Introduction to

Information Retrieval

Lecture 5: Index Compression

Inverted index

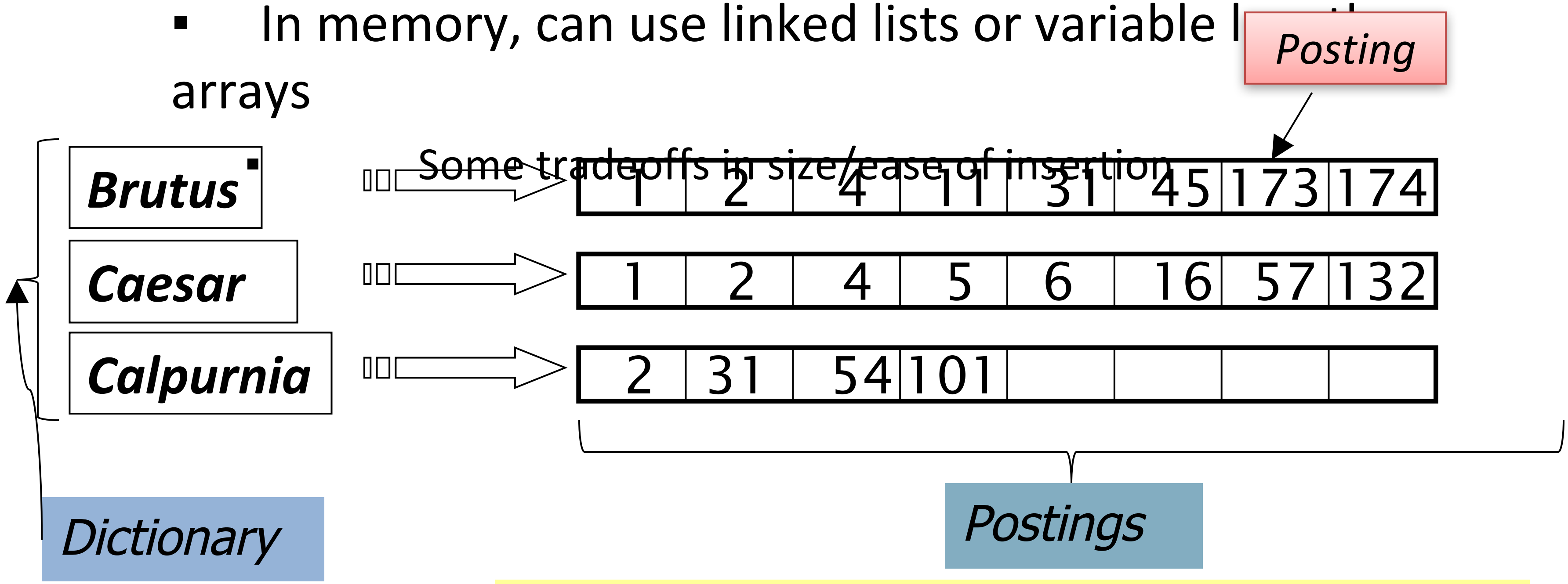
- For each term *t*, we must store a list of all documents that contain *t*.
  - Identify each doc by a **docID**, a document serial number
- Can we used fixed-size arrays for this?

Brutus →	1	2	4	11	31	45	173	174
Caesar →	1	2	4	5	6	16	57	132
Calpurnia →	2	31	54	101				

What happens if the word ***Caesar*** is added to document 14?

Inverted index

- We need variable-size postings lists
  - On disk, a continuous run of postings is normal and best
  - In memory, can use linked lists or variable length arrays
    - Some tradeoffs in size/ease of insertion



Sorted by docID (more later on why).

