ELE20: Basis Elektro - Week 6

Hogeschool Rotterdam - Opleiding Elektrotechniek

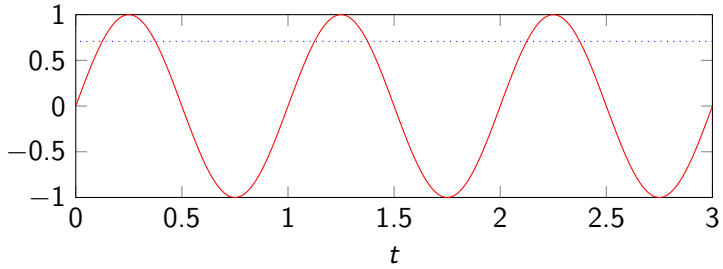
Vandaag

1. Gemiddelde en Effectieve waarde
2. Phasor notatie
3. Herhaling complexe getallen
4. Complexe rekenwijze
5. Complexe KVL en KCL methode
6. Huiswerk

Gemiddelde en Effectieve waarde

Sinusvormig signaal:

- $v(t) = V_m \cdot \cos(\omega \cdot t + \theta)$
- $\omega = 2 \cdot \pi \cdot f$ en $\omega \cdot T = 2 \cdot \pi$
- Frequentie $f = \frac{1}{T}$
- θ is de fase of starthoek



Gemiddelde / Effectieve waarde

Effectieve waarde komt overeen met de gelijkspanning die hetzelfde vermogen levert.

$$p(t) = v(t) \cdot i(t) = \frac{v^2(t)}{R} = \frac{V_m^2 \cdot \sin^2(\omega t)}{R}$$

$$P_{eff} = P_{avg} = \frac{1}{T} \cdot \int_0^T \frac{V_m^2 \cdot \sin^2(\omega t)}{R} \cdot dt = \frac{V_m^2}{2R}$$

$$P_{avg} = \frac{V_{rms}^2}{R} = \frac{V_m^2}{2R}$$

Dus geldt dat:

$$V_{eff} = V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{V_{peak}}{\sqrt{2}}$$

Dit geldt voor een sinusvormig signaal. Voor andere signalen (driehoek, zaagtand, etc.) wordt de factor $\sqrt{2}$ anders.

Exercise 5.1

Exercise 5.1: $v(t) = 100 \cdot \cos(100\pi t), R = 50\,\Omega$.
Hoe groot is V_{rms}, P_{avg}, f, T en V_{peak}?

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Hoe groot is V_{rms}, P_{avg}, f, T en V_{peak}?

$$V(t) = 100 \cdot \cos(100 \cdot \pi t)$$
$$f = 50\,\text{Hz}, T = 20\,\text{ms}$$
$$V_{peak} = 100\,\text{V}$$
$$V_{rms} = \frac{100}{\sqrt{2}} = 70.71\,\text{V}$$
$$P_{avg} = \frac{(\frac{100}{\sqrt{2}})^2}{50} = 100\,\text{W}$$

Phasor notatie

Spanningen en stromen kunnen we als een vector beschrijven. Dit vereenvoudigt veel berekeningen.

- Afspraak: Altijd **cosinus** gebruiken voor de vectornotatie!
- Gebruik: $\sin(x) = \cos(x - 90^\circ)$
- Definitie: $v(t) = V_1 \cdot \cos(\omega \cdot t + \theta_1) \rightarrow V_1 \angle \theta_1$
- Ook complex weer te geven: $V_1 \angle \theta_1 = V_1 \cdot e^{j \cdot (\omega \cdot t + \theta_1)}$

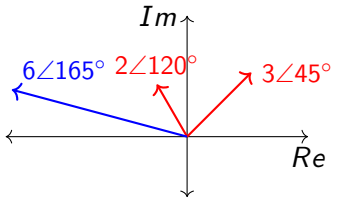
Voorbeeld:

$$v(t) = 10 \cdot \sin\left(300\pi t + \frac{\pi}{3}\right) = 10 \cdot \cos\left(300\pi t - \frac{\pi}{6}\right) = 10 \angle -\frac{\pi}{6}$$

