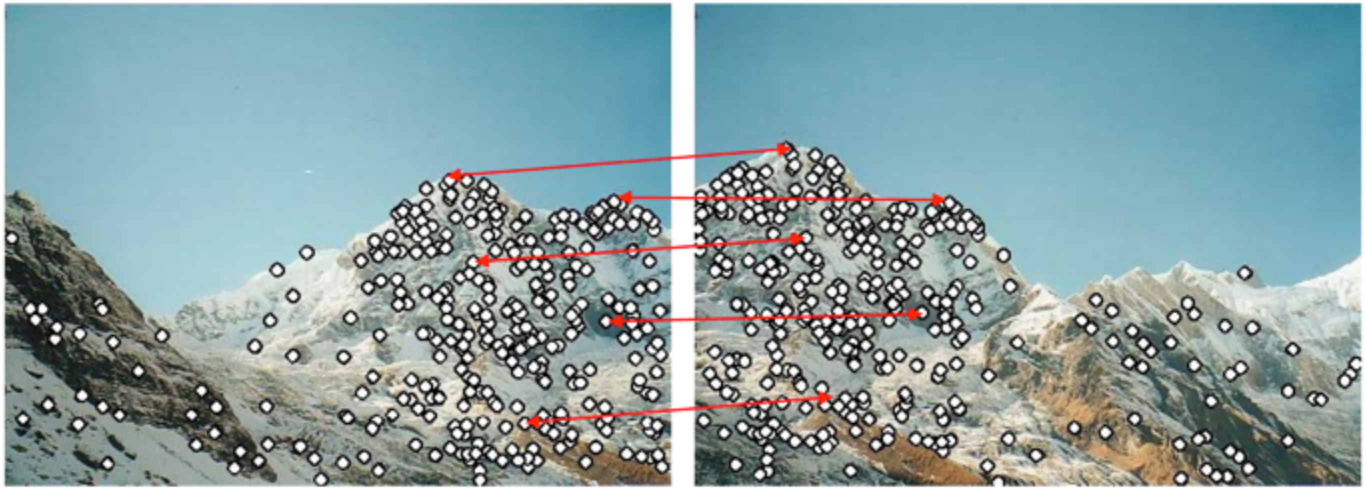


COMP 9517 Computer Vision

Feature Matching

Feature Matching

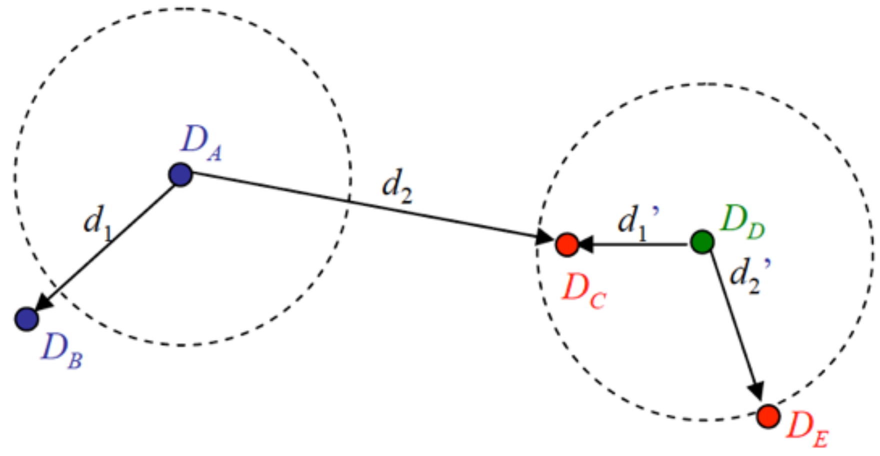
- Simple approach: one feature matches to another if those features are nearest neighbours and their distance is below some thresholds
  - Threshold is difficult to set
  - Non-distinctive features could have lots of close matches, only one of which is correct

