九、着色（下）

吴雨婷

北京交通大学 软件学院

ytwu1@bjtu.edu.cn

Reference: GAMES101:https://sites.cs.ucsb.edu/~lingqi/teaching/games101.html

Interpolation Across Triangles: Barycentric Coordinates

(重心坐标)

Interpolation Across Triangles

Why do we want to interpolate?

- Specify values **at vertices**
- Obtain smoothly varying values **across triangles**

What do we want to interpolate?
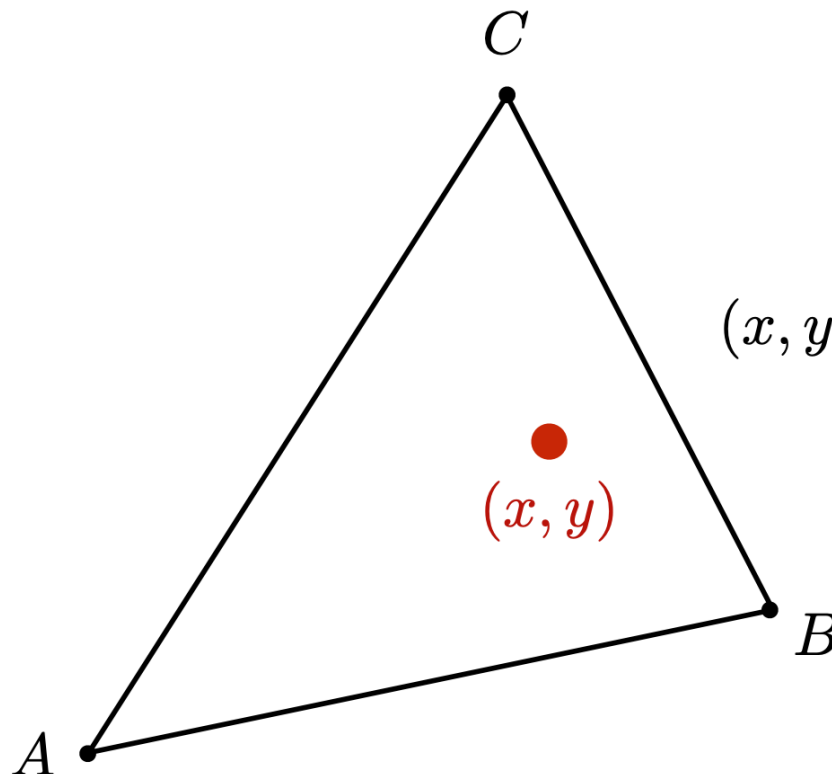
- Texture coordinates, colors, normal vectors, …

How do we interpolate?

- **Barycentric coordinates**

Barycentric Coordinates

A coordinate system for triangles (α, β, γ)

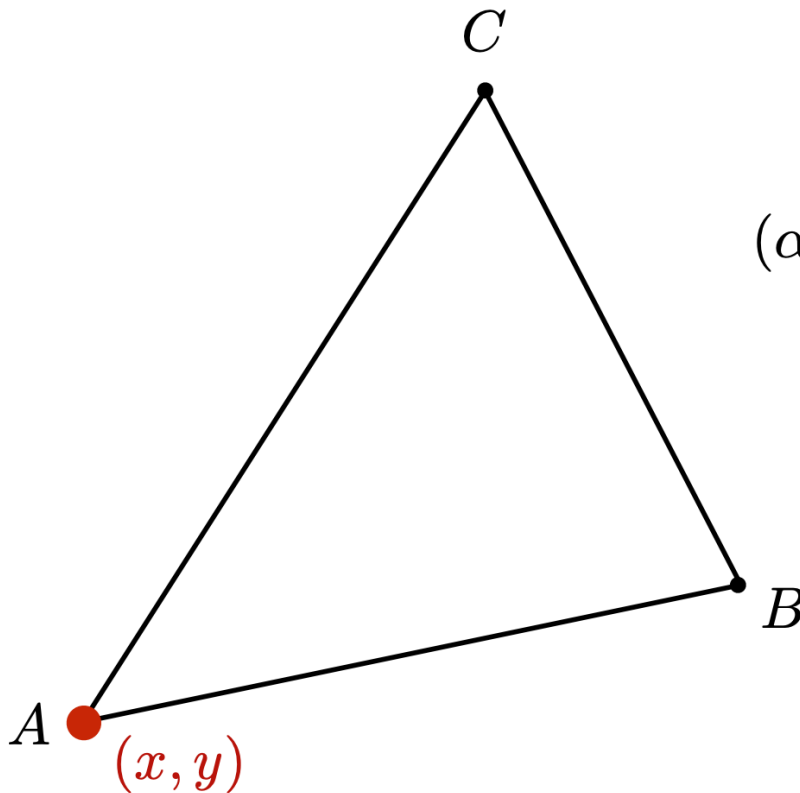$$(x, y) = \alpha\, A + \beta\, B + \gamma\, C$$

$$\alpha + \beta + \gamma = 1$$

Inside the triangle if all three coordinates are non-negative

Reference: GAMES101:https://sites.cs.ucsb.edu/~lingqi/teaching/games101.html

4

Barycentric Coordinates

What's the barycentric coordinate of A?