Herhaling complexe getallen

Complexe getallen $Z_1 = a + jb$ en $Z_2 = c + jd$

- Optellen: $Z_1 + Z_2 = a + c + j(b + d)$
- Vermenigvuldigen: $Z_1 \cdot Z_2 = ac - bd + j(ad + bc)$
  - Lengten vermenigvuldigen: $\|Z_1.Z_2\| = \|Z_1\| \cdot \|Z_2\|$
  - Hoeken optellen: $\angle Z_1 + \angle Z_2 = \angle(Z_1 \cdot Z_2)$
- Delen: $\frac{Z_1}{Z_2} = \frac{ac+bd+j(bc-ad)}{c^2+d^2}$
  - Lengten op elkaar delen: $\|\frac{Z_1}{Z_2}\| = \frac{\|Z_1\|}{\|Z_2\|}$
  - Hoeken aftrekken: $\angle Z_1 - \angle Z_2 = \angle \frac{Z_1}{Z_2}$
- Omrekenen naar hoekcoördinaten: $Z = \|L\| \angle \theta$
  - $L = \sqrt{a^2 + b^2}$
  - $\theta = \arctan\left(\frac{b}{a}\right)$
- Omrekenen naar Re en Im coördinaten: $Z = a + jb$
  - $a = \|L\| \cdot \cos(\theta)$
  - $b = \|L\| \cdot \sin(\theta)$



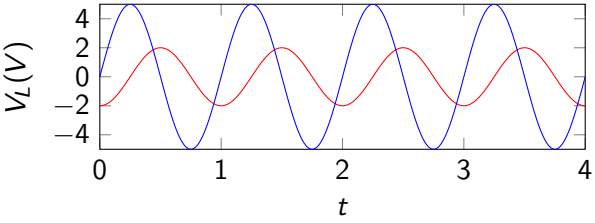
Complexe rekenwijze bij een Spoel

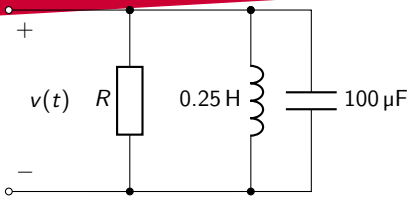
Gevraagd: de stroom I_L door de spoel.
$v(t) = v_L(t) = v_C(t) = v_R(t) = 100 \cdot \cos(200t)$
$V = 100\angle 0°$

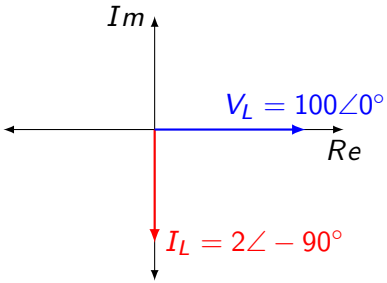
Oplossing:

$$Z_L = j\omega L = j \cdot 200 \cdot 0.25 = 50j = 50\angle 90°$$

$$I_L = \frac{V_L}{Z_L} = \frac{100\angle 0°}{50\angle 90°} = 2\angle(-90°)$$







De stroom loopt dus 90° achter op de spanning!

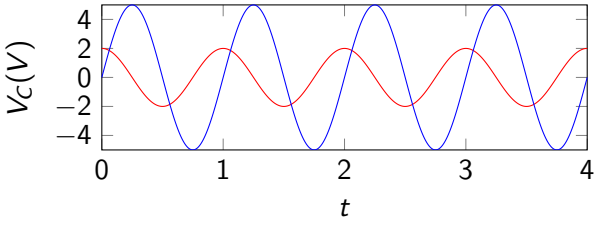
Complexe rekenwijze bij een Condensator

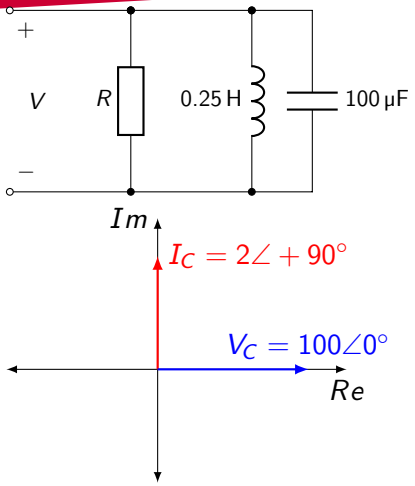
Gevraagd: de stroom I_C door de condensator.
$V = V_L = V_C = V_R = 100 \cdot \cos(200t) = 100\angle 0°$

