Assignment 1 CSU33081 October 2020

Please answer where asked by entering A to E as appropriate and upload this document along with your typewritten solutions (as a separate document) via Blackboard. Both documents should be in .pdf format.

Q1.

How would we represent the summation of the following two polynomials in MATLAB?

$$2x^2 + 2x - 6$$

and

$$x^3 + 2x - 4$$

Choose your answer from the following:

A. [-6 2 2]+[-4 2 1]
B. [2 2 -6]+[1 2 4]
C. [2 2 2 -6]+[1 0 2 -4]
D. [2 2 -6]+[1 2 -4]
E. None of these

Answer: E

The right answer is [2 2 -6] + [1 0 2 -4]

Q2.

What is the final value of the matrix A when the following MATLAB commands are executed?

```
A=eye(3,3);

for x=1:2:3

A(1,x)=1;

End
```

Choose your answer from the following:

A. $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$

B. $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

C. $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

D. $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

E. None of these

Answer: B

When I executed the above command in MATLAB, I got the value of B.

Q3.

What is the displayed result when the following MATLAB script file is executed?

x=[6:8;-1:1;567];

y=x(:,3);

size(y')

Choose your answer from the following:

A. 1 1
B. 3 1
C. 1 3
D. 3 3
E. None if these

Answer: C

When I typed the above command, I got the value of C.

sample1.m | newton.m | newton2.m | gauss_seiden.m | secant.m

```
% x=linspace(0.5,5,100);
% A=x.^2+pi*((10-4*x)./(2*pi)).^2;
% plot(x,A);
% xlabel('x'), ylabel('y')

% p1 = [500 400 300 200 100 50];
% p2 = [3 2 1 5];
% [zeros(1, size(p1,2)-size(p2,2)) p2] + [zeros(1, size(p2,2)-size(p1,2)) p1]


% A=eye(3);
% for x=1:2:3
% A(1,x)=1;
% end

x=[6:8;-1:1;5 6 7];
y=x(:,3);
size(y')
```

Command Window

New to MATLAB? See resources for Getting Started.

```
>>
>>
>>
>> sample1

ans =

     1     3

>>
```

Q4.

Calculate the Truncation Error, $f(x) - P_2(x)$ at $x = 2.5$, in approximating the function $f(x) = 3 - 17x^3$.
For the approximation use the Taylor Series polynomial approximation of degree two, $P_2(x)$, expanded about the point $x_0 = 2.0$.

Choose your answer from the following:

A. -7.182755
B. -7.645227
C. -4.358405
D. -7.994173
E. None of these

Answer: E

f(x) = 3-17x^3 = -17x^3 + 3

f'(x) = -51x^2

f''(x) =-102x

At X0 = 2.0

P2(x) = f(x0) + f'(x0)/1! (x-x0) + f''(x0)/2!(x-x0)^2

f(2.0) = 3 – 17(2.0)^3 = -133

f'(2.0) = -51 (2.0)^2 = -204

f''(2.0) = -102(2.0) = -204

Thus,

P2(x) = f(x0) + f'(2.0)/1! (x-2.0) + f''(2.0)/2!(x-2.0)^2

= -133 +(-204)/1! (x-2.0) +(-204)/2!(x-2.0)^2

= -133 – 204x + 408 – 102x^2 + 408x – 408

= -102x^2 + 204x -133

f(2.5) = 3 -17(2.5)^3 = -262.625

P2(2.5) = -102(2.5)^2 + 204(2.5) -133 = -260.5

Therefore,

Truncation error = f(2.5) - P2(2.5) = -260.5 – (-262.625) = 2.125

Q5.

