COMP 9517 Computer Vision

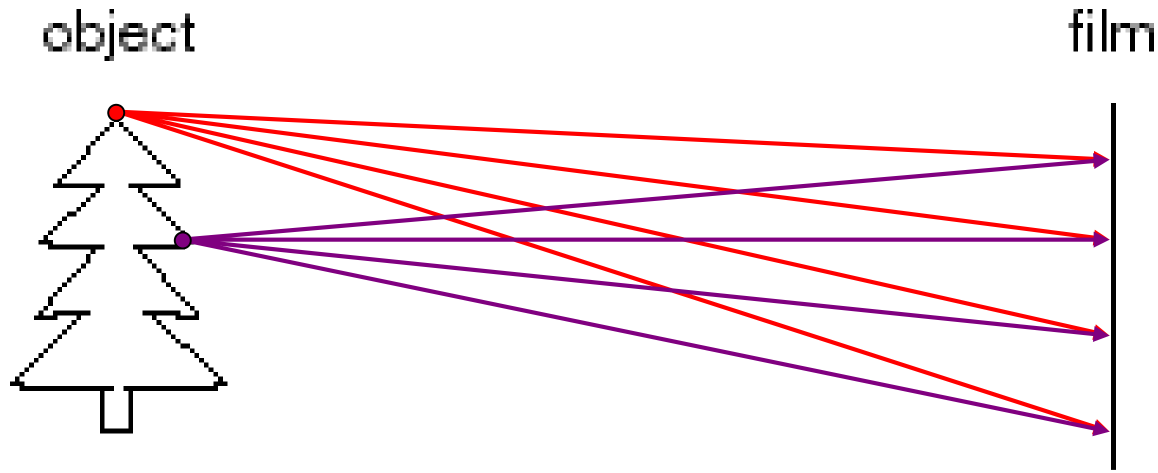
Image Formation

Geometry of Image Formation

Mapping between image and world coordinates

- Pinhole camera model
- Projective geometry
  - Vanishing points and lines
- Projection matrix

Image formation

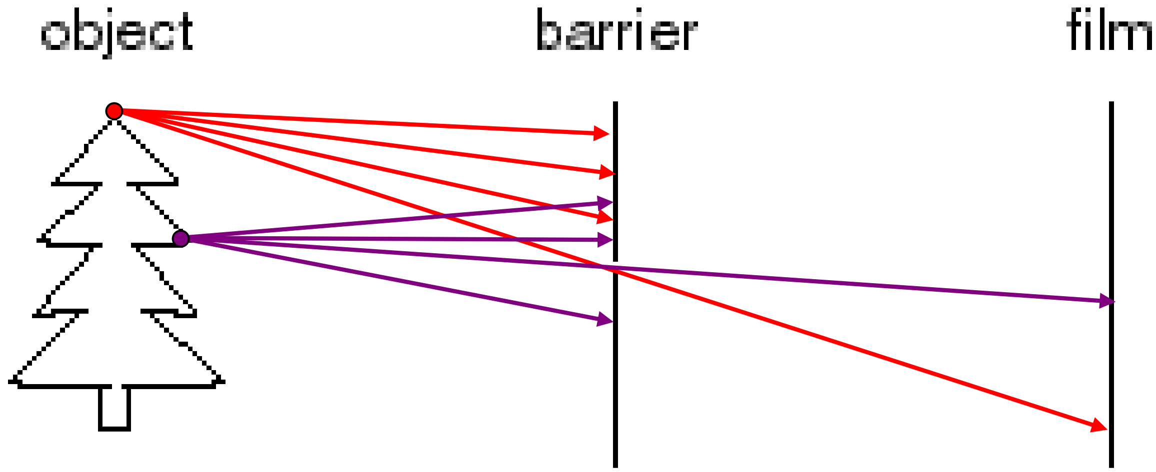


Let us design a camera

- Idea 1: put a piece of film in front of an object
- Do we get a reasonable image?

Slide source: Seitz

Pinhole camera

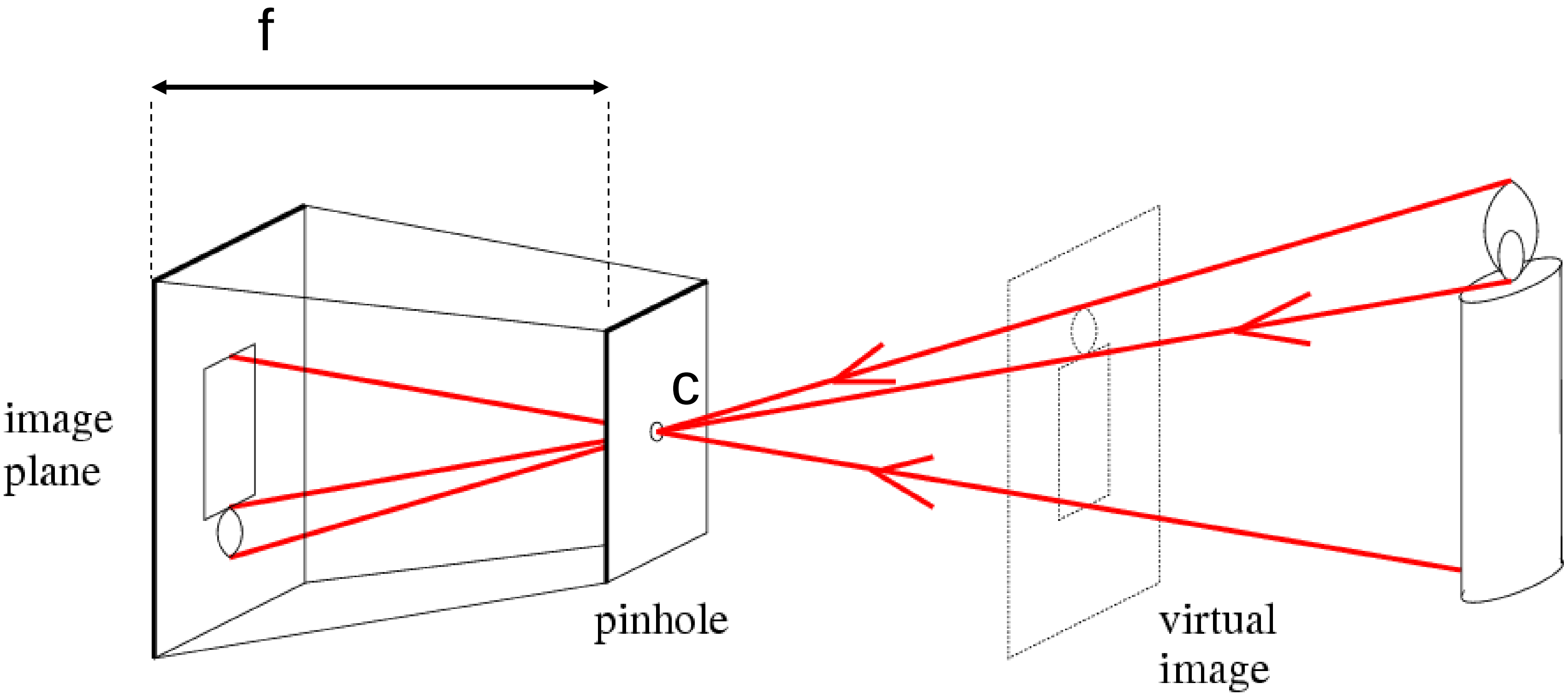


Idea 2: add a barrier to block off most of the rays

– This reduces blurring
– The opening known as the **aperture**

Slide source: Seitz

Pinhole camera



f = focal length
c = centre of the camera

Figure from Forsyth

Camera obscura: the pre-camera

- Known during classical period in China and Greece (e.g. Mo-Ti, China, 470BC to 390BC)

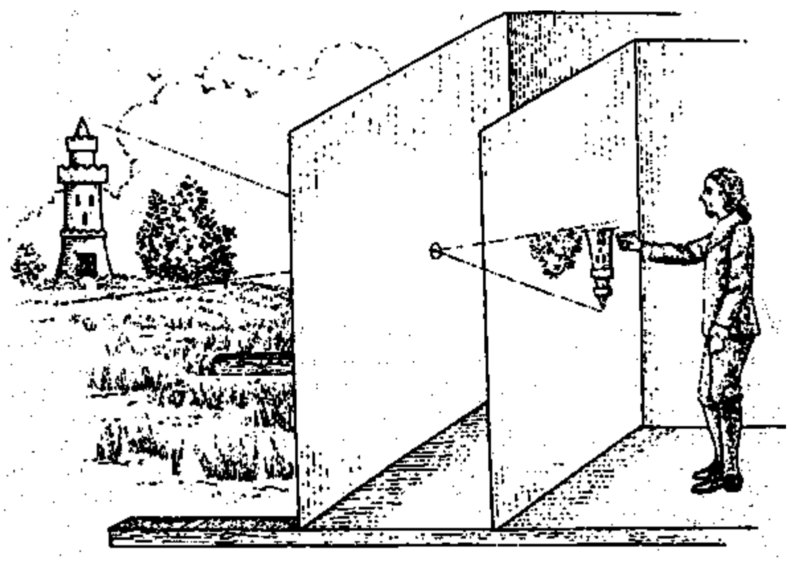
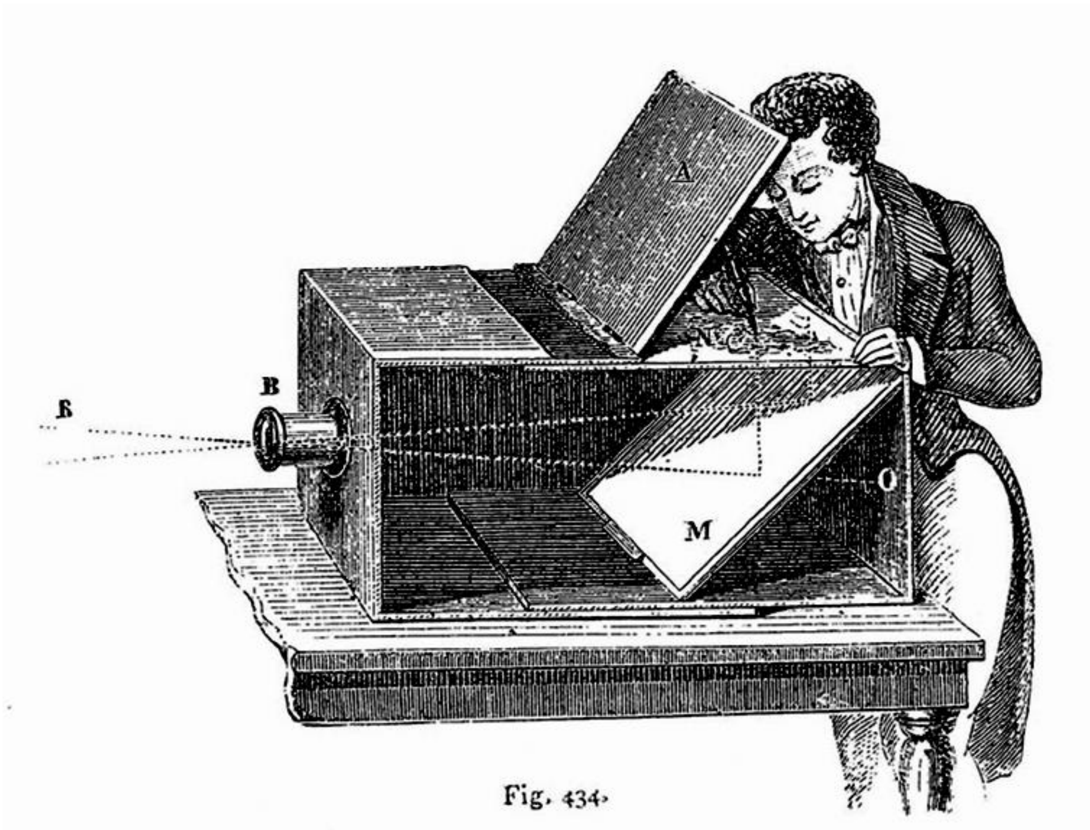


Illustration of Camera Obscura

Freestanding camera obscura at UNC Chapel Hill

Photo by Seth Ilys

Camera Obscura used for Tracing



Lens Based Camera Obscura, 1568

Accidental Cameras

Accidental Pinhole and Pinspeck Cameras
Revealing the scene outside the picture.
Antonio Torralba, William T. Freeman

First Photograph

Oldest surviving photograph

- Took 8 hours on pewter plate

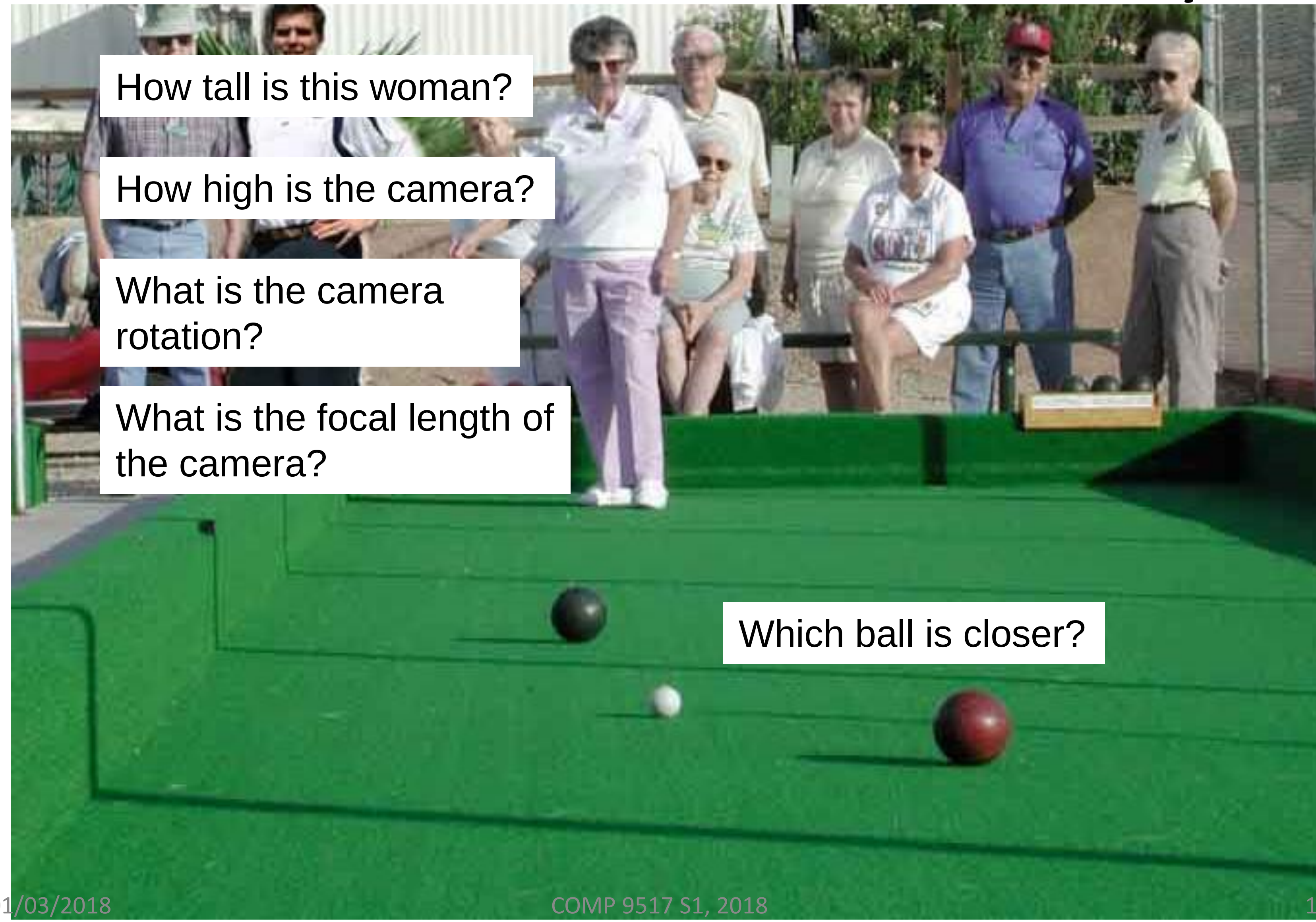
Photograph of the first photograph

Joseph Niepce, 1826

Stored at UT Austin

Niepce later teamed up with Daguerre, who eventually created Daguerrotypes

Camera and World Geometry



How tall is this woman?

How high is the camera?

What is the camera rotation?

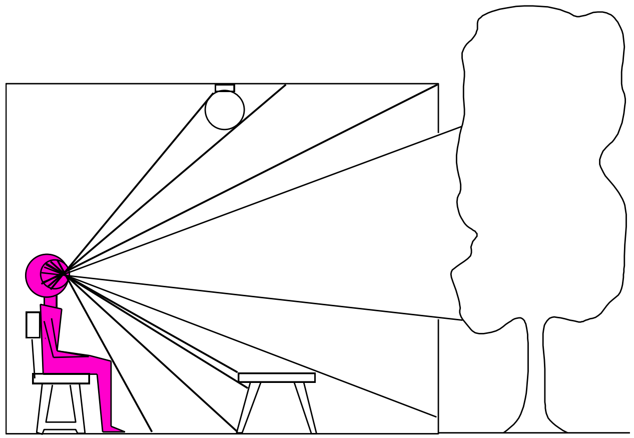
What is the focal length of the camera?

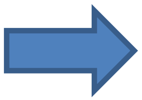
Which ball is closer?

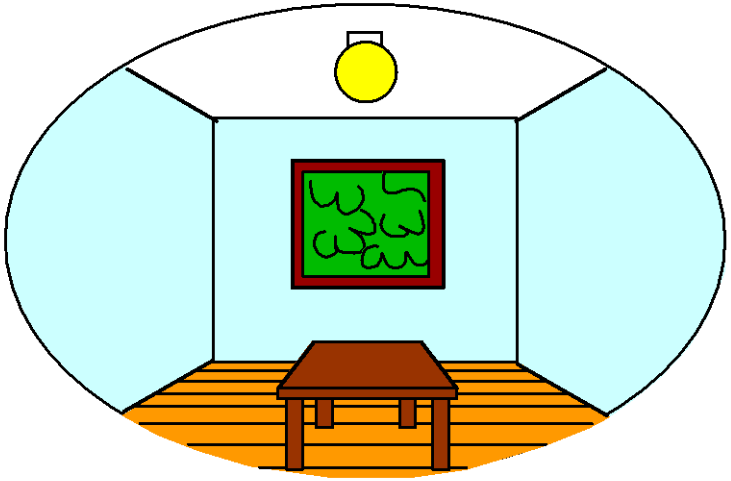
Dimensionality Reduction Machine (3D to 2D)

3D world

2D image







Point of observation

Slide source: Seitz

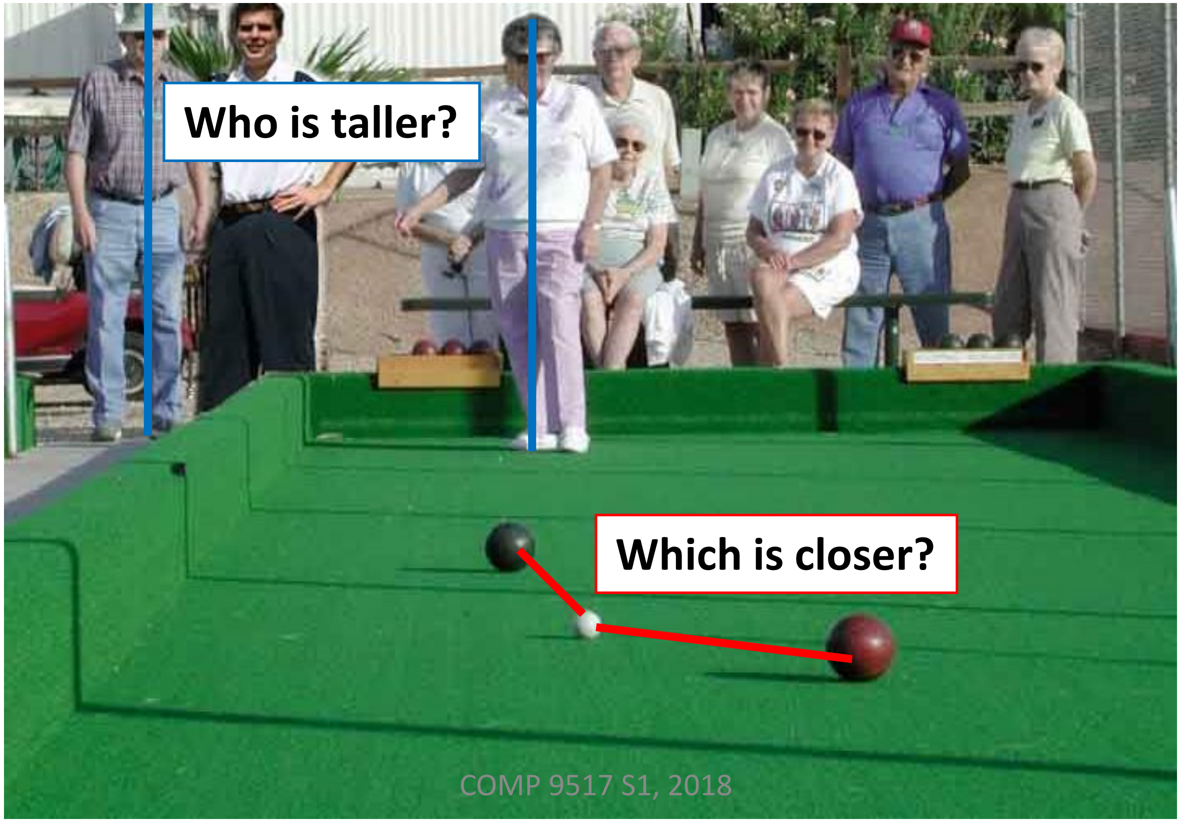
Projection can be tricky…

Projection can be tricky…

Projective Geometry

What is lost?

- Length



Length and area are not preserved

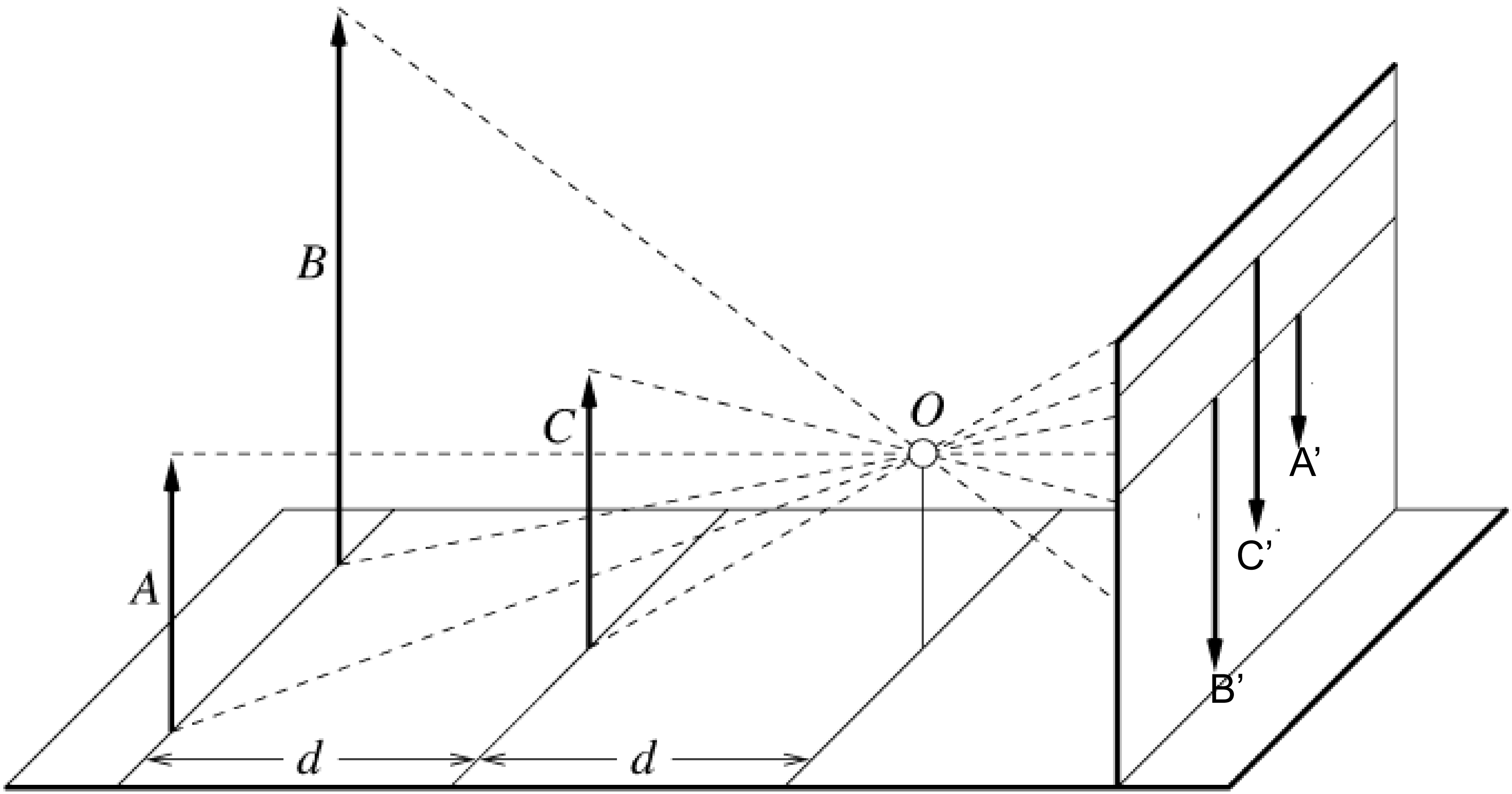
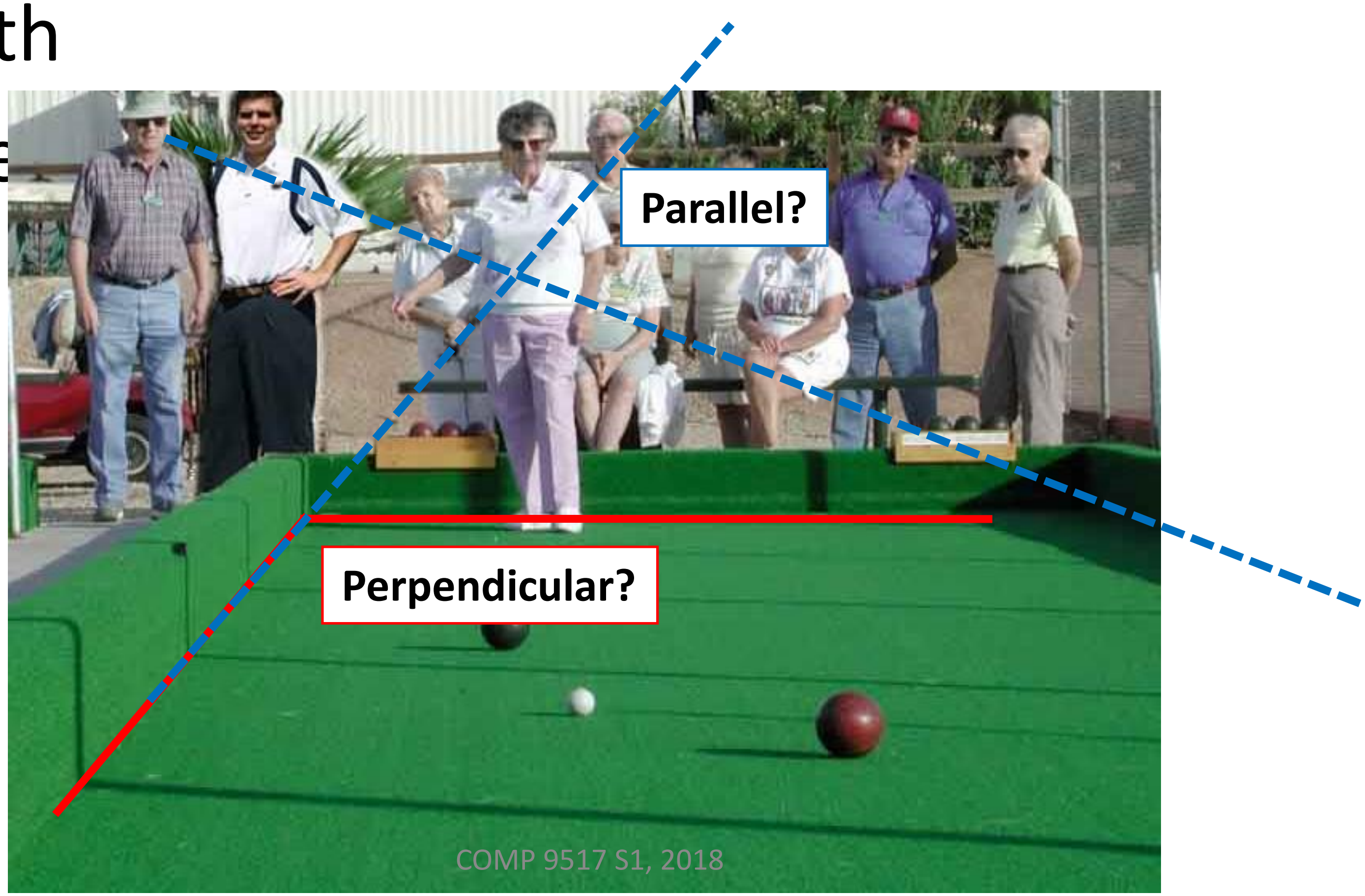


Figure by David Forsyth

Projective Geometry

What is lost?

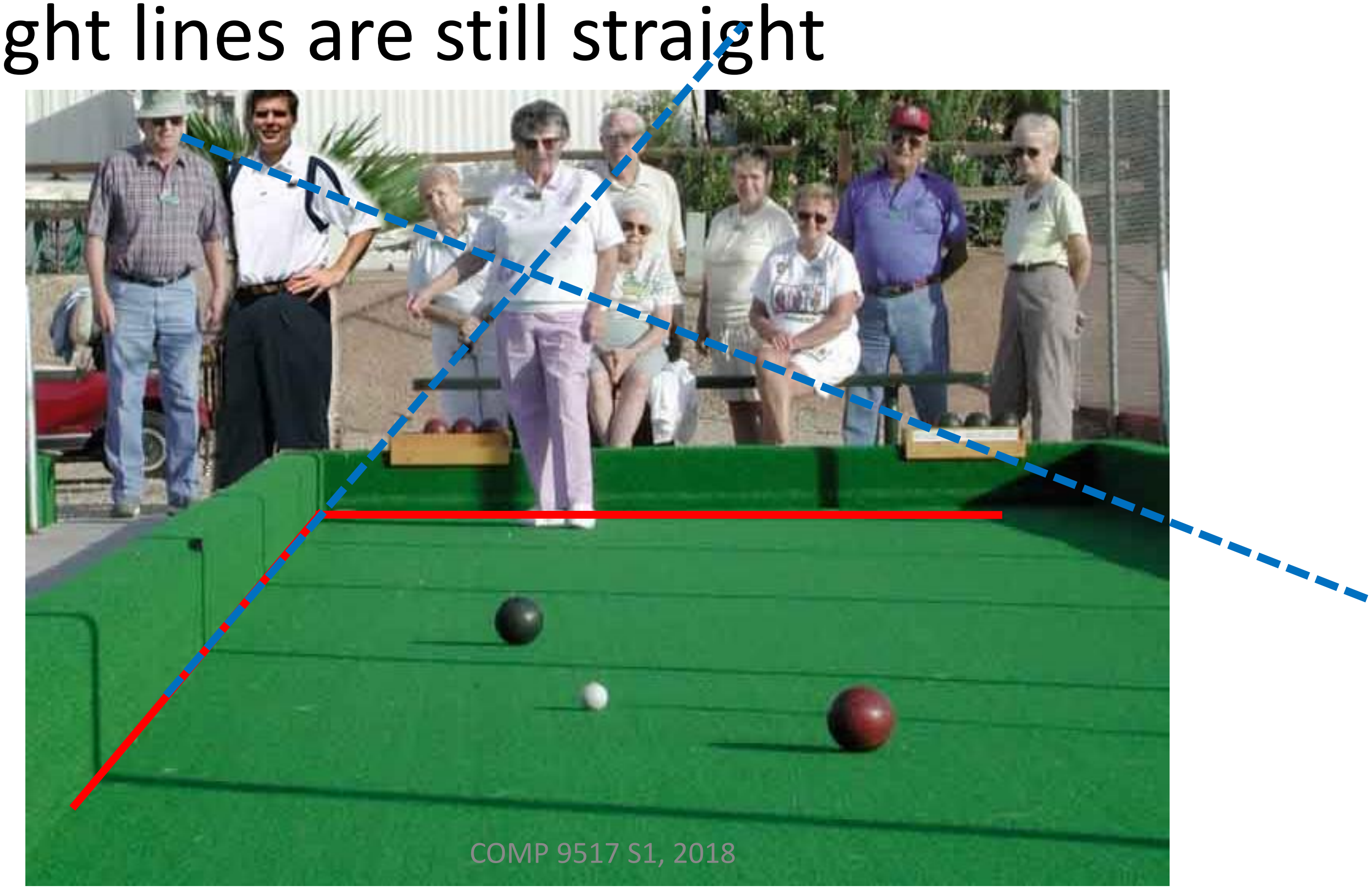
- Length
- Angle



Projective Geometry

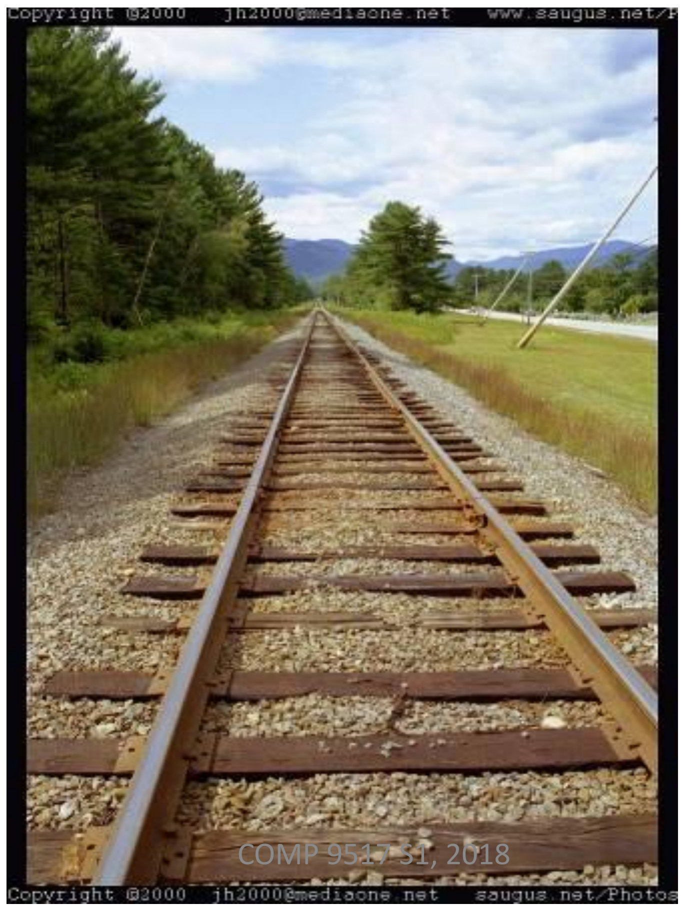
What is preserved?

- Straight lines are still straight

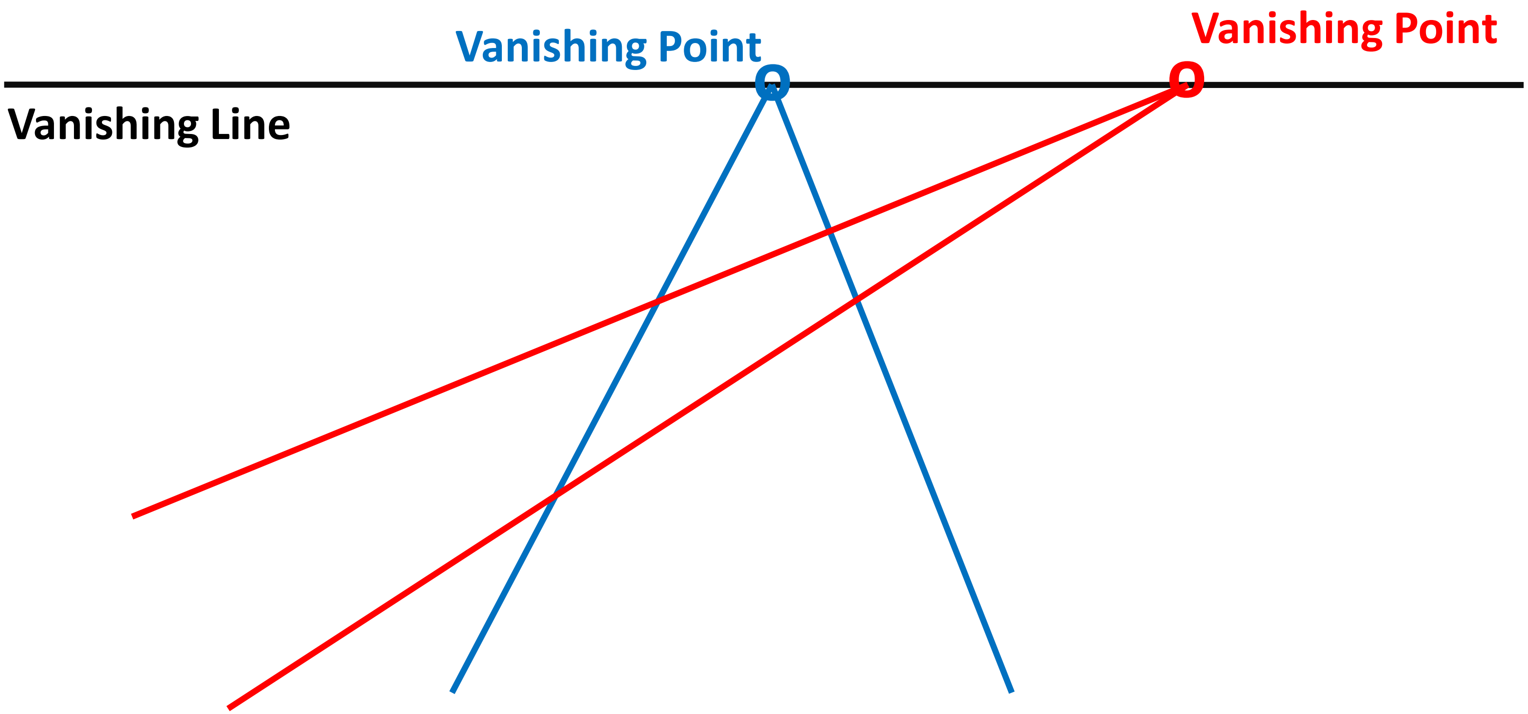


Vanishing points and lines

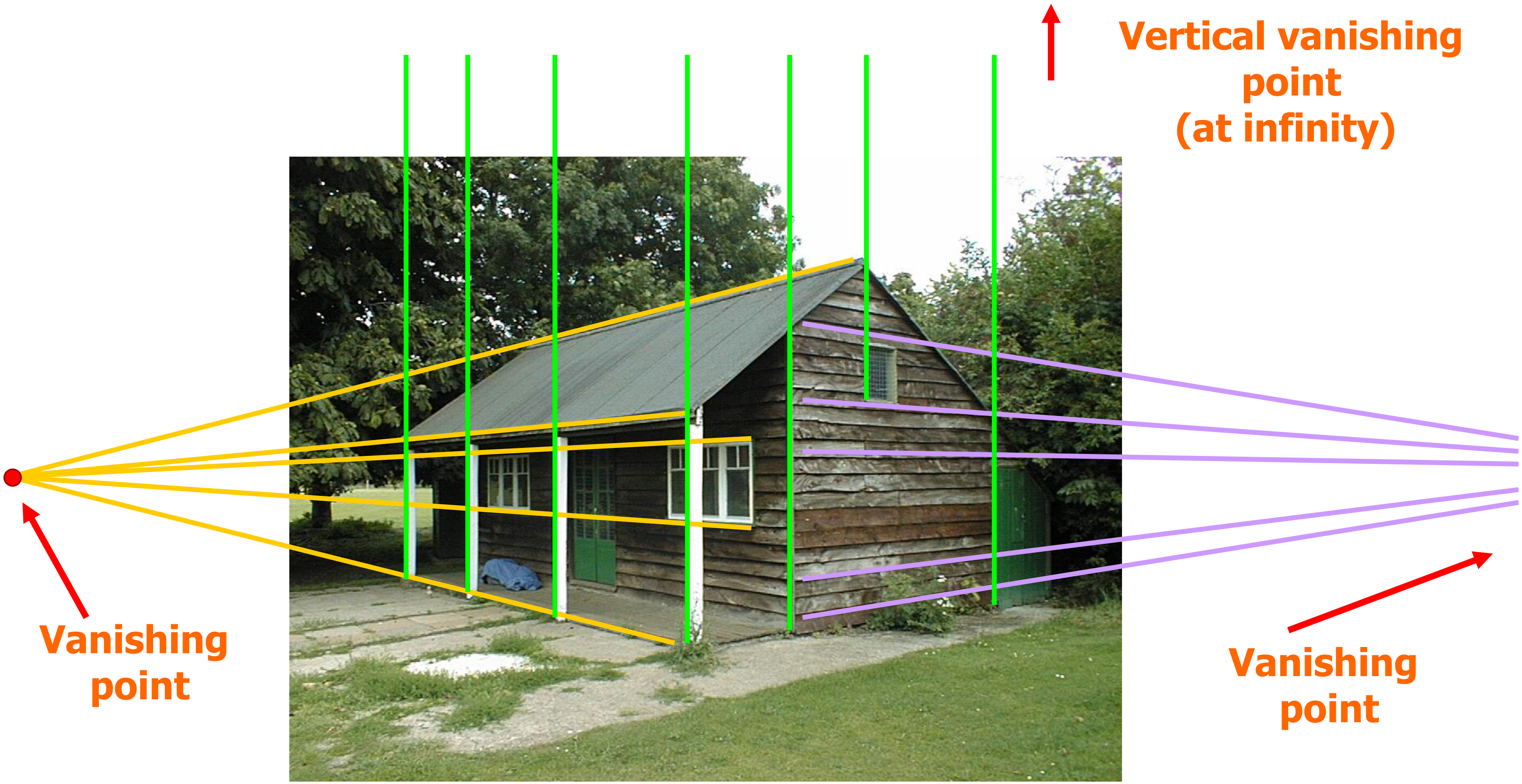
Parallel lines in the world intersect in the image at a "vanishing point"



Vanishing points and lines

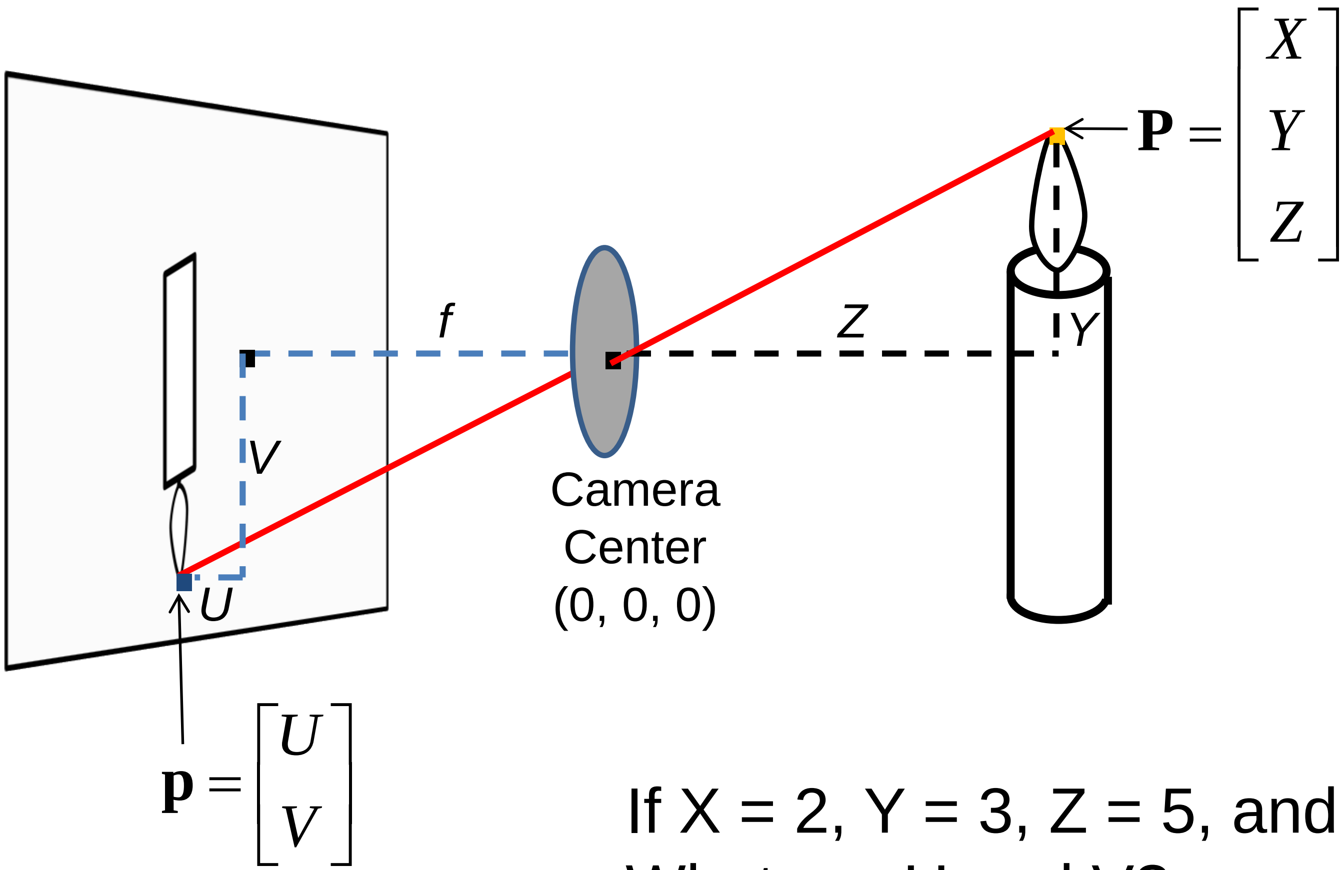


Vanishing points and lines



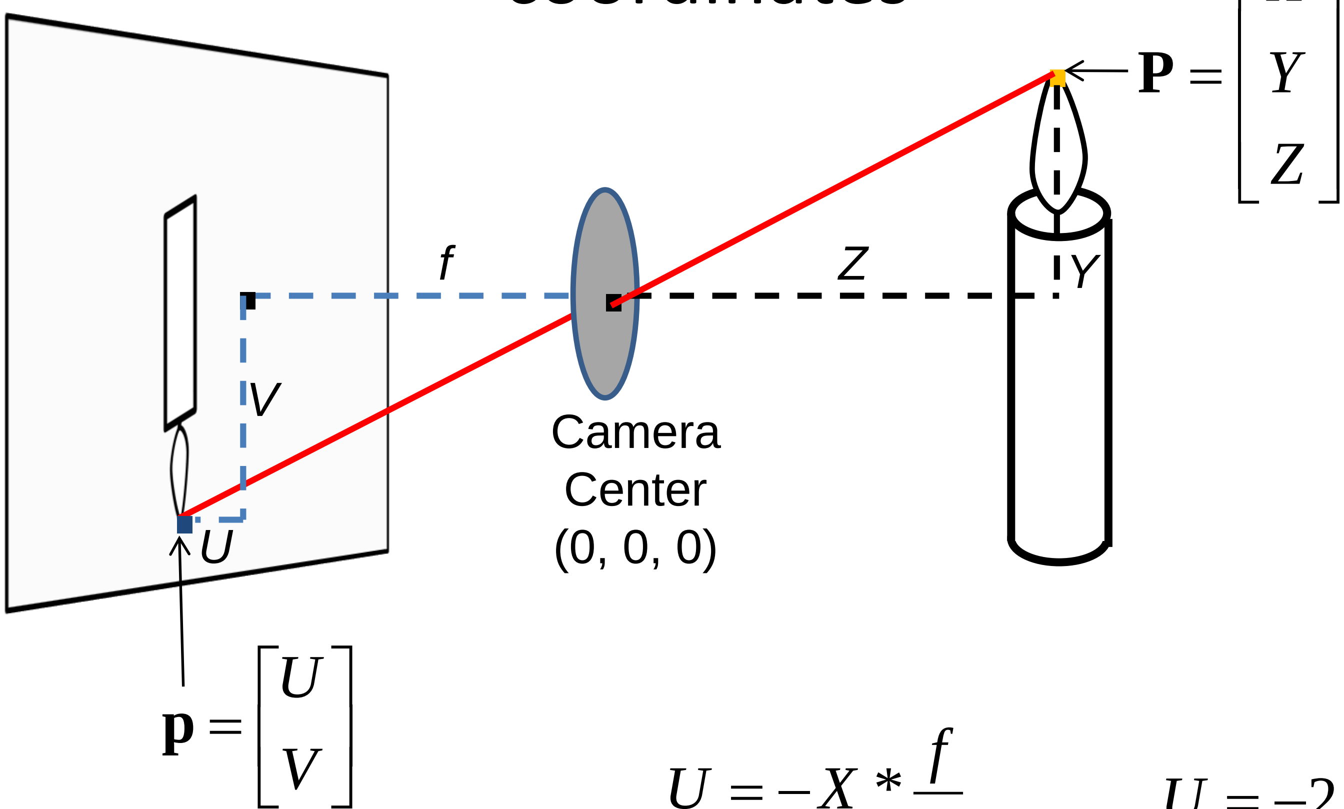
Slide from Efros, Photo from Criminisi

Projection: world coordinates→image coordinates



If X = 2, Y = 3, Z = 5, and f = 2
What are U and V?

Projection: world coordinates→image coordinates

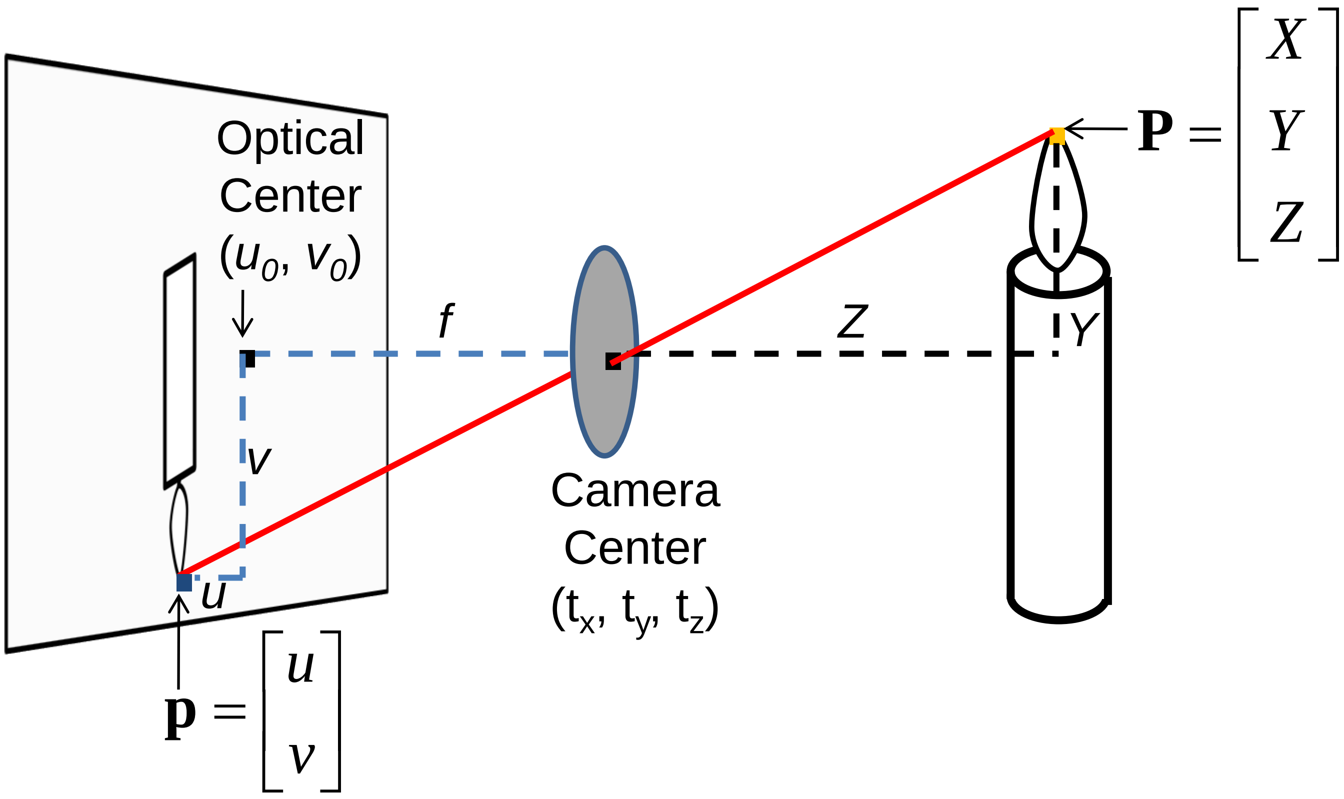


$$U = -X * \frac{f}{Z} \qquad U = -2 * \frac{2}{5}$$

$$V = -Y * \frac{f}{Z} \qquad V = -3 * \frac{2}{5}$$

Sanity check, what if f and Z are equal?

Projection: world coordinates→image coordinates



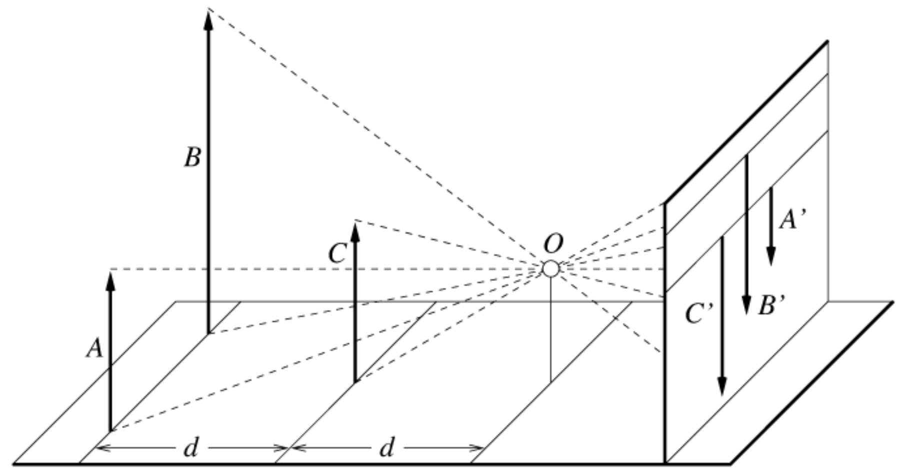
Perspective Projection

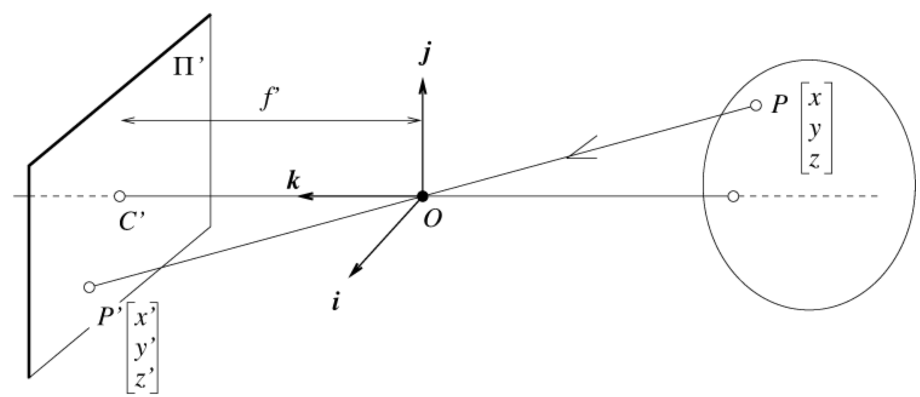
- Apparent size of object depends on its distance: far objects appear smaller
- By similar triangles

$$(x', y', z') \rightarrow (f\frac{x}{z}, f\frac{y}{z}, -f)$$

- Ignore the third coordinate, and get

$$(x', y') \rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

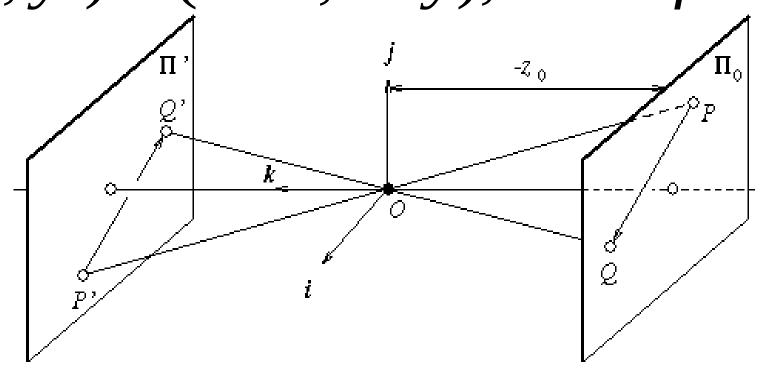




Affine Projection

- Suitable when scene depth is small relative to the average distance from the camera
- Let magnification $m = -f'/z_0$ be positive constant, since z_0 is negative, i.e. treat all points in the scene as at constant distance from camera
- Leads to weak perspective projection

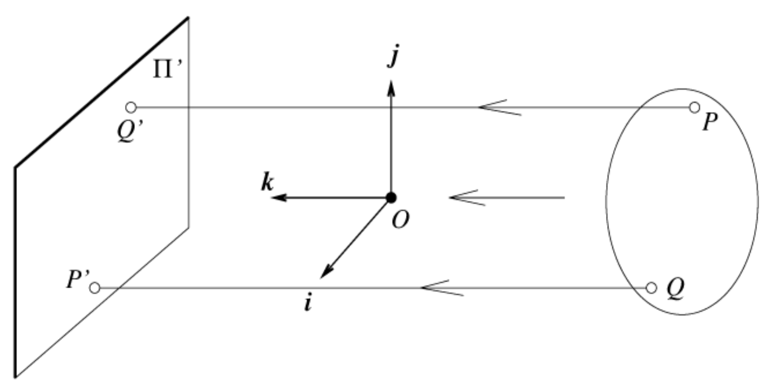
$$(x', y') = (-mx, -my), m = -f'/z_0$$



Affine Projection- ctd

- Camera always remains at roughly constant distance from the scene
- Orthographic projection when m normalised to -1

$$(x', y') = (x, y), m = -1$$



Homogeneous coordinates

Converting to *homogeneous* coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene coordinates

Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Homogeneous coordinates

Invariant to scaling

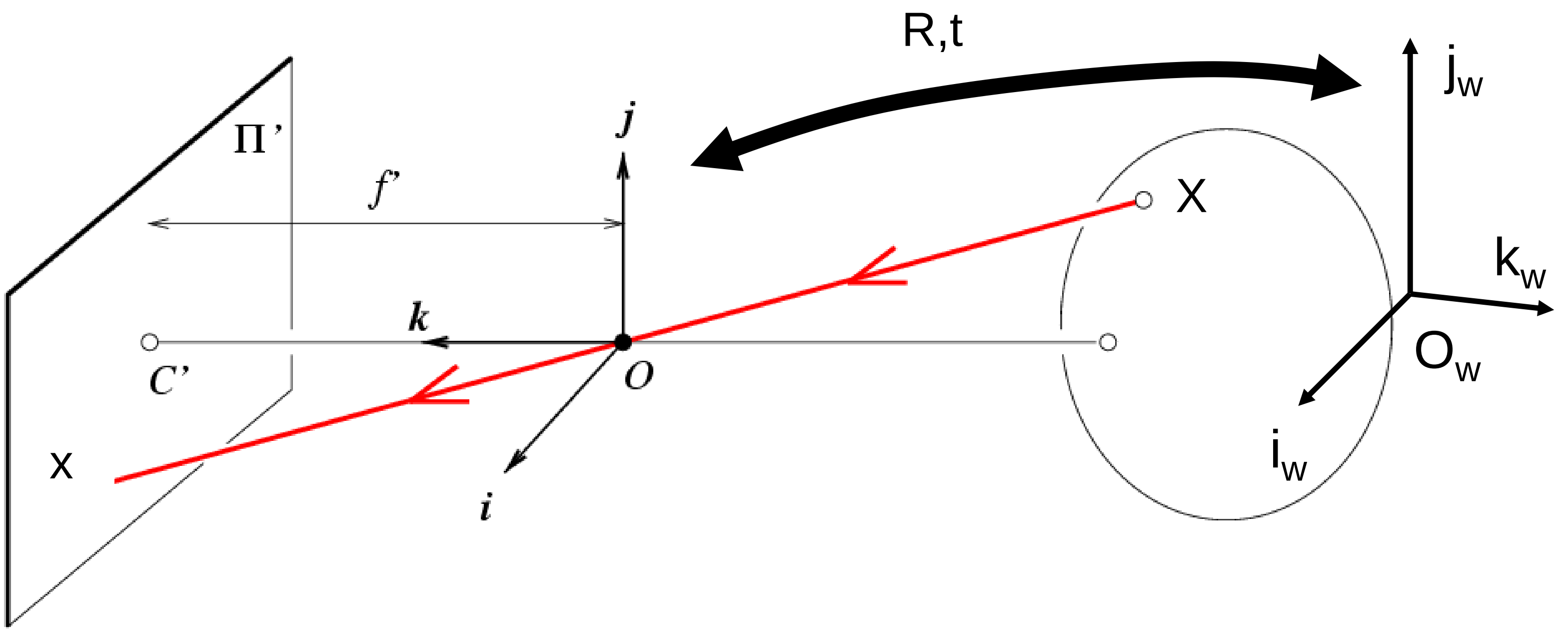
$$k\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} kx \\ ky \\ kw \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{kx}{kw} \\ \frac{ky}{kw} \end{bmatrix} = \begin{bmatrix} \frac{x}{w} \\ \frac{y}{w} \end{bmatrix}$$

Homogeneous Coordinates

Cartesian Coordinates

Point in Cartesian is ray in Homogeneous

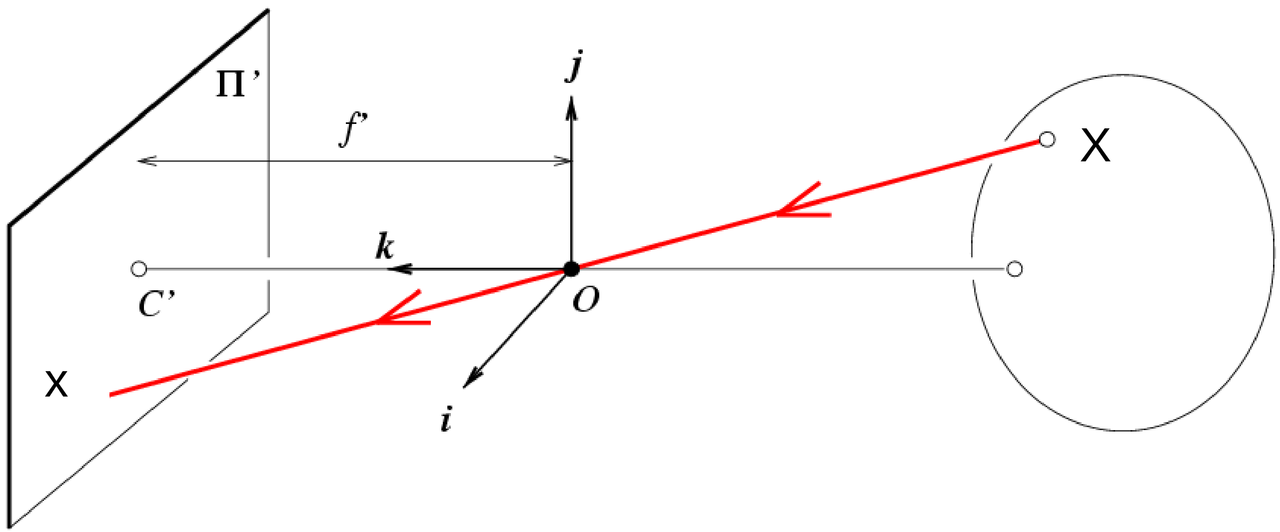
Projection matrix



$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

x: Image Coordinates: (u,v,1)
K: Intrinsic Matrix (3x3)
R: Rotation (3x3)
t: Translation (3x1)
X: World Coordinates: (X,Y,Z,1)

Projection matrix



Intrinsic Assumptions

- Unit aspect ratio
- Optical center at (0,0)
- No skew

Extrinsic Assumptions

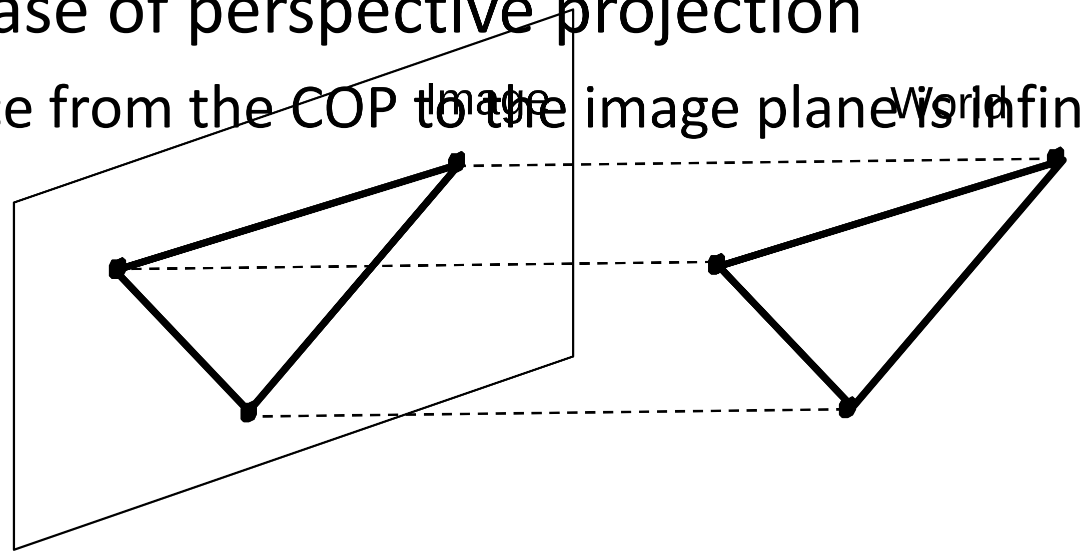
- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{X} \;\Rightarrow\; w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \boxed{\begin{matrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{matrix}}^{\,\mathbf{K}} & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Slide Credit: Savarese

Orthographic Projection

- Special case of perspective projection
  - Distance from the COP to the image plane is infinite

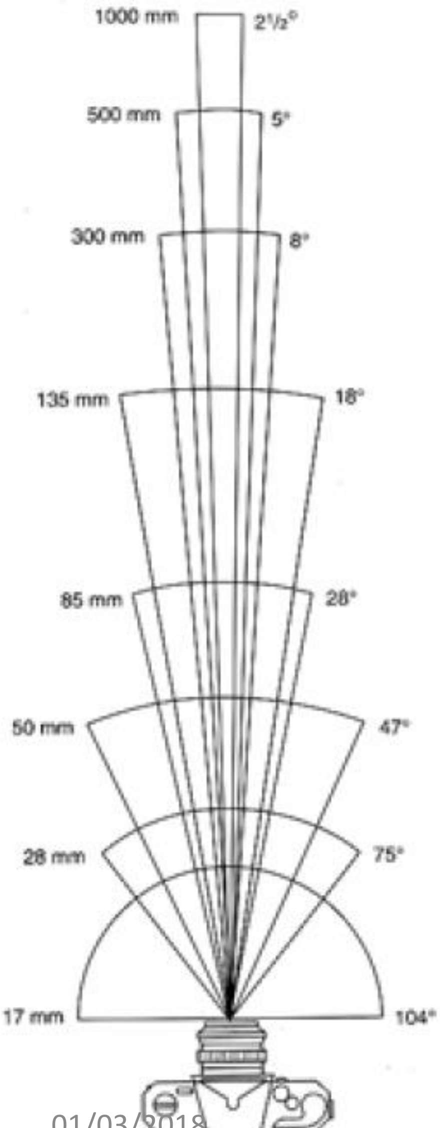


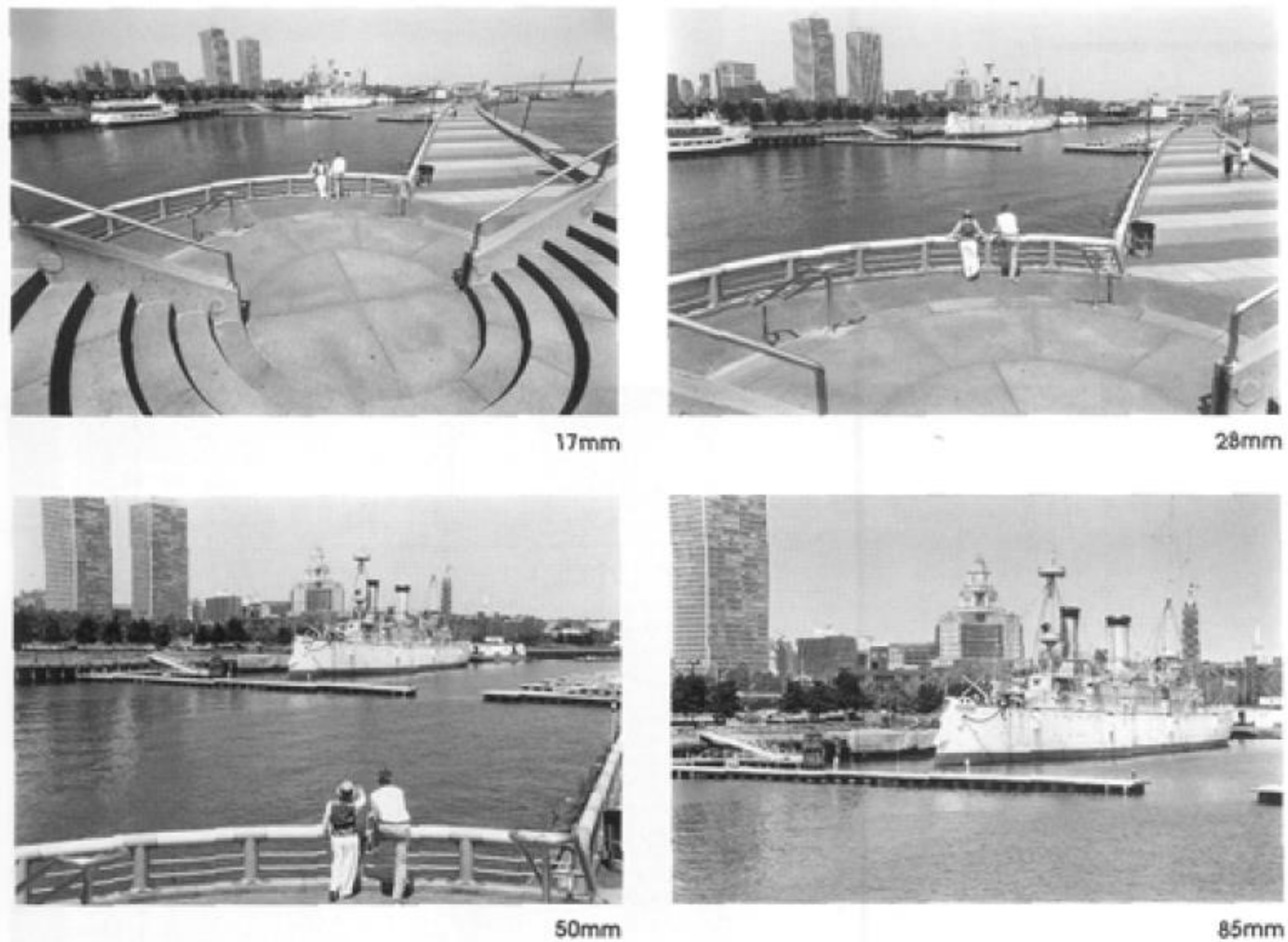
  - Also called “parallel projection”
  - What’s the projection matrix?

$$w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Slide by Steve Seitz

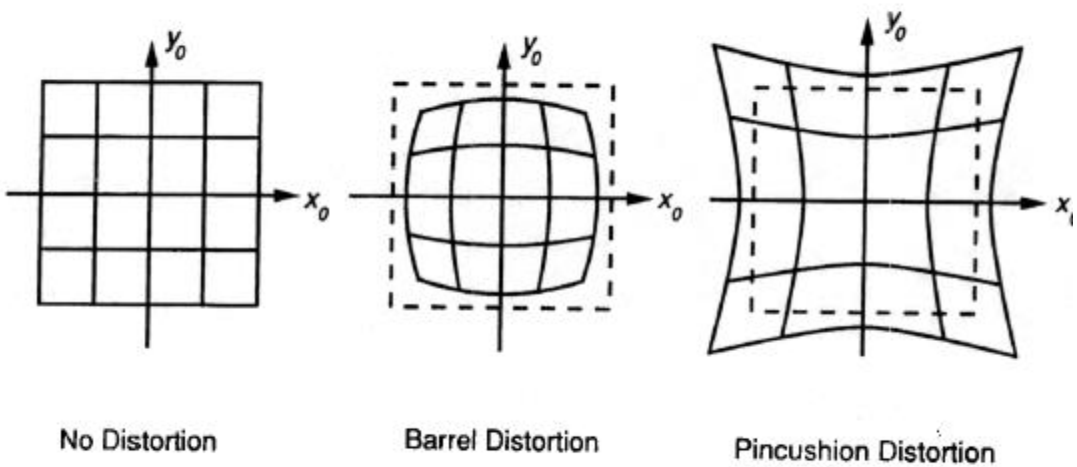
Field of View (Zoom, focal length)





From London and Upton

Beyond Pinholes: Radial Distortion

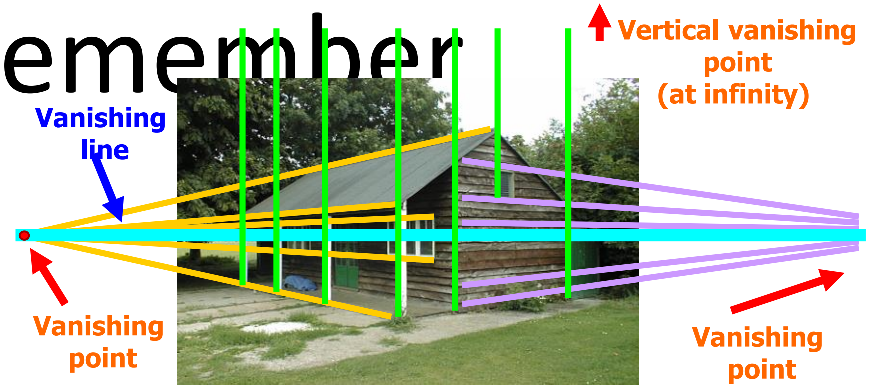


Corrected Barrel Distortion

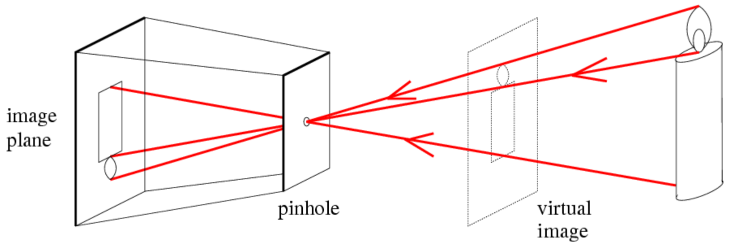
Image from Martin Habbecke

Things to remember

- Vanishing points and vanishing lines



- Pinhole camera model and camera projection matrix



$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

- Homogeneous coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

For Reading

- Szeliski Chapter 2.1

Acknowledgement

- Slides from Derek Hoiem, Alexei Efros, Steve Seitz, and David Forsyth
- Image sources credited where possible