Oplossing:

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j \cdot 200 \cdot 100.10^{-6}} = -50j = 50\angle -90°$$

$$I_C = \frac{V_C}{Z_C} = \frac{100\angle 0°}{50\angle -90°} = 2\angle(+90°)$$





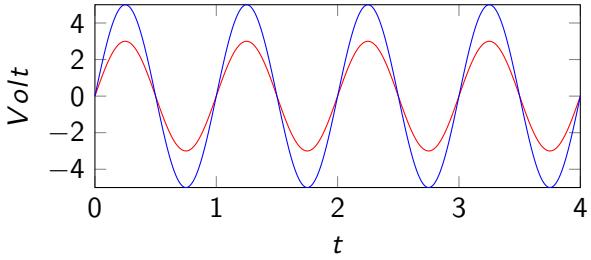
De spanning loopt dus 90° achter op de stroom!

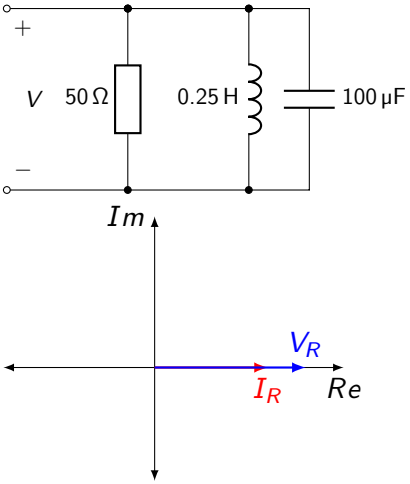
Complexe rekenwijze bij een Weerstand

Gevraagd: de stroom I_R door de weerstand.
$V = V_L = V_C = V_R = 100 \cdot \cos(200t) = 100\angle 0^\circ$

Oplossing: De spanning en stroom zijn in fase:

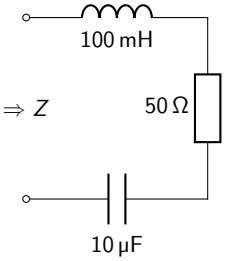
$$I_R = \frac{V_R}{Z_R} = \frac{100\angle 0^\circ}{50\angle 0^\circ} = 2\angle 0^\circ$$





Complexe rekenwijze: Opgave 1

Opdracht: Bepaal de impedantie Z als $f = 127.324\,\text{Hz}$



$$\omega = 2\pi f = 800\,\text{rad}\,\text{s}^{-1}$$

Complexe rekenwijze: Opgave 1

Opdracht: Bepaal de impedantie Z als $f = 127.324\,\text{Hz}$

$$Z_R = 50\angle 0^\circ = 50 + 0j$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j \cdot 800 \cdot 10.10^{-6}} = 0 - 125j = 125\angle(-90^\circ)$$

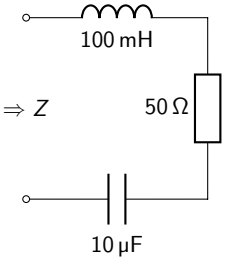
$$Z_L = j\omega L = j \cdot 800 \cdot 0.1 = 0 + 80j = 80\angle(+90^\circ)$$

$$Z = Z_R + Z_C + Z_L = 50 + 0j + 0 - 125j + 0 + 80j = 50 - 45j$$

$$\angle = \arctan\frac{-45}{50} \approx \angle(-42^\circ)$$

$$lengte = \sqrt{50^2 + 45^2} = 67.3$$

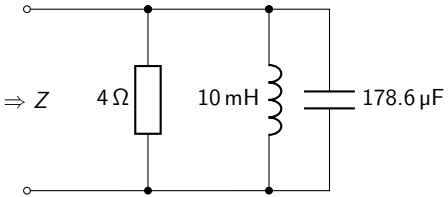
$$Z = 50 - 45j = 67.3\angle(-42^\circ)$$