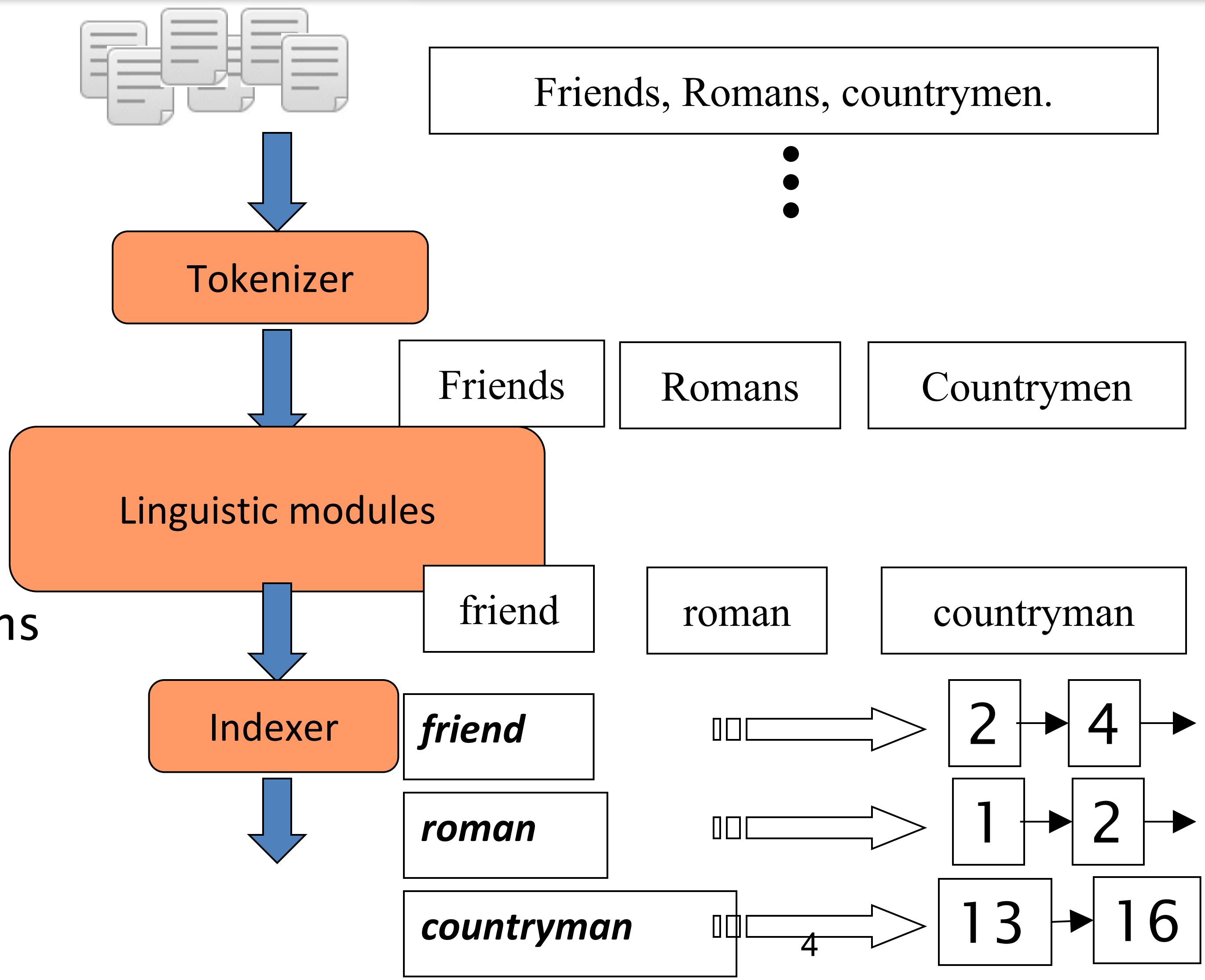
Inverted index construction

Documents to be indexed

Token stream

Modified tokens

Inverted index



Where do we pay in storage?

term	doc. freq.	→	postings lists
ambitious	1	→	2
be	1	→	2
brutus	2	→	1 → 2
capitol	1	→	1
caesar	2	→	1 → 2
did	1	→	1
enact	1	→	1
hath	1	→	2
i	1	→	1
i'	1	→	1
it	1	→	2
julius	1	→	1
killed	1	→	1
let	1	→	2
me	1	→	1
noble	1	→	2
so	1	→	2
the	2	→	1 → 2
told	1	→	2
you	1	→	2
was	2	→	1 → 2
with	1	→	2

Terms and counts

Lists of docIDs

Pointers

Today

BRUTUS	⟶	1	2	4	11	31	45	173	174	
CAESAR	⟶	1	2	4	5	6	16	57	132	...
CALPURNIA	⟶	2	31	54	101					

- Why compression?
- Collection statistics in more detail (with RCV1)
  - How big will the dictionary and postings be?
- Dictionary compression
- Postings compression

Why compression (in general)?

- Use less disk space
  - Saves a little money
- Keep more stuff in memory
  - Increases speed
- Increase speed of data transfer from disk to memory
  - [read compressed data | decompress] is faster than [read uncompressed data]
  - Premise: Decompression algorithms are fast
    - True of the decompression algorithms we use

Why compression for inverted indexes?

- Dictionary
  - Make it small enough to keep in main memory
  - Make it so small that you can keep some postings lists in main memory too
- Postings file(s)
  - Reduce disk space needed
  - Decrease time needed to read postings lists from disk
  - Large search engines keep a significant part of the postings in memory.
    - Compression lets you keep more in memory
- We will devise various IR-specific compression schemes

RCV1: Our collection for this lecture

- Shakespeare's collected works definitely aren't large enough for demonstrating many of the points in this course.
- The collection we'll use isn't really large enough either, but it's publicly available and is at least a more plausible example.
- As an example for applying scalable index construction algorithms, we will use the Reuters RCV1 collection.
- This is one year of Reuters newswire (part of 1995 and 1996)

Reuters RCV1 statistics

- **symbol statistic value**
- N documents 800,000
- L avg. # tokens per doc 200
- M terms (= word types) ~400,000
- avg. # bytes per token 6 (incl. spaces/punct.)
- avg. # bytes per token 4.5 (without spaces/punct.)
- avg. # bytes per term 7.5

Index parameters vs. what we index

(details *IIR* Table 5.1, p.80)

size of	word types (terms)			non-positional postings			positional postings		
	dictionary			non-positional index			positional index		
	Size (K)	∆%	cumul %	Size (K)	∆ %	cumul %	Size (K)	∆ %	cumul %
Unfiltered	484			109,971			197,879		
No numbers	474	-2	-2	100,680	-8	-8	179,158	-9	-9
Case folding	392	-17	-19	96,969	-3	-12	179,158	0	-9
30 stopwords	391	-0	-19	83,390	-14	-24	121,858	-31	-38
150 stopwords	391	-0	-19	67,002	-30	-39	94,517	-47	-52
stemming	322	-17	-33	63,812	-4	-42	94,517	0	-52

Exercise: give intuitions for all the '0' entries. Why do some zero entries correspond to big deltas in other columns?

Lossless vs. lossy compression

- Lossless compression: All information is preserved.
  - What we mostly do in IR.
- Lossy compression: Discard some information
- Several of the preprocessing steps can be viewed as lossy compression: case folding, stop words, stemming, number elimination.
- Chap/Lecture 7: Prune postings entries that are unlikely to turn up in the top *k* list for any query.
  - Almost no loss quality for top *k* list.

