Factorial Methods

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Factorial Methods

- Find the isomorph transformation from original space

  keeps the adjacency relationships among variables

- Results expressed in a ficticious space

- Might produce interpretation problems

- Methods
  - PCA (Principal components analysis)
  - Simple correspondence analysis
  - Multiple correspondence analysis

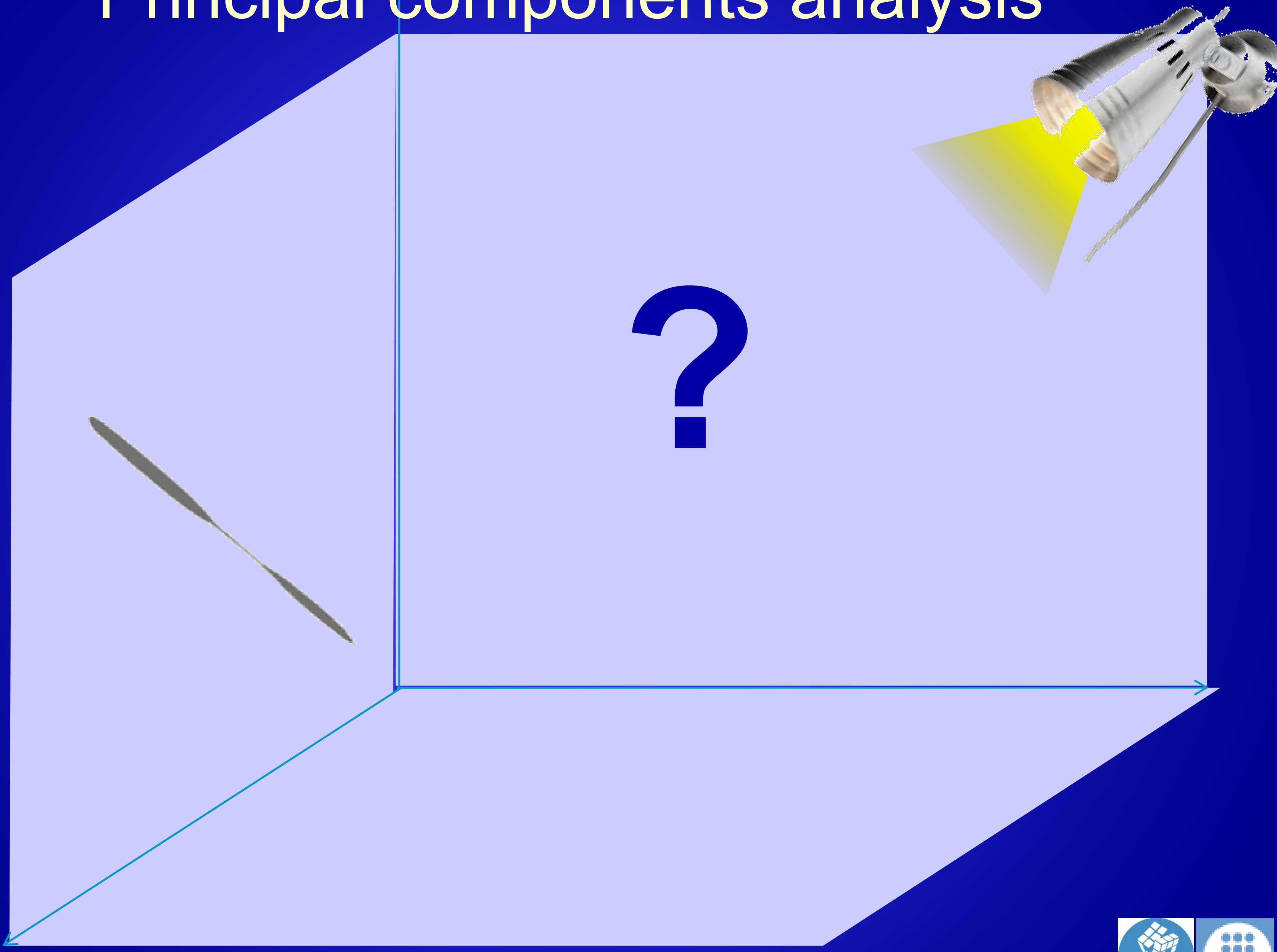
Factorial Methods

- Principal Components Analysis
  - Only numerical variables
  - Find the most informative projection planes

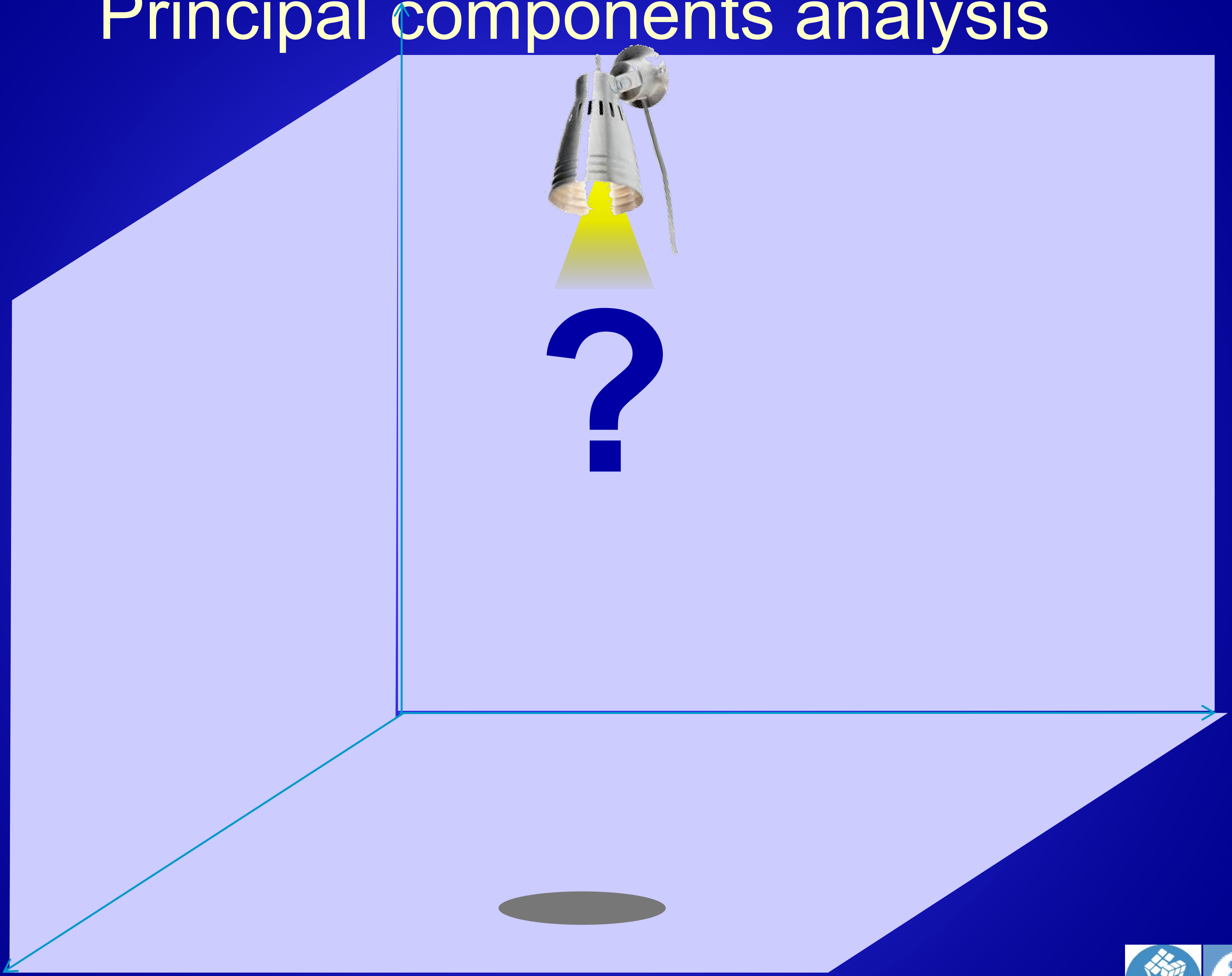
    (factorial planes)

Example “Copas”

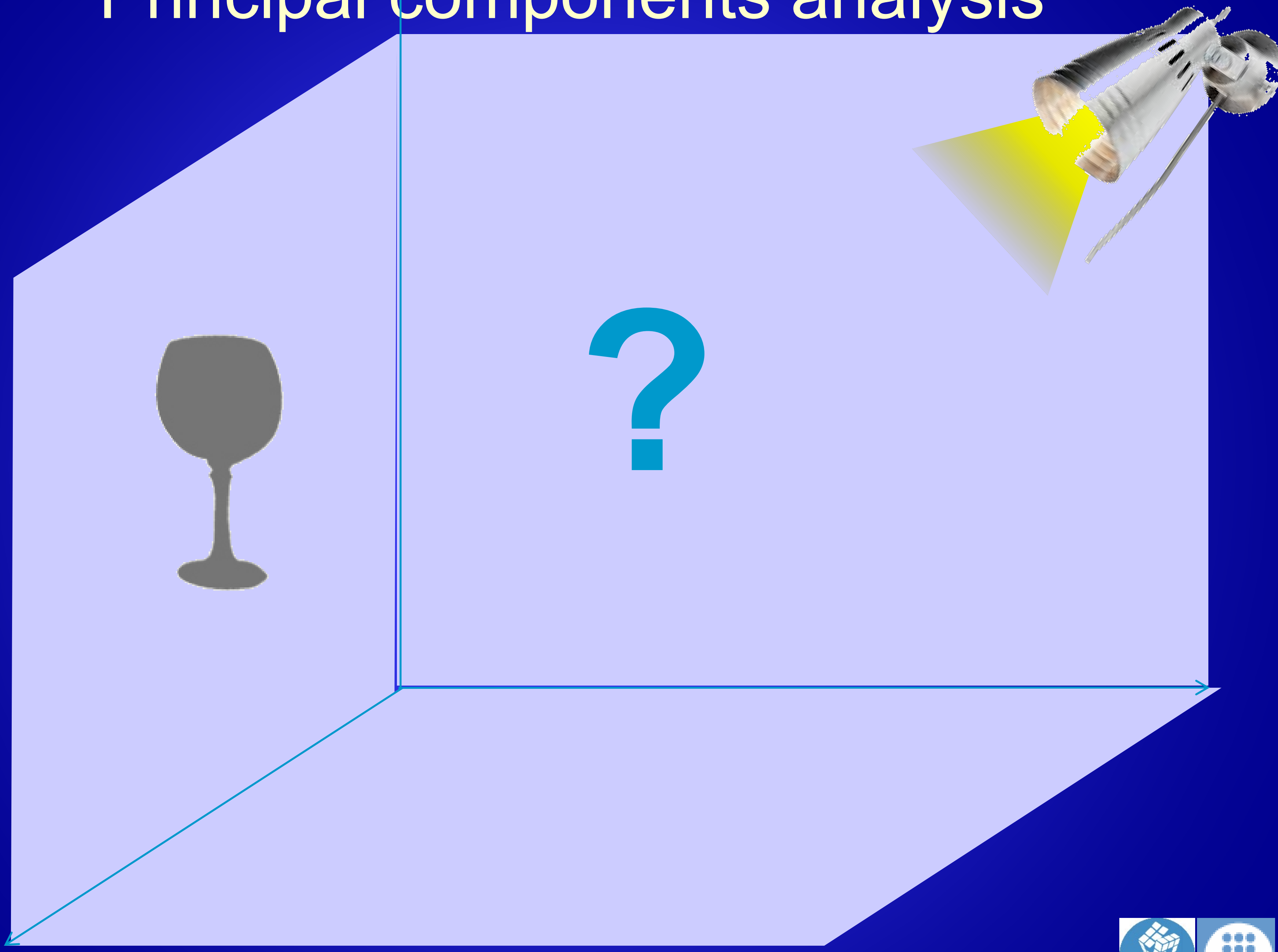
Principal components analysis



Principal components analysis



Principal components analysis



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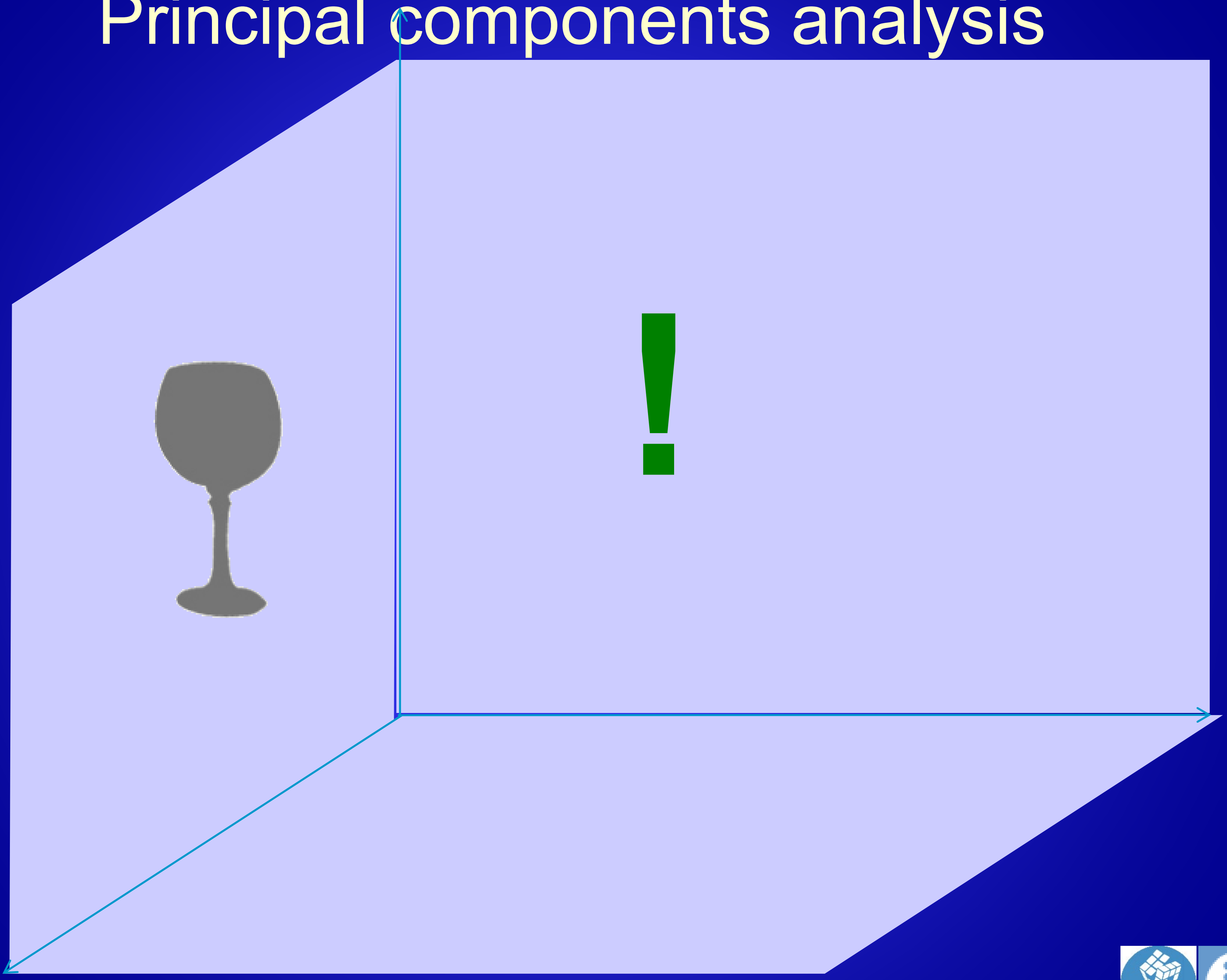

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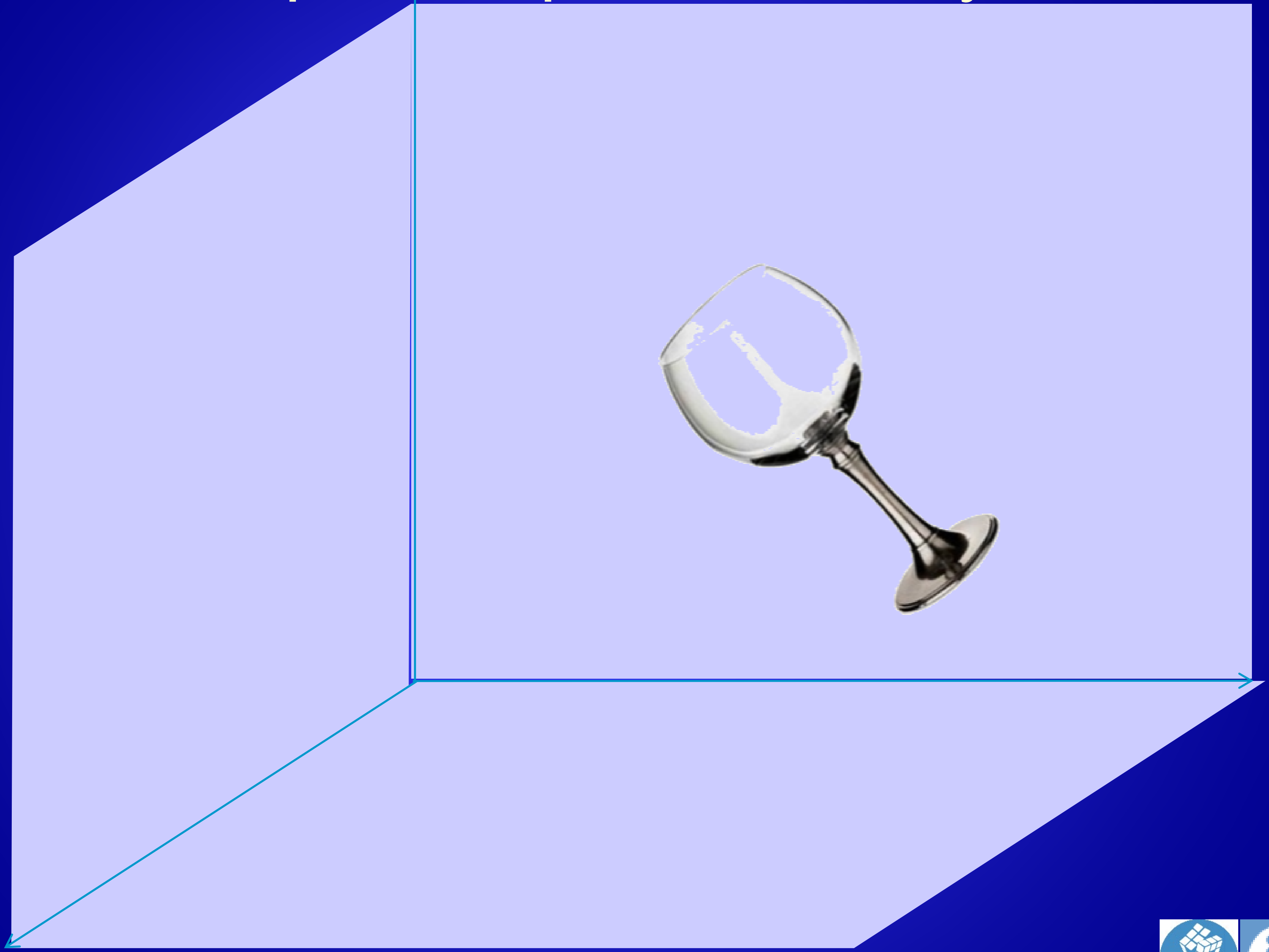
Principal components analysis



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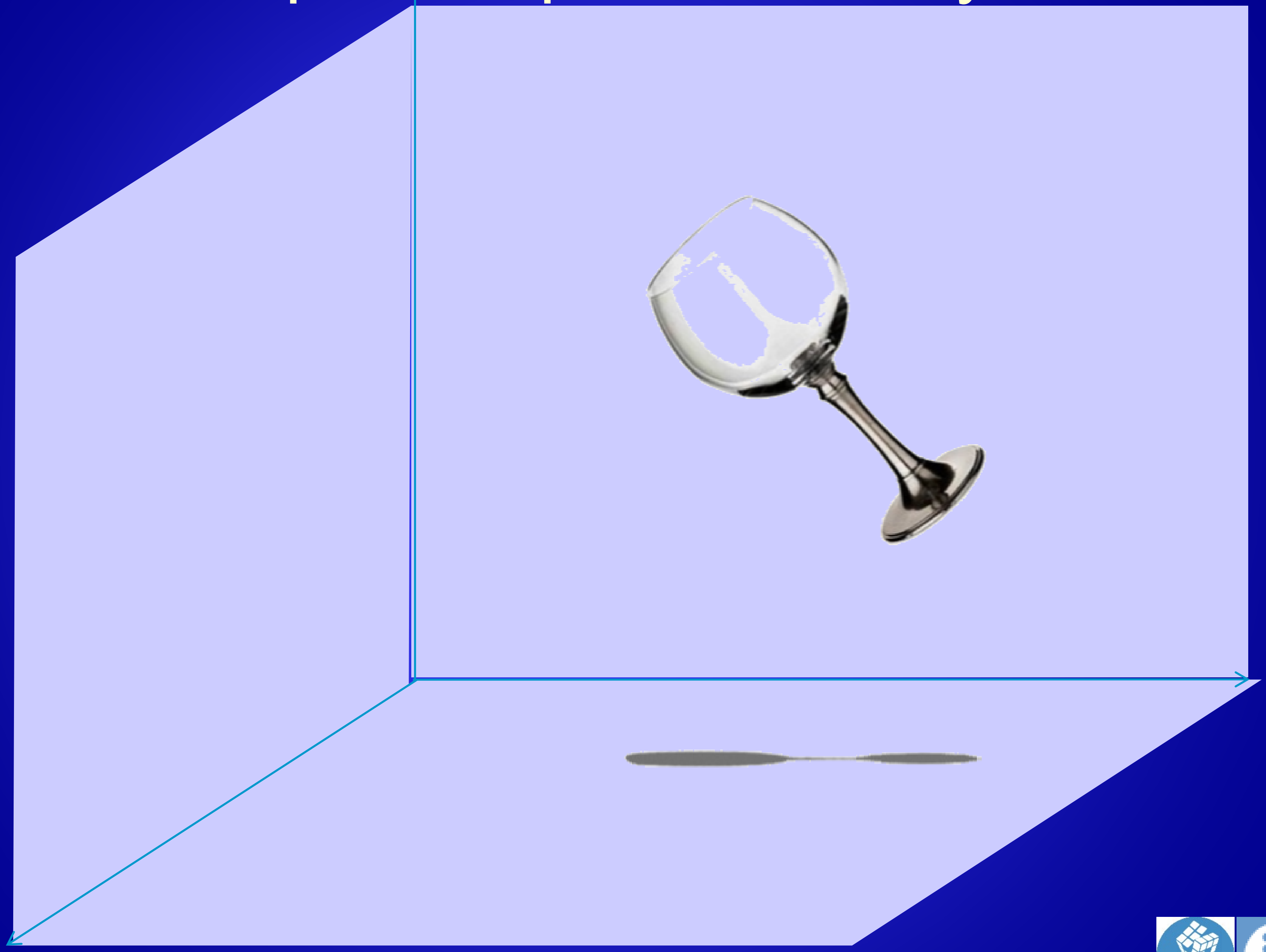
Principal components analysis