$$\omega = 2\pi f = 800\,\text{rad}\,\text{s}^{-1}$$

Complexe rekenwijze: Opgave 2

Opdracht: Bepaal de impedantie Z als $f = 127.324\,\text{Hz}$



$$\omega = 2\pi f = 800\,\text{rad}\,\text{s}^{-1}$$

Complexe rekenwijze: Opgave 2

Opdracht: Bepaal de impedantie Z als $f = 127.324\,\text{Hz}$

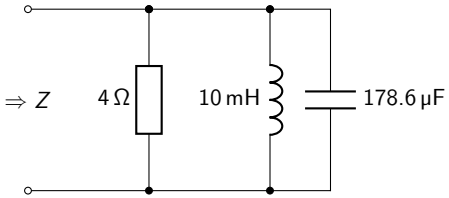
$$Z_R = 4\angle 0^\circ = 4 + 0j$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j \cdot 800 \cdot 178.6.10^{-6}} = 0 - 7j = 7\angle(-90^\circ)$$

$$Z_L = j\omega L = j \cdot 800 \cdot 0.01 = 0 + 8j = 8\angle(+90^\circ)$$

$$Z = \frac{1}{\frac{1}{R} + \frac{1}{Z_C} + \frac{1}{Z_L}} = \frac{1}{\frac{1}{4} + \frac{1}{-7j} + \frac{1}{8j}} = \frac{1}{\frac{1}{4} + \frac{j}{7} - \frac{j}{8}}$$

$$Z = \frac{1}{\frac{1}{4} + \frac{j}{56}} = \frac{1\angle 0^\circ}{0.25\angle 4.09^\circ} = 4\angle -4.09^\circ \approx 3.99 - 0.29j$$



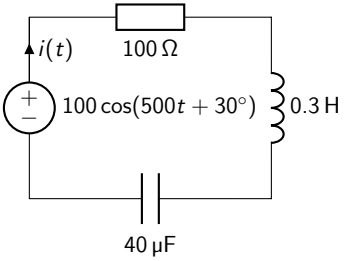
$$\omega = 2\pi f = 800\,\text{rad}\,\text{s}^{-1}$$

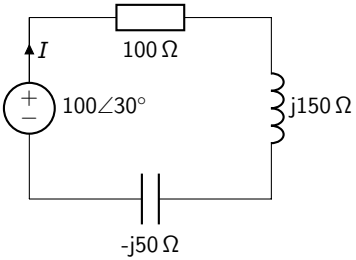
Complexe rekenwijze met KVL en KCL

KVL en KCL ook bruikbaar bij complexe impedanties:

- Tijd -> complex (frequentie moet overal gelijk zijn!)
- Weerstand Z is complex getal
- KVL en KCL gewoon toepassen
- Alle spanningen en stromen zijn dan ook complex

Oefening: Bepaal de complexe stroom I en de stroom i(t)





Complexe rekenwijze met KVL en KCL

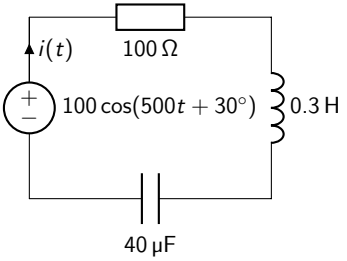
$$Z = R + Z_L + Z_C = 100 + j150 - j50 = 100 + 100j$$

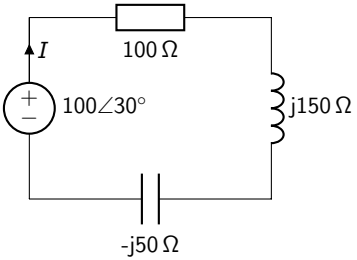
$$Z = 141.4\angle 45^\circ$$

$$I = \frac{V}{Z} = \frac{100\angle 30^\circ}{141.4\angle 45^\circ} = 0.71\angle(-15^\circ) = 0.69 - j0.18$$

Hiermee wordt $i(t)$ gelijk aan:

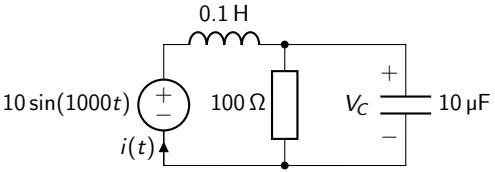
$$i(t) = 0.71\cos(500t - 15^\circ)$$

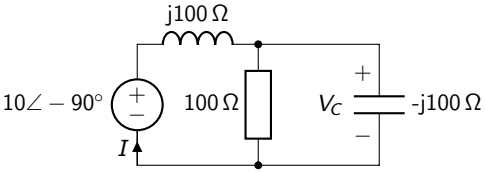


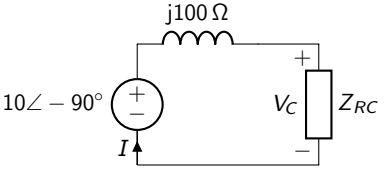


Complexe rekenwijze met KVL en KCL: opdracht

Bepaal ω, Z_L, I, I_R en I_C en teken het phasor diagram.







Complexe rekenwijze met KVL en KCL: opdracht

Bepaal ω, Z_L, I, I_R en I_C en teken het phasor diagram.

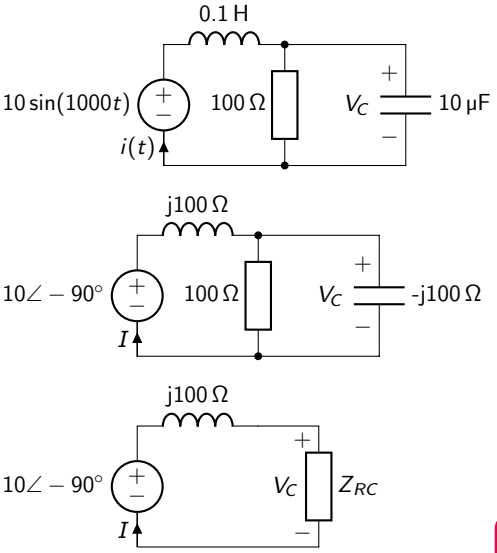
$$Z_{RC} = \frac{100 \cdot -100j}{100 + -100j} = \frac{-100j}{1-j} = \frac{100\angle(-90^\circ)}{1.41\angle(-45^\circ)}$$

$$Z_{RC} = 70.1\angle -45^\circ = 50 - 50j$$

Z_{RC} met L in serie levert:

$$Ztot = Z_{RC} + Z_L = 50 - 50j + 100j = 50 + 50j$$

$$Z_{tot} \approx 70.71\angle(45^\circ)$$



Complexe rekenwijze met KVL en KCL: opdracht

Bepaal ω, Z_L, I, I_R en I_C en teken het phasor diagram.

$$Z_{RC} = 70.1\angle -45^\circ$$

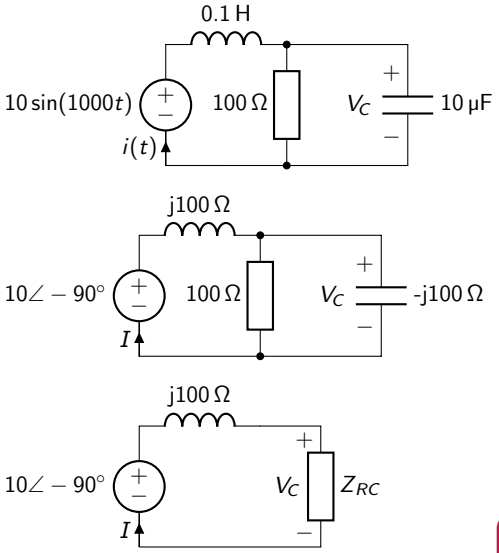
$$Z_{tot} \approx 70.71\angle(45^\circ)$$

$$V_s = 10\sin(1000t) = 10\cos(1000t - 90^\circ))$$

$$V_s = 10\angle(-90^\circ)$$

$$V_C = \frac{Z_{RC}}{Z_{tot}} \cdot V_s = \frac{70.71\angle(-45^\circ)}{70.71\angle(45^\circ)} 10\angle(-90^\circ)$$