Feature Matching

- Nearest Neighbour Distance Ration

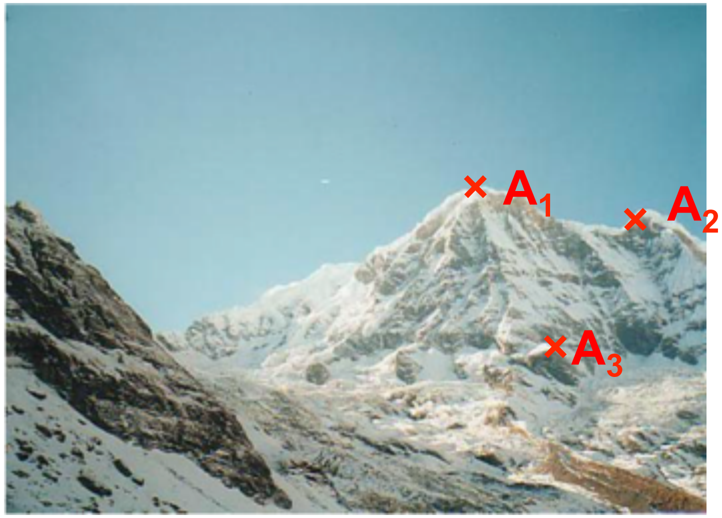
$$NNDR = \frac{d_1}{d_2} = \frac{\left\| D_A - D_B \right\|}{\left\| D_A - D_C \right\|}$$

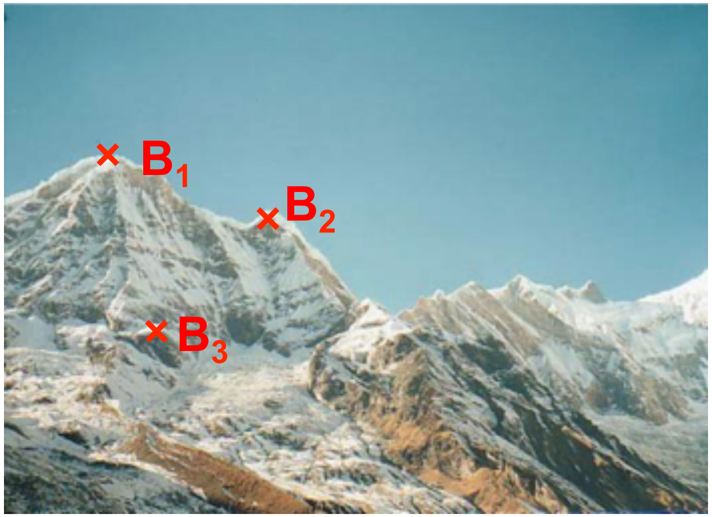
  - d1 is the distance to the first nearest neighbour
  - d2 is the distance to the second nearest neighbour
  - Neighbour in feature space



Feature Matching

- Providing a set of matched candidates, how do we decide which features really match?





Alignment

- Alignment: find the parameters of the transformation that best align matched points

- Fitting: find the parameters of a model that best fit the data

Transformations

original

translation

rotation

scale

affine

perspective

Fitting and Alignment

- Transformation

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & \alpha_x \\ \alpha_y & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Shear

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\Theta & -\sin\Theta \\ \sin\Theta & \cos\Theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Rotate

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Affine

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Projective

Transformations

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & \alpha_x \\ \alpha_y & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

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Rotate

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

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Projective

Fitting and Alignment

- Lease squares fit

$$E_{LS} = \sum_i \|r_i\|^2 = \sum_i \left\| f(x_i; p) - x_i^{'} \right\|^2$$

  - Most problems in computer vision do not have a simple linear relationship
- Non-linear lease squares
  - Iteratively find an update Δp to the current parameter estimate p by mininising

$$E_{NLS}\left(\Delta p\right) = \sum_i \left\| f(x_i; p + \Delta p) - x_i^{'} \right\|^2$$

