

Tabel 1: Eigenschappen van de Laplacetransformatie

Eigenschap of operatie	Signaal	Laplacetransformatie
Transformatie	$f(t)$	$F(s) = \int_0^\infty f(t)e^{-st} dt$
Inverse transformatie	$f(t) = \frac{1}{2\pi j} \lim_{\omega \rightarrow \infty} \int_{\sigma-j\omega}^{\sigma+j\omega} F(s)e^{st} ds$	$F(s)$
Lineariteit	$c_1 \cdot f_1(t) + c_2 \cdot f_2(t)$	$c_1 \cdot F_1(s) + c_2 \cdot F_2(s)$
Verschuiving in s -domein	$e^{-at}f(t)$	$F(s+a)$
Verschuiving in t -domein	$f(t+a)$	$F(s)e^{-as}$
Afgeleiden	$\frac{d^n}{dt^n}f(t)$	$s^n F(s) - f(0)$
Primitieven	$\int_0^t f(t) dt$	$\frac{1}{s}F(s)$
Beginwaardestelling	$f(0) = \lim_{t \rightarrow 0} f(t)$	$f(0) = \lim_{t \rightarrow \infty} s \cdot F(s)$
Eindwaardestelling	$f(\infty) = \lim_{t \rightarrow \infty} f(t)$	$f(\infty) = \lim_{t \rightarrow 0} s \cdot F(s)$

Tabel 2: Standaard Laplacetransformaties

functie in tijd	Laplace getrans.
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$
$\sin(\omega t)$	$\frac{\omega}{s^2 + \omega^2}$
$\cos(\omega t)$	$\frac{s}{s^2 + \omega^2}$
at^n	$a \frac{n!}{s^{n+1}}$

Bode Diagram regels:

1. Zet de overdracht om in de Bode vorm.
2. Sorteert (complexe paren) nulpunten en polen op hun (kantel)frequentie.
3. Bepaal magnitude/fase van $K_o(j\omega)^n$ term.
4. Ga de nulpunten en polen in oplopende frequentie af om te zien hoe magnitude/fase veranderen.

$$i_c = C \frac{dv_c}{dt}$$

$$i_l = \frac{1}{L} \int v_l dt$$

$$E_{\text{stat}} = \frac{100\%}{1 + K_{\text{lus}}}$$

$$PI(s) = K_r \frac{\tau_i s + 1}{\tau_i s}$$

$$PD(s) = K_r (\tau_d s + 1)$$

Eerste orde systeem:

$$H(s) = \frac{K_{\text{stat}}}{\tau s + 1}$$

Tweede orde systeem:

$$H(s) = \frac{K_{\text{pn}}}{s^2 + 2\beta\omega_o s + \omega_o^2} = \frac{K_{\text{stat}}}{\frac{s^2}{\omega_o^2} + \frac{2\beta s}{\omega_o} + 1} = \frac{K_{\text{stat}}\omega_o^2}{s^2 + 2\beta\omega_o s + \omega_o^2}$$

$$\omega_g = \omega_o \sqrt{1 - \beta^2}$$

$$t_p = \frac{\pi}{\omega_g}$$

$$D_{\%} = e^{-\pi \frac{\beta}{\sqrt{1-\beta^2}}} \cdot 100\% = e^{-\pi |\frac{\lambda}{\omega_g}|} \cdot 100\%$$

$$t_{s(2\%)} \approx \frac{4}{\beta\omega_o} = \frac{4}{|\lambda|}$$

$$t_{s(5\%)} \approx \frac{3}{\beta\omega_o} = \frac{3}{|\lambda|}$$

Integrator:

$$H(s) = \frac{1}{\tau_i s}$$

Differentiator:

$$H(s) = \tau_d s$$

Poolbanen

$$\alpha_s = \frac{\sum_{i=1}^n p_i - \sum_{i=1}^m z_i}{n - m}$$

$$\alpha_h = \frac{180^\circ + k \cdot 360^\circ}{n - m} \quad (k = 0, 1, \dots, n - m - 1)$$

$$\sum_{i=1}^n \frac{1}{s - p_i} = \sum_{i=1}^m \frac{1}{s - z_i}$$

Overdrachtsfuncties

Evans vorm:

$$H(s) = K_{\text{pn}} \frac{\prod_{i=1}^m (s - z_i)}{\prod_{i=1}^n (s - p_i)}$$

Bode vorm:

$$H(j\omega) = K_o(j\omega)^n \frac{\left(\frac{j\omega}{\omega_0} + 1\right) \left(\frac{j\omega}{\omega_1} + 1\right) \dots}{\left(\frac{j\omega}{\omega_a} + 1\right) \left(\frac{j\omega}{\omega_b} + 1\right) \dots}$$

State-Space

$$\frac{Y(s)}{U(s)} = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D}$$

$$H(s) = \frac{\det \begin{bmatrix} s\mathbf{I} - \mathbf{A} & -\mathbf{B} \\ \mathbf{C} & D \end{bmatrix}}{\det(s\mathbf{I} - \mathbf{A})}$$

$$\mathcal{O} = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix} \quad \mathcal{C} = \begin{bmatrix} B & AB & A^2B & \dots & A^{n-1}B \end{bmatrix}$$

$$\det(s\mathbf{I} - \mathbf{A} + \mathbf{B}\mathbf{K}) = 0$$

$$\dot{\mathbf{x}} = \begin{bmatrix} -a_1 & -a_2 & \dots & \dots & -a_n \\ 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & 1 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u$$

$$H(s) = \frac{b_1 s^{n-1} + b_2 s^{n-2} + \dots + b_n}{s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_n}$$

$$y = \begin{bmatrix} b_1 & b_2 & \dots & \dots & b_n \end{bmatrix} \mathbf{x}$$

$$\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} \right)^{-1} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \frac{1}{\det(da - cb)}$$

$$\dot{\mathbf{x}} = \begin{bmatrix} -a_1 & 1 & 0 & \dots & 0 \\ -a_2 & 0 & 1 & \ddots & \vdots \\ \vdots & 0 & 0 & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & 1 \\ -a_n & 0 & 0 & \dots & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ \vdots \\ b_n \end{bmatrix} u$$

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}u$$

$$y = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \end{bmatrix} \mathbf{x}$$

$$\mathbf{K} = \begin{bmatrix} 0 & \dots & 0 & 1 \end{bmatrix} \mathcal{C}^{-1} \alpha_c(A)$$

$$\alpha_c(A) = A^n + \alpha_1 A^{n-1} + \alpha_2 A^{n-2} + \dots + \alpha_n I$$