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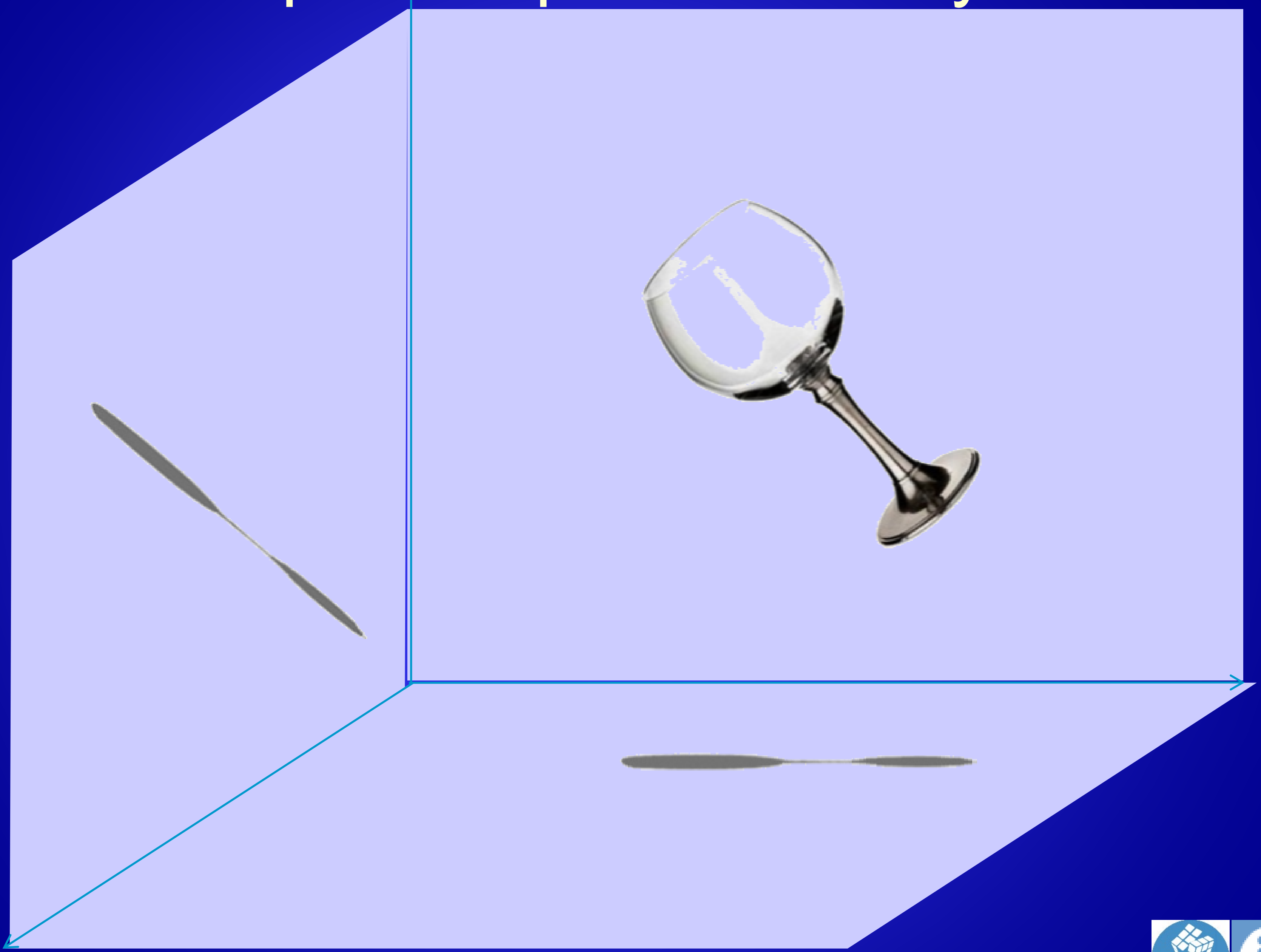
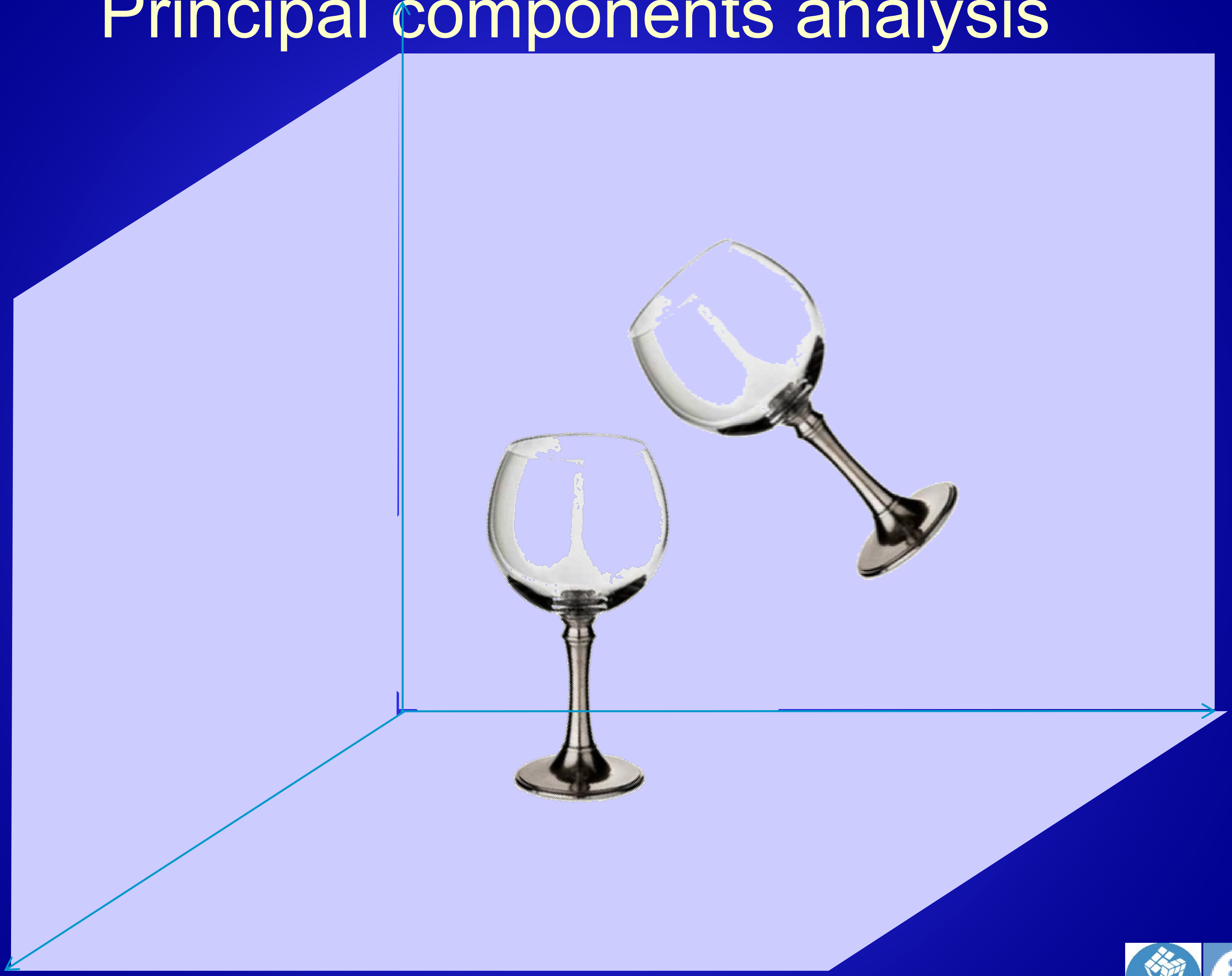
Principal components analysis



Principal components analysis



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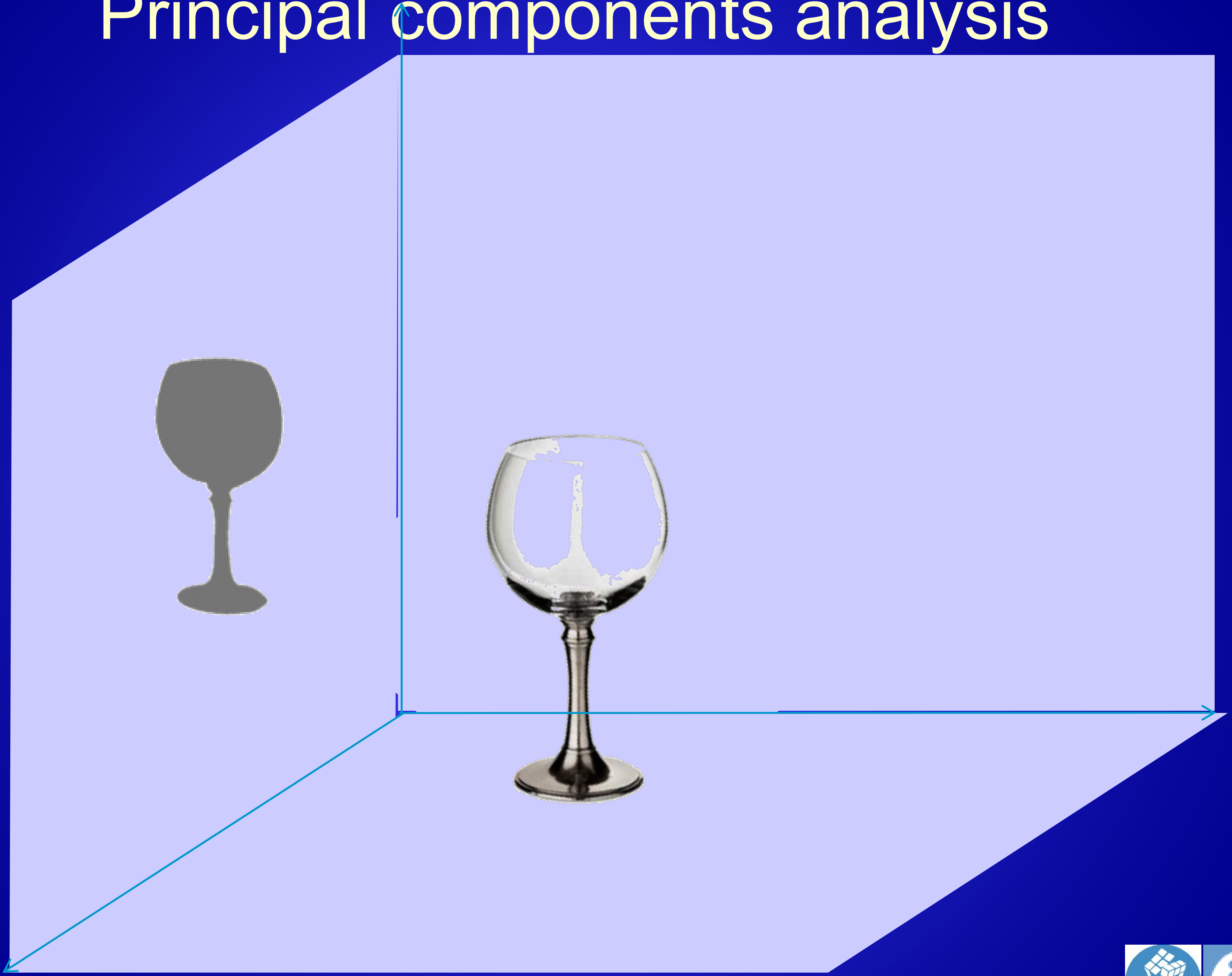

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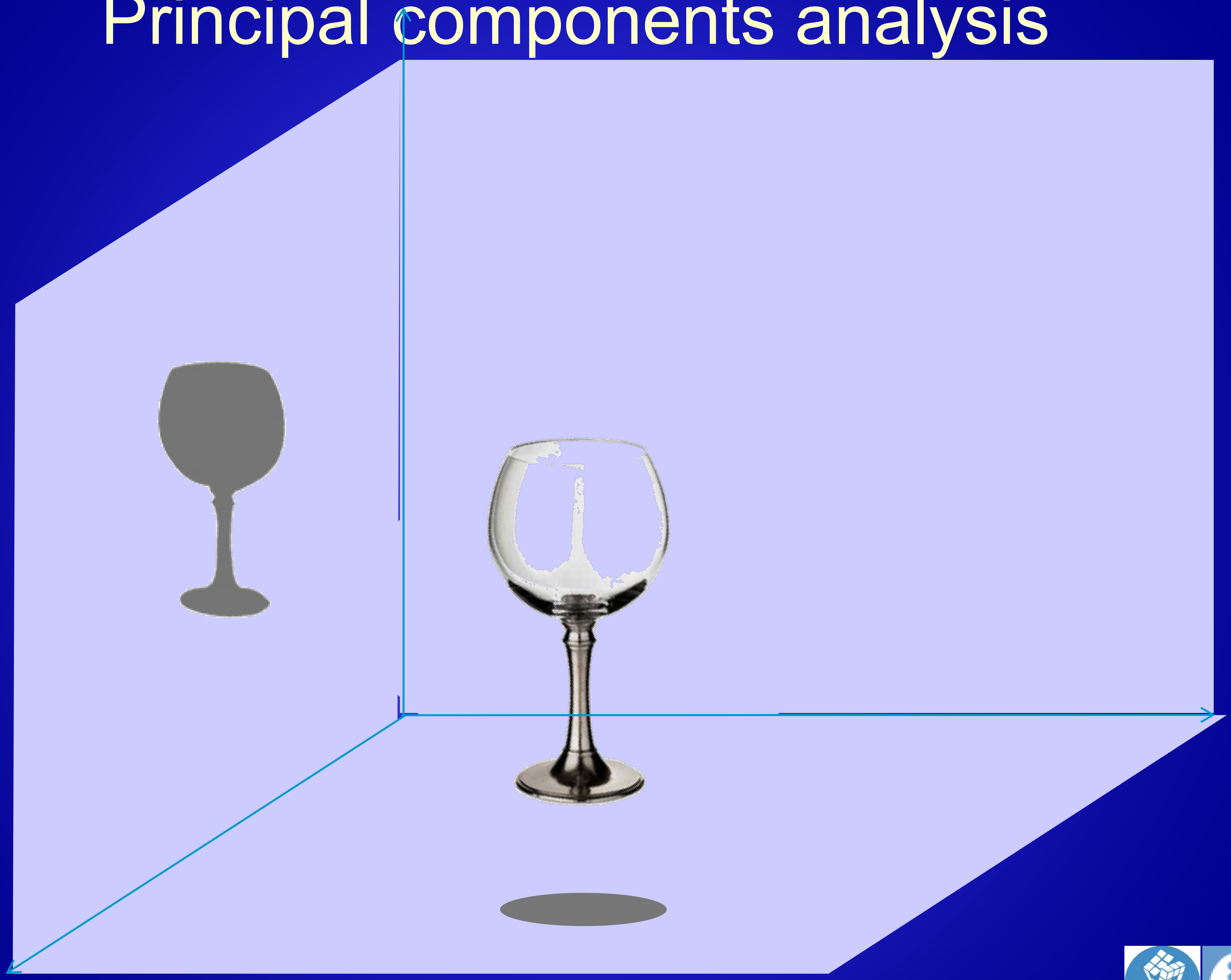

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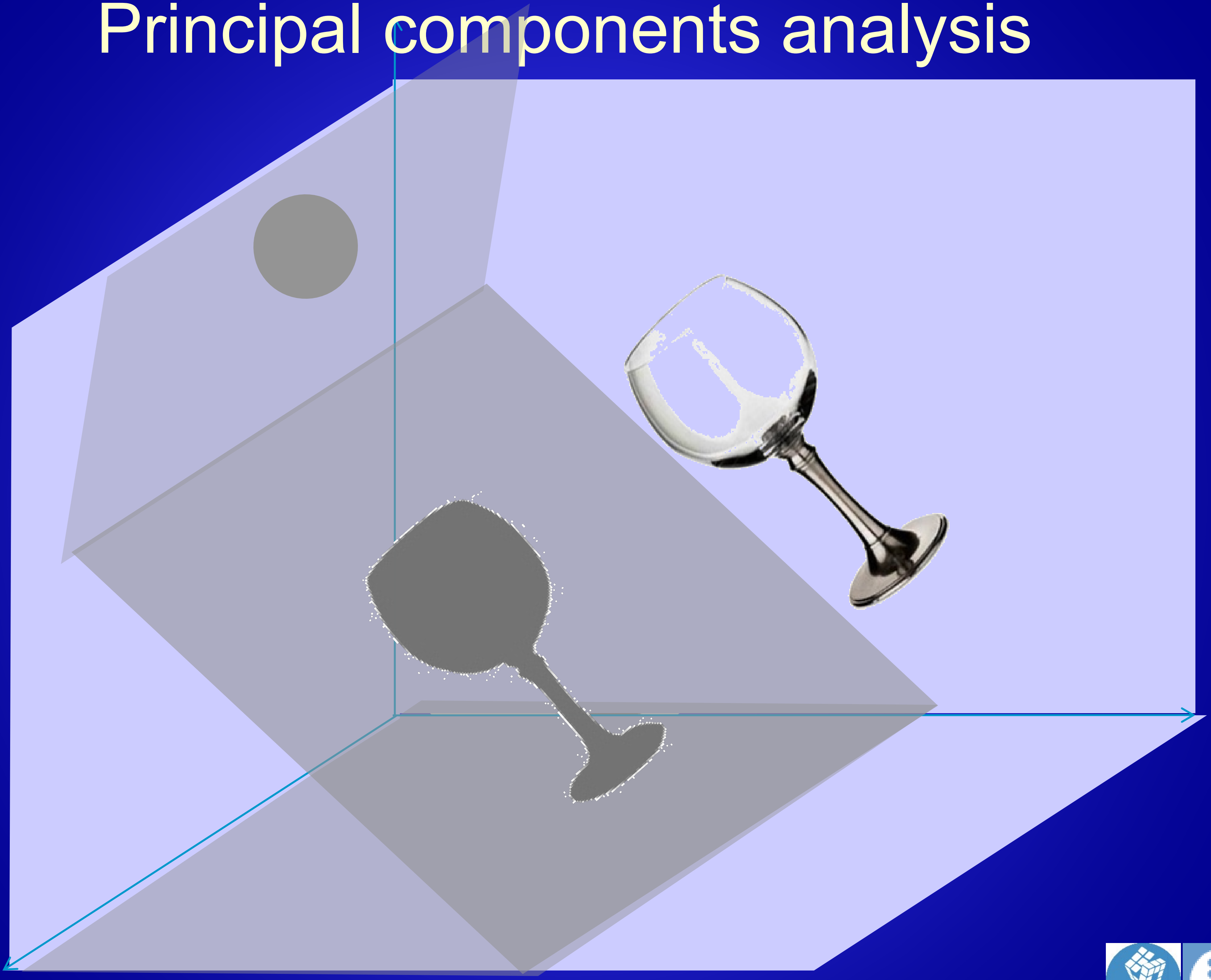