Fitting and Alignment

- In the case of outliers among the correspondences
- Robust lease squares fit

$$E_{RLS} = \sum_i \rho\left(\left\|r_i\right\|\right)$$

Fitting and Alignment

- RANdom SAmple Consensus (RANSAC)
  - Starting with too many outliers will prevent from covering to the global optimum
  - Use RANSAC to fit a geometric transformation to a small subset of all possible matches

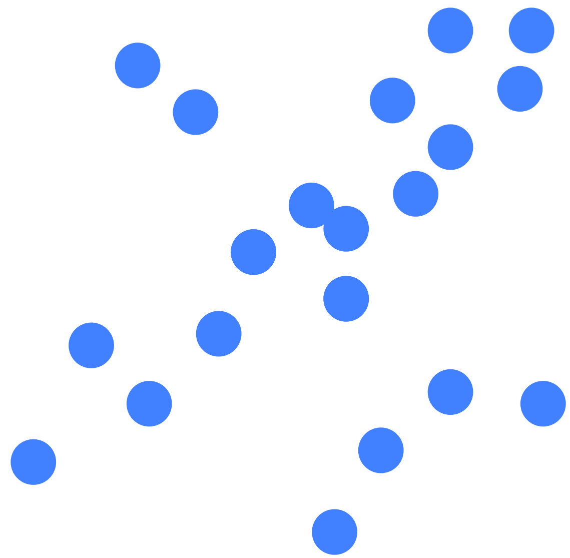
Algorithm:

1. **Sample** (randomly) the number of points required to fit the model
2. **Solve** for model parameters using samples
3. **Score** by the fraction of inliers within a preset threshold of the model

Repeat 1-3 until the best model is found with high confidence

Fitting and Alignment

- RANSAC



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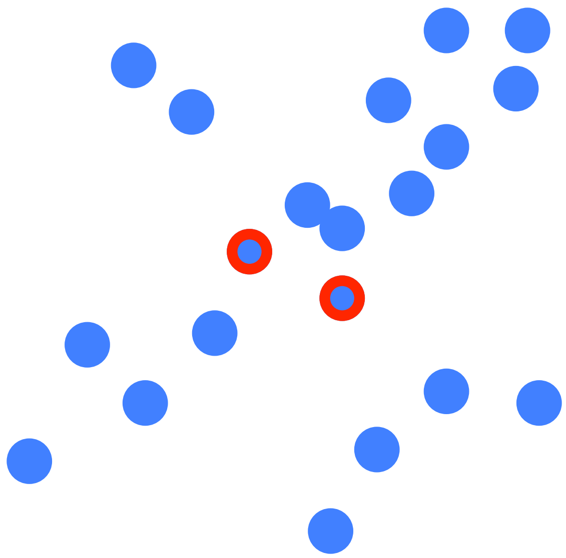
Fitting and Alignment

- RANSAC



Algorithm:

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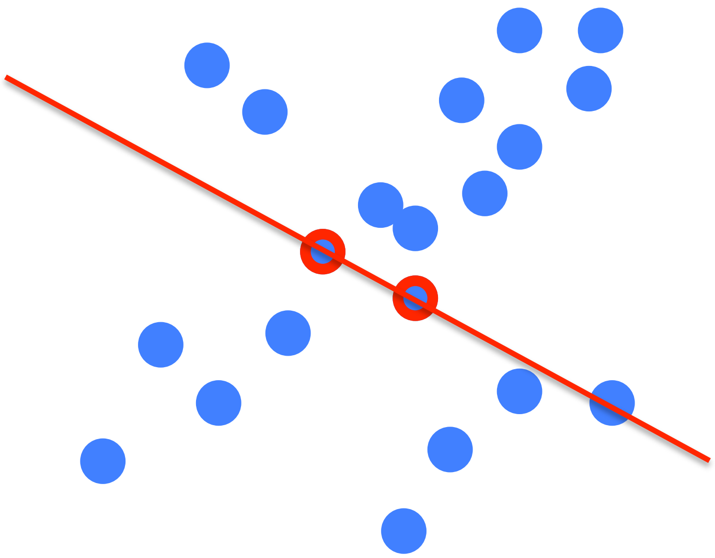
Fitting and Alignment