Vocabulary vs. collection size

- How big is the term vocabulary?
  - That is, how many distinct words are there?
- Can we assume an upper bound?
  - Not really: At least $70^{20} = 10^{37}$ different words of length 20
- In practice, the vocabulary will keep growing with the collection size
  - Especially with Unicode ☺

Vocabulary vs. collection size

- Heaps' law: $M = kT^b$
- M is the size of the vocabulary, T is the number of tokens in the collection
- Typical values: $30 \le k \le 100$ and $b \approx 0.5$
- In a log-log plot of vocabulary size M vs. T, Heaps' law predicts a line with slope about ½
  - It is the simplest possible relationship between the two in log-log space
  - An empirical finding ("empirical law")

Heaps' Law

Fig 5.1 p81

For RCV1, the dashed line

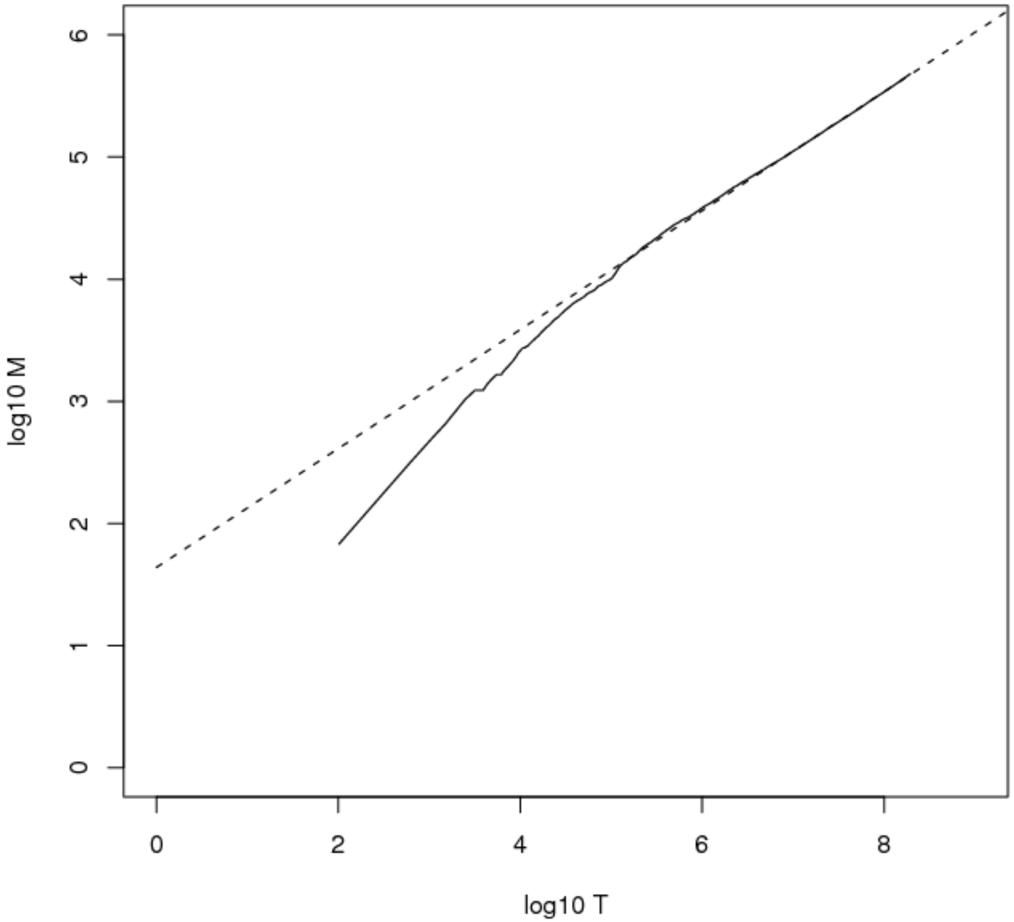
$\log_{10}M = 0.49 \log_{10}T + 1.64$

is the best least squares fit.

Thus, $M = 10^{1.64}T^{0.49}$ so $k = 10^{1.64} \approx 44$ and $b = 0.49$.

Good empirical fit for Reuters RCV1 !

For first 1,000,020 tokens, law predicts 38,323 terms; actually, 38,365 terms



Exercises

- What is the effect of including spelling errors, vs. automatically correcting spelling errors on Heaps' law?
- Compute the vocabulary size *M* for this scenario:
  - Looking at a collection of web pages, you find that there are 3000 different terms in the first 10,000 tokens and 30,000 different terms in the first 1,000,000 tokens.
  - Assume a search engine indexes a total of 20,000,000,000 (2×10^{10}) pages, containing 200 tokens on average
  - What is the size of the vocabulary of the indexed collection as predicted by Heaps' law?