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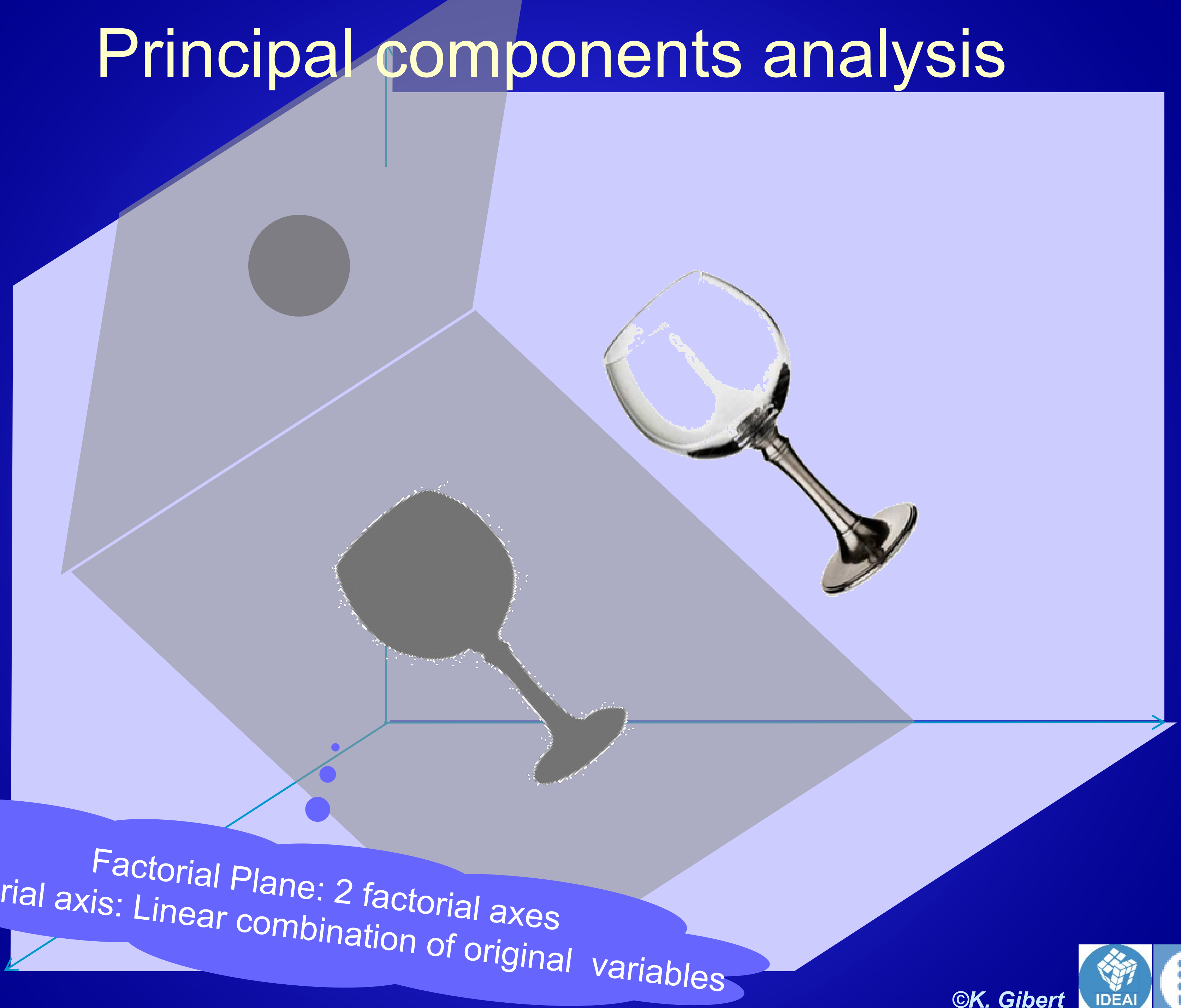
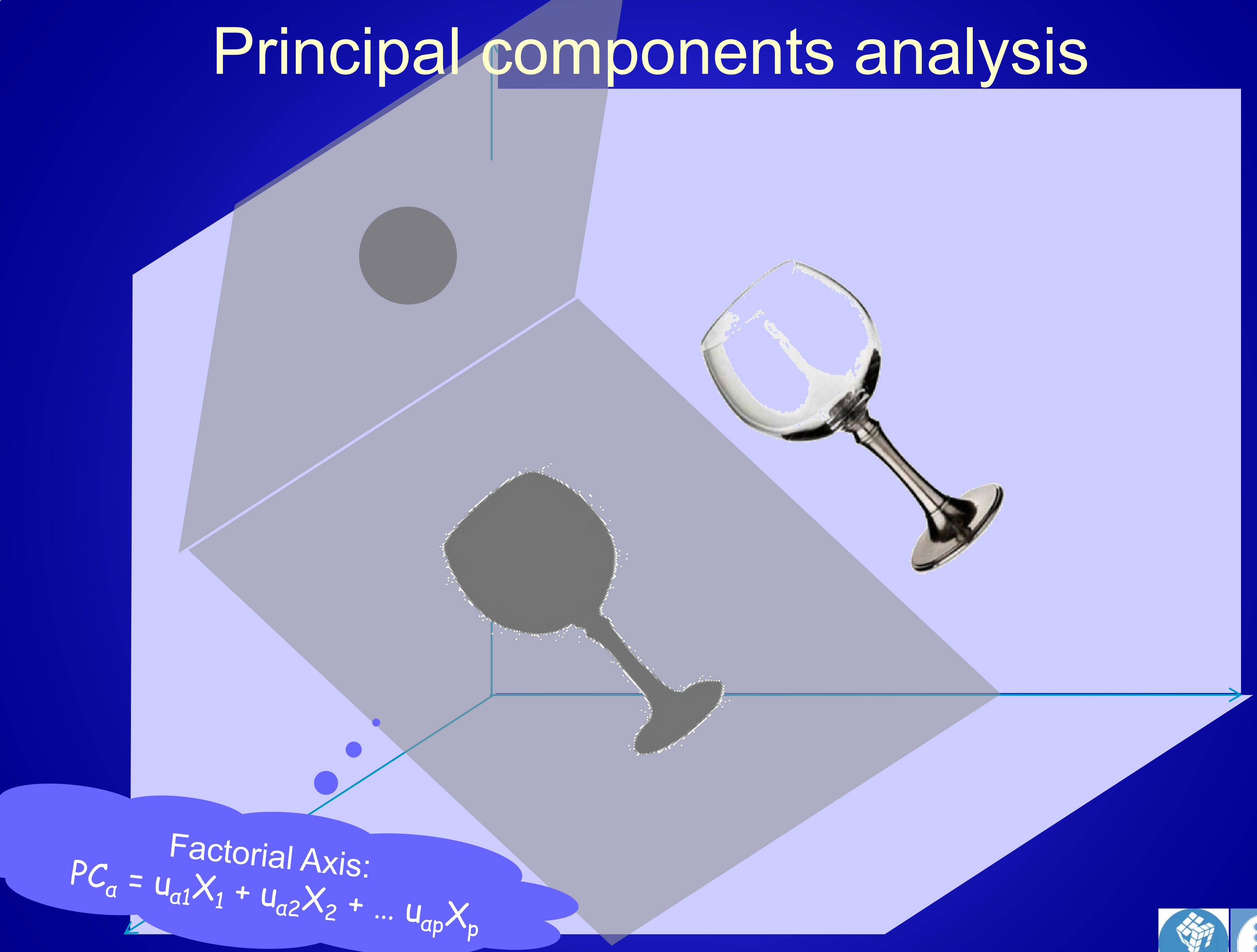
Principal components analysis



Factorial Plane: 2 factorial axes

Factorial axis: Linear combination of original variables

Principal components analysis



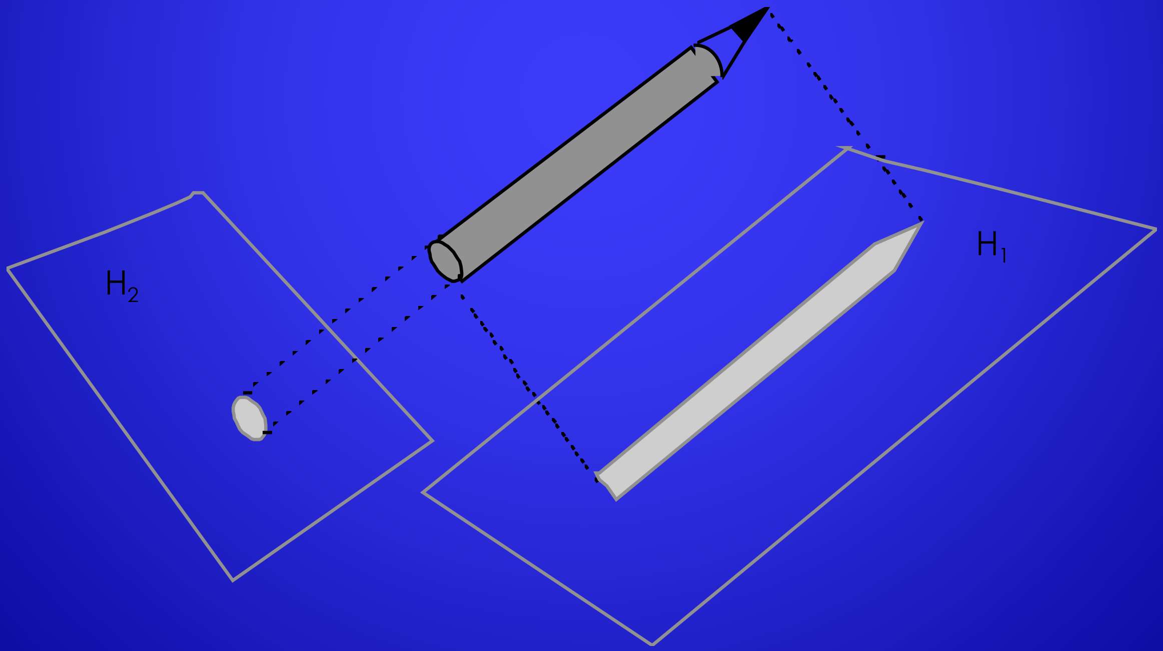
Factorial Axis:

$PC_\alpha = u_{\alpha 1}X_1 + u_{\alpha 2}X_2 + \ldots u_{\alpha p}X_p$

Principal components analysis

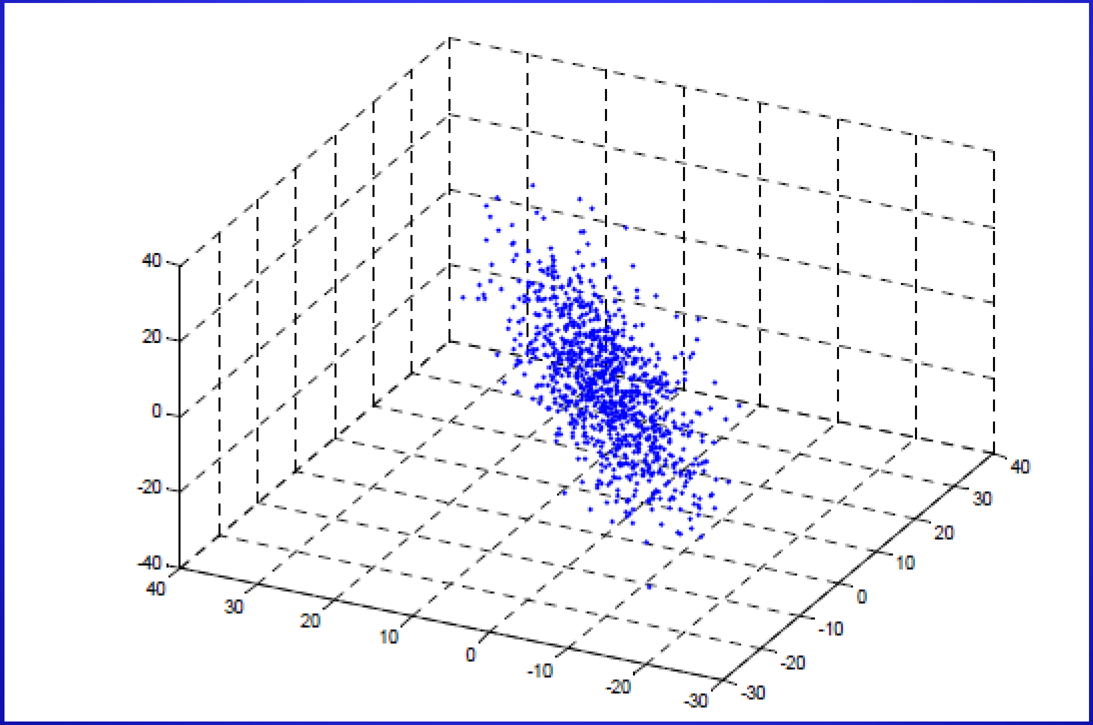
- Purpose:
  - To project the cloud of points upon a subspace (plane) retaining as much original cloud information.

    (see video)



Principal components analysis

- Find the most informative projection planes of data cloud *(factorial planes)*



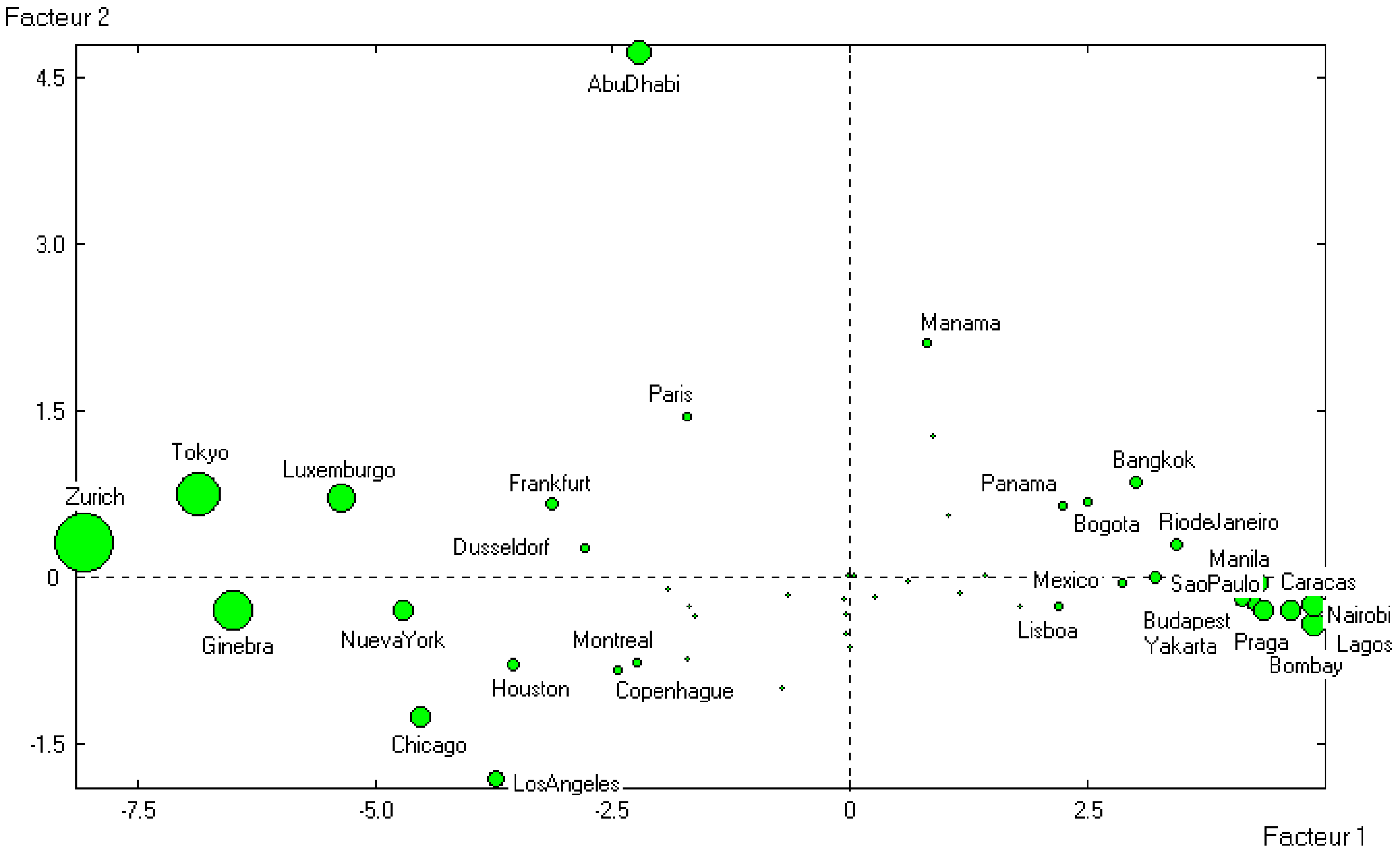
Factorial Methods

- Output: K factors rotating original X variables
- Factors: Linear combinations of original variables

Several uses:

- As an associative data mining method:
  analyze relationships among variables
  Project variables and modalities and find associations

Visualisation of international cities according their salaries. USB 1994.



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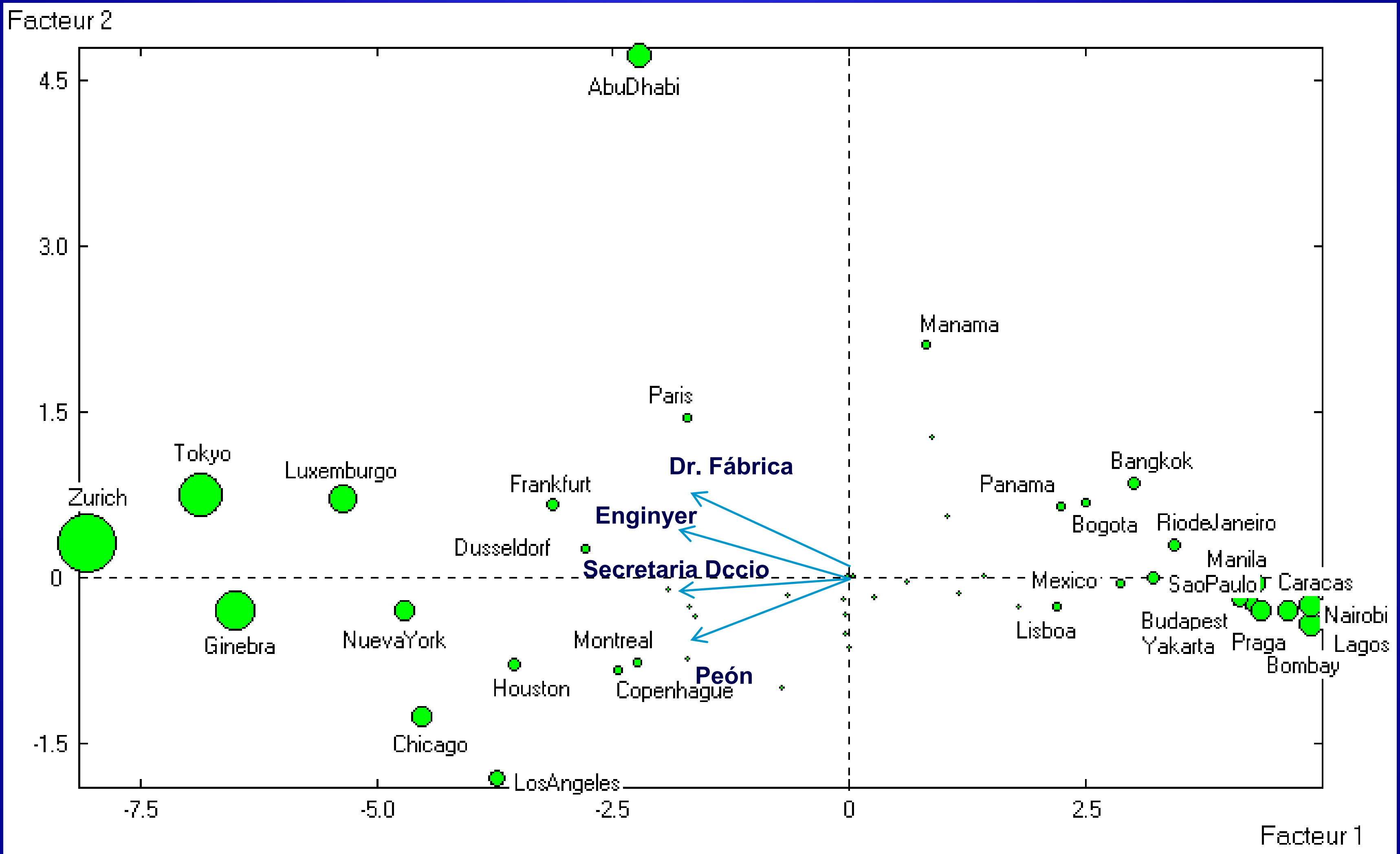
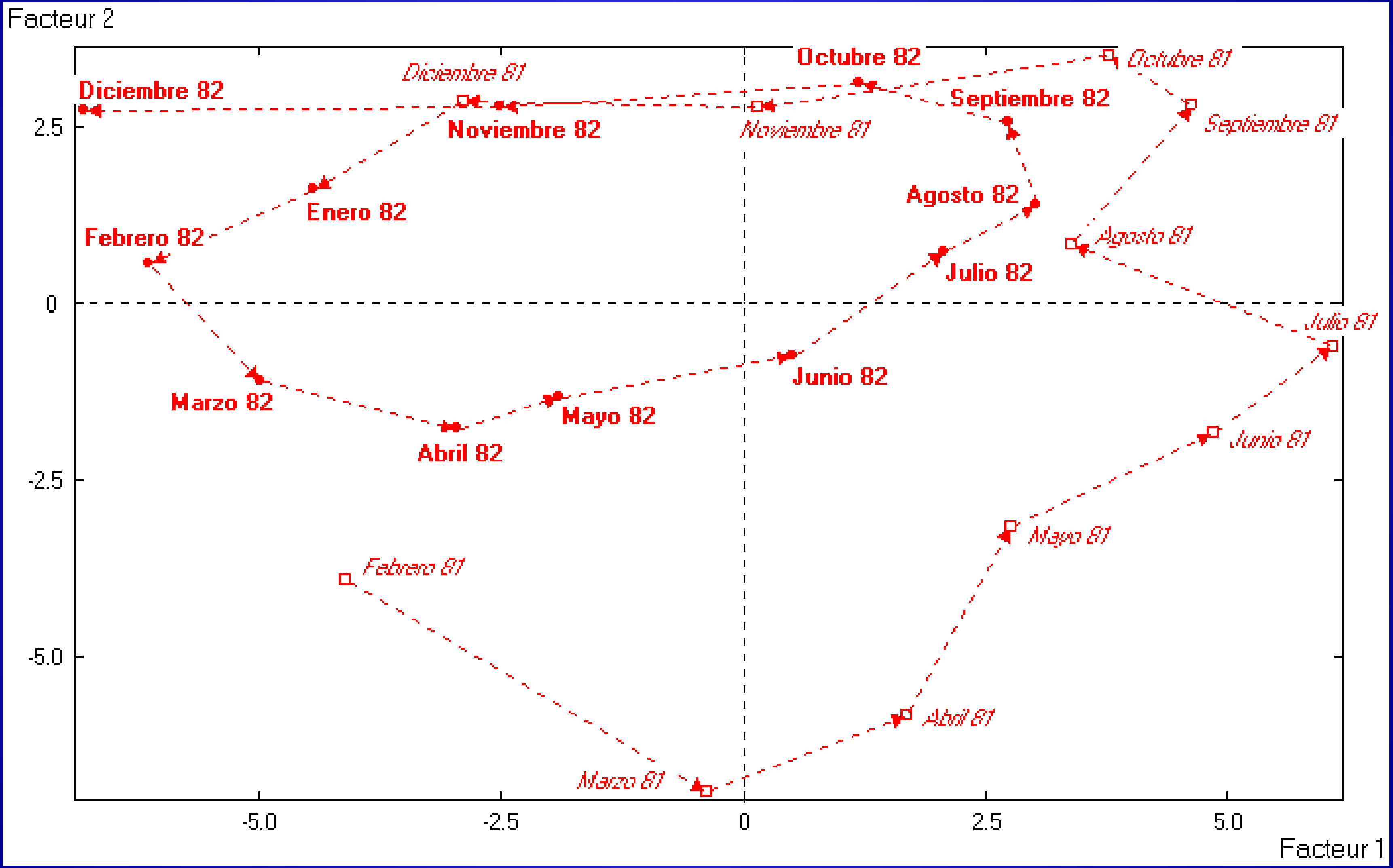


Monitoring of the inner temperatures of Lascaux cave (France)

https://www.facebook.com/grupoajau/posts/1875392015926077



Factorial Methods

- Output: K factors rotating original X variables
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Several uses:

- As an associative data mining method to analyze relationships among variables

  Project variables and modalities and find associations

- As a preprocessing method for elicitation of latent variables

  Project active and illustrative variables/individuals on first/second factorial plane and interpret factors (find latent variables)

