

Q8 Maximum subarray sum. (Kadane's Algorithm)

I/P $\rightarrow \{5, 4, -1, 7, 8\}$

O/P $\rightarrow 23$

Sub arrays can be defined as part of array which is contiguous.

All possible sub-arrays

$\{5\} \rightarrow 5$

$\{5, 4\} \rightarrow 9$

$\{5, 4, -1\} \rightarrow 8$

$\{5, 4, -1, 7\} \rightarrow 15$

$\{5, 4, -1, 7, 8\} \rightarrow 23$] maximum sum.

$\{4\} \rightarrow 4$

$\{4, -1\} \rightarrow 3$

$\{4, -1, 7\} \rightarrow 10$

$\{4, -1, 7, 8\} \rightarrow 18$

$\{-1\} \rightarrow -1$

$\{-1, 7\} \rightarrow 6$

$\{-1, 7, 8\} \rightarrow 14$

$\{7\} \rightarrow 7$

$\{7, 8\} \rightarrow 15$

$\{8\} \rightarrow 8$

The above approach is the brute force as we are finding all the possible sub-arrays. We need to find the optimized approach as here 3 loops would be required & hence $O(n^3)$ is the time complexity. We need to find the solution in $O(n)$ approach & here Kadane's algorithm is required.

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Dry run + Approach

$\{-2, 1, -3, 4, -1, 2, 1, -5, 4\}$

$i \nearrow$

$\text{arr}[0]$

Initially sum = 0, $\max_i = -2$

1) $i = 0$

$$\text{sum} = \text{sum} + \text{arr}[i]$$

$$\text{sum} = 0 + (-2) = -2$$

Now update \max_i

$$\max_i = \max(\text{sum}, \max_i) = \max(-2, -2) = -2$$

Now as $\text{sum} < 0$, make $\text{sum} = 0$.

2) $i = 1$

$$\text{sum} = \text{sum} + \text{arr}[i]$$

$$\text{sum} = 0 + 1 = 1$$

$$\max_i = \max(\text{sum}, \max_i) = \max(-2, 1) = 1$$

$\leftarrow \max_i$

$\text{sum} < 0 \rightarrow \text{False} \& \text{ hence do nothing}$

3) $i = 2$

$$\text{sum} = \text{sum} + \text{arr}[i]$$

$$\text{sum} = 1 + (-3) = -2$$

$$\max_i = \max(1, -2) = 1$$

$\text{sum} < 0 \rightarrow \text{True} \& \text{ hence } \text{sum} = 0$

4) $i = 3$

$$\text{sum} = 0 + 4 = 4$$

$$\max_i = \max(1, 4) = 4$$

$\text{sum} < 0 \rightarrow \text{False} \& \text{ hence do nothing}$

5) $i = 4$

$$\text{sum} = 4 + (-1) = 3$$

$$\text{maxi} = \max(4, 3) = 4$$

$\text{sum} < 0 \rightarrow \text{False}$ & hence do nothing

6) $i = 5$

$$\text{sum} = 3 + 2 = 5$$

$$\text{maxi} = \max(4, 5) = 5$$

$\text{sum} < 0 \rightarrow \text{False}$ & hence do nothing

7) $i = 6$

$$\text{sum} = 5 + 1 = 6$$

$$\text{maxi} = \max(5, 6) = 6$$

$\text{sum} < 0 \rightarrow \text{False}$ & hence do nothing

8) $i = 7$

$$\text{sum} = 6 - 5 = 1$$

$$\text{maxi} = \max(6, 1) = 6$$

$\text{sum} < 0 \rightarrow \text{False}$ & hence do nothing

9) $i = 8$

$$\text{sum} = 1 + 4 = 5$$

$$\text{maxi} = \max(6, 5) = 6$$

$\text{sum} < 0 \rightarrow \text{False}$ & hence do nothing.

Hence we have traversed the whole array & at last just return maxi.

Steps

1) $\text{sum} = 0, \text{maxi} = \text{nums}[0];$

2) Traverse array

* $\text{sum} = \text{sum} + \text{arr}[i]$

* $\text{maxi} = \max(\text{sum}, \text{maxi})$

* $\text{sum} < 0 \rightarrow \text{sum} = 0$

3) Return maxi.

Code

```
int maxSubArraySum (vector <int> &nums) {  
    //Initial steps  
    int sum = 0;  
    int maxi = nums[0];  
    //Traversing array  
    for (int i=0 ; i<nums.size() ; i++) {  
        // Step-2 (1st step)  
        sum = sum + nums[i];  
        // Step-2 (2nd step)  
        maxi = max (sum, maxi);  
        // Step-1 (3rd step)  
        if (sum < 0) {  
            sum = 0;  
        }  
    } //Step-3  
    return maxi;  
}
```

Time complexity = $O(n)$

Space complexity = $O(1)$