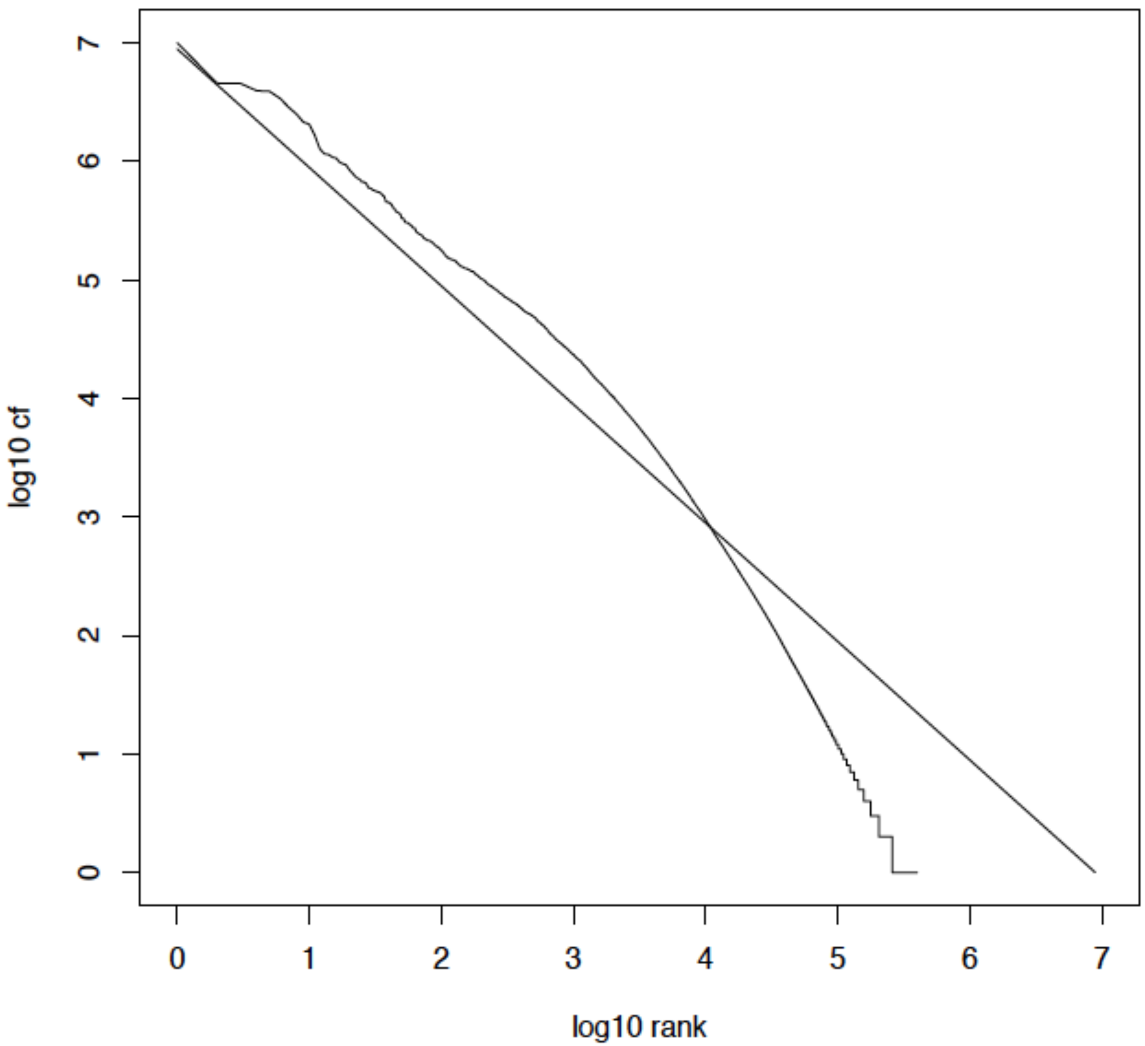
Zipf's law

- Heaps' law gives the vocabulary size in collections.
- We also study the relative frequencies of terms.
- In natural language, there are a few very frequent terms and very many very rare terms.
- Zipf's law: The *i*th most frequent term has frequency proportional to $1/i$.
- $\mathrm{cf}_i \propto 1/i = K/i$ where K is a normalizing constant
- cf_i is <u>collection frequency</u>: the number of occurrences of the term t_i in the collection.

Zipf consequences

- If the most frequent term (*the*) occurs cf_1 times
  - then the second most frequent term (*of*) occurs $cf_1/2$ times
  - the third most frequent term (*and*) occurs $cf_1/3$ times …
- Equivalent: $cf_i = K/i$ where K is a normalizing factor, so
  - $\log cf_i = \log K - \log i$
  - Linear relationship between $\log cf_i$ and $\log i$
- Another power law relationship

Zipf’s law for Reuters RCV1



Compression

- Now, we will consider compressing the space for the dictionary and postings
  - Basic Boolean index only
  - No study of positional indexes, etc.
  - We will consider compression schemes

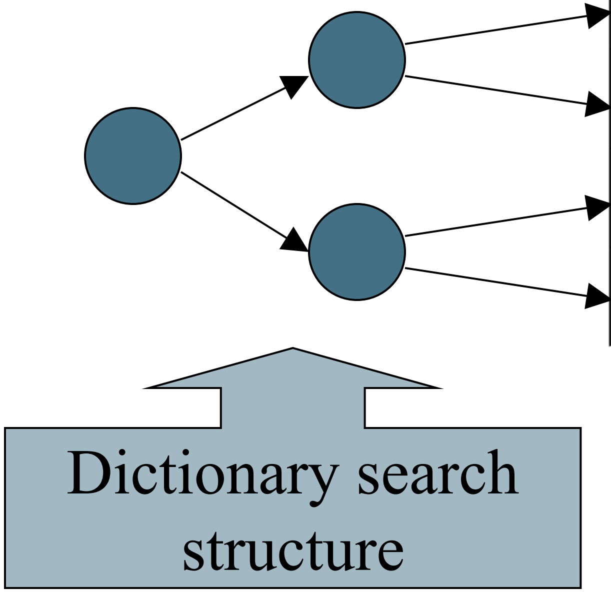
DICTIONARY COMPRESSION

Why compress the dictionary?

- Search begins with the dictionary
- We want to keep it in memory
- Memory footprint competition with other applications
- Embedded/mobile devices may have very little memory
- Even if the dictionary isn't in memory, we want it to be small for a fast search startup time
- So, compressing the dictionary is important

Dictionary storage - first cut

- Array of fixed-width entries
  - ~400,000 terms; 28 bytes/term = 11.2 MB.