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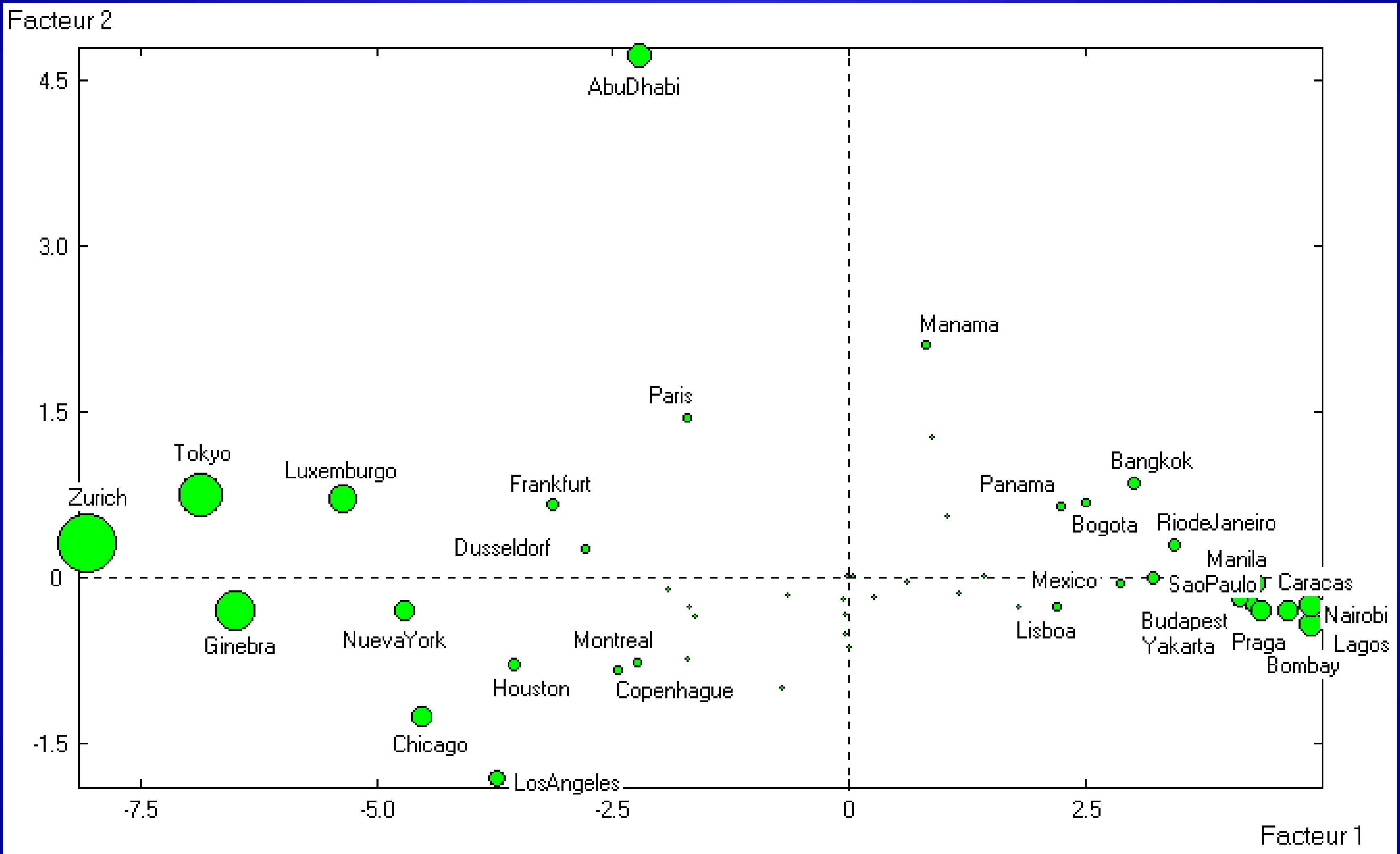

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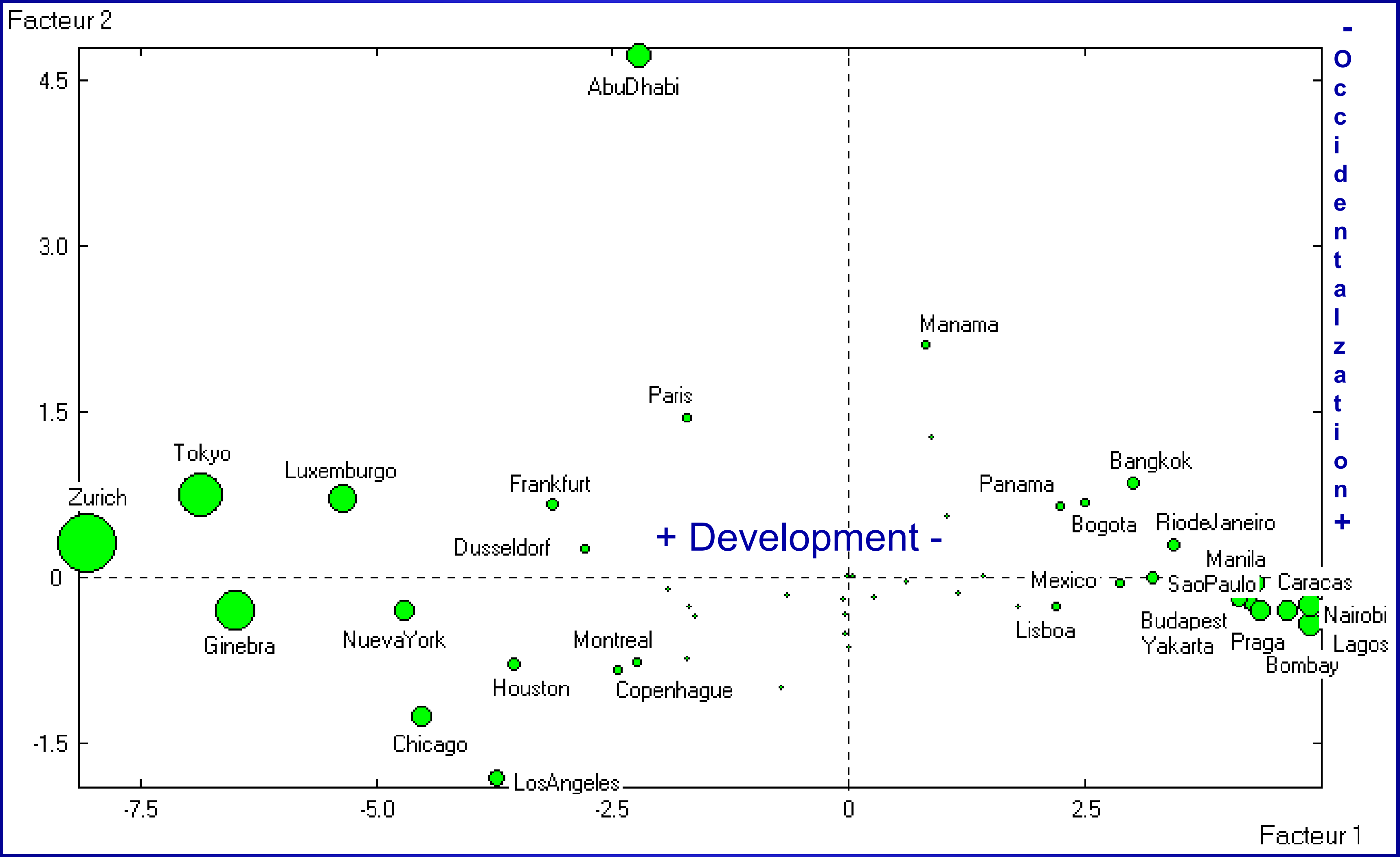
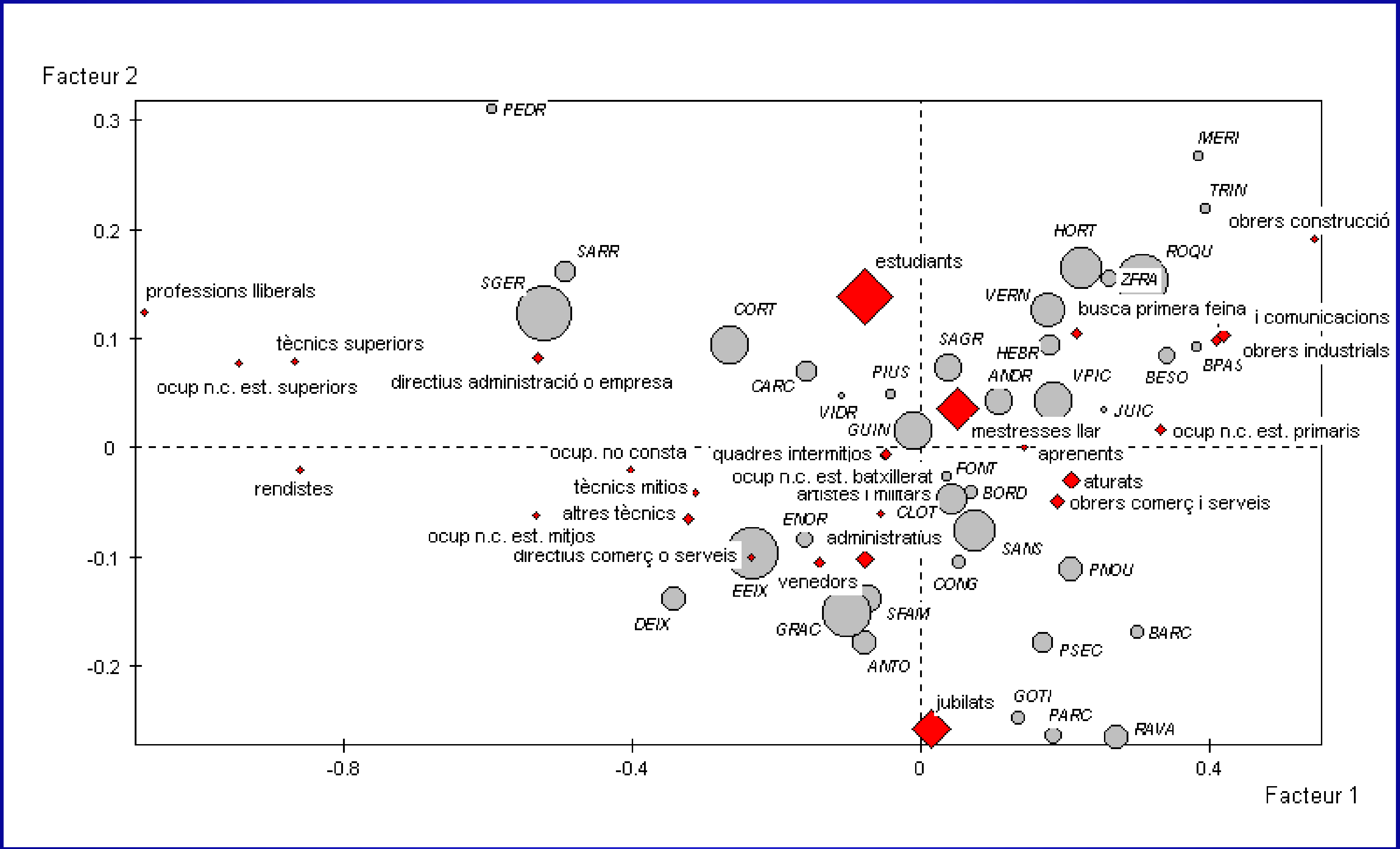


Visualization of the table
BCN Quarters x *Profession of inhabitants*



Visualization of the table
BCN Quarters x *Profession of inhabitants*

Ciutat Meridiana
B-20
C-31
Parc de Montigalà
Torre Baró
BP-1417
Nou Barris
Santa Coloma de Gramenet
Badalona
La Guineueta
C-33
Planes
Horta - Guinardó
Horta
Sant Andreu
C-31
Sant Adrià de Besòs
Av. Meridiana
La Verneda i la Pau
La Teixonera
El Guinardó
El Carmel
B-20
Sarrià - Sant Gervasi
Gràcia
La Sagrada Família
Clot
Sant Martí
Asturies
Sarrià
El Poblenou
B-10
Pedralbes
Av. Diagonal
L'Eixample
A
Les Corts
Barcelona
Les Corts
Sants
N-340
El Poble Sec
-20
L'Hospitalet de Llobregat
Jardins de Joan Maragall
B-10
Barcelona, ES - Génova, IT
Sants - Montjuïc
Puerto de Barcelona
Barcelona - Alcúdia
Porto Torres, IT - Barcellona, ES
Ronda Litoral
Palma de Mallorca - Barcelona
Ciutadella - Barcelona
Mahón - Barcelona
Zone Franca - Port
Mercabarna
Barcelona - San Anto
Carrer A
Carrer Sis

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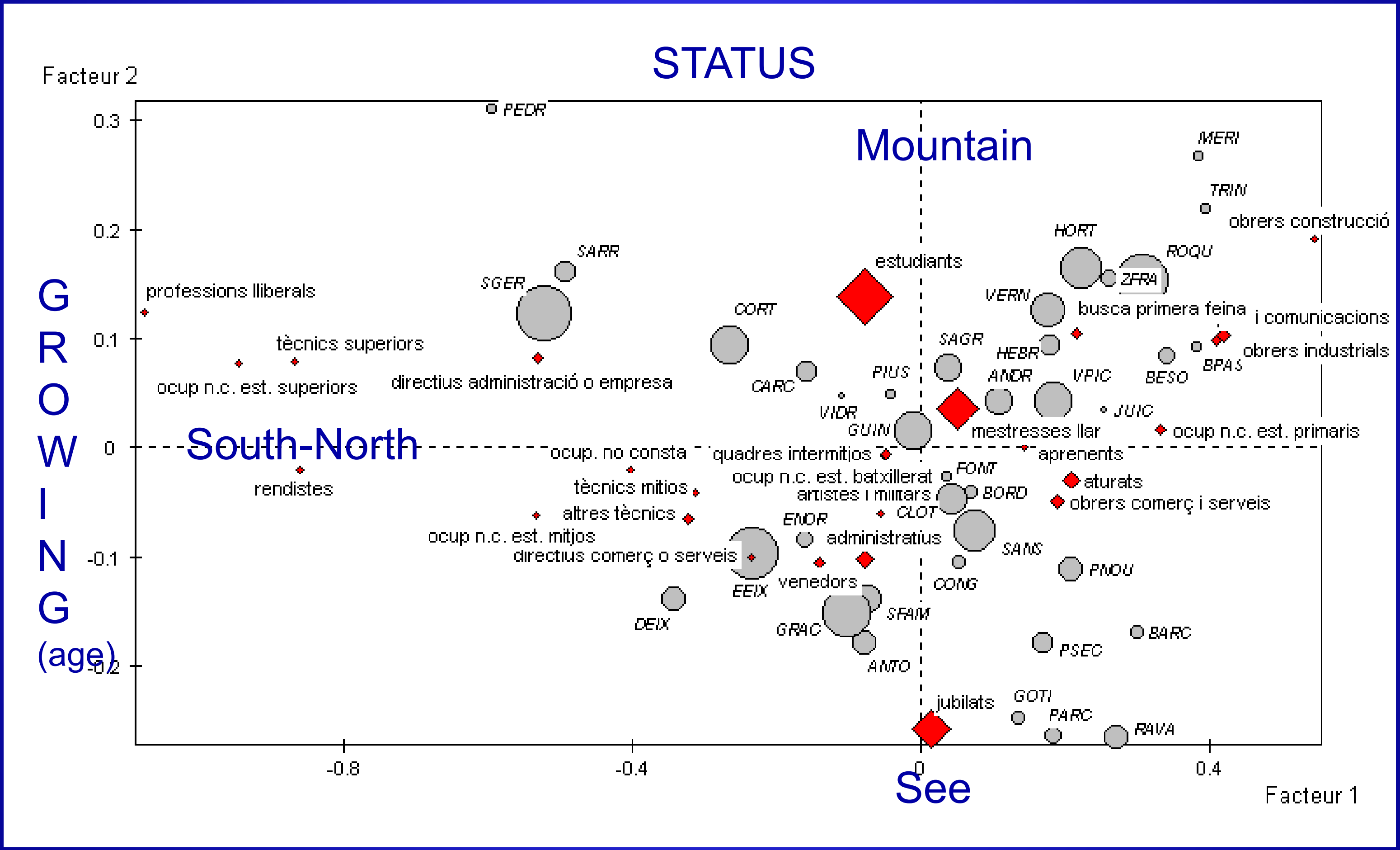
Visualization of the table
BCN Quarters x *Profession of inhabitants*



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Factorial Methods

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- Factors: Linear combinations of original variables

Several uses:

- As an associative data mining method to analyze relationships among variables
  Project variables and modalities and find associations

- As a preprocessing method for elicitation of latent variables
  Project active and illustrative variables/individuals on first/second factorial plan and interpret factors (find latent variables)

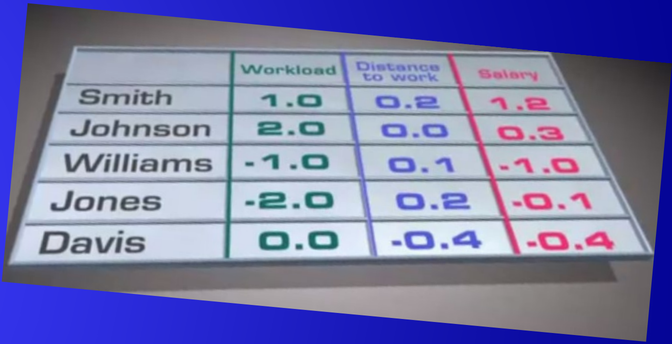
-As a preprocessing method for multidimensionality reduction

Factorial Methods

Data	Factorial Method
Continuous variables	Principal Component Analysis PCA
Contingency table	(Simple) Correspondence Analysis CA
Categorical variables	Multiple Correspondence Analysis MCA

Factorial Methods

- Principal Components Analysis
  - Only numerical variables
  - Find the most informative projection planes (factorial planes, maximize projected inertia)



	Workload	Distance to work	Salary
Smith	1.0	0.2	1.2
Johnson	2.0	0.0	0.3
Williams	-1.0	0.1	-1.0
Jones	-2.0	0.2	-0.1
Davis	0.0	-0.4	-0.4

Given <X,M,D>

- A data matrix X (nxp) centered
- A matrix of individuals weights D (nxn)
- Assume euclidean metrics to compare individuals (M= $\mathbb{I}_p$)

Si les dades estan centrades l'angle entre dues variables projectades coincideix amb la correlació entre elles

Matrix $M^{1/2} X'DXM^{1/2}$

- Product of data with the two metrics
- Simetric,
- Semidefinite
- Catches relationships and opositions of data

Factorial Methods

Given triplet <X,M,D>, diagonalize $M^{1/2}X'DXM^{1/2}$

Data	Factorial Method	X	M	D
Continuous variables	PCA	Centered data matrix	$\mathbb{I}_p$	$\mathbb{I}_n$
Contingency table (n_{ij})	CA	$F=(n_{ij}/n_i)$	$\mathrm{diag}(1/f_j)$	$\mathrm{diag}\,(f_i)$
		$G=(n_{ij}/n_j)$	$\mathrm{diag}(1/f_i)$	$\mathrm{diag}\,(f_j)$
Categorical variables	MCA	$F=(f_{ij}/(f_i/\sqrt{f_j}))$	$\mathbb{I}_p$	$\mathrm{diag}\,(f_i)$
		Burt table	$\mathbb{I}_{n+p}$	$\mathrm{diag}(n_{ij})$

Factorial Methods

- Principal Components Analysis

$M^{½}X'DXM^{½}$ catches well the data structures

Rang($M^{½}X'DXM^{½}$) =r, r=rang(X) r positive vaps and p-r null vaps

Trace($M^{½}X'DXM^{½}$)=$\sum_{\alpha=1}^{r} \lambda_{\alpha}$ (λ_{α}, the r non null vaps)

M= $\mathbb{I}_p$: $M^{½}X'DXM^{½}$ = X'DX

X centered and D diagonal : X'DX= Cov(X)

X standardized and D diagonal : X'DX= Corr(X)

(preferred, big variabilities do not dominate analysis)

Build variances and covariances matrix: X'DX

Diagonalize X'DX (i.e. solving the equation) $X'DXu= \lambda u$

provides eigen values λ_{α} and

eigenvectors $u_{\alpha} = (u_{\alpha 1} \dots u_{\alpha p})$

Factorial Methods

- Principal Components Analysis

Diagonalize X'DX (i.e. solving the equation) $X'DXu = \lambda u$ (1)

det(X'DX-λ)=0 (find roots of characteristic polynomial)

provides eigen values λ_α *(α =1:r, r=rang(X))*

substituting in (1) provides eigenvectors $u_\alpha = (u_{\alpha 1} \dots u_{\alpha p})$

$u^{-1}X'DXu = \lambda$ *is a diagonal matrix*

(X'DX becomes diagonal when pre/post multiplied by u)

$u^{-1}=u'$ *in orthonormal basis:* $u'X'DXu = \lambda$

X'DX decompose in a product by a diagonal matrix $X'DX = u\lambda u'$

$$X'DX = u\lambda u' = u\lambda^{1/2}\lambda^{1/2} u' = u\lambda^{1/2}\mathbb{I}\lambda^{1/2} u' = u\lambda^{1/2} u'u\, \lambda^{1/2} u' = A^{1/2} A^{1/2}$$

X'DX decompose in a product of something by itself (A square root)

Trace(X'DX)=Trace(λ) (property of diagonalization)

Factorial Methods

- Given <X,M,D>

Diagonalize correlations matrix (with normalized data X'DX)

Get r eigen values λ_α and sort decreasingly

$$\{\lambda_\alpha\}_{\alpha=1:r} \quad \lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \ldots \geq \lambda_r$$

Corresponding eigenvectors $u_\alpha = (u_{\alpha 1} \ldots u_{\alpha p})$

$$|u_\alpha|=1$$

$$u_\alpha u_{\alpha'}=0$$

$\{u_\alpha\}_{\alpha=1:r}$ *orthonormal base for individuals*

The subspace generated by $\{u_\alpha\}_{\alpha=1:r}$ is the same as the subspace generated by the rows of X

Factorial Methods

- Given <X,M,D>

Diagonalize Correlations matrix X'DX

Get r eigen values λ_α *and sort decreasingly*

$\{\lambda_\alpha\}_{\alpha=1:r}$ $\quad \lambda_1 \geq \lambda_2 \geq \lambda_3 \geq_{.....} \geq \lambda_r$

Corresponding eigenvectors $u_\alpha = (u_{\alpha 1} .... u_{\alpha p})$

for M= $\mathbb{I}_p$: $u^*_\alpha = u_\alpha$; *for* M≠ $\mathbb{I}_p$: $u^*_\alpha = M^{-½} u_\alpha$

$\{u^*_\alpha\}_{\alpha=1:r}$ *orthonormal base for individuals*

u^*_α *are the principal factors of X : good rotation directions*

$U^*=([u^*_1]\ [u^*_2] .....\ [u^*_r])$ *is the basis for the projection space*

Factorial Methods

- Given <X,M,D>

In general *Diagonalize* $M^{\frac{1}{2}}X'DXM^{\frac{1}{2}}$

Get r eigen values λ_{α} *and sort decreasingly (vaps are conserved!!!!)*

$$\{\lambda_{\alpha}\}_{\alpha=1:r} \qquad \lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \ldots \geq \lambda_r$$