Use the Secant Method to find a root of the function

$$f(x) = 16x^5 - 73x^2 - 133$$

accurate to within an error of $\epsilon = x_n - x_{n-1} = 0.001$, where x_n is the value of x at the nth iteration. Use starting points $x_0 = 3$ and $x_1 = 2.5$

Choose your answer from the following:

A. 0.982274
B. 0.342803
C. 1.900475
D. 1.513896
E. None of these

Answer: C

$x_3 = 2.068259249$

$f(x_3) = 16(2.068259249)^5 - 73(2.068259249)^2 - 133$

$= 160.2709875$

$$x_4 = x_3 - \frac{f(x_3)(x_3 - x_3)}{f(x_2) - f(x_3)}$$

$$= 2.068259249 - \frac{160.2709875(2.270973056 - 2.068259249)}{456.9668534 - 160.2709875}$$

$= 1.958756065$

$x_4 = 1.958756065$

$f(x_4) = 16(1.958756065)^5 - 73(1.958756065)^2 - 133$

$= 48.25973389$

$$x_5 = x_4 - \frac{f(x_4)(x_3 - x_4)}{f(x_3) - f(x_4)}$$

$$= 1.958756065 - \frac{(2.068259249 - 1.958756065)48.25973389}{160.2709875 - 48.25973389}$$

$= 1.911576925$

$x_5 = 1.911576925$

$f(x_5) = 16(1.911576925)^5 - 73(1.911576925)^2 - 133$

$= 8.64233$

$$x_6 = x_5 - \frac{f(x_5)(x_4 - x_5)}{f(x_4) - f(x_5)}$$

$$= 1.911576925 - \frac{8.64233(1.958756 - 1.911576925)}{48.25973389 - 8.64233}$$

$= 1.901285038$

$f(x) = 16x^5 - 73x^2 - 133$

$x_0 = 3 \quad f(x_0) = f(3) = 16(3)^5 - 73(3)^2 - 133 = 3098$

$x_1 = 2.5 \quad f(x_1) = f(2.5) = 16(2.5)^5 - 73(2.5)^2 - 133 = 973.25$

$$x_2 = x_1 - \frac{f(x_1)(x_0 - x_1)}{f(x_0) - f(x_1)}$$

$$= 2.5 - \frac{f(2.5)(3 - 2.5)}{f(3) - f(2.5)}$$

$$= 2.5 - \frac{973.25(3 - 2.5)}{3098 - 973.25}$$

$= 2.270973056$

$f(x_2) = 16(2.270973056)^5 - 73(2.270973056)^2 - 133 = 456.9668534$

$$x_3 = x_2 - \frac{f(x_2)(x_1 - x_2)}{f(x_1) - f(x_2)}$$

$$= 2.270973056 - \frac{456.9668534(2.5 - 2.270973056)}{973.25 - 456.9668534}$$

$= 2.068259249$

$x_6 = 1.90128538$

$f(x^6) = 16(1.90128538)^5 - 73(1.90128538)^2 - 133$

$= 0.63084 \quad .32509$

$x_7 = x_6 - \dfrac{f(x_6)(x_5 - x_6)}{f(x_5) - f(x_6)}$

$= 0.63084 - \dfrac{0.63084 \quad (1.9115742^5 - 1.901285^5)}{.32509 \quad - 0.63084 \quad .32509}$

$= \dfrac{0.63084 \quad 8.647233 \quad - 0.63084 \quad .32509}{.32509}$

$= 1.900474689$

$x_9 - x_6 = 1.90047468 9 - 1.901285038$

$= 8.10691 \times 10^{-4} < 0.001$

$\therefore$ answer $= 1.900474689 \doteq C$

Q6.

Use Newton-Raphson's Method to find a root of the equation:

$f(x) = x^6 - x - 1$ accurate to within an error of $\epsilon = x_n - x_{n-1} = 0.001$, where x_n is the value of x at the nth iteration. Use a starting point of $x_0 = 1.5$.