Terms	Freq.	Postings ptr.
a	656,265	
aachen	65	
….	….	
zulu	221	

20 bytes

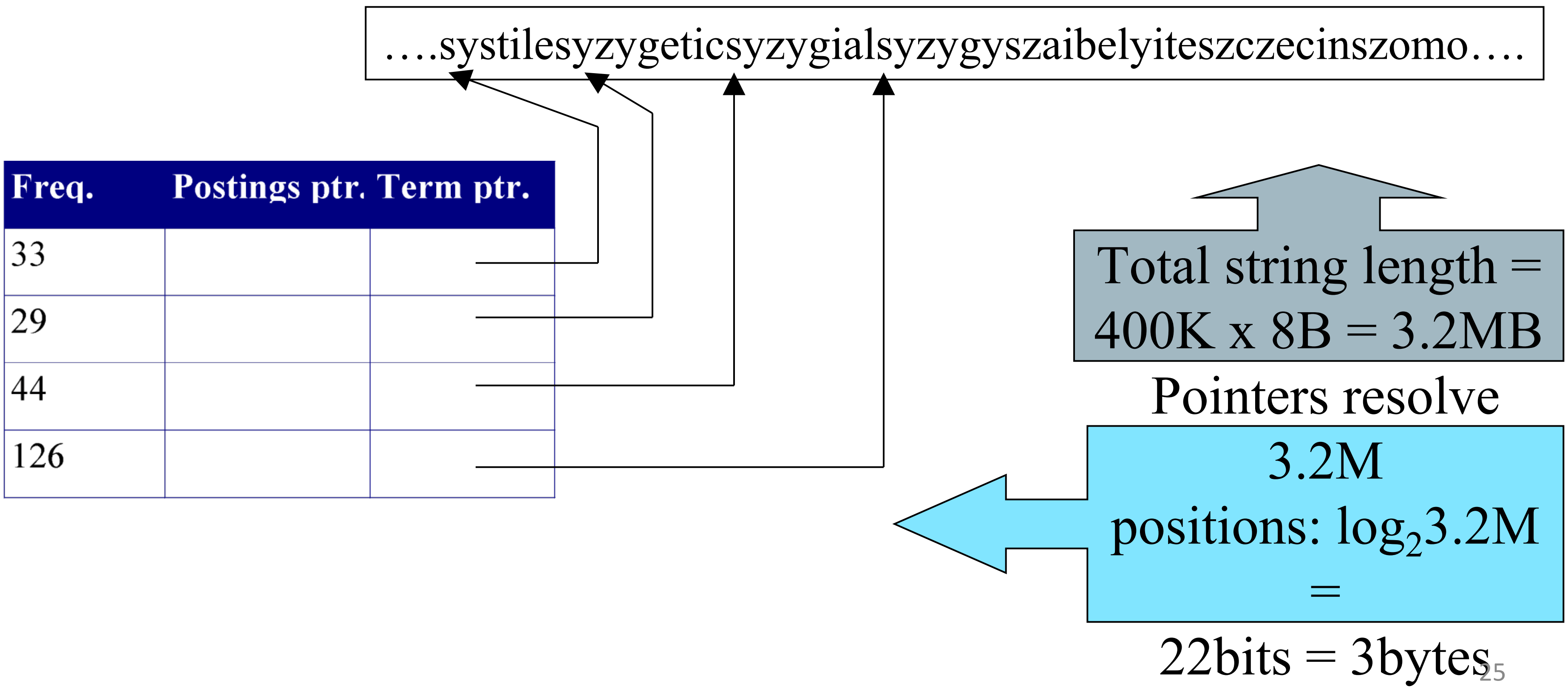
4 bytes each

Fixed-width terms are wasteful

- Most of the bytes in the **Term** column are wasted – we allot 20 bytes for 1 letter terms.
  - And we still can't handle *supercalifragilisticexpialidocious* or *hydrochlorofluorocarbons.*
- Written English averages ~4.5 characters/word.
  - Exercise: Why is/isn't this the number to use for estimating the dictionary size?
- Ave. dictionary word in English: ~8 characters
  - How do we use ~8 characters per dictionary term?
- Short words dominate token counts but not type average.

Compressing the term list: Dictionary-as-a-String

- Store dictionary as a (long) string of characters:
  - Pointer to next word shows end of current word
  - Hope to save up to 60% of dictionary space.



Freq.	Postings ptr.	Term ptr.
33		
29		
44		
126		

Space for dictionary as a string

- 4 bytes per term for Freq.
- 4 bytes per term for pointer to Postings.
- 3 bytes per term pointer
- Avg. 8 bytes per term in term string
- 400K terms x 19 ➔ 7.6 MB (against 11.2MB for fixed width)

Now avg. 19 bytes/term, not 28.

Blocking

- Store pointers to every *k*th term string.
  - Example below: *k*=4.
- Need to store term lengths (1 extra byte)

….*7systile9syzygetic8syzygial6syzygy11szaibelyite8szczecin9szomo*….

Freq.	Postings ptr.	Term ptr.
33		
29		
44		
126		
7		

Save 9 bytes on 3 pointers.

Lose 4 bytes on term lengths.

Net

- Example for block size $k = 4$
- Where we used 3 bytes/pointer without blocking
  - 3 x 4 = 12 bytes,

now we use 3 + 4 = 7 bytes.

Shaved another ~0.5MB. This reduces the size of the dictionary from 7.6 MB to 7.1 MB.
We can save more with larger *k*.

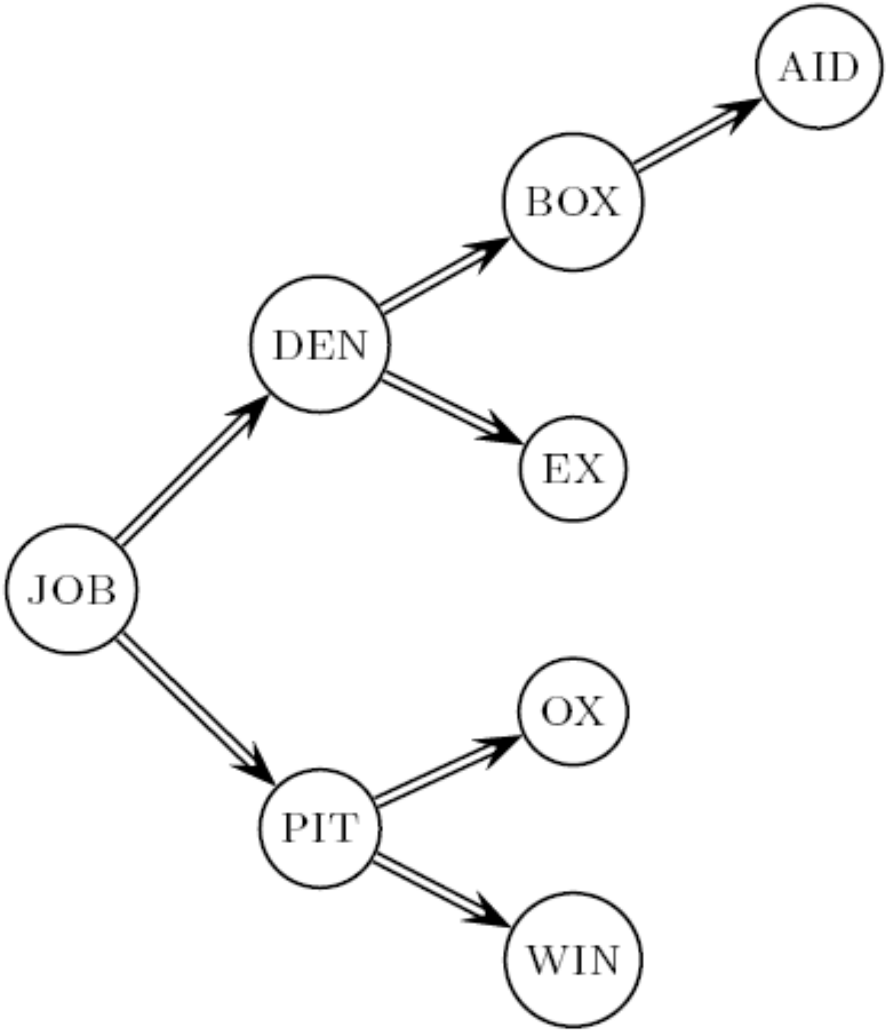
Why not go with larger *k*?