Corresponding eigenvectors $u^*_{\alpha} = (u^*_{\alpha 1} \ldots u^*_{\alpha p})$

by algebraic properties, u^*_{α} *can be found from* u^*

$$u^*_{\alpha} = M^{-\frac{1}{2}} u_{\alpha}$$

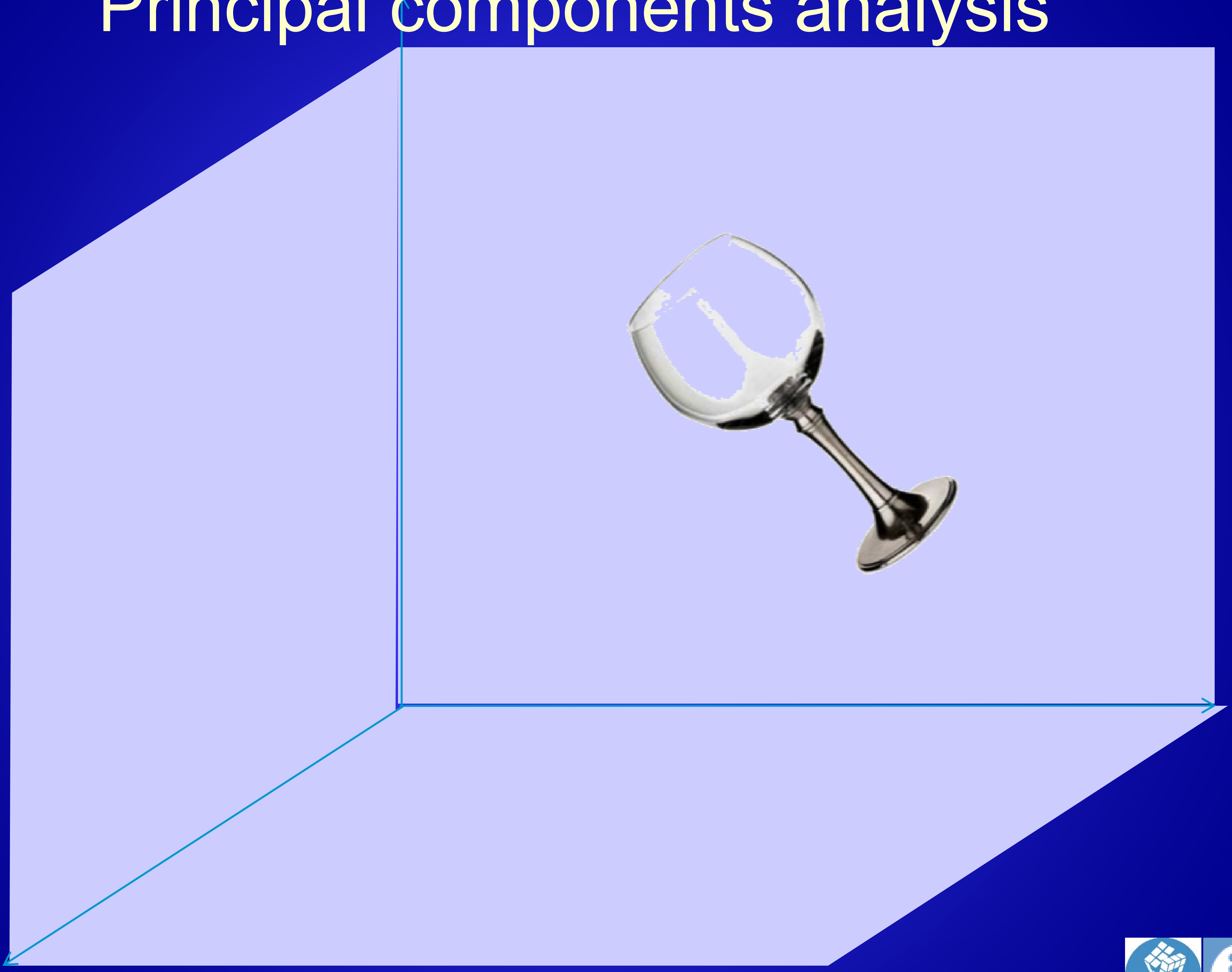
$\{u^*_{\alpha}\}_{\alpha=1:r}$ *orthonormal base for individuals*

$$|u^*_{\alpha}|_M = 1 : \quad u^{*\prime}_{\alpha} M u^*_{\alpha} = u'_{\alpha} M^{-\frac{1}{2}} M M^{-\frac{1}{2}} u_{\alpha} = 1$$

$$u^*_{\alpha} M u^*_{\alpha'} = 0 : \quad u^*_{\alpha} M u^*_{\alpha'} = u'_{\alpha} M^{-\frac{1}{2}} M M^{-\frac{1}{2}} u_{\alpha'} = 0$$

Subspace generated by $\{u^*_{\alpha}\}_{\alpha=1:r}$ = *Subspace generated by X rows*

Principal components analysis



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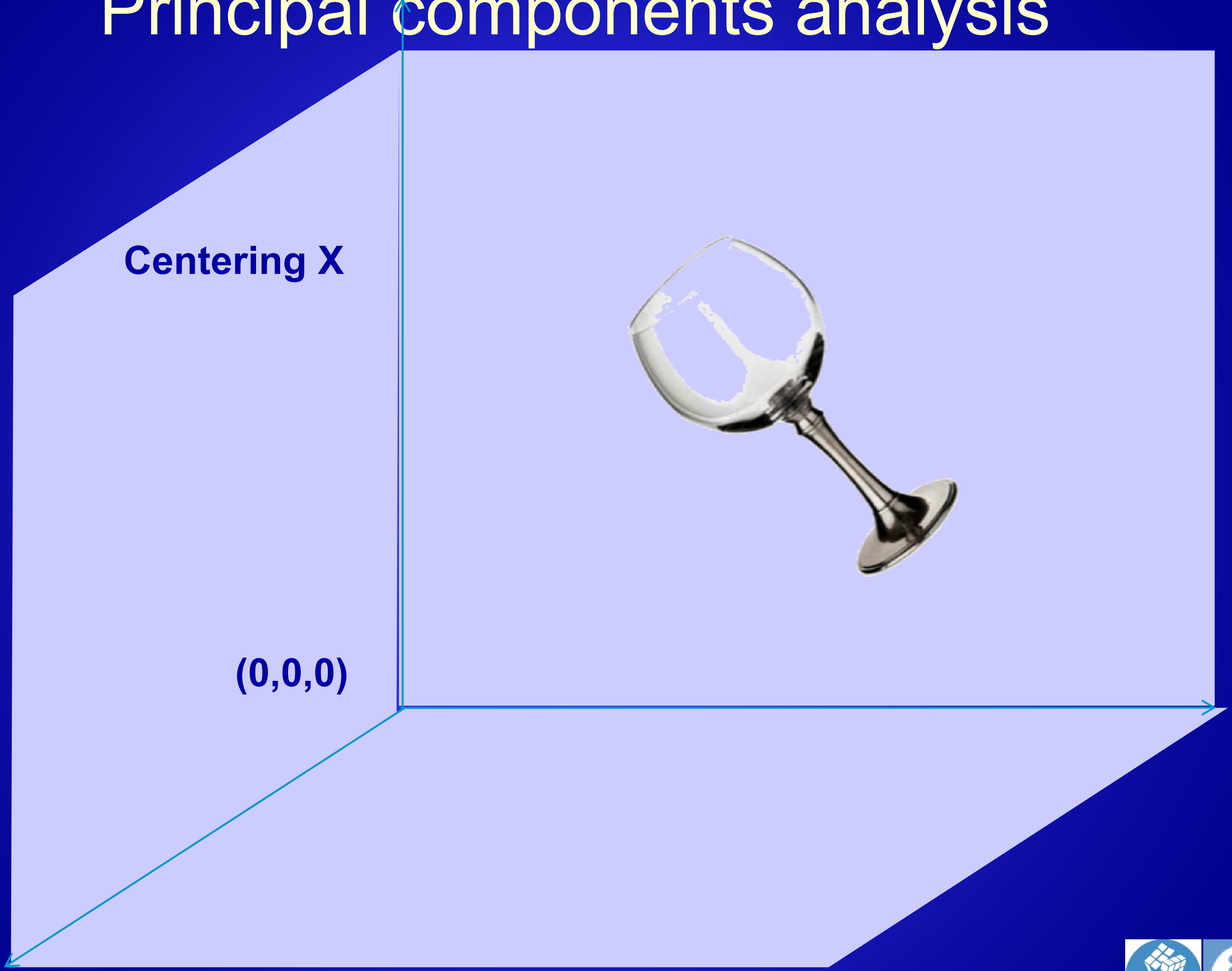
Principal components analysis



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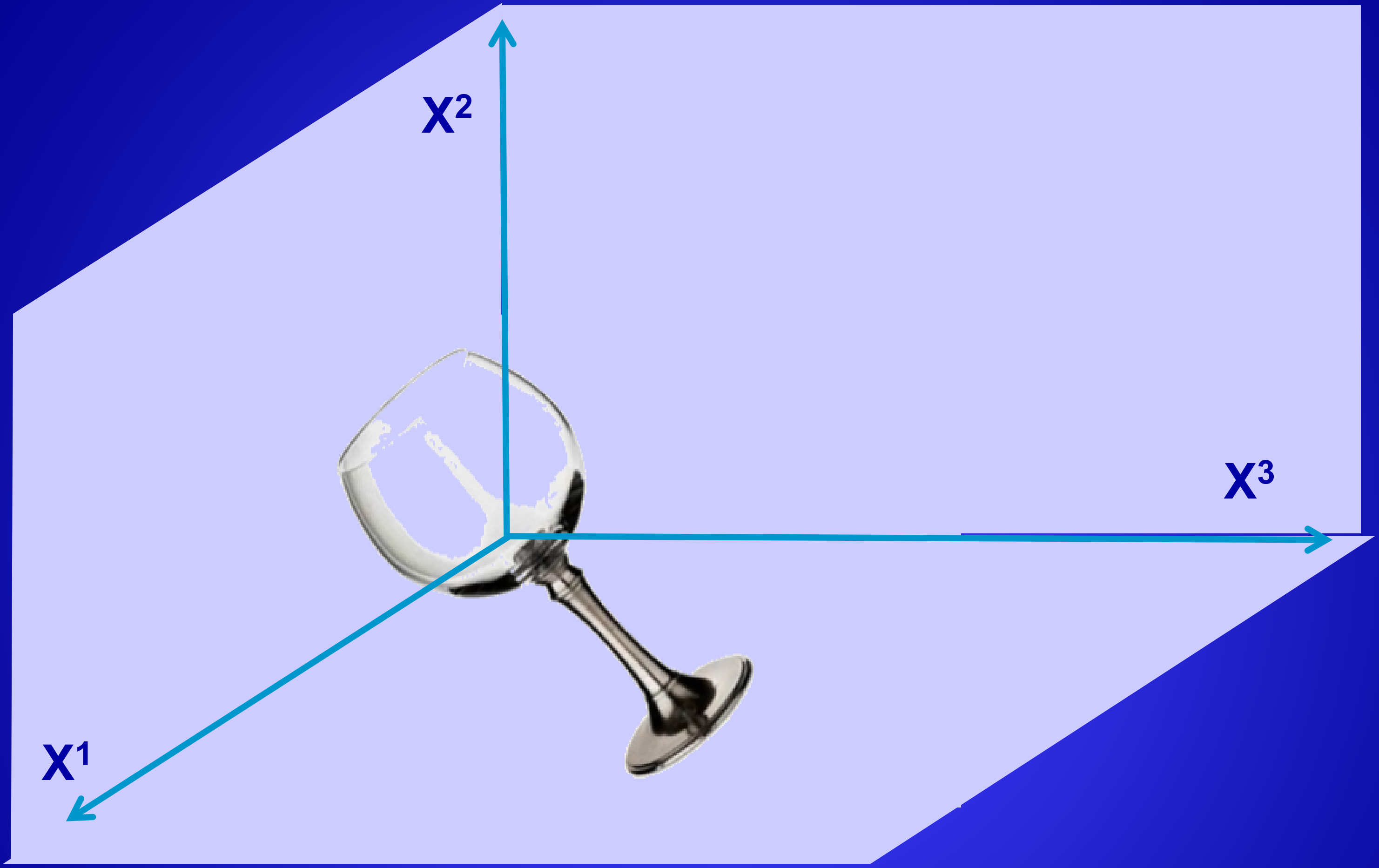

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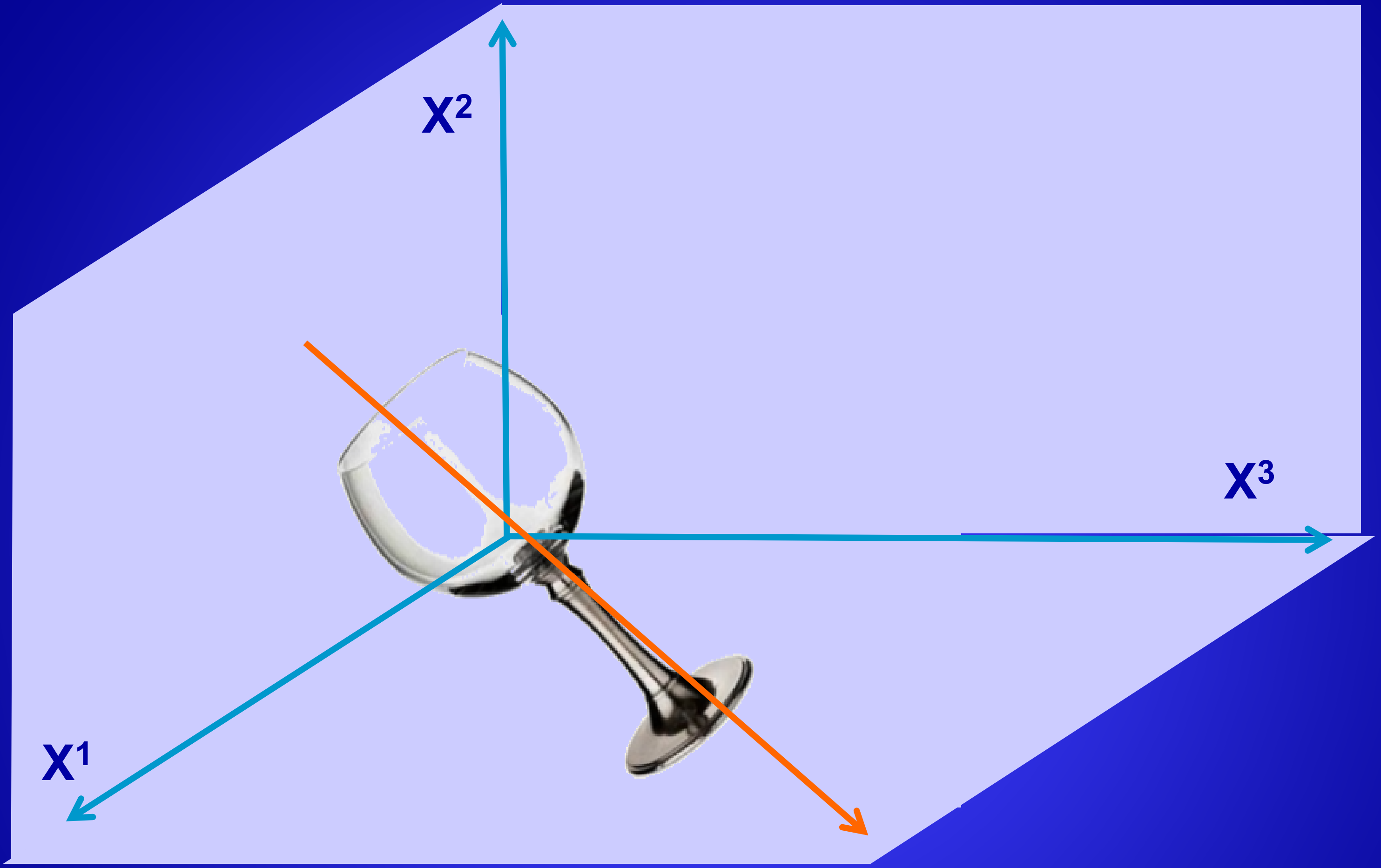

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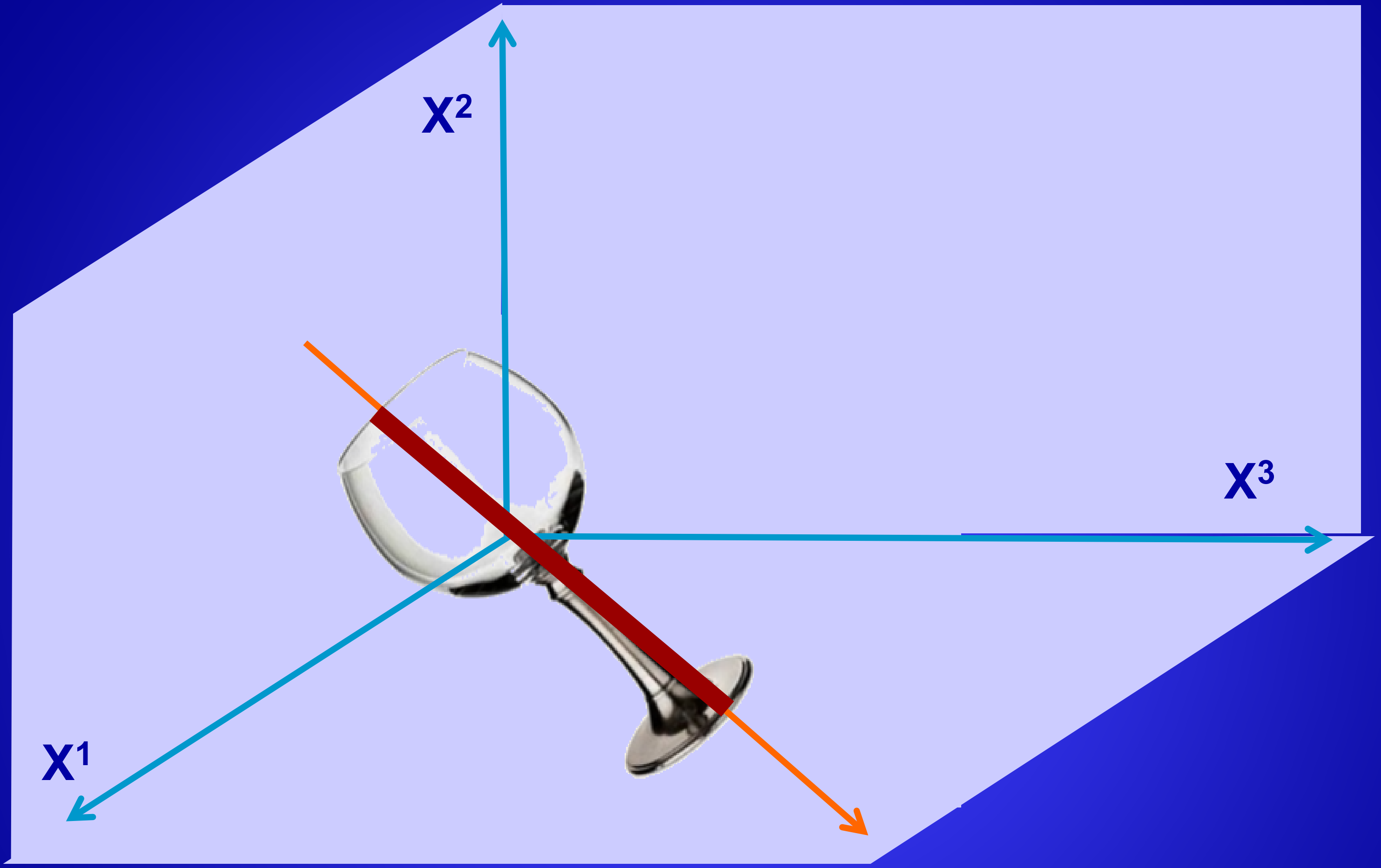

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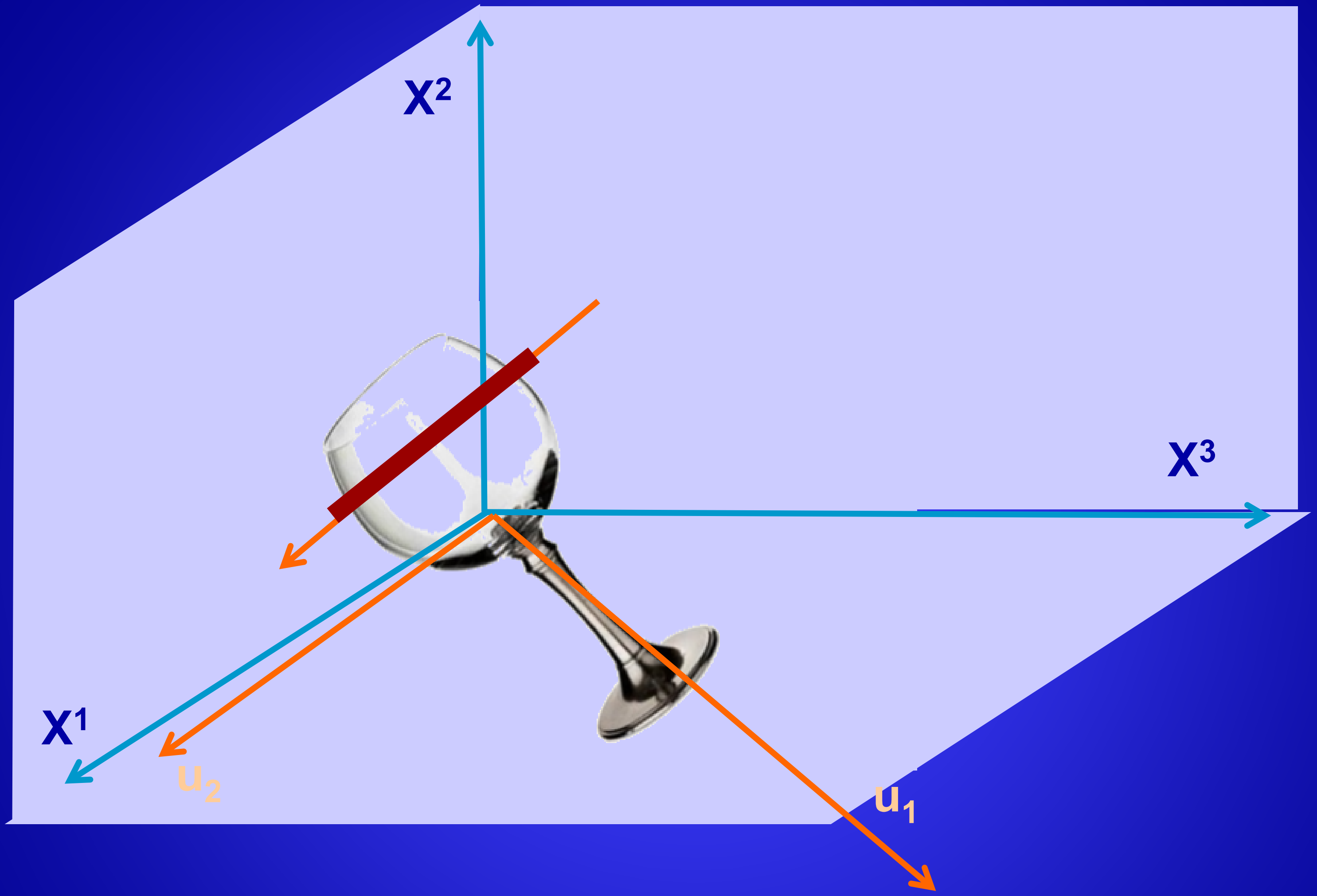