Choose your answer from the following:

A. 1.134778
B. 0.616384
C. 1.505056
D. 1.160489
E. None of these

Answer: A

$i=1$, $x_1 = 1.300490884$

$$x_2 = x_1 - \frac{(x_1)^6 - (x_1) - 1}{6(x_1)^5 - 1}$$

$$= 1.300490884 - \frac{(1.300490884)^6 - (1.300490884) - 1}{6(1.300490884)^5 - 1}$$

$$= 1.181480416$$

$i=2$ $x_2 = 1.181480416$

$$x_3 = x_2 - \frac{(x_2)^6 - (x_2) - 1}{6(x_2)^5 - 1}$$

$$= 1.181480416 - \frac{(1.181480416) - (1.181480416) - 1}{6(1.181480416)^5 - 1}$$

$$= 1.13945559$$

$i=3$ $x_3 = 1.13945559$

$$x_4 = x_3 - \frac{(x_3)^6 - (x_3) - 1}{6(x_3)^5 - 1}$$

$$= 1.13945559 - \frac{(1.13945559)^6 - (1.13945559) - 1}{6(1.13945559)^5 - 1}$$

$$= 1.134777626$$

$i=4$ $x_4 = 1.134777626$

$$x_5 = x_4 - \frac{(x_4)^6 - (x_4) - 1}{6(x_4)^5 - 1}$$

$$= 1.134777626 - \frac{(1.134777626)^6 - (1.134777626) - 1}{6(1.134777626)^5 - 1}$$

$$= 1.134724145$$

$$x_4 - x_5 = 1.134777626 - 1.134724145$$

$$= 5.3481 \times 10^{-5} < 0.001$$

$1+2e$

Q 6 $f(x) = x^6 - x - 1$

$f'(x) = 6x^5 - 1$ $\quad x_0 = 1.5$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$= x_i - \frac{(x_i)^6 - (x_i) - 1}{6(x_i)^5 - 1}$$

$i=0$, $x_0 = 1.5$

$$x_1 = x_0 - \frac{(x_0)^6 - (x_0) - 1}{6(x_0)^5 - 1}$$

$$= 1.5 - \frac{(1.5)^6 - (1.5) - 1}{6(1.5)^5 - 1}$$

$$= 1.300490884$$

Q7.

Use Newton's Method to solve the following equations for x and y. Perform three iterations.

$$x^2 + xy = 10$$

$$y + 3xy^2 = 57$$

You should use an initial guess of $x = 1.5$ and $y = 3.5$. Perform three iterations.

Choose your answer from the following:

A. $x = 3, y = 3$
B. $x = 2, y = 3$
C. $x = 2, y = 2$
D. $x = 3, y = 2$
E. None of these

Answer: E

$f_1: x^2 + xy = 10$

$f_2: y + 3xy^2 = 57$

$\frac{\partial f_1}{\partial x} = 2x + y \quad , \quad \frac{\partial f_1}{\partial y} = 1$

$\frac{\partial f_2}{\partial x} = 3 \quad , \quad \frac{\partial f_2}{\partial y} = 6y + 1$

$J(f_1, f_2) = \det \begin{bmatrix} 2x+1 & 1 \\ 3 & 6y+1 \end{bmatrix}$

$= (2x+1)(6y+1) - 3$

$= 12xy + 2x + 6y - 2$

$\Delta x = \frac{-\overset{-70}{(x^2 + xy)}(6y+1) + \overset{-57}{(y + 3xy^2)}(1)}{12xy + 2x + 6y - 2}$

$\Delta y = \frac{-\overset{-57}{(y + 3xy^2)}(2x+1) + \overset{-70}{(x^2 + xy)}(3)}{12xy + 2x + 6y - 2}$

Q8.

Find the inverse of the following matrix using the Gauss-Jordan Method:

$x_0 = 1.5$, $y_0 = 3.5$

$$x_1 = 1.5 + \frac{-(1.5^2 + 1.5 \cdot 3.5 - 0)(6 \cdot 3.5 + 1) + (3.5 + 3 \cdot 1.5 \cdot 3.5^2)}{12 \cdot 1.5 \cdot 3.5 + 2 \cdot 1.5 + 6 \cdot 3.5 - 2}$$