Exercise

- Estimate the space usage (and savings compared to 7.6 MB) with blocking, for block sizes of $k = 4, 8$ and 16.

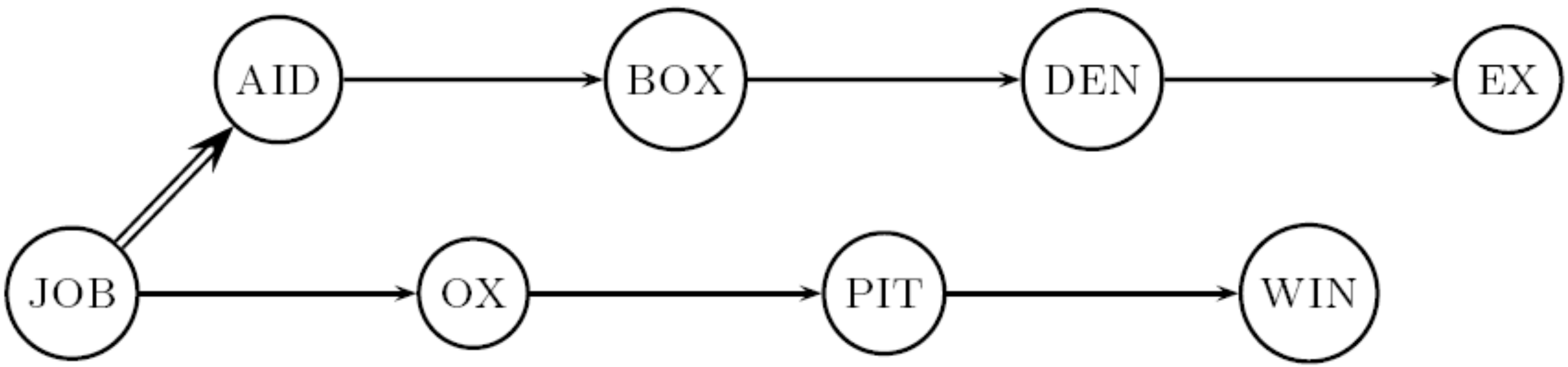
Dictionary search without blocking

- Assuming each dictionary term equally likely in query (not really so in practice!), average number of comparisons = $(1+2\cdot2+4\cdot3+4)/8 \sim 2.6$



Exercise: what if the frequencies of query terms were non-uniform but known, how would you structure the dictionary search tree?

Dictionary search with blocking



- Binary search down to 4-term block;
  - Then linear search through terms in block.
- Blocks of 4 (binary tree), avg. = $(1+2 \cdot 2+2 \cdot 3+2 \cdot 4+5)/8 = 3$ compares

Exercise

- Estimate the impact on search performance (and slowdown compared to k=1) with blocking, for block sizes of $k = 4, 8$ and 16.

Front coding

- Front-coding:
  - Sorted words commonly have long common prefix – store differences only
  - (for last *k-1* in a block of *k*)

8***automata***8***automate***9***automatic***10***automation***

◊8***automat*a***1◊***e***2◊***ic***3◊***ion***

Encodes ***automat***

Extra length beyond ***automat.***

Begins to resemble general string compression.

Front Encoding [Witten, Moffat, Bell]

- Sorted words commonly have long common prefix
  - store differences only
- Complete front encoding
  - (prefix-len, suffix-len, suffix)
- Partial 3-in-4 front encoding
  - No encoding/compression for the first string in a block
  - Enables binary search

Assume previous string is "auto"

String	Complete Front Encoding	Partial 3-in-4 Front Encoding
8, automata	4, 4, mata	, 8, automata
8, automate	7, 1, e	7, 1, e
9, automatic	7, 2, ic	7, 2, ic
10, automation	8, 2, on	8, , on

RCV1 dictionary compression summary

Technique	Size in MB
Fixed width	11.2
Dictionary-as-String with pointers to every term	7.6
Also, blocking k = 4	7.1
Also, Blocking + front coding	5.9

POSTINGS COMPRESSION

Postings compression

- The postings file is much larger than the dictionary, factor of at least 10.
- Key desideratum: store each posting compactly.
- A posting for our purposes is a docID.
- For Reuters (800,000 documents), we would use 32 bits per docID when using 4-byte integers.
- Alternatively, we can use $\log_2 800{,}000 \approx 20$ bits per docID.
- Our goal: use a lot less than 20 bits per docID.