- RANSAC



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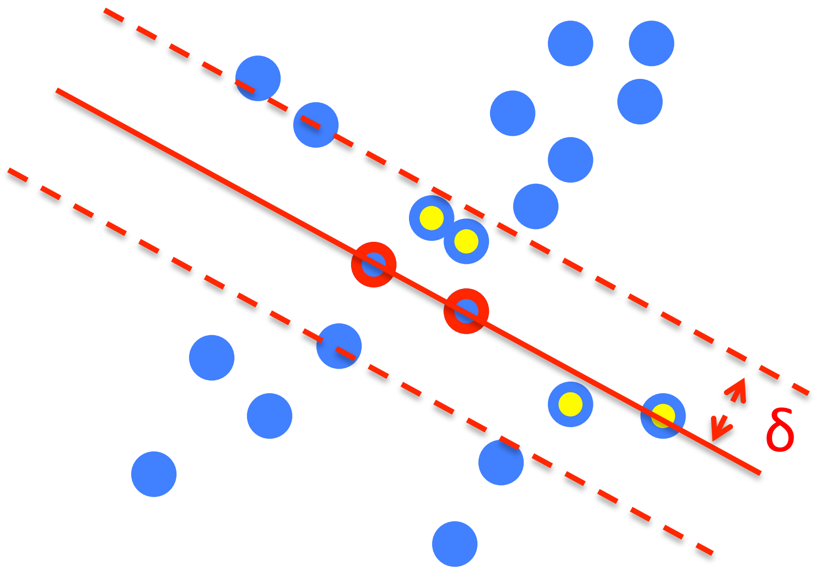
Fitting and Alignment



Algorithm:

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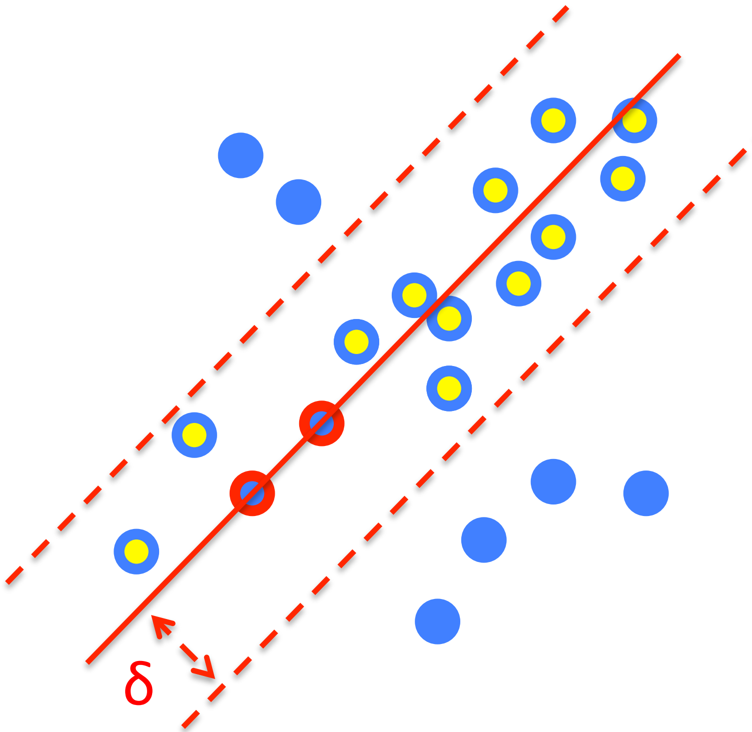
Fitting and Alignment

- RANSAC



Algorithm:

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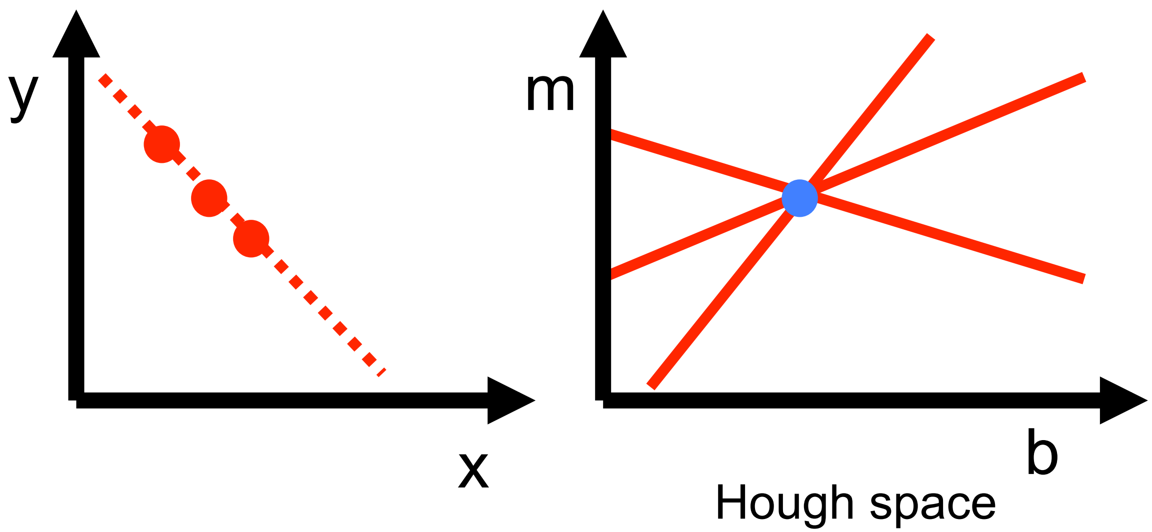
2. **Solve** for model parameters using samples
3. **Score** by the fraction of inliers within a preset threshold of the model

Repeat 1-3 until the best model is found with high confidence

Fitting and Alignment

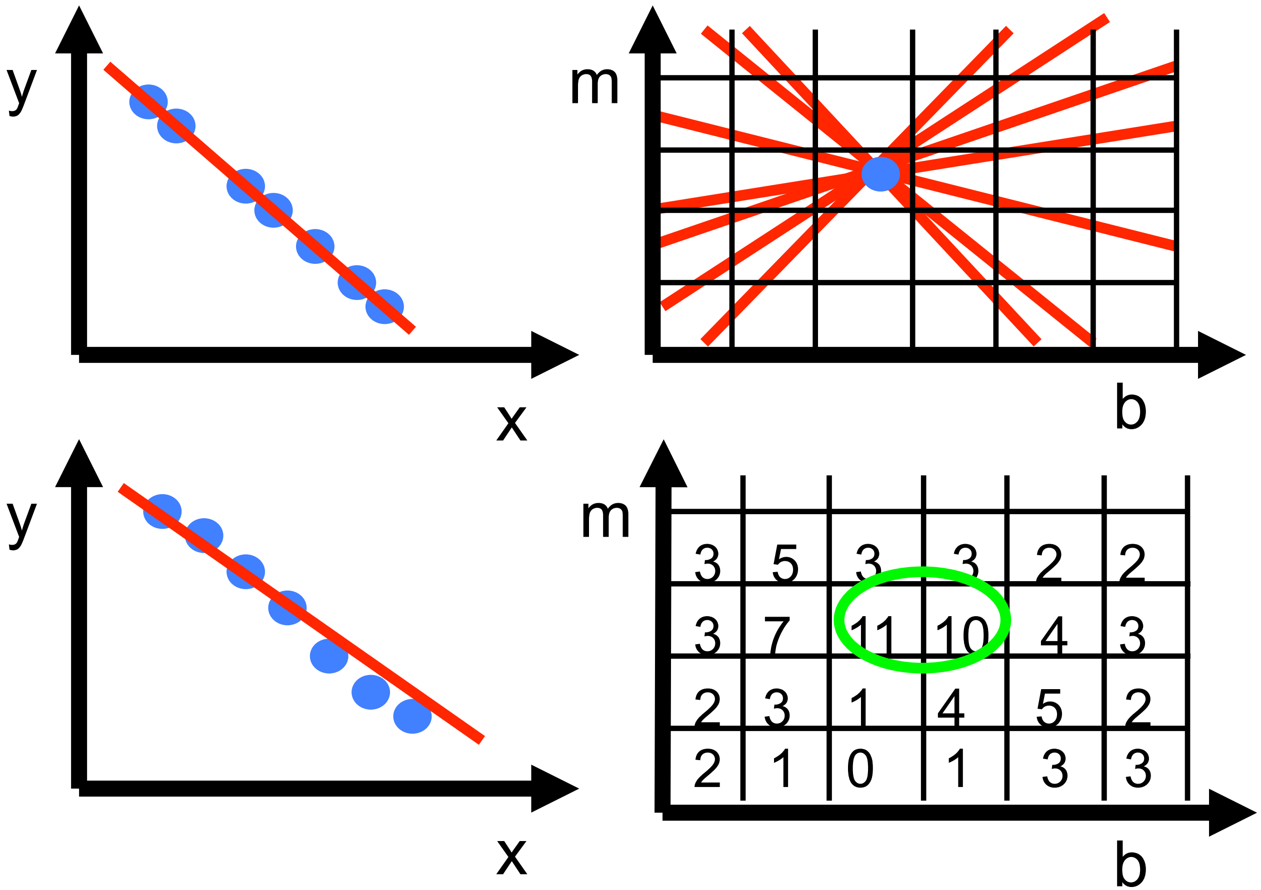
- Hough transform
  - Given a set of points, find the curve or line that explains the data points best