$= 2.166176$

$$y_1 = 3.5 + \frac{-(3.5 + 3 \cdot 1.5 \cdot 3.5^2 - 57)(2 \cdot 1.5 + 1) + (1.5^2 + 1.5 \cdot 3.5 - 0)(3)}{12 \cdot 1.5 \cdot 3.5 + 2 \cdot 1.5 + 6 \cdot 3.5 - 2}$$

$= 3.335294$

$$x_2 = 2.166176 + \frac{-(2.166176^2 + 2.166176 \cdot 3.335294 - 0)(6 \cdot 3.335294 + 1) + (3.335294 + 3 \cdot 2.166176 \cdot 3.335294^2)}{12 \cdot 2.166176 \cdot 3.335294 + 2 \cdot 2.166176 + 6 \cdot 3.335294 - 2}$$

$= 1.967568$

$$y_2 = 3.335294 + \frac{-(3.335294 + 3 \cdot 2.166176 \cdot 3.335294^2 - 57)(2 \cdot 2.166176 + 1) + (2.166176^2 + 2.166176 \cdot 3.335294 - 0)(3)}{12 \cdot 2.166176 \cdot 3.335294 + 2 \cdot 2.166176 + 6 \cdot 3.335294 - 2}$$

$= 2.477188$

$$x_3 = 1.967568 + \frac{-(1.967568^2 + 1.967568 \cdot 2.477188 - 0)(6 \cdot 2.477188 + 1) + (2.477188 + 3 \cdot 1.967568 \cdot 2.477188^2 - 57)}{12 \cdot 1.967568 \cdot 2.477188 + 2 \cdot 1.967568 + 6 \cdot 2.477188 - 2}$$

$= 1.988839$

$$y_3 = 2.477188 + \frac{-(2.477188 + 3 \cdot 1.967568 \cdot 2.477188^2 - 57)(2 \cdot 1.967568 + 1) + (1.967568^2 + 1.967568 \cdot 2.477188 - 0)(3)}{12 \cdot 1.967568 \cdot 2.477188 + 2 \cdot 1.967568 + 6 \cdot 2.477188 - 2}$$

$= 3.626853$

$\vdots$

$x_n = 2.0000$

$y_n = 3.0000$

If it continues infinite time, $x = 2$ and $y = 3$.

However If it continues only 3 iterations,

$x = 1.988839$ and $y = 3.626853$

$$A = \begin{pmatrix} 0 & -3 & -2 \\ 1 & -4 & -2 \\ -3 & 4 & 1 \end{pmatrix}$$

Choose your answer form the following:

A. $\begin{pmatrix} -4 & 5 & 2 \\ 5 & -6 & 2 \\ 8 & 9 & -3 \end{pmatrix}$

B. $\begin{pmatrix} -4 & 5 & 2 \\ -5 & 6 & 2 \\ -8 & -9 & 3 \end{pmatrix}$

C. $\begin{pmatrix} -4 & -5 & 2 \\ 5 & -6 & 2 \\ -8 & 9 & 3 \end{pmatrix}$

D. $\begin{pmatrix} 4 & -5 & -2 \\ 5 & -6 & -2 \\ -8 & 9 & 3 \end{pmatrix}$

E. None of these

Answer: D

$$= \begin{bmatrix} 1 & 0 & \frac{2}{3} & -\frac{4}{3} & 1 & 0 \\ 0 & 1 & \frac{2}{3} & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & -\frac{4}{3} & 3 & 1 \end{bmatrix} \times 3$$

$$= \begin{bmatrix} 1 & 0 & \frac{2}{3} & -\frac{4}{3} & 1 & 0 \\ 0 & 1 & \frac{2}{3} & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 & -8 & 9 & 3 \end{bmatrix} \begin{matrix} -\frac{2}{3}[0\ 0\ 1\ -8\ 9\ 3] \\ -\frac{2}{3}[0\ 0\ 1\ -8\ 9\ 3] \\ \end{matrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 4 & -5 & -2 \\ 0 & 1 & 0 & 5 & -1 & -2 \\ 0 & 0 & 1 & -8 & 9 & 3 \end{bmatrix}$$