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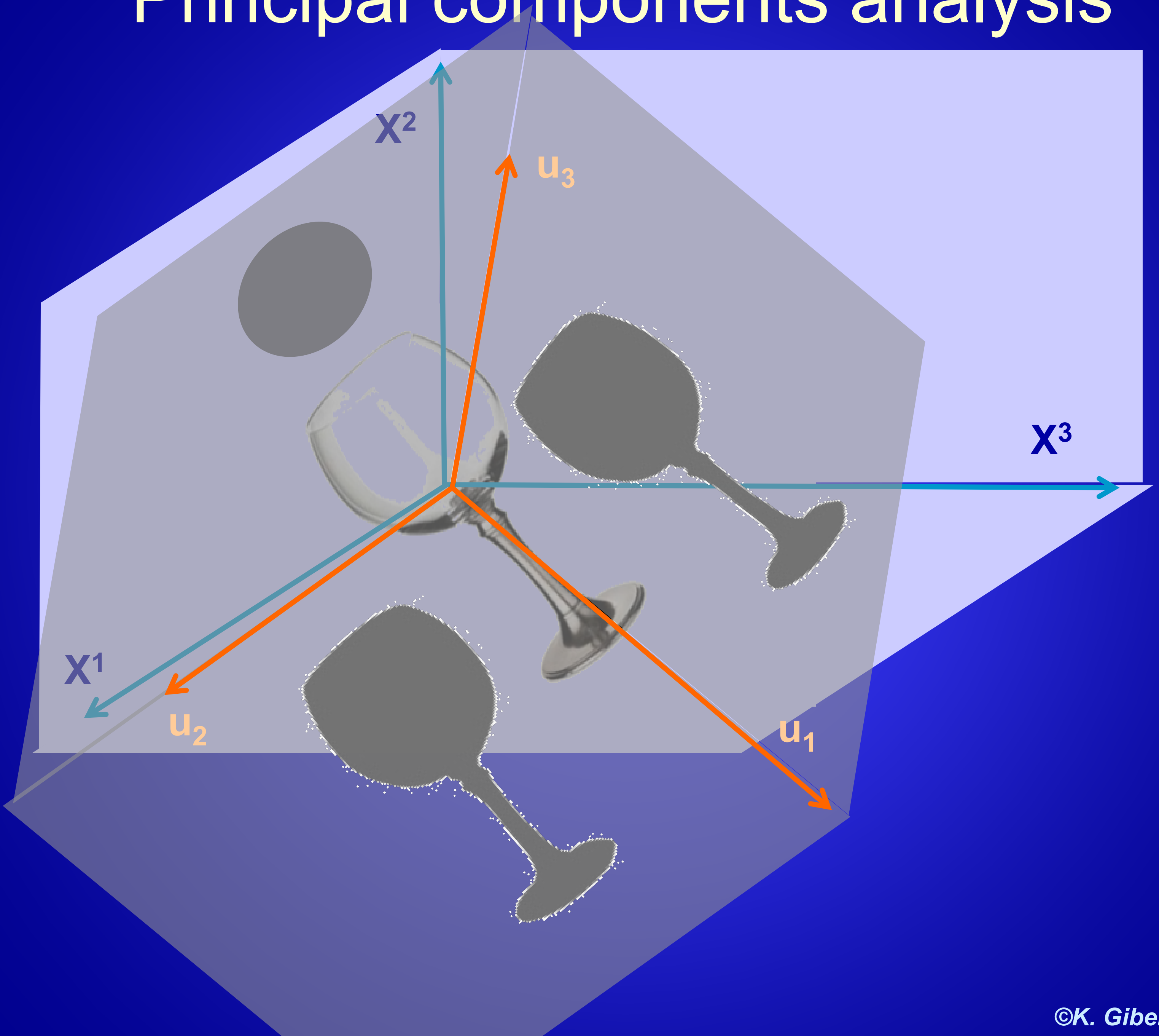

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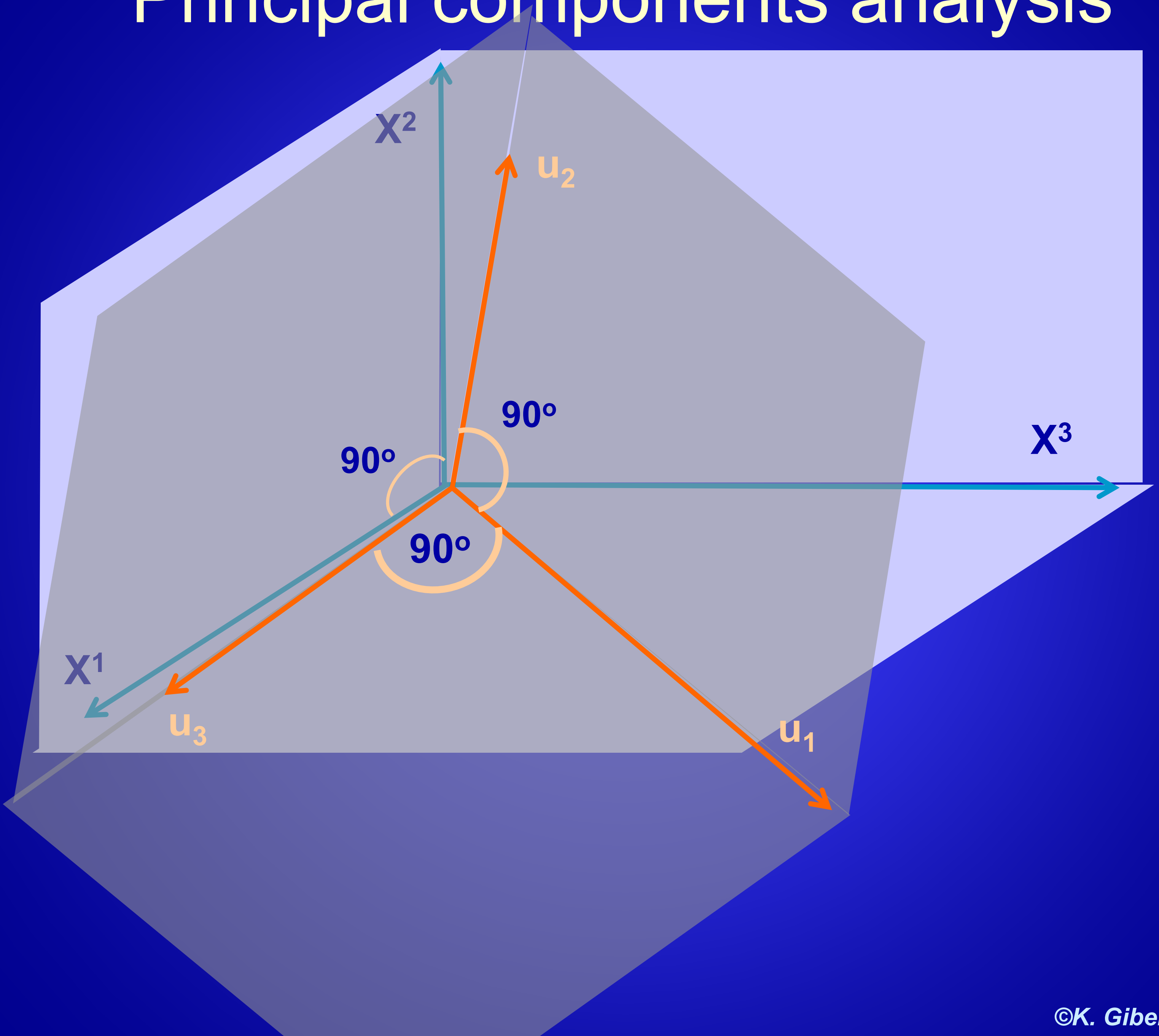

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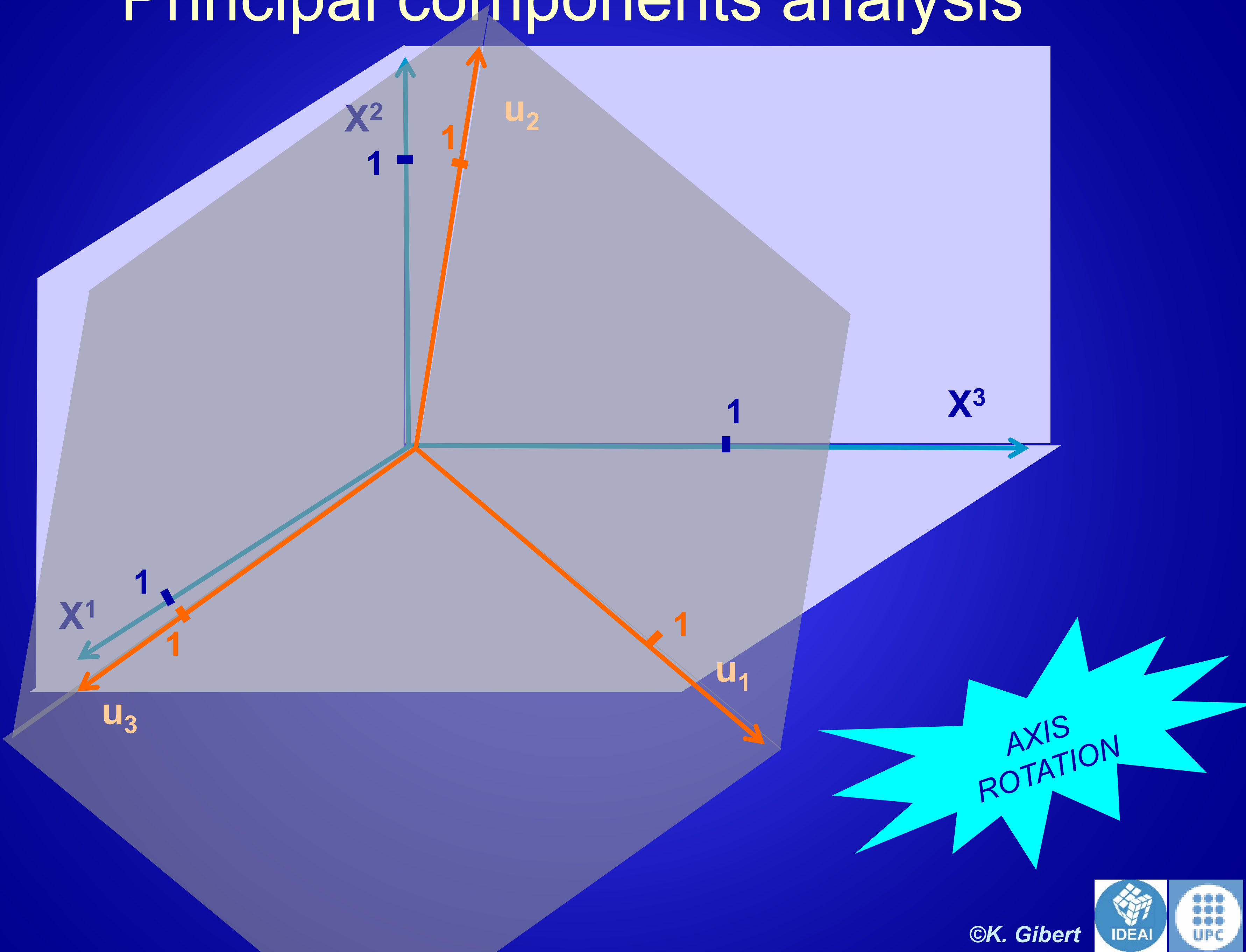

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Factorial Methods

- Given <X,M,D>

Diagonalize Correlations matrix X'DX

Get r eigen values λ_α *and sort decreasingly*

$\{\lambda_\alpha\}_{\alpha=1:r} \quad \lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \ldots\ldots \geq \lambda_r$

Corresponding eigenvectors $u_\alpha = (u_{\alpha 1} \ldots\ldots u_{\alpha p})$

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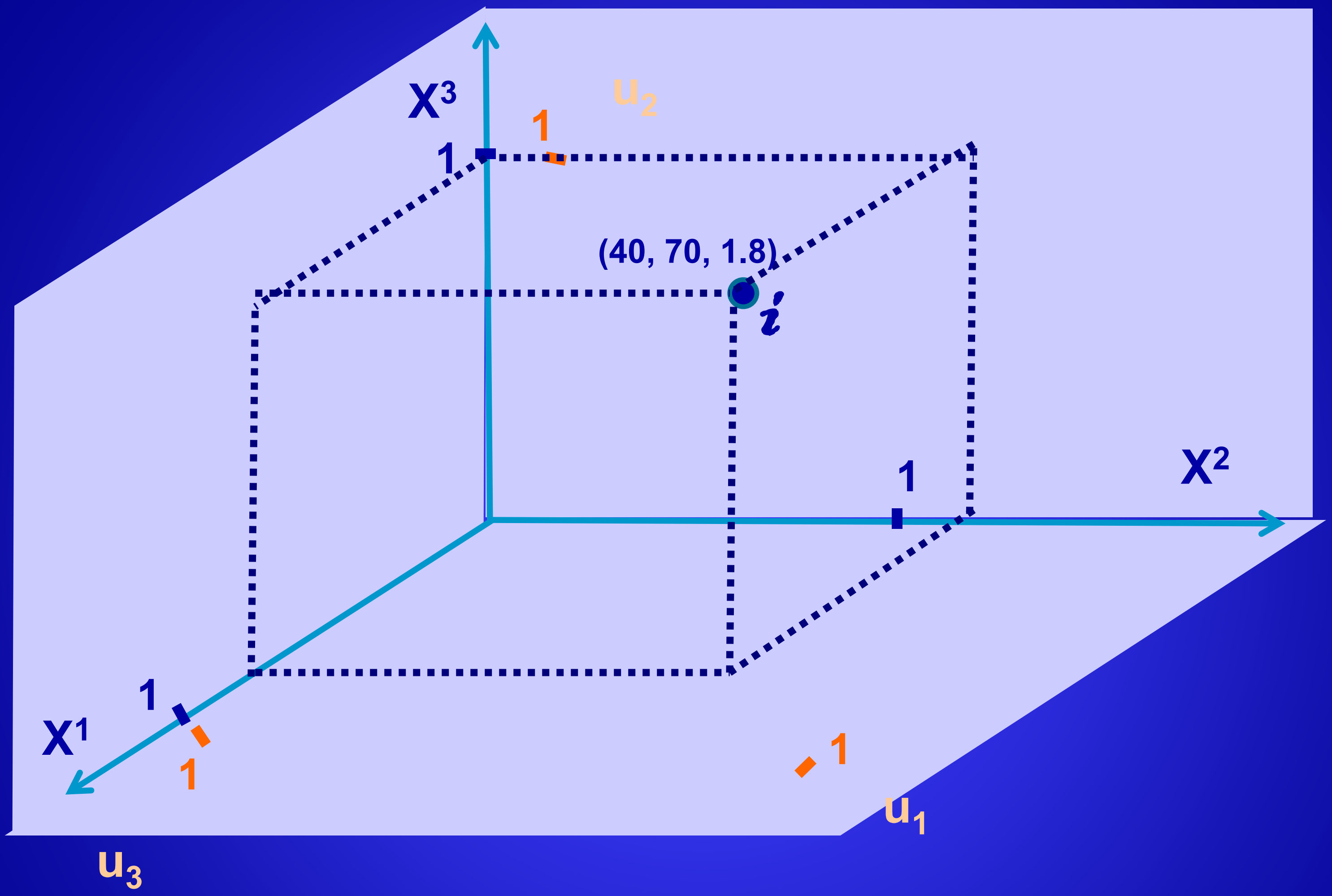
$\{u^*_\alpha\}_{\alpha=1:r}$ *orthonormal base for individuals*

u^*_α *are the principal factors of X : good rotation directions*

$U^*=([u^*_1]\ [u^*_2] \ldots\ldots [u^*_r])$ *is the basis for the projection space*

How is i expressed in rotated space?

Principal components analysis



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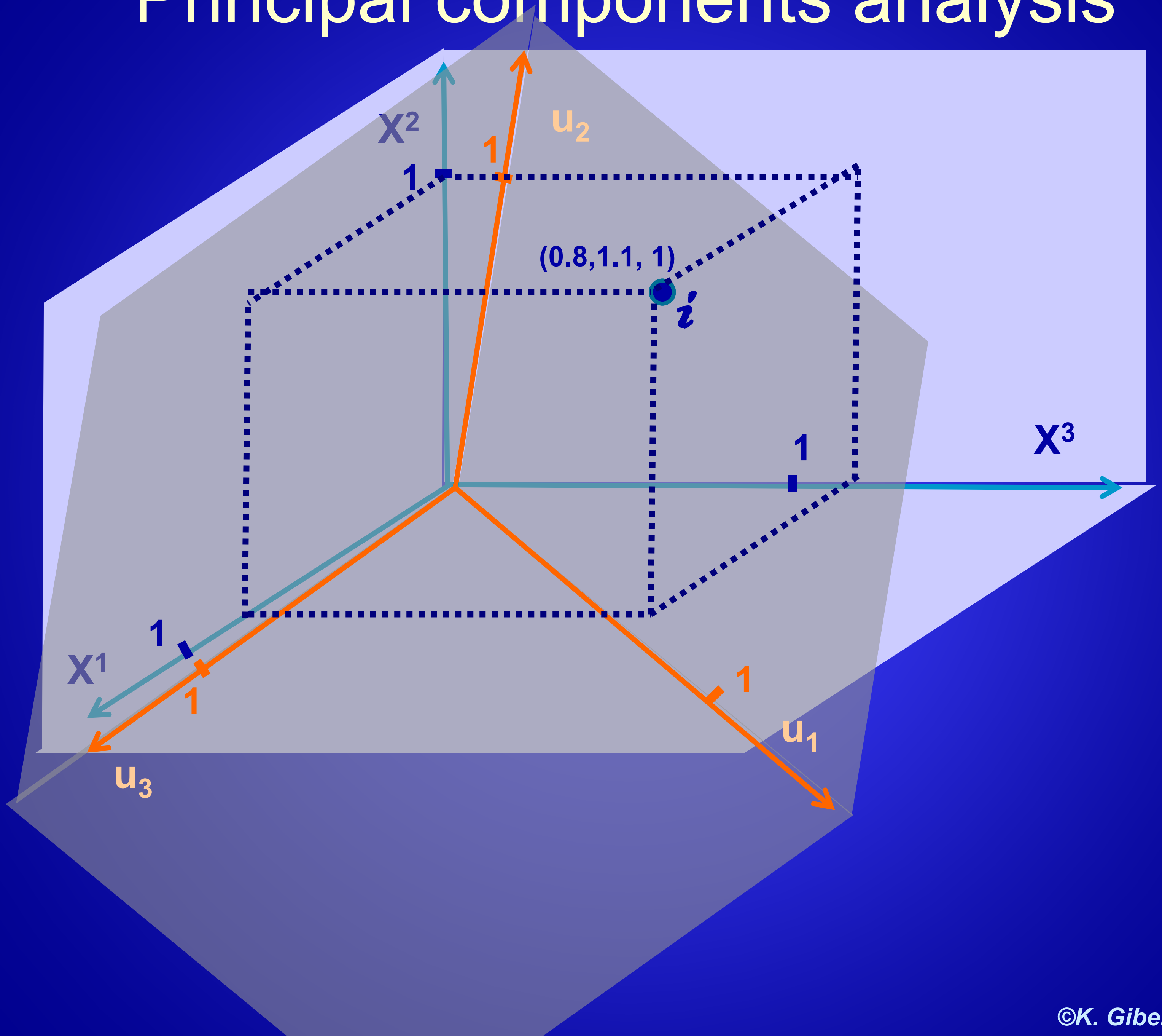

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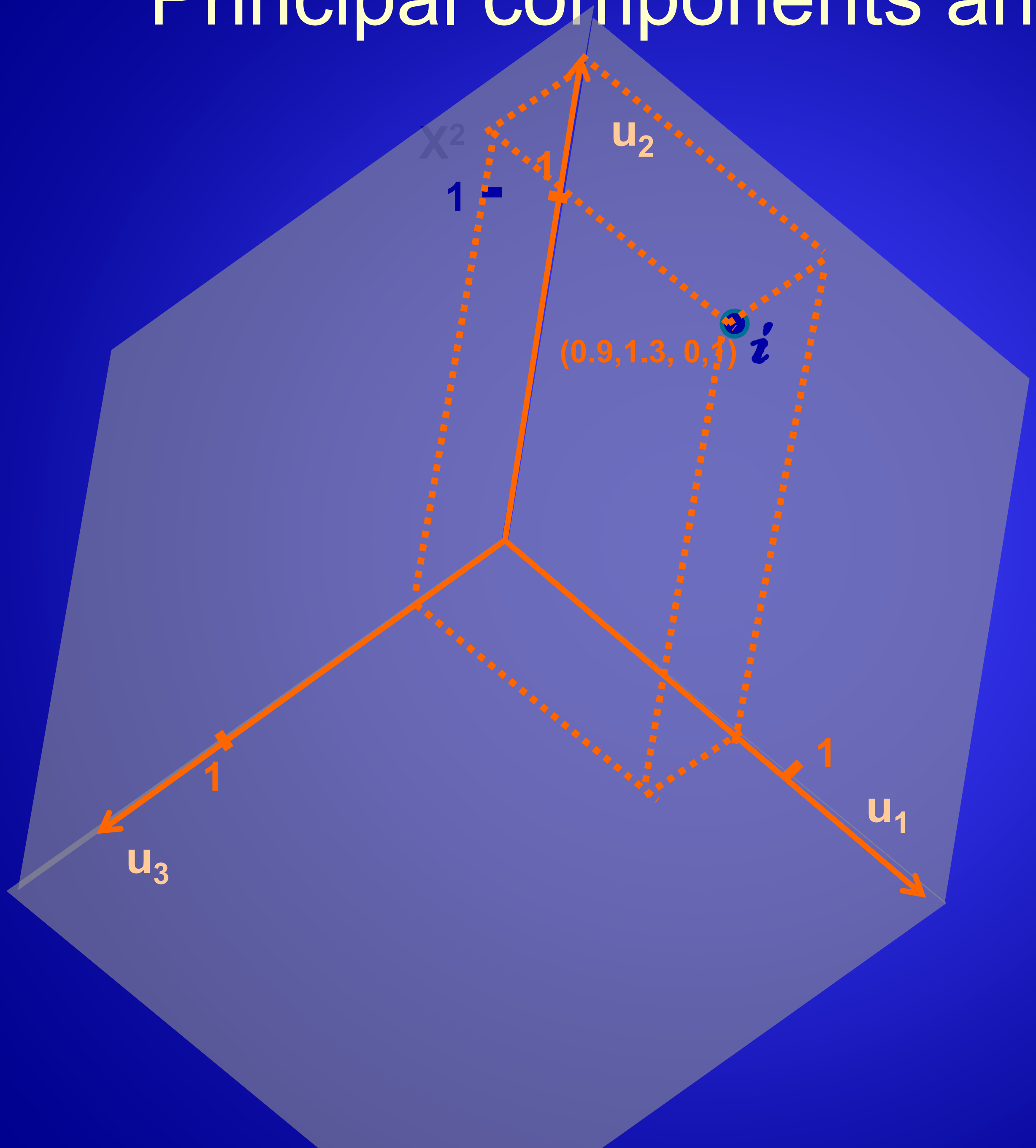