$$y = m x + b$$

Fitting and Alignment

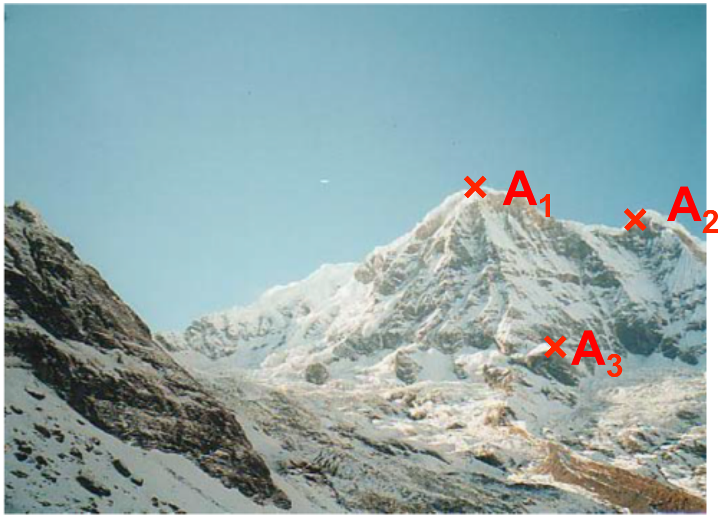
- Hough transform

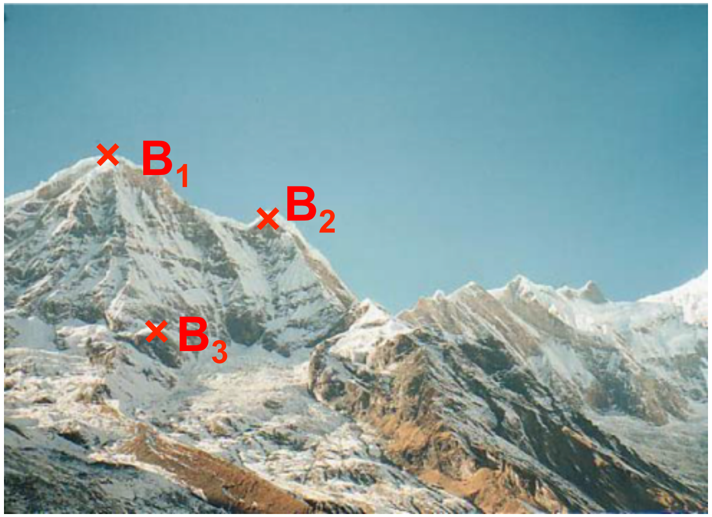


Fitting and Alignment

- Given matched points A and B, estimate the translation

$$\begin{bmatrix} x_i^B \\ y_i^B \end{bmatrix} = \begin{bmatrix} x_i^A \\ y_i^A \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