$$A = \begin{pmatrix} 0 & -3 & -2 \\ 1 & -4 & -2 \\ -3 & 4 & 1 \end{pmatrix}$$

$$\left[\begin{array}{ccc|ccc} 0 & -3 & -2 & 1 & 0 & 0 \\ 1 & -4 & -2 & 0 & 1 & 0 \\ -3 & 4 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & -4 & -2 & 0 & 1 & 0 \\ 0 & -3 & -2 & 1 & 0 & 0 \\ -3 & 4 & 1 & 0 & 0 & 1 \end{array}\right] \begin{matrix} \\ \\ +3[1\ -4\ -2\ 0\ 1\ 0] \end{matrix}$$

$$= \left[\begin{array}{ccc|ccc} 1 & -4 & -2 & 0 & 1 & 0 \\ 0 & -3 & -2 & 1 & 0 & 0 \\ 0 & -8 & -5 & 0 & 3 & 1 \end{array}\right] \quad * -\frac{1}{3}$$

$$= \left[\begin{array}{ccc|ccc} 1 & -4 & -2 & 0 & 1 & 0 \\ 0 & 1 & \frac{2}{3} & -\frac{1}{3} & 0 & 0 \\ 0 & -8 & -5 & 0 & 3 & 1 \end{array}\right] \begin{matrix} +4[0\ 1\ \frac{2}{3}\ -\frac{1}{3}\ 0\ 0] \\ \\ +8[0\ 1\ \frac{2}{3}\ -\frac{1}{3}\ 0\ 0] \end{matrix}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & \frac{2}{3} & -\frac{4}{3} & 1 & 0 \\ 0 & 1 & \frac{2}{3} & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & -\frac{4}{3} & 3 & 1 \end{array}\right]$$

Q9.

Using $x_1 = 1, x_2 = 3, x_3 = 5$ as an initial guess at the solution, determine the values of x_1, x_2 and x_3 that result from three iterations of the Gauss-Seidel method applied to this matrix equation:

$$\begin{pmatrix} 12 & 7 & 3 \\ 1 & 5 & 1 \\ 2 & 7 & -11 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=\begin{pmatrix} 2 \\ -5 \\ 6 \end{pmatrix}$$

Choose your answer from the following:

A. $x_1 = -2.833, x_2 = -1.4333, x_3 = -1.9727$
B. $x_1 = 1.4959, x_2 = -0.90464, x_3 = -0.84914$
C. $x_1 = 0.90666, x_2 = -1.0115, x_3 = -1.0243$
D. $x_1 = 1.2148, x_2 = -0.72060, x_3 = -0.82451$
E. None of these

Answer: C

$$\begin{pmatrix} 12 & 7 & 3 \\ 1 & 5 & 1 \\ 2 & 7 & -11 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \\ 6 \end{pmatrix}$$

$$x_1 = [2 - (7x_2 + 3x_3)] / 12$$
$$x_2 = [-5 - (x_1 + x_3)] / 5$$
$$x_3 = [6 - (2x_1 + 7x_2)] / -11$$

$$x_1 = 1,\ x_2 = 3,\ x_3 = 5$$

1st iteration

$$x_1 = [2 - (7 \cdot 3 + 3 \cdot 5)] / 12 = -2.8333\,4$$
$$x_2 = [-5 - (1 + 5)] / 5 = -1.4\,3333$$
$$x_3 = [6 - (2 \cdot 1 + 7 \cdot 3)] / -11 = -1.97273$$

2nd iteration

$$x_1 = [2 - (7 \cdot (-1.43333) + 3 \cdot (-1.97273))] / 12 = 1.49596$$
$$x_2 = [-5 - (-2.83334 + (-1.97273))] / 5 = -0.90465$$
$$x_3 = [6 - (2 \cdot (-2.83334) + 7 \cdot (-1.43333))] / -11 = -0.84915$$

3rd iteration

$$x_1 = [2 - (7 \cdot (1.47596) + 3 \cdot (-0.84915))] / 12 = 0.90465$$
$$x_2 = [-5 - (1.49596 + (-0.84915))] / 5 = -1.01150$$
$$x_3 = [6 - (2 \cdot 1.49596 + 7 \cdot (-0.90465))] / -11 = -1.024$$

Q10.