$$(\alpha, \beta, \gamma) = (1, 0, 0)$$
$$(x, y) = \alpha\, A + \beta\, B + \gamma\, C$$
$$= A$$

Reference: GAMES101:https://sites.cs.ucsb.edu/~lingqi/teaching/games101.html

Barycentric Coordinates

Geometric viewpoint — proportional areas
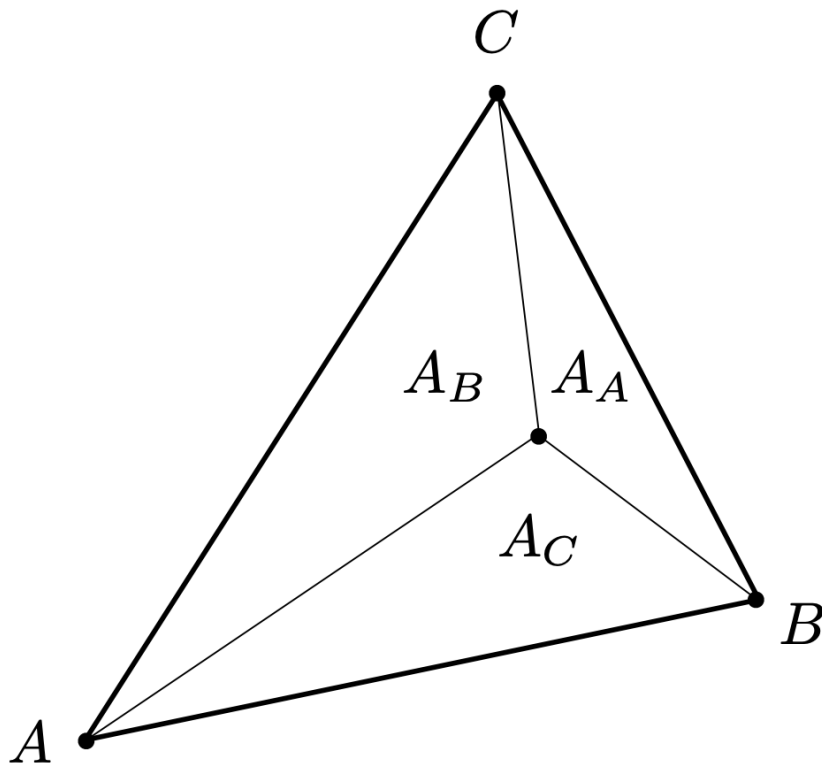
$$\alpha = \frac{A_A}{A_A + A_B + A_C}$$

$$\beta = \frac{A_B}{A_A + A_B + A_C}$$

$$\gamma = \frac{A_C}{A_A + A_B + A_C}$$

Reference: GAMES101:https://sites.cs.ucsb.edu/~lingqi/teaching/games101.html

Barycentric Coordinates

What's the barycentric coordinate of the centroid?

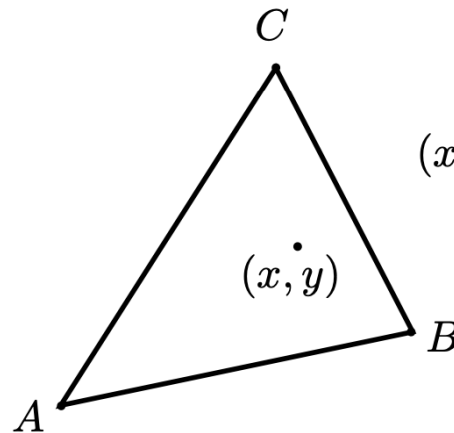
$$(\alpha, \beta, \gamma) = \left(\tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}\right)$$
$$(x, y) = \tfrac{1}{3}A + \tfrac{1}{3}B + \tfrac{1}{3}C$$