$$V_C = 10\angle(-180\,) = -10\cos(1000t)$$

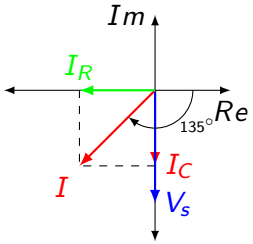


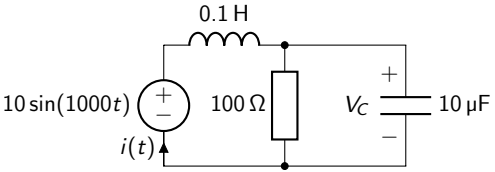
Complexe rekenwijze met KVL en KCL: opdracht

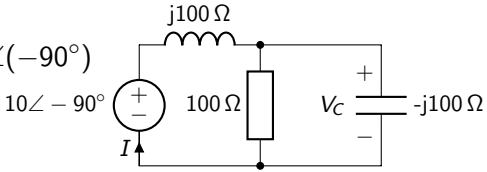
$$I = \frac{V_s}{Z_{tot}} = \frac{10\angle(-90°)}{70.71\angle(45°)} = 0.14\angle(-135°)$$

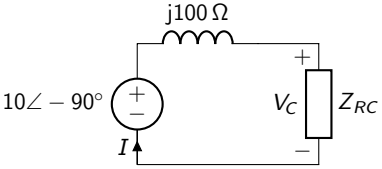
$$I_R = \frac{V_C}{R} = \frac{10\angle(-180°)}{100\angle(0°)} = 0.1\angle(-180°)$$

$$I_C = \frac{V_C}{Z_C} = \frac{10\angle(-180°)}{-100j} = \frac{10\angle(-180°)}{100\angle(-90°)} = 0.1\angle(-90°)$$

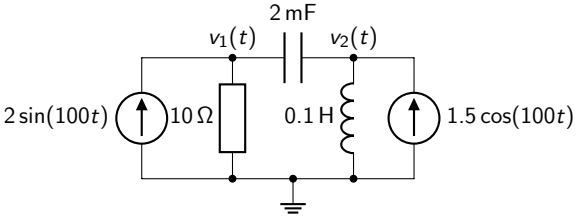


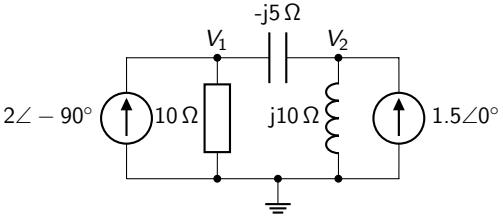




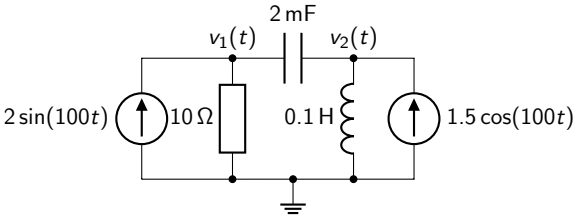


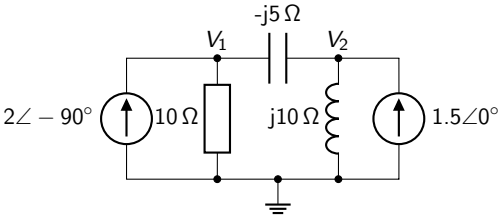
Complexe knooppuntmethode KCL





Complexe knooppuntmethode KCL





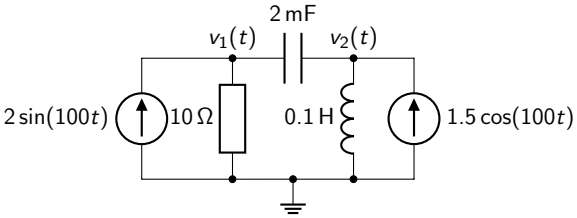
KP1: $\dfrac{v_1 - 0}{10} + \dfrac{v_1 - v_2}{-5j} = 2\angle -90° = 0 - 2j$