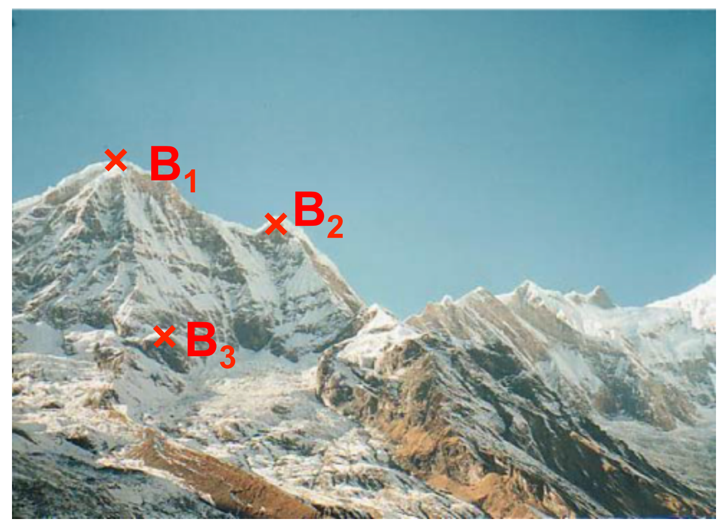




Alignment by Least Squares

1. Write down objective function
2. Derived solution
   a) Compute derivative
   b) Compute solution
3. Computational solution
   a) Write in form Ax=b
   b) Solve using pseudo-inverse or eigenvalue decomposition

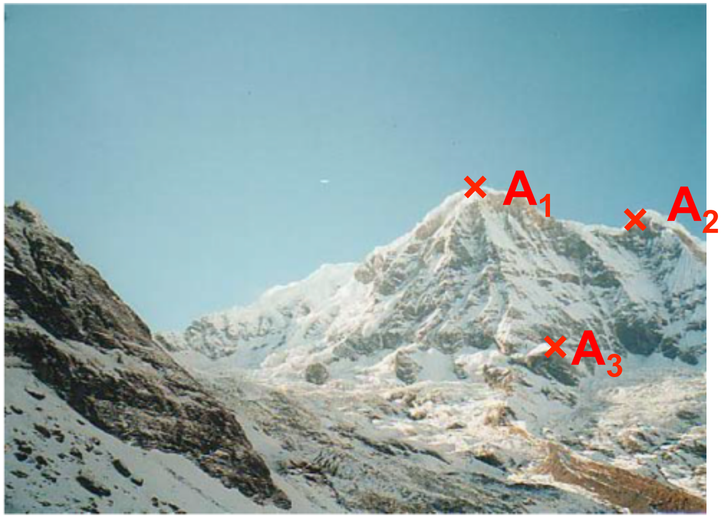
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \vdots & \vdots \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} t_x \\ t_y \end{bmatrix} = \begin{bmatrix} x_1^B - x_1^A \\ y_1^B - y_1^A \\ \vdots \\ x_n^B - x_n^A \\ y_n^B - y_n^A \end{bmatrix}$$