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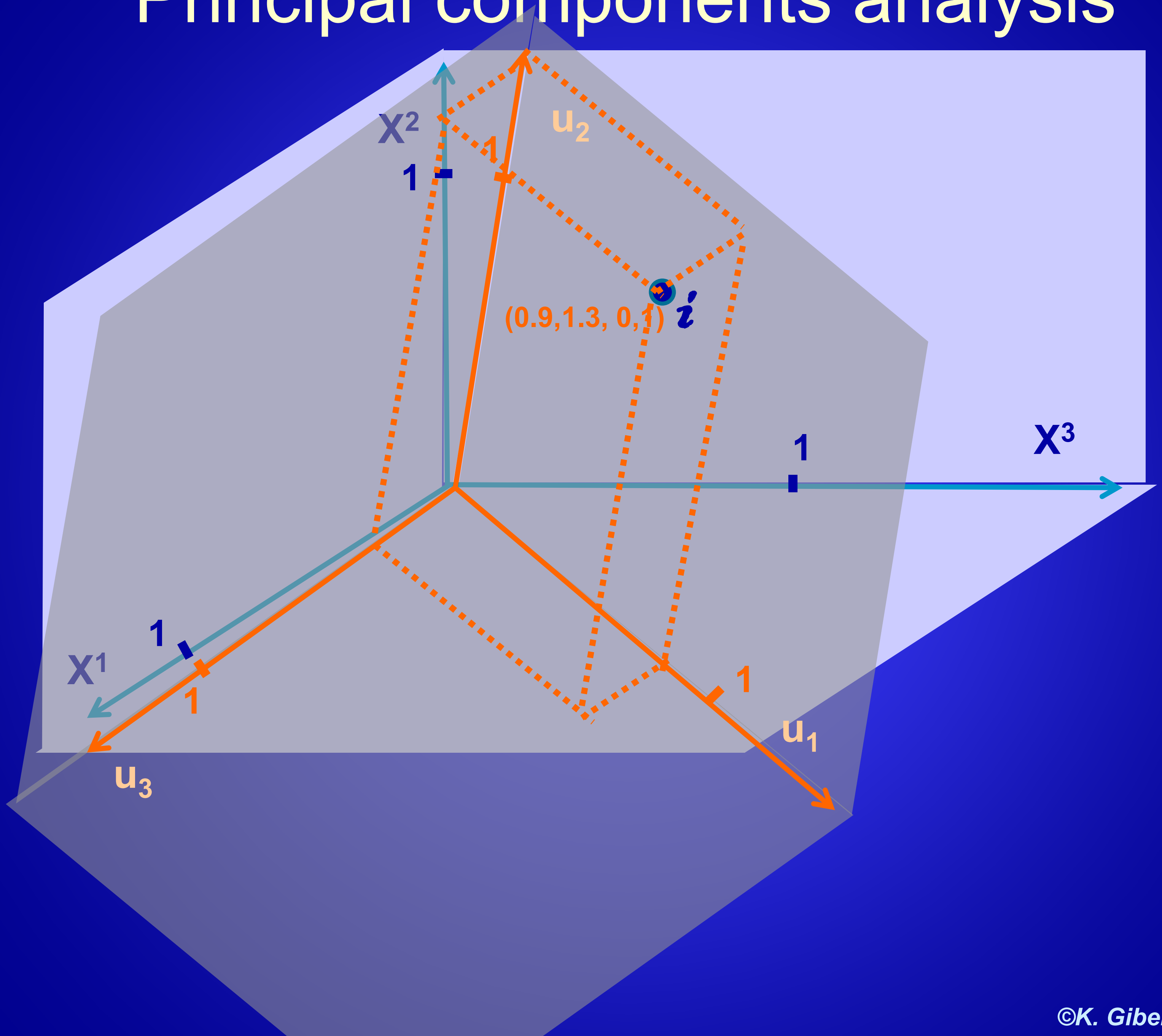

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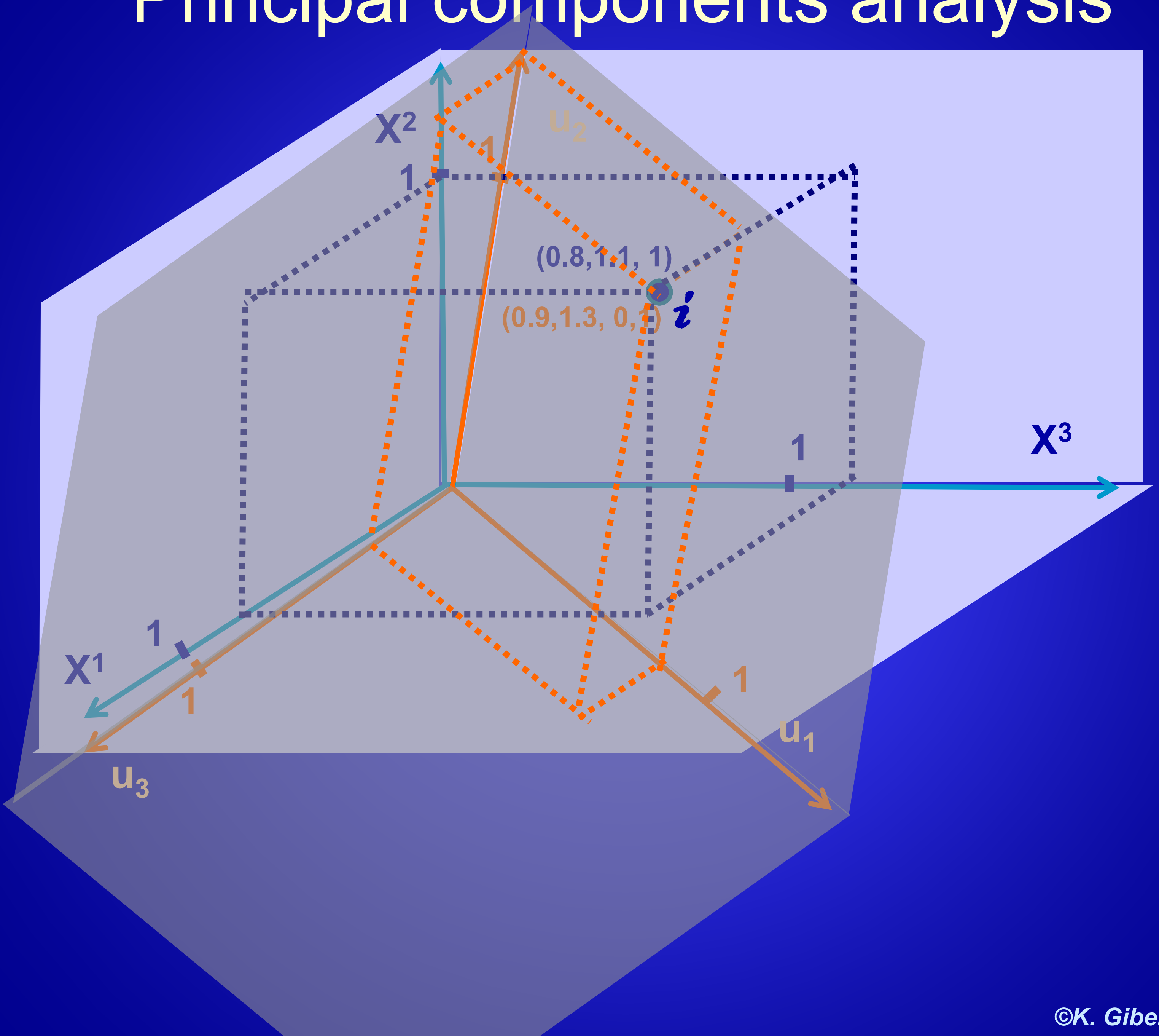
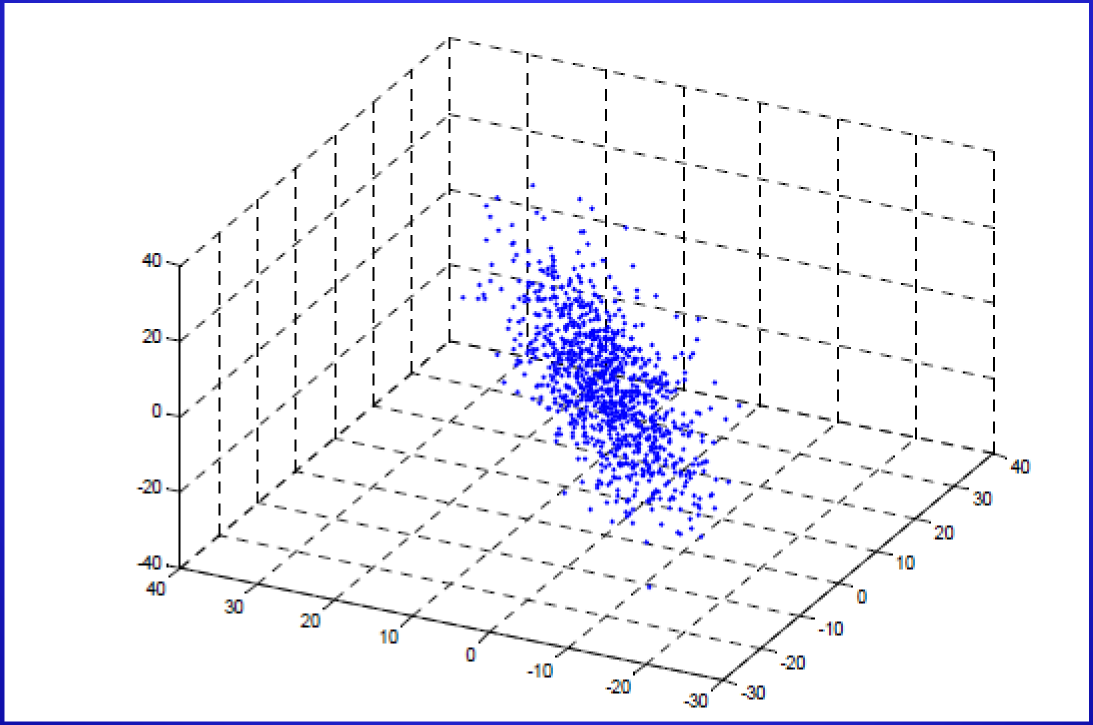


Principal components analysis

- Find the most informative projection planes of data cloud *(factorial planes)*



Factorial Methods

- Given <X,M,D>

Can we find coordinates in rotated space from original ones?

The projection matrix $P = U^*_k U^{*\prime}_k M$

Projection of a single individual: $Pr(i)= U^*_k U^{*\prime}_k M x_i$

Projection of all individuals: $Pr(X)= U^*_k U^{*\prime}_k M X'$

Get a matrix with projections in ROWS: $Pr(X)'= XMU^*_k U^{*\prime}_k$

Projections expressed in original vectorial space

The best possible projection over k dimensions

Factorial Methods

- Given <X,M,D>

Matrix $XMU^*_k U^{*\prime}_k$ provides the best possible k-projection of X

Silver-Smidth norm: $||X||^2_{MD} = \sum_{\alpha=1}^{r} \lambda_\alpha$

Measures variability, information contained in X

Property: $||XMU^*_k U^{*\prime}_k||^2_{MD} = ||X||^2_{MD}$

Any other k-projection of X

- Provides smallest values of Silver-Smidth norm
- Has less variability
- Keeps smallest information from X

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Factorial Methods

- Given <X,M,D>

Diagonalize correlations matrix (with normalized data)

eigenvectors $u_{\alpha} = (u_{\alpha 1} .... u_{\alpha p})$ *(direction of factor α, α =1:p)*

$u_{\alpha p}$: contribution of variable p to the factor α

$(u_1 .... u_k)$ *ortonormal*

eigen values λ_k *(quantity of information converved by factor k)*

(Projected inertia)

$\{\lambda_\alpha\}_{\alpha=1:r}$ $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq ..... \geq \lambda_r$

$\Sigma_{\forall\alpha} \lambda_\alpha$ = Total inertia of X (information in data)

Close objects project close

proximity linked with association

Factorial Methods

- Given <X,M,D>

eigenvectors $u_\alpha = (u_{\alpha 1}.... u_{\alpha p})$ *(direction of factor k)*

$u_{\alpha p}$: contribution of variable p to the factor α

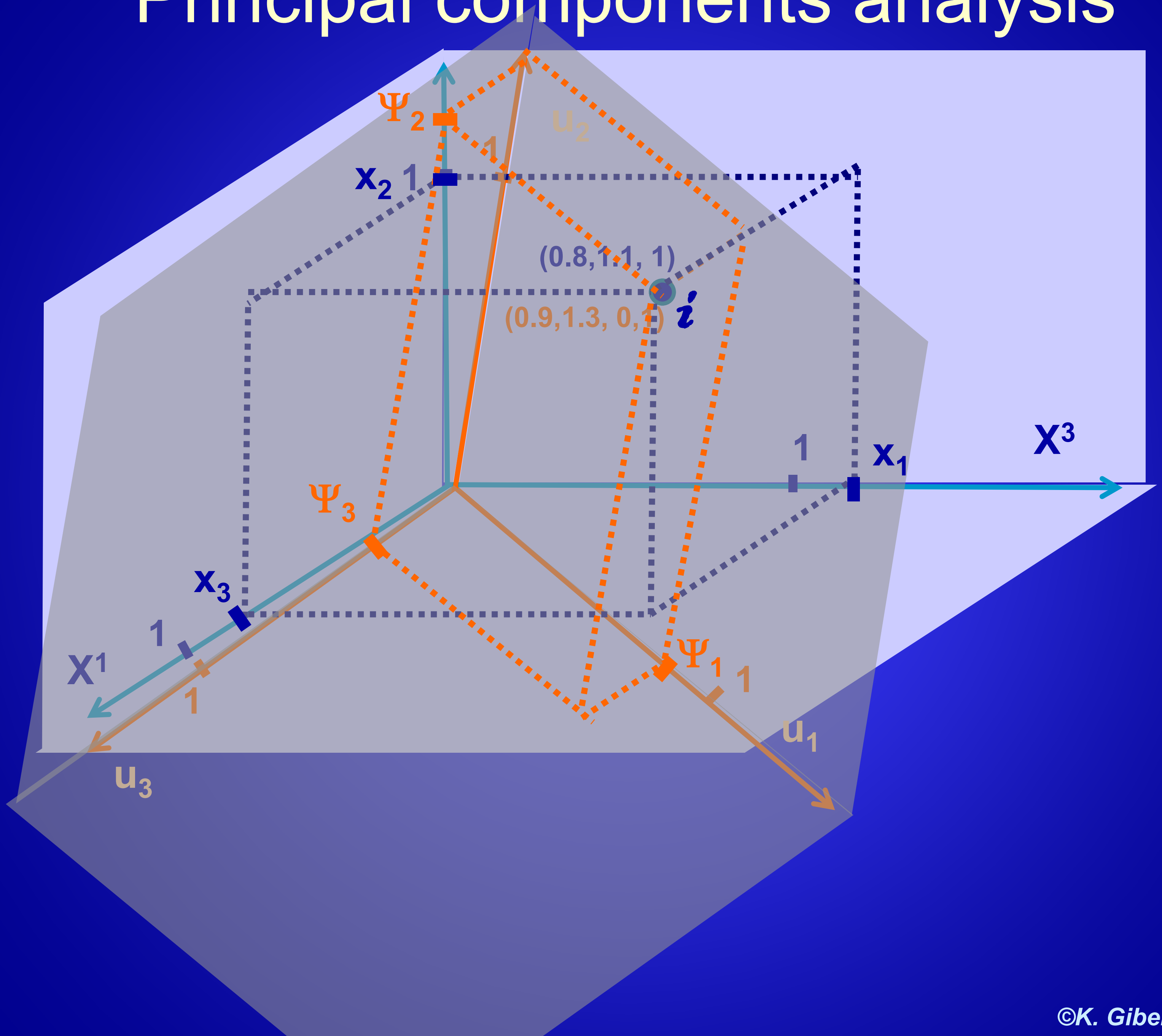
eigen values λ_α *(quantity of information conserved by factor* α*)*

(Projected inertia)

$\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq ..... \geq \lambda_r$

$\Sigma_{\forall\alpha}\ \lambda_\alpha$ = Total inertia of X (information of data)

Principal components analysis



Factorial Methods

- Given <X,M,D>

$$i = (x_{1i}, \dots, x_{pi})$$

Points in projected space: $i = (\Psi_{1i}, \dots, \Psi_{\alpha i}, \dots, \Psi_{ri})$ (often r=p)

$$\Psi_{\alpha i} = x_{1i}u_{\alpha 1} + x_{2i}u_{\alpha 2} + \cdots + x_{pi}u_{\alpha p} \qquad \psi_\alpha = Xu_\alpha$$

Then $\Psi_\alpha^{'} D\Psi_\alpha = \lambda_\alpha$

Illustrative points z also projectable

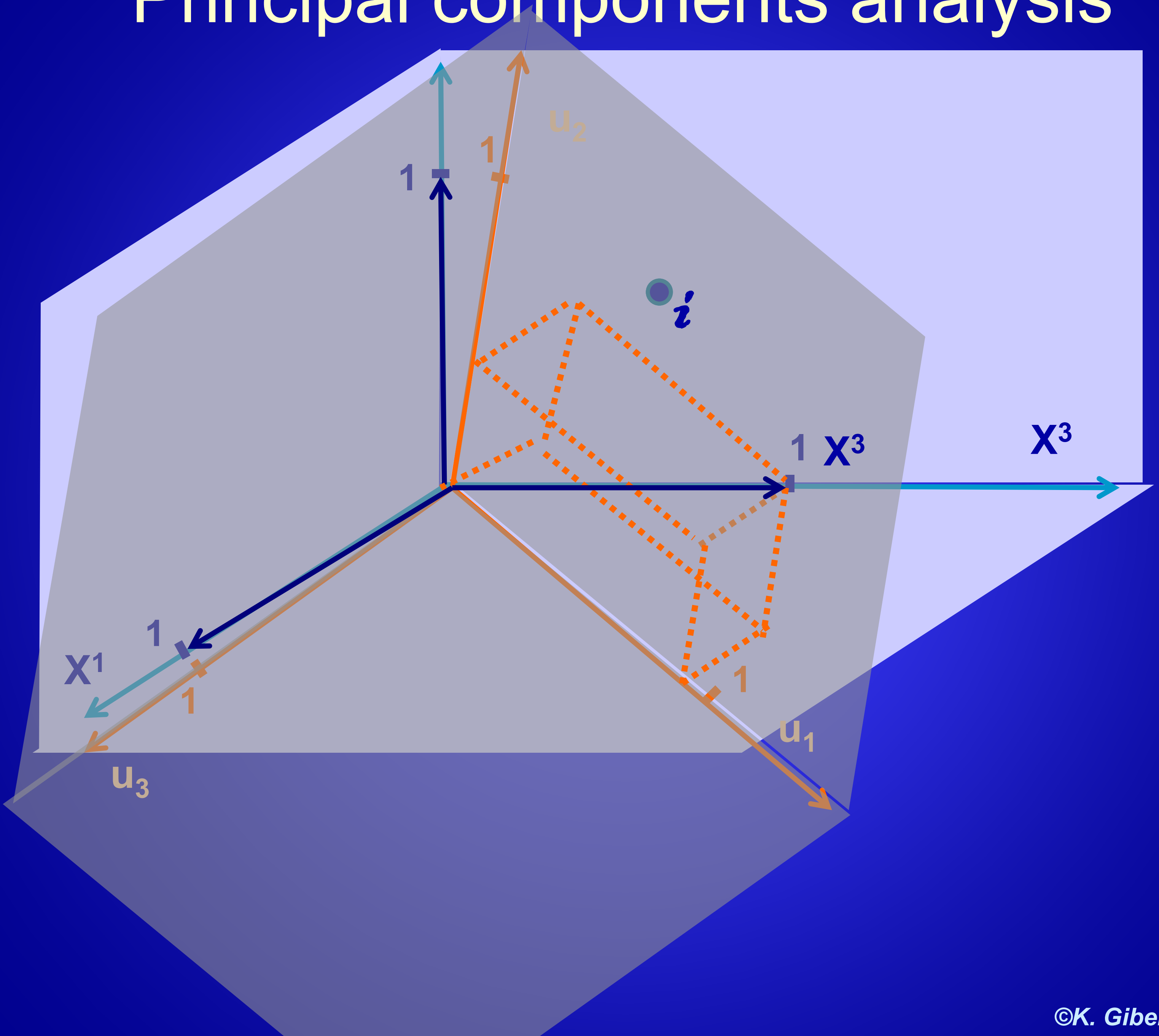
$$\Psi_{\alpha z} = x_{1z}u_{\alpha 1} + x_{2z}u_{\alpha 2} + \cdots + x_{pz}u_{\alpha p}$$