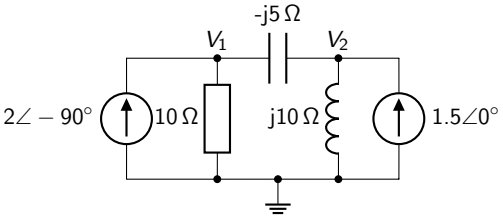
$$jv_1 - 2v_1 + 2v_2 = 20 \Rightarrow (j-2)\,v_1 + 2v_2 = 20$$

KP2: $\dfrac{v_2 - 0}{10j} + \dfrac{v_2 - v_1}{-5j} = 1.5\angle 0° = 1.5 + 0j$

$$v_2 - 2v_2 + 2v_1 = 15j \Rightarrow 2v_1 - v_2 = 15j$$

Complexe knooppuntmethode KCL





KP1: $\dfrac{v_1 - 0}{10} + \dfrac{v_1 - v_2}{-5j} = 2\angle -90° = 0 - 2j$

$$jv_1 - 2v_1 + 2v_2 = 20 \Rightarrow (j-2)\,v_1 + 2v_2 = 20$$

KP2: $\dfrac{v_2 - 0}{10j} + \dfrac{v_2 - v_1}{-5j} = 1.5\angle 0° = 1.5 + 0j$

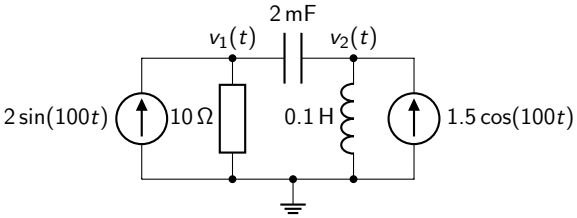
$$v_2 - 2v_2 + 2v_1 = 15j \Rightarrow 2v_1 - v_2 = 15j$$

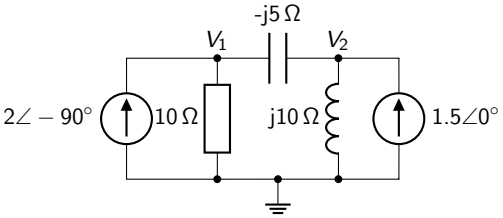
$$\begin{pmatrix} j-2 & 2 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 20 \\ 15j \end{pmatrix}$$

$$v_1 = \frac{D_1}{D} = \frac{-20 - 30j}{-2 - j} = \frac{20 + 30j}{2 + j}$$

$$v_1 = \frac{36.06\angle 56.31°}{2.24\angle 26.57°} = 16.1\angle 29.7°$$

Complexe knooppuntmethode KCL





KP1: $\dfrac{v_1 - 0}{10} + \dfrac{v_1 - v_2}{-5j} = 2\angle -90° = 0 - 2j$

$$jv_1 - 2v_1 + 2v_2 = 20 \Rightarrow (j-2)\,v_1 + 2v_2 = 20$$

KP2: $\dfrac{v_2 - 0}{10j} + \dfrac{v_2 - v_1}{-5j} = 1.5\angle 0° = 1.5 + 0j$

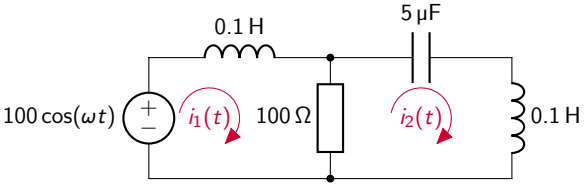
$$v_2 - 2v_2 + 2v_1 = 15j \Rightarrow 2v_1 - v_2 = 15j$$