Reference: GAMES101:https://sites.cs.ucsb.edu/~lingqi/teaching/games101.html
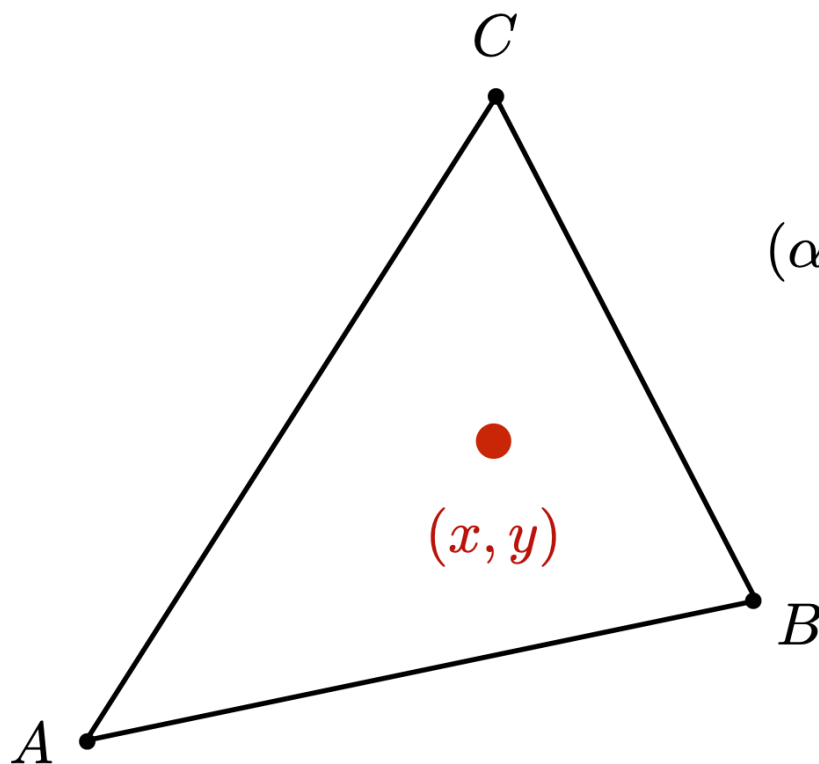
Barycentric Coordinates: Formulas

$$(x, y) = \alpha A + \beta B + \gamma C$$
$$\alpha + \beta + \gamma = 1$$

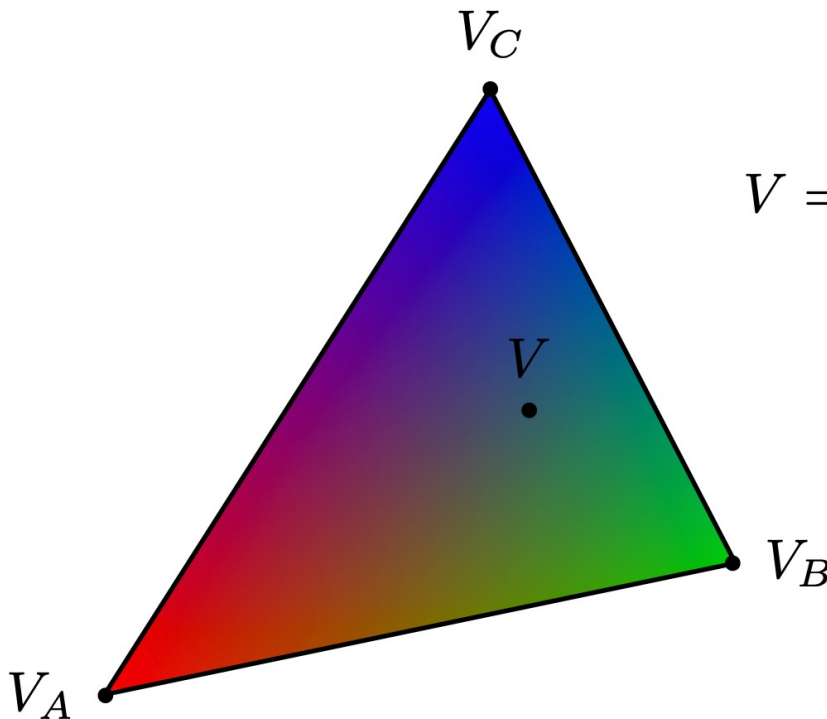$$\alpha = \frac{-(x - x_B)(y_C - y_B) + (y - y_B)(x_C - x_B)}{-(x_A - x_B)(y_C - y_B) + (y_A - y_B)(x_C - x_B)}$$

$$\beta = \frac{-(x - x_C)(y_A - y_C) + (y - y_C)(x_A - x_C)}{-(x_B - x_C)(y_A - y_C) + (y_B - y_C)(x_A - x_C)}$$

$$\gamma = 1 - \alpha - \beta$$

Reference: GAMES101:https://sites.cs.ucsb.edu/~lingqi/teaching/games101.html

Using Barycentric Coordinates

Linearly interpolate values at vertices

$$V = \alpha V_A + \beta V_B + \gamma V_C$$

V_A, V_B, V_C can be positions, texture coordinates, color, normal, depth, material attributes...

However, barycentric coordinates are not invariant under projection!

Reference: GAMES101:https://sites.cs.ucsb.edu/~lingqi/teaching/games101.html

9

感谢大家的倾听！

吴雨婷

北京交通大学 软件学院

ytwu1@bjtu.edu.cn