Postings: two conflicting forces

- A term like ***arachnocentric*** occurs in maybe one doc out of a million – we would like to store this posting using $\log_2$ 1M ~ 20 bits.
- A term like ***the*** occurs in virtually every doc, so 20 bits/posting is too expensive.
  - Prefer 0/1 bitmap vector in this case

Postings file entry

- We store the list of docs containing a term in increasing order of docID.
  - ***computer***: 33,47,154,159,202 …
- Consequence: it suffices to store *gaps*.
  - 33,14,107,5,43 …
- Hope: most gaps can be encoded/stored with far fewer than 20 bits.

Three postings entries

	encoding	postings list									
THE	docIDs	...		283042		283043		283044		283045	...
	gaps				1		1		1		...
COMPUTER	docIDs	...		283047		283154		283159		283202	...
	gaps				107		5		43		...
ARACHNOCENTRIC	docIDs	252000		500100							
	gaps	252000	248100								

Variable length encoding

- Aim:
  - For ***arachnocentric***, we will use ~20 bits/gap entry.
  - For ***the***, we will use ~1 bit/gap entry.
- If the average gap for a term is *G*, we want to use ~$\log_2 G$ bits/gap entry.
- Key challenge: encode every integer (gap) with about as few bits as needed for that integer.
- This requires a *variable length encoding*
- Variable length codes achieve this by using short codes for small numbers

Variable Byte (VB) codes

- For a gap value G, we want to use close to the fewest bytes needed to hold $\log_2 G$ bits
- Begin with one byte to store G and dedicate 1 bit in it to be a continuation bit c
- If $G \leq 127$, binary-encode it in the 7 available bits and set $c = 1$
- Else encode G's lower-order 7 bits and then use additional bytes to encode the higher order bits using the same algorithm
- At the end set the continuation bit of the last byte to 1 ($c = 1$) – and for the other bytes $c = 0$.

Hex(824)=0x0338
Hex(214577)=0x00034631

Example

docIDs	824	829	215406
gaps		5	214577
VB code	00000110 10111000	10000101	00001101 00001100 10110001

Postings stored as the byte concatenation
000001101011100010000101000011010000110010110001

Key property: VB-encoded postings are uniquely prefix-decodable.

For a small gap (5), VB uses a whole byte.

Other variable unit codes

- Instead of bytes, we can also use a different “unit of alignment”: 32 bits (words), 16 bits, 4 bits (nibbles).
- Variable byte alignment wastes space if you have many small gaps – nibbles do better in such cases.
- Variable byte codes:
  - Used by many commercial/research systems
  - Good low-tech blend of variable-length coding and sensitivity to computer memory alignment matches (vs. bit-level codes, which we look at next).
- There is also recent work on word-aligned codes that pack a variable number of gaps into one word (e.g., simple9)

Simple9

- Encodes as many gaps as possible in one DWORD
- 4 bit selector + 28 bit data bits
  - Encodes 9 possible ways to "use" the data bits

Selector	# of gaps encoded	Len of each gap encoded	Wasted bits
0000	28	1	0
0001	14	2	0
0010	9	3	1
0011	7	4	0
0100	5	5	3
0101	4	7	0
0110	3	9	1
0111	2	14	0
1000	1	28	0

Unary code

- Represent *n* as *n* 1s with a final 0.
- Unary code for 3 is 1110.
- Unary code for 40 is
  11111111111111111111111111111111111111110 .
- Unary code for 80 is:
  111111111111111111111111111111111111111111
  111111111111111111111111111111111111110

- This doesn't look promising, but….

Bit-Aligned Codes

- Breaks between encoded numbers can occur after any bit position
- *Unary* code
  - Encode *k* by *k* 1s followed by 0
  - 0 at end makes code unambiguous

Number	Code
0	0
1	10
2	110
3	1110
4	11110
5	111110

Unary and Binary Codes

- Unary is very efficient for small numbers such as 0 and 1, but quickly becomes very expensive
  - 1023 can be represented in 10 binary bits, but requires 1024 bits in unary
- Binary is more efficient for large numbers, but it may be ambiguous

Elias-γ Code

- To encode a number *k*, compute
  - $k_d = \lfloor \log_2 k \rfloor$ unary
  - $k_r = k - 2^{\lfloor \log_2 k \rfloor}$ binary
  - k_d is number of binary digits, encoded in unary

Number (k)	k_d	k_r	Code
1	0	0	0
2	1	0	10 0
3	1	1	10 1
6	2	2	110 10
15	3	7	1110 111
16	4	0	11110 0000
255	7	127	11111110 1111111
1023	9	511	1111111110 111111111

Elias-δ Code

- Elias-γ code uses no more bits than unary, many fewer for k > 2
  - 1023 takes 19 bits instead of 1024 bits using unary
- In general, takes $2\lfloor \log_2 k \rfloor + 1$ bits
- To improve coding of large numbers, use Elias-δ code
  - Instead of encoding k_d in unary, we encode $k_d + 1$ using Elias-γ
  - Takes approximately $2 \log_2 \log_2 k + \log_2 k$ bits

Elias-δ Code

- Split $(k_d + 1)$ into:

$$k_{dd} = \lfloor \log_2 (k_d + 1) \rfloor$$

$$k_{dr} = (k_d + 1) - 2^{\lfloor \log_2 (k_d + 1) \rfloor}$$

  - encode k_{dd} in unary, k_{dr} in binary, and k_r in binary

Number (k)	k_d	k_r	k_{dd}	k_{dr}	Code
1	0	0	0	0	0
2	1	0	1	0	10 0 0
3	1	1	1	0	10 0 1
6	2	2	1	1	10 1 10
15	3	7	2	0	110 00 111
16	4	0	2	1	110 01 0000
255	7	127	3	0	1110 000 1111111
1023	9	511	3	2	1110 010 111111111

```
#
# Generating Elias-gamma and Elias-delta codes in Python
#

import math

def unary_encode(n):
 return "1" * n + "0"

def binary_encode(n, width):
    r = ""
    for i in range(0,width):
     if ((1<<i) & n) > 0:
     r = "1" + r
     else:
     r = "0" + r
    return r

def gamma_encode(n):
    logn = int(math.log(n,2))
    return unary_encode( logn ) + " " + binary_encode(n, logn)

def delta_encode(n):
 logn = int(math.log(n,2))
if n == 1:
 return "0"
 else:
 loglog = int(math.log(logn+1,2))
 residual = logn+1 - int(math.pow(2, loglog))
        return unary_encode( loglog ) + " " + binary_encode( residual, loglog ) + " " + binary_encode(n, logn)

if __name__ == "__main__":
    for n in [1,2,3, 6, 15,16,255,1023]:
        logn = int(math.log(n,2))
        loglogn = int(math.log(logn+1,2))
        print n, "d_r", logn
        print n, "d_dd", loglogn
        print n, "d_dr", logn + 1 - int(math.pow(2,loglogn))
        print n, "delta", delta_encode(n)
        #print n, "gamma", gamma_encode(n)
        #print n, "binary", binary_encode(n)
```

Gamma code properties

- G is encoded using $2\lfloor \log G \rfloor + 1$ bits
  - Length of offset is $\lfloor \log G \rfloor$ bits
  - Length of length is $\lfloor \log G \rfloor + 1$ bits
- All gamma codes have an odd number of bits
- Almost within a factor of 2 of best possible, $\log_2 G$

- Gamma code is uniquely prefix-decodable, like VB
- Gamma code can be used for any distribution
- Gamma code is parameter-free

Gamma seldom used in practice

- Machines have word boundaries – 8, 16, 32, 64 bits
  - Operations that cross word boundaries are slower
- Compressing and manipulating at the granularity of bits can be slow
- Variable byte encoding is aligned and thus potentially more efficient
- Regardless of efficiency, variable byte is conceptually simpler at little additional space cost

Shannon Limit

- Is it possible to derive codes that are optimal (under certain assumptions)?
- What is the optimal average code length for a code that encodes each integer (gap lengths) independently?
- Lower bounds on average code length: Shannon entropy
  - $H(X) = - \Sigma_{x=1}^{n} \Pr[X=x] \log \Pr[X=x]$
- Asymptotically optimal codes (finite alphabets): arithmetic coding, Huffman codes

How to design an optimal code for geometric distribution?

Global Bernoulli Model

- Assumption: term occurrence are Bernoulli events
- Notation:
  - n: # of documents, m: # of terms in vocabulary
  - N: total # of (unique) occurrences
- Probability of a term t_j occurring in document d_i: p = N/nm
- Each term-document occurrence is an independent event
- Probability of a gap of length x is given by the geometric distribution $\Pr[X = x] = (1-p)^{x-1} \cdot p$

It can also be deemed as a generalization of the unary code.

Golomb Code

- Golomb Code (Golomb 1966): highly efficient way to design optimal Huffman-style code for geometric distribution
  - Parameter b
  - For given x ≥ 1, computer integer quotient
  - and remainder

$$q = \lfloor (x-1)/b \rfloor$$
$$r = (x-1) - q \cdot b$$

- Assume b = 2^k
  - Encode q in unary, followed by r coded in binary
  - A bit complicated if b != 2^k. See wikipedia.
- First step: (q+1) bits
- Second step: log(b) bits

Golomb Code & Rice Code

- How to determine optimal b*?
- Select minimal b such that

$$(1-p)^b + (1-p)^{b+1} \le 1$$

- Result due to Gallager and Van Voorhis 1975: generates an optimal prefix code for geometric distribution
- Small p approximation:

$$b^* \approx \ln 2/p = 0.69 \cdot avg\_val$$

- Rice code: only allow b = 2^k

Local Bernoulli Model

- If length of posting lists is known, then a Bernoulli model on each individual inverted list can be used
- Frequent words are coded with smaller b, infrequent words with larger b
- Term frequency need to be encoded (use gamma-code)
- Local Bernoulli outperforms global Bernoulli model in practice (method of practice!)

RCV1 compression

Data structure	Size in MB
dictionary, fixed-width	11.2
dictionary, term pointers into string	7.6
with blocking, k = 4	7.1
with blocking & front coding	5.9
collection (text, xml markup etc)	3,600.0
collection (text)	960.0
Term-doc incidence matrix	40,000.0
postings, uncompressed (32-bit words)	400.0
postings, uncompressed (20 bits)	250.0
postings, variable byte encoded	116.0
postings, γ–encoded	101.0

Google's Indexing Choice

- Index shards partition by doc, multiple replicates
- Disk-resident index
  - Use outer parts of the disk
  - Use different compression methods for different fields: Rice_k (a special kind of Golomb code) for gaps, and Gamma for positions.
- In-memory index
  - All positions; No docid
    - Keep track of document boundaries
  - Group-variant encoding
    - Fast to decode

Source: [Jeff Dean's WSDM 2009 Keynote]

Other details

- Gap = $docid_n$- $docid_{n-1}$ **- 1**
- Freq = freq – 1
- Pos_Gap = pos_n- pos_{n-1} - 1

- C.f., Jiangong Zhang, Xiaohui Long and Torsten Suel: Performance of Compressed Inverted List Caching in Search Engines. WWW 2008.

Index compression summary

- We can now create an index for highly efficient Boolean retrieval that is very space efficient
- Only 4% of the total size of the collection
- Only 10-15% of the total size of the text in the collection
- However, we've ignored positional information
- Hence, space savings are less for indexes used in practice
  - But techniques substantially the same.

Resources for today's lecture

- *IIR* 5
- *MG* 3.3, 3.4.
- F. Scholer, H.E. Williams and J. Zobel. 2002. Compression of Inverted Indexes For Fast Query Evaluation. *Proc. ACM-SIGIR 2002*.
  - Variable byte codes
- V. N. Anh and A. Moffat. 2005. Inverted Index Compression Using Word-Aligned Binary Codes. *Information Retrieval* 8: 151–166.
  - Word aligned codes