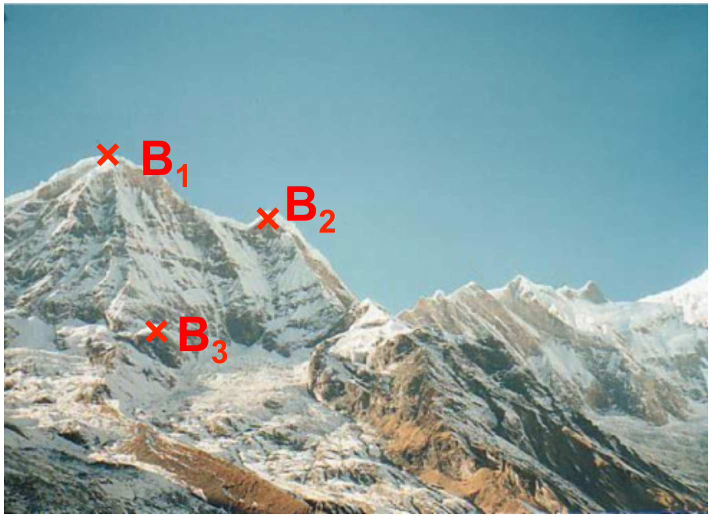


Alignment by RANCAC

1. Sample a set of matching points (1 pair)
2. Solve for transformation parameters
3. Score parameters with number of inliers
4. Repeat steps 1-3 N times

$$\begin{bmatrix} x_i^B \\ y_i^B \end{bmatrix} = \begin{bmatrix} x_i^A \\ y_i^A \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

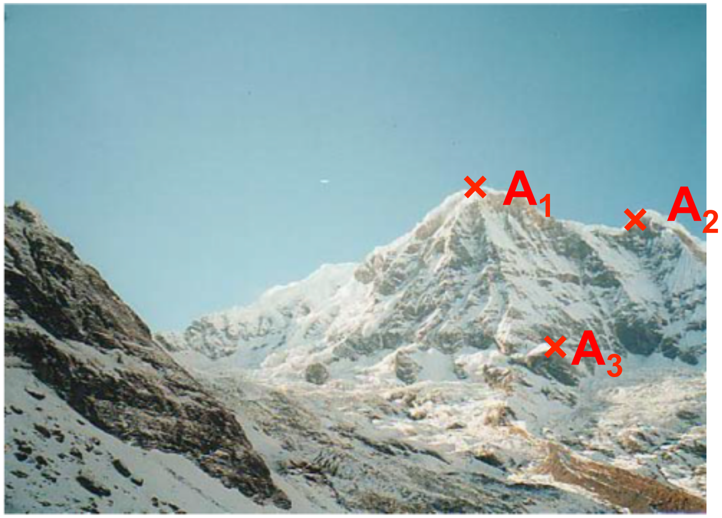


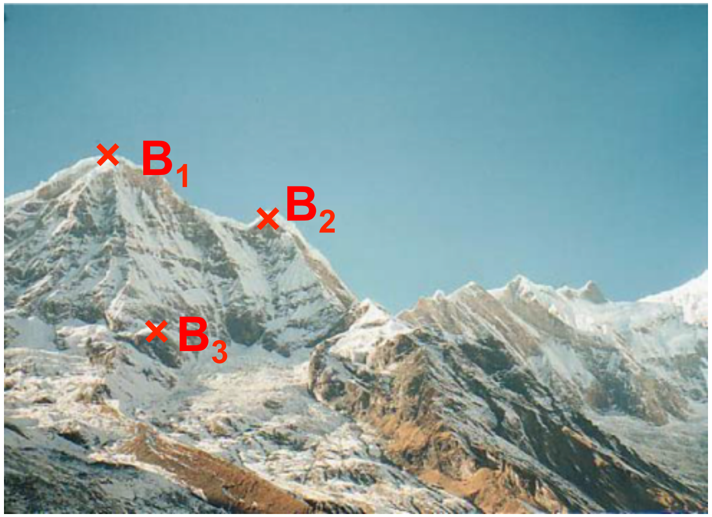


Alignment by Hough Transform

1. Initialize a grid of parameter values
2. Each matched pair casts a vote for consistent values
3. Find the parameters with the most votes
4. Solve using least squares with inliers

$$\begin{bmatrix} x_i^B \\ y_i^B \end{bmatrix} = \begin{bmatrix} x_i^A \\ y_i^A \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$





References and Acknowledgements

- Szeliski, Chapter 6
- Some content are extracted from the above resource and James Hays slides