Factors are linear combinations of original variables

Original variables project as VECTORS over factorial space

angle and lenght important

Principal components analysis



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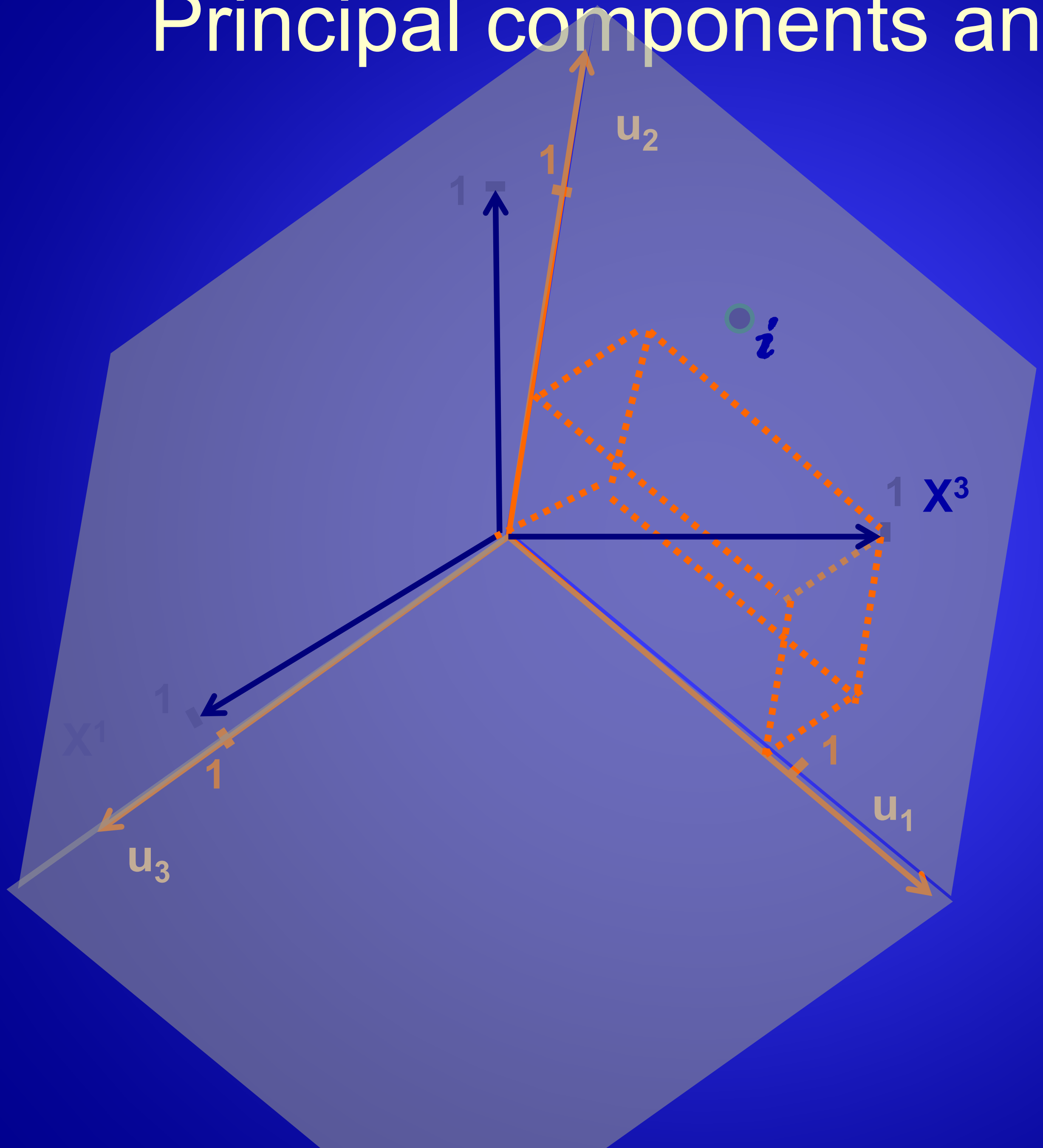
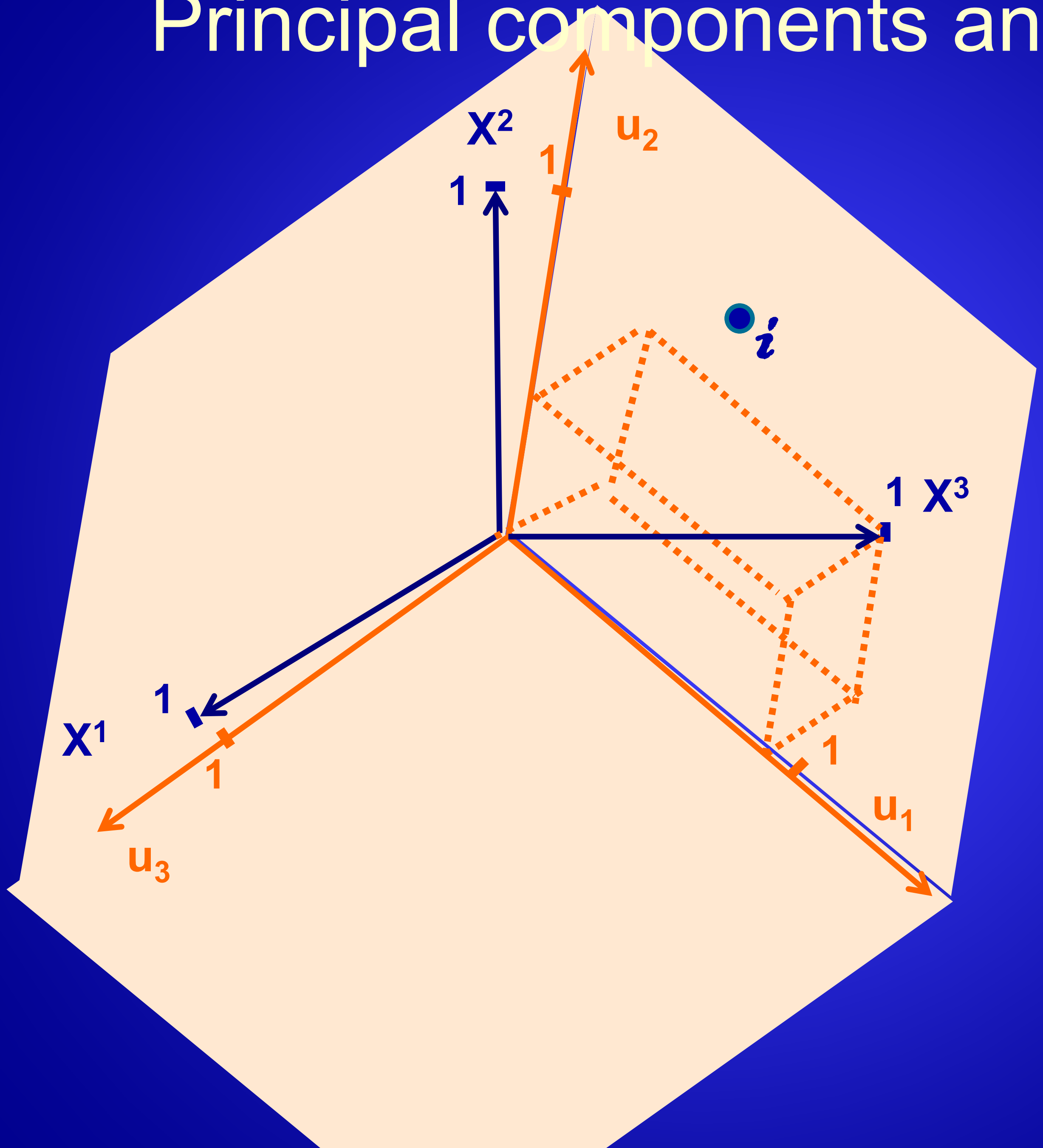
Principal components analysis



Principal components analysis



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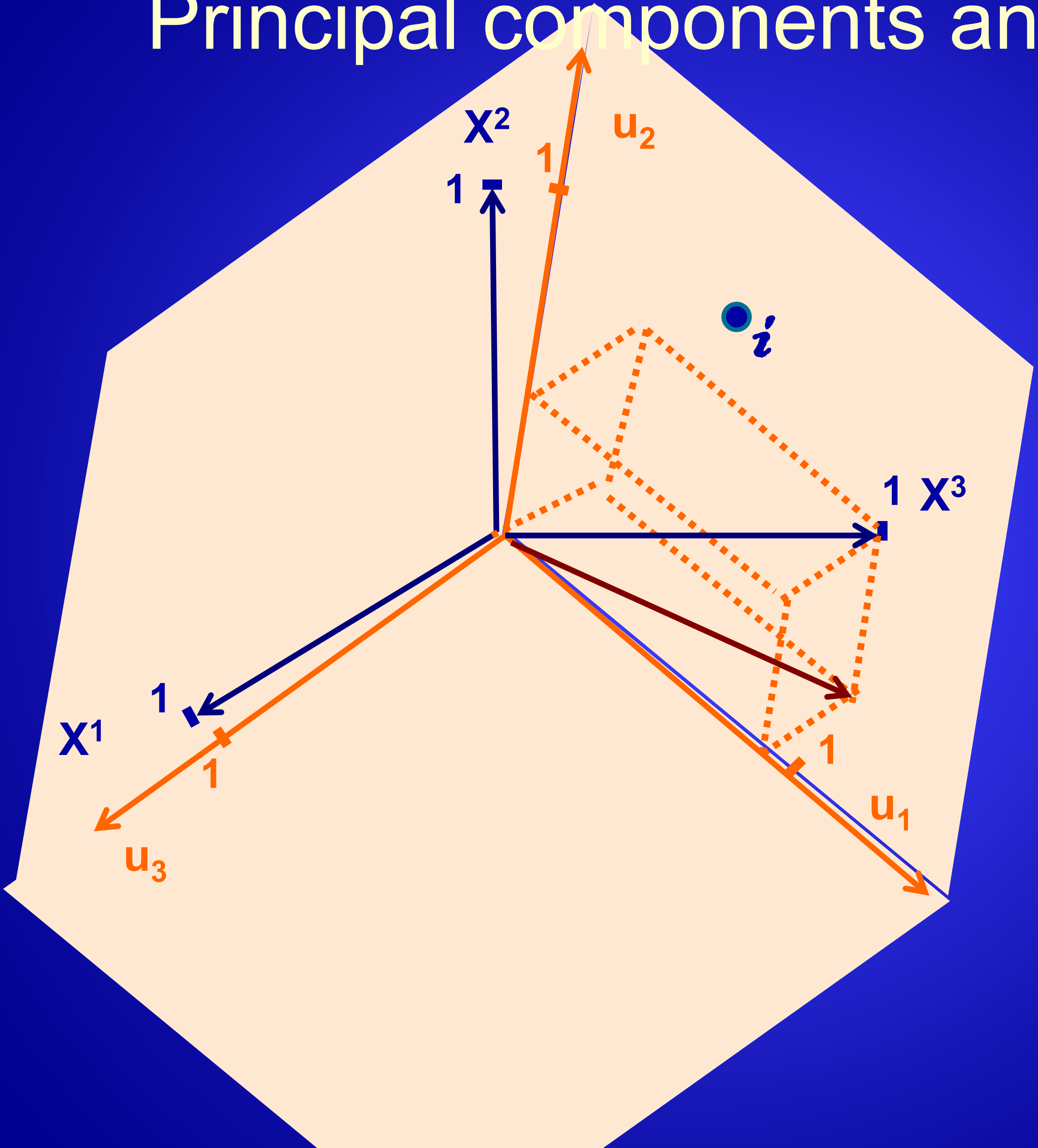
Principal components analysis



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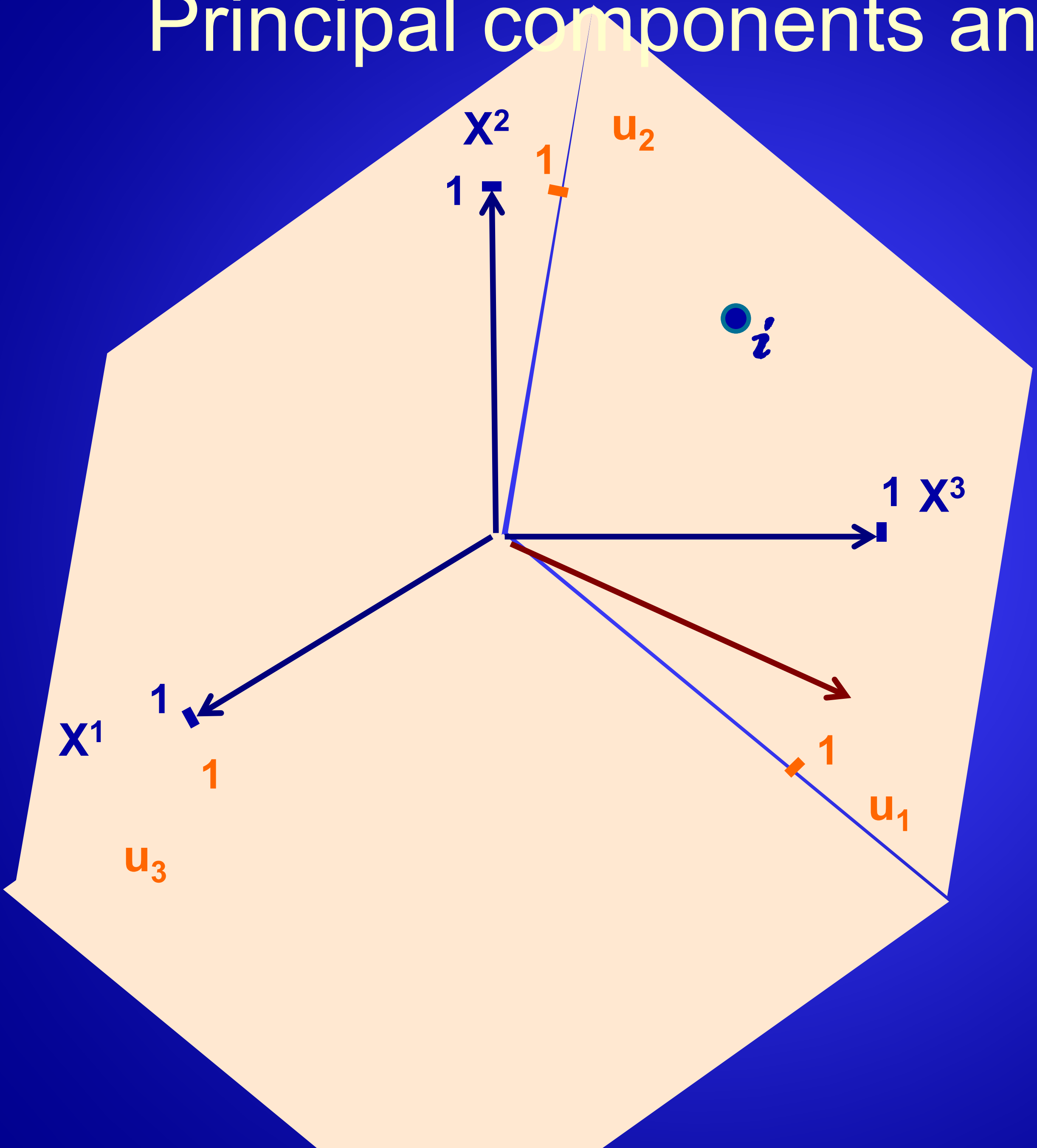
Principal components analysis



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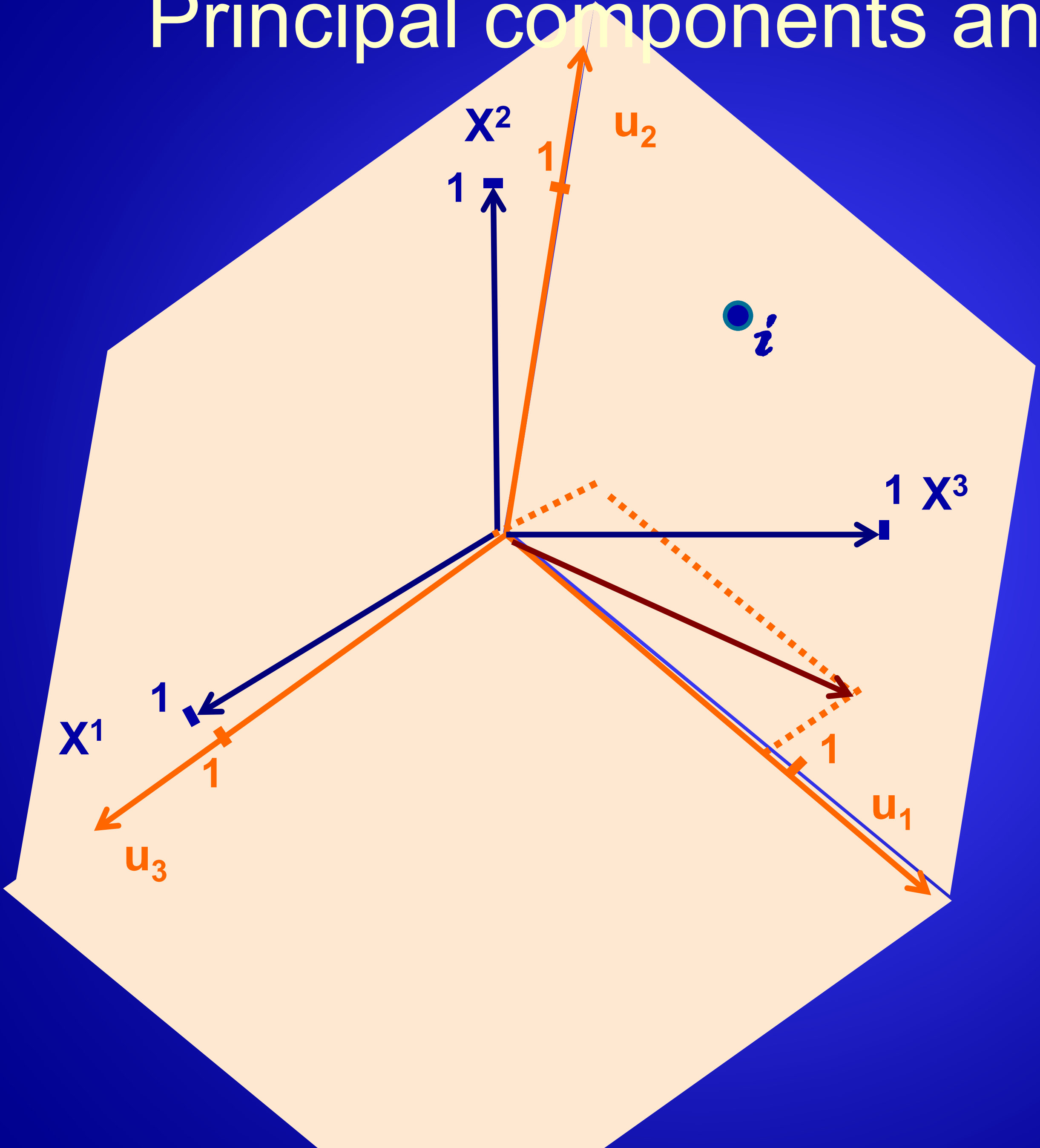
Principal components analysis



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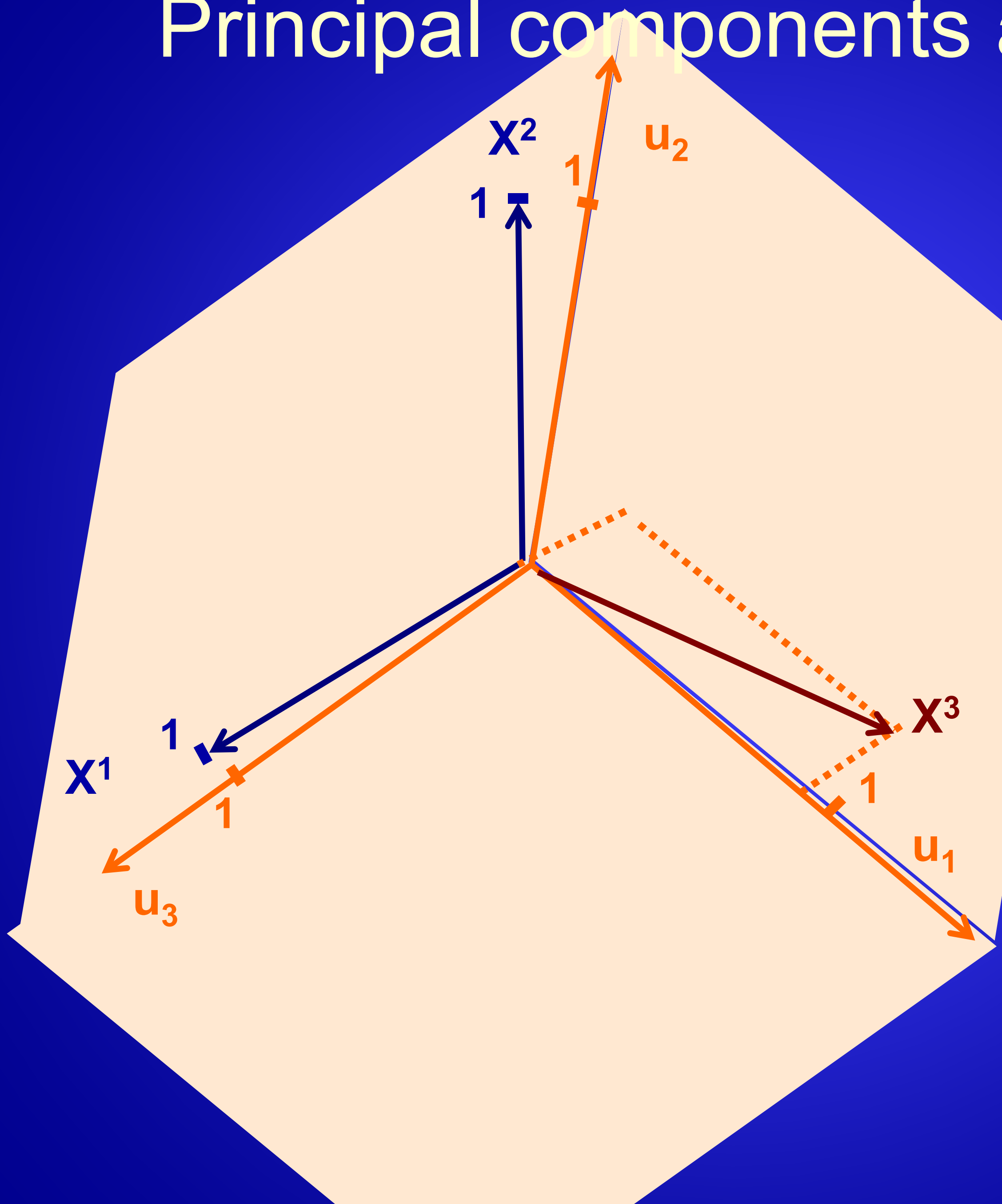
Principal components analysis



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