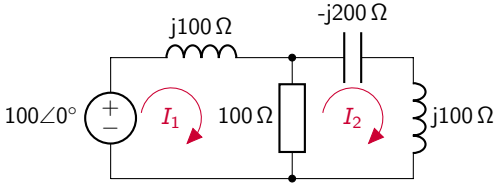
$$\begin{pmatrix} j-2 & 2 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 20 \\ 15j \end{pmatrix}$$

$$v_2 = \frac{D_2}{D} = \frac{-55 - 30j}{-2 - j} = \frac{55 + 30j}{2 + j}$$

$$v_2 = \frac{62.65\angle 28.61°}{2.24\angle 26.57°} = 28.0\angle 2°$$

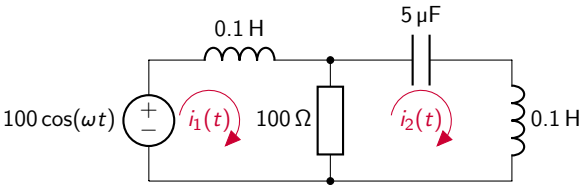
Complexe maasmethode KVL

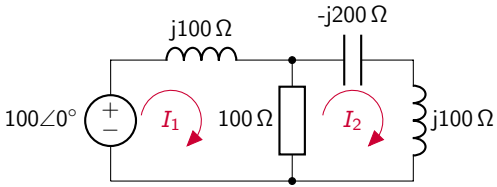




$\omega = 1000\,\mathrm{rad\,s^{-1}}$

Complexe maasmethode KVL





$\omega = 1000\,\text{rad}\,\text{s}^{-1}$

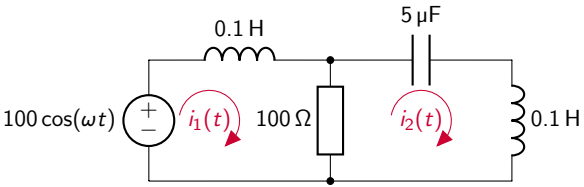
Lus 1: $-100\angle 0^\circ + I_1 100j + (I_1 - I_2)100 = 0$

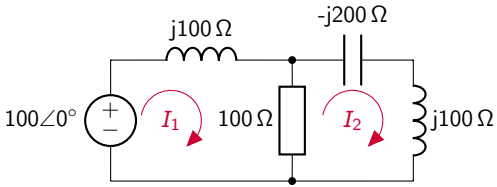
$I_1(100j+100) - 100I_2 = 100 \Rightarrow I_1(j+1) - I_2 = 1$

Lus 2: $100(I_2 - I_1) - 100jI_2 = 0$

$-I_1 + (1-j)I_2 = 0$

Complexe maasmethode KVL





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Lus 2: $100(I_2 - I_1) - 100jI_2 = 0$

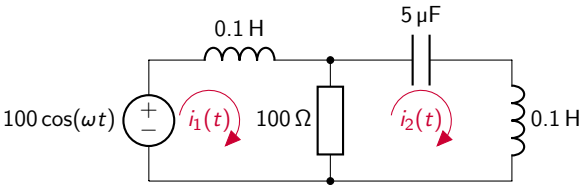
$-I_1 + (1-j)I_2 = 0$

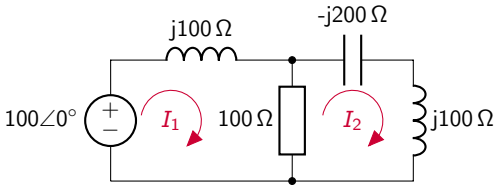
$$\begin{pmatrix} j+1 & -1 \\ -1 & 1-j \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$I_1 = \frac{D_1}{D} = \frac{1-j}{2-1} = 1-j$$

$$I_1 = 1.41\angle -45^\circ$$

Complexe maasmethode KVL





$\omega = 1000\,\text{rad}\,\text{s}^{-1}$

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Lus 2: $100(I_2 - I_1) - 100jI_2 = 0$

$-I_1 + (1-j)I_2 = 0$

$$\begin{pmatrix} j+1 & -1 \\ -1 & 1-j \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$I_2 = \frac{D_2}{D} = \frac{1}{1} = 1$$

$$I_2 = 1\angle 0^\circ$$

Huiswerk

- Hambley H5 opgaven P5.5, 5.6, 5.8, 5.9, 5.23, 5.25, 5.31, 5.32, 5.33, 5.34, 5.40, 5.41, 5.43, 5.47 t/m 5.59 en van de appendix: PA.1 t/m PA.7
- Hambley H6 opgaven P6.8, 6.11, 6.26
- Zie ook de cursushandleiding