Solve the following equation for $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ using LU Decomposition:

$$\begin{pmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 13 \\ 4 \end{pmatrix}$$

Choose your answer from the following:

A. $x = \begin{pmatrix} 3 \\ 13 \\ 4 \end{pmatrix}$

B. $x = \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix}$

C. $x = \begin{pmatrix} 3 \\ 13 \\ -4 \end{pmatrix}$

D. $x = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix}$

E. None of these

Answer: B

Q.10 $\begin{pmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 13 \\ 4 \end{pmatrix}$

Step 1 : Calculate the 1st column of [L]

for i = 1 ... 3 $L_{i1} = a_{i1}$;

$L_{11} = 1$, $L_{21} = 3$, $L_{31} = 2$

Step 2 : Substituting 1s in diagonal of [U]

$U_{11} = 1$, $U_{22} = 1$, $U_{33} = 1$

Step 3 : Calculating the 1st row elements in [U]

for j = 2, 3 $U_{1j} = a_{1j} / L_{11}$;

$U_{12} = a_{12} / L_{11} = 2 / 1 = 2$

$U_{13} = a_{13} / L_{11} = 4 / 1 = 4$

Step 4 : Calculate the rest of elements

for i = 2, 3

for j = 2, 3

$L_{ij} = a_{ij} - \sum_{k=1}^{k=j-1} L_{ik} U_{kj}$

for j = (i+1)

$U_{ij} = \frac{a_{ij} - \sum_{k=1}^{k=j-1} L_{ik} U_{kj}}{L_{ij}}$

for i = 2

j = 2 : $L_{22} = a_{22} - \sum_{k=1}^{k=j-1} L_{ik} U_{kj}$

$= 8 - (3 \cdot 2)$

$= 8 - 6 = 2$

j = 3 : $U_{23} = \frac{a_{23} - \sum_{k=1}^{k=j-1} L_{ik} U_{kj}}{L_{ij}}$

$= \frac{14 - (L_{21} \cdot U_{13})}{L_{22}}$

$= \frac{14 - (3 \cdot 4)}{2}$

$= \frac{14 - 12}{2} = 1$

for i = 3

j = 2 : $L_{32} = a_{32} - \sum_{k=1}^{k=j-1} L_{ik} U_{kj}$

$= a_{32} - L_{31} U_{12}$

$= 6 - (2 \cdot 2)$

$= 6 - 4 = 2$

j = 3 : $L_{33} = a_{33} - \sum_{k=1}^{k=j-1} L_{ik} U_{kj}$

$= a_{33} - (L_{31} U_{13} + L_{32} U_{23})$

$= 13 - (2 \cdot 4 + 2 \cdot 1)$

$= 13 - (8 + 2)$

$= 3$

Therefore,

$$L = \begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 2 & 0 \\ 2 & 2 & 3 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & U_{12} & U_{13} \\ 0 & 1 & U_{23} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 3 & 2 & 0 \\ 2 & 2 & 3 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 13 \\ 4 \end{pmatrix}$$

$y_1 = 3 / 1 = 3$

$y_2 = (13 - 3y_1) / 2 = (13 - 9) / 2 = 4/2 = 2$

$y_3 = (4 - 2y_1 - 2y_2) / 3 = (4 - 6 - 4) / 2$

$= -6 / 3 = -2$

$$\begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix}$$

$x_3 = -3 / 1 = -2$

$x_2 = 2 - x_3 = 2 - (-2) = 4$

$x_1 = 3 - 2x_2 - 4x_3 = 3 - (-8)$

$= 3$

$= 3$

$\therefore x_1 = 3, \; x_2 = 4 \quad x_3 = -2$