COMP9418 Assignment 1

Advanced Topics in Statistical Machine Learning, 18s2, UNSW Sydney

Last Update: Friday 10$^{\text{th}}$ August, 2018 at 09:51

Submission deadline: Monday August 20th, 2018 at 23:59:59

Late Submission Policy: 20% marks will be deducted from the total for each day late, up to a total of four days. If five or more days late, a zero mark will be given.

Form of Submission: You should submit your solution in one single file in pdf format with the name `solution.pdf`. No other formats will be accepted (scanned versions of legible handwritten answers are accepted). There is a maximum file size cap of 5MB so make sure your submission does not exceed this size.

Submit your files using give. On a CSE Linux machine, type the following on the command-line:

```
$ give cs9418 ass1 solution.pdf
```

Alternative, you can submit your solution via the course website
https://webcms3.cse.unsw.edu.au/COMP9418/18s2/resources/20267

Recall the guidance regarding plagiarism in the course introduction: this applies to this homework and if evidence of plagiarism is detected it may result in penalties ranging from loss of marks to suspension.

1 [100 Marks] Inference in Directed Graphical Models

Consider the Bayesian network in fig. 1 and a corresponding valid Junction Tree in fig. 2.

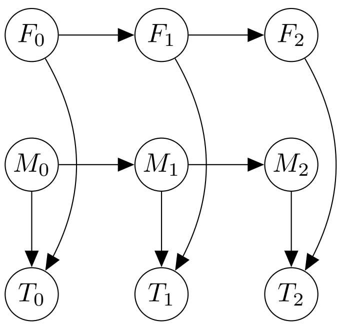

Figure 1: Bayesian network.

This graphical model corresponds to a simplified instance of the model proposed by Williams et al. (2006) for condition monitoring in a neonatal intensive care unit. You can read more about the problem in the reference provided but tracking this and other references will not help you solve this assignment. The network is composed of *true states* F_i, *artifactual states* M_i and *observations* T_i for $i = 0, 1, 2$. The main task in these types of models is, given observations

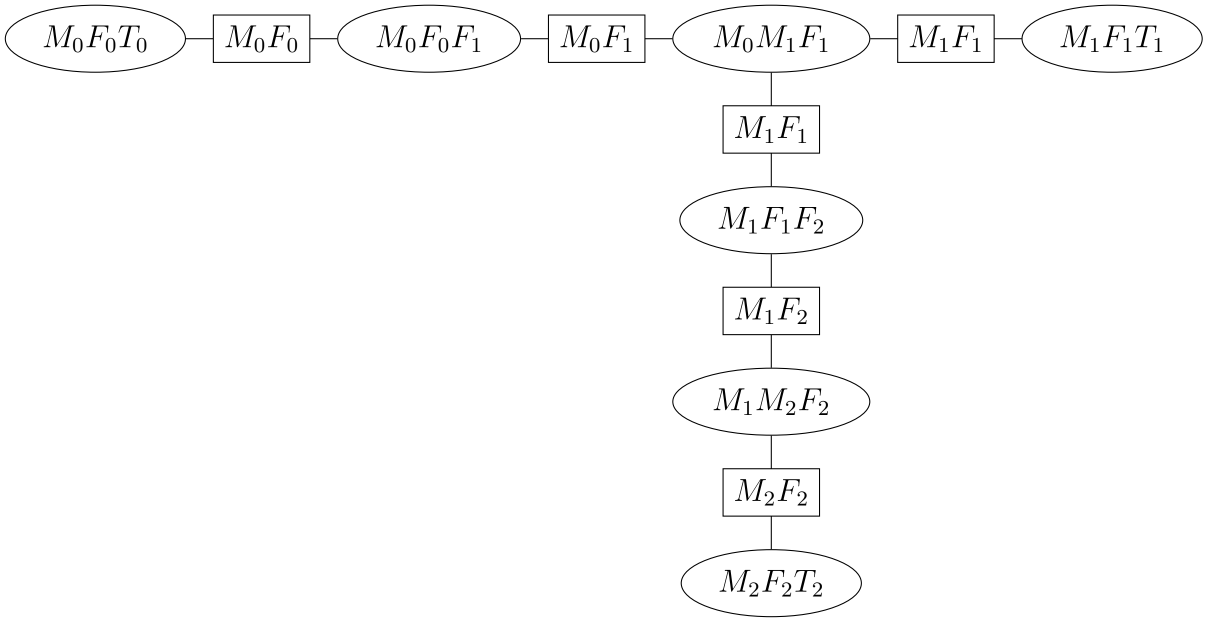

Figure 2: A valid Junction Tree corresponding to the Bayesian network given in fig. 1.

at time i, to infer the configuration of the underlying true state and how much it has been obscured by artifacts. The variables T_i are discrete with states $\{\text{low}, \text{medium}, \text{high}\}$ and F_i, M_i are binary variables taking on values in $\{\text{true}, \text{false}\}$.
You are also given the following conditional probability tables (CPTs):

$$P(F_0 = \text{true}) = 0.25 \qquad P(M_0 = \text{true}) = 0.12$$
$$P(F_1 = \text{true}|F_0 = \text{false}) = 0.1 \qquad P(M_1 = \text{true}|M_0 = \text{false}) = 0.05$$
$$P(F_1 = \text{true}|F_0 = \text{true}) = 0.9 \qquad P(M_1 = \text{true}|M_0 = \text{true}) = 0.95$$
$$P(F_2 = \text{true}|F_1 = \text{false}) = 0.33 \qquad P(M_2 = \text{true}|M_1 = \text{false}) = 0.25$$
$$P(F_2 = \text{true}|F_1 = \text{true}) = 0.2 \qquad P(M_2 = \text{true}|M_1 = \text{true}) = 0.68$$

Table 1: CPT for $P(T_i = t_i|M_i = m_i, F_i = f_i)$ for $i = 0, 1, 2$.

		t_0				t_1				t_2	
m_0	f_0	low	medium	m_1	f_1	low	medium	m_2	f_2	low	medium
false	false	0.2	0.4	false	false	0.25	0.15	false	false	0.33	0.4
false	true	0.1	0.8	false	true	0	0.6	false	true	0.7	0.25
true	false	0.95	0	true	false	1	0	true	false	0.88	0.05
true	true	0.5	0	true	true	1	0	true	true	0.2	0.75

In the questions below, unless otherwise stated explicitly, you must **show all your working**. Omission of details or derivations may yield a reduction in the corresponding marks.

a) [10 marks] Write down the corresponding joint distribution $P(M_0, M_1, M_2, F_0, F_1, F_2, T_0, T_1, T_2)$.

b) [20 marks] Calculate $P(M_0|T_1 = \text{low})$ efficiently using Variable (i.e. Bucket) Elimination.

c) [20 marks] Using the Junction Tree provided in fig. 2 along with the message-passage mechanism in the Junction Tree Algorithm (JTA), calculate $P(F_1|T_0 = \text{low}, T_1 = \text{medium})$.

d) [20 marks] Provide an elimination order of nodes in the Bayesian network such that the created cliques give rise to the the Junction Tree of fig. 2. Note that you need to moralise the graph first.

e) [15 marks] Calculate $P(M_0, M_2 | T_1 = \text{medium})$ using the JTA.

f) [15 marks] Suppose edges $T_0 \to T_1$ and $T_1 \to T_2$ are added to the original Bayesian network. Construct a valid Junction Tree corresponding to this new Bayesian network.

References

Williams, C., Quinn, J., and McIntosh, N. (2006). Factorial switching Kalman filters for condition monitoring in neonatal intensive care. In *Advances in Neural Information Processing Systems*, pages 1513–1520.