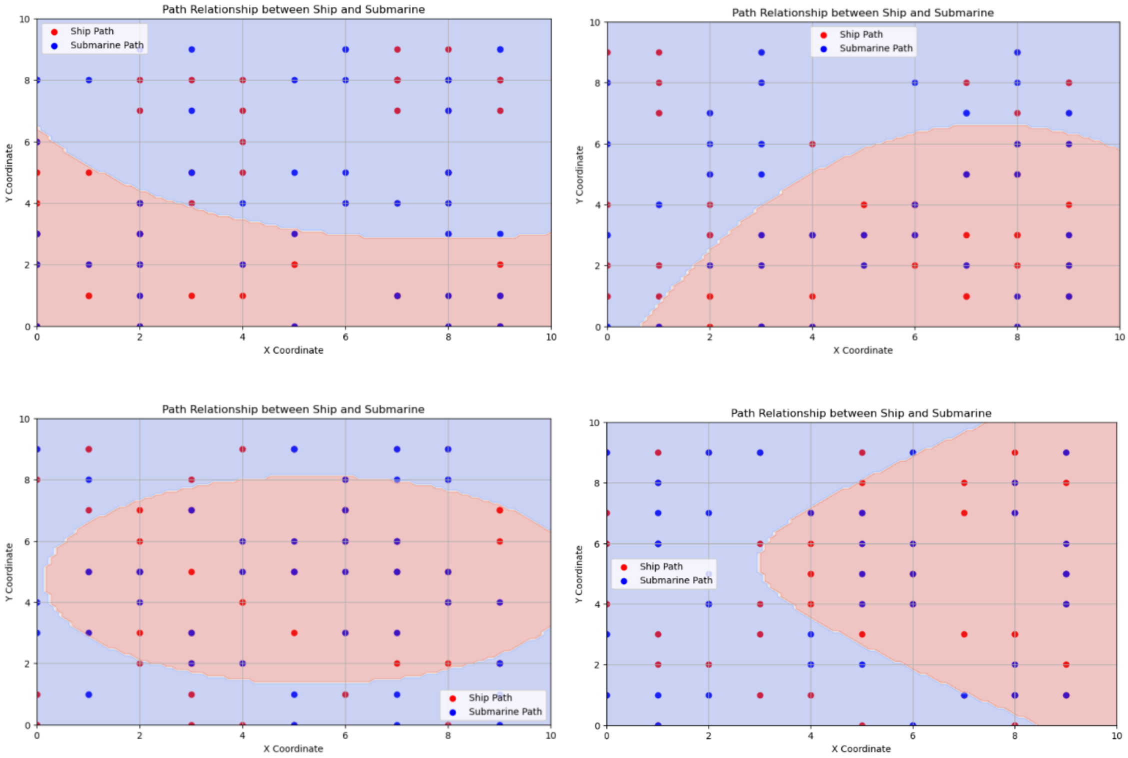


Figure 16. Grid prediction data

7 Extrapolate

7.1 Problem Analysis

In this regard, we need to expand the model to other tourist destinations, such as the Caribbean Sea, etc., that is, we need to carry out environmental migration of the model, and also consider the submersible movement model. In the case of multiple submarines, it is necessary to introduce multi-target Kalman filter to track and predict multiple submarines, and the rescue should first start from the rescue area with the greatest probability.

7.2 Preparation of the Model

7.2.1 Model Assumption

- ✧ Assume that uncertainties such as weather in the area where the submersible and the main vessel are located are ignored.
- ✧ Assume that the routes of multiple submersibles do not conflict with each other.
- ✧ Assume that the submersible has sufficient fuel and normal communication equipment.

7.2.2 Processing Procedure

✧ **Data collection and preprocessing**

The path data of main ships and submersibles in the Caribbean were collected first, and pre-processed and cleaned.

✧ **Extract Eigenvalues**

Due to the need to account for changes in the sea area, we need to add more environmental factors, such as the complexity of the seabed topography, the influence of ocean currents and wind direction, and possible diving sites.

✧ **Model selection and training**

For the case of multiple submersible moving in the same area, a common algorithm is Deep-Q-Network algorithm (MADQN) in multi-agent reinforcement training learning. MADQN is often used to solve the problem of cooperation and competition among multiple agents.

✧ **Model evaluation and Optimization**

The hyperparameters or structure of the model may need to be adjusted to meet the specific conditions in the Caribbean.

7.3 Establishment of the Model

Faced with the above scenarios and needs, we decided to address the two main directions of location migration and multi-agent reinforcement learning respectively

7.3.1 Location transfer

Location migration will lead to increased uncertainties in the navigation process of the main ship and submersible, among which the most influential factor is the ocean current interference of the submersible during diving. Therefore, the dynamic prediction model we established in the first question will have uncertainties. In view of the ocean current interference of the submersible, we need to establish a submersible model under the influence of ocean current to improve the submersible The control performance.

The submersible model affected by ocean currents is sorted into the following submersible dynamic model:

$$M_{RB}\dot{V} + M_A\dot{V}_r + C_{RB}(V)V + C_A(V_r)V_r + D(V_r)V_r + G(P) = F + d + d_c \tag{31}$$

Now we can determine the current disturbance of the submersible:

$$d_c = M_{RB}\dot{V} + M_A\dot{V}_r + C_{RB}(V)V + C_A(V_r)V_r + D(V_r)V_r + G(P) - F - d \tag{32}$$

7.3.2 Multi-agent reinforcement learning

We first modeled the environment of the Caribbean Sea as a multi-agent environment, treated each submersible as an agent, selected actions according to the environmental state, and adjusted the strategy according to the reward signal. The Deep-Q-Network algorithm was initially determined for training, and multiple submersible movements in the environment were simulated. And train the agent's behavior based on the reward signal returned by the environment. Each agent has its own DQN model and chooses actions and learning strategies independently. In each episode, the agent performs an action and observes feedback from the environment, then updates its own Q function based on that feedback.

In reinforcement learning, an episode usually refers to a series of actions and state transitions that begin with the initial state of the environment and end with an episode. In each episode, the agent interacts with the environment, takes actions, observes the state and rewards of the environment, and tries to improve its behavioral strategy through learning; At the end of each episode, the sequence number of the episode and the rewards received by the agent in that episode are printed.

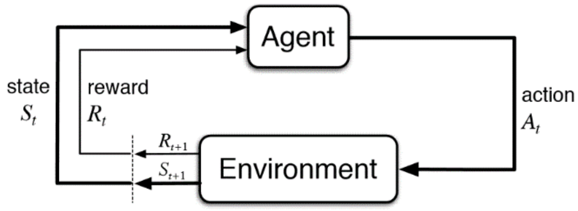
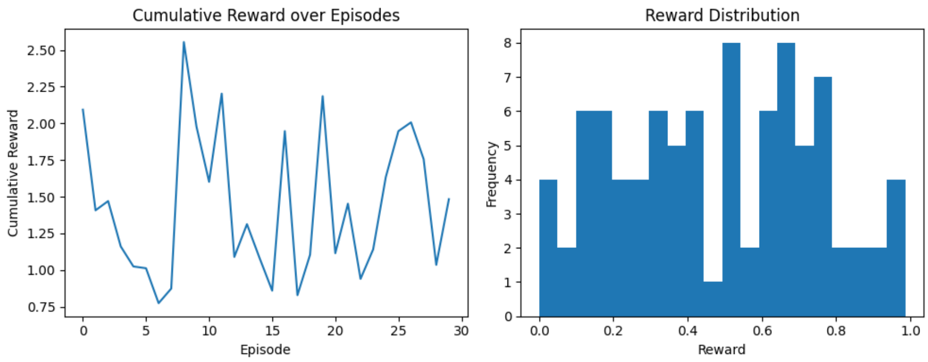
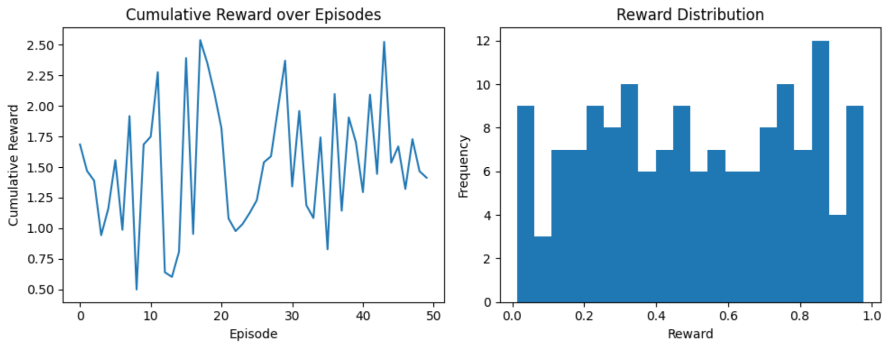


Figure 17. Grid prediction data

The results of 30 rounds of training are as follows:



The results of 50 rounds of training are as follows:



The results of 70 rounds of training are as follows:

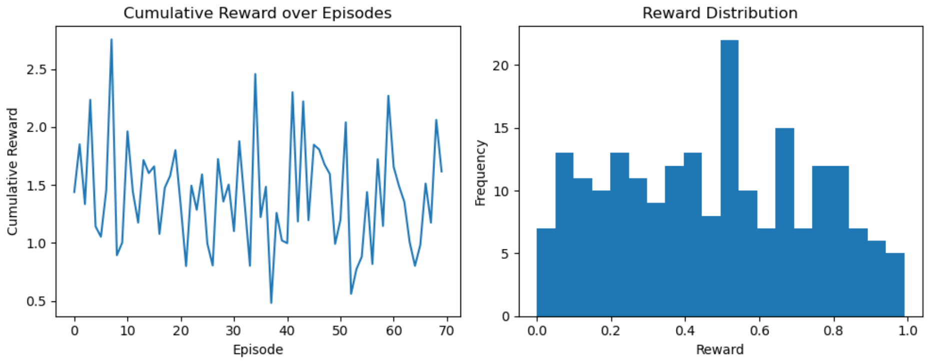


Figure 18. Reward curve and histogram

8 .Sensitivity Analysis

- ✧ The dynamic prediction model based on the improved Kalman filter algorithm to simulate the position of the submersible needs to be updated in real-time in practical applications. In general, the model prediction is consistent with the given path, and the volatility is small.
- ✧ Deep-Q-Learning algorithm is used to deal with the movement problem of multiple submersives, taking into account the cooperation and competition between submersives, to better optimize the overall search efficiency and positioning accuracy. If the parameters continue to be optimized, the accuracy of the model will be higher after a certain number of iterations.

9 Evaluation and Extension of the Model

9.1 Advantages

- ✧ **Lightweight:** The model structure is not complicated, compatible with many practical factors, appropriate introduction of engineering design theory, high optimization
- ✧ **Creativity:** We create metrics that can be verified quantitatively, which also makes our models original
- ✧ **Accuracy:** The model conclusions are consistent with the corresponding simulation data, mutual confirmation, and the logic is consistent and practical
- ✧ **Stability:** The model foundation has been tested by sensitivity analysis, and the error is acceptable, so the model is stable

9.2 Limitations and Extension of the Model

Our model has the following limitations and related improvements:

- ✧ The application of the interference of uncertain factors such as extreme weather, ocean currents, and terrain in extreme weather conditions needs further discussion
- ✧ The model parameters are simulated and not trained on the real dataset, and the accuracy can be improved.
- ✧ We only optimize the initialization model to a certain extent, and the number of model iterations is small, so the practicability needs to be examined and improved.
- ✧ Some of the models may be limited, for example, the cost-benefit ratios, when simulated using the Monte Carlo algorithm, were not given the exact requirements, resulting in less applicability of the subsequent models, which needs to be improved.

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Memorandum

To：Greek government
From: Team 2427171
Subject: Submarine Evolution: Breaking deep sea location and rescue bottlenecks
Date: February 5, 2024

Dear officials of the Greek government,

First of all, on behalf of Maritime Cruises Mini-Submarines (MCMS), allow me to express our deepest respect to you and the members of your department. MCMS, a submersible company dedicated to carrying humans to the deepest parts of the ocean, would like to send you this letter to express our strong desire to take advantage of an innovative and very romantic project. The project aims to explore deep-sea shipwrecks by submarine, offering visitors the adventure of a lifetime, hopefully with the support and permission of your country.

As we all know, Greece is a country rich in Marine resources and historical heritage, and its unique geographical location makes it an ideal place for underwater sightseeing activities. Exploring the Ionian sea floor, you can find many shipwrecks, these ancient ruins lying quietly on the bottom of the sea, telling stories of the past, attracting divers to explore. Around the shipwrecks, Marine life has established its habitat, turning these historic sites into vibrant undersea landscapes. We believe that by working with your country, we can jointly develop a new type of tourism program that will not only provide a unique experience for tourists, which will attract more international tourists to Greece, but will promote the development of tourism in Greece and have a positive impact on the local economy, but also increase people's awareness and responsibility for Marine conservation.

We are well aware of the risks involved in deep-sea exploration activities, so before the official launch of the project, we have invested considerable time and resources in developing a comprehensive submersible safety procedure to ensure the personal safety of every visitor.

The following are the core elements of our submersible safety procedures:

1. Safety standards and compliance: Our submarine design, positioning, and search and rescue functions are very complete, and the materials, construction, and operation of subversives and various other equipment comply with relevant national standards to ensure that the highest safety requirements are met.

2. Professional training and qualification: All employees operating submarines are required to receive professional training and obtain the corresponding qualification certification. In addition, we will conduct regular safety training and emergency response drills for our employees.

3. Safety Inspection and Maintenance: The submarine will undergo a comprehensive safety inspection before each departure to ensure that all systems are in optimal condition. At the same time, we will implement a regular maintenance schedule to prevent any potential

technical problems。

4. Emergency response measures: We have formulated a detailed emergency response plan, involving close cooperation between the main ship, rescue ship and the lost submarine, and rescue operations on surface and underwater and even deep sea. At the same time, the submarine will be equipped with advanced communications equipment to ensure real-time contact with surface support teams, and high-precision positioning equipment to ensure that the submarine can get accurate position information whether on the surface or in the deep sea.

5. Environmental protection: We undertake to take all necessary measures during the implementation of the project to protect the Marine environment and avoid any damage to the wreck site and the surrounding ecology.

We sincerely request the Greek government to review our project proposal and approve it. We believe that through our joint efforts, we can ensure the safe and smooth progress of the project, while bringing more international attention and tourism benefits to Greece."

We look forward to further communication with the Greek government to jointly advance this project. We look forward to the opportunity to further discuss the details of this project with your country and explore how we can work together to make this innovative project a reality."

Thank you for taking the time to read this letter. We look forward to hearing from you and hope to embark on this exciting journey of underwater exploration together in the near future.

Yours sincerely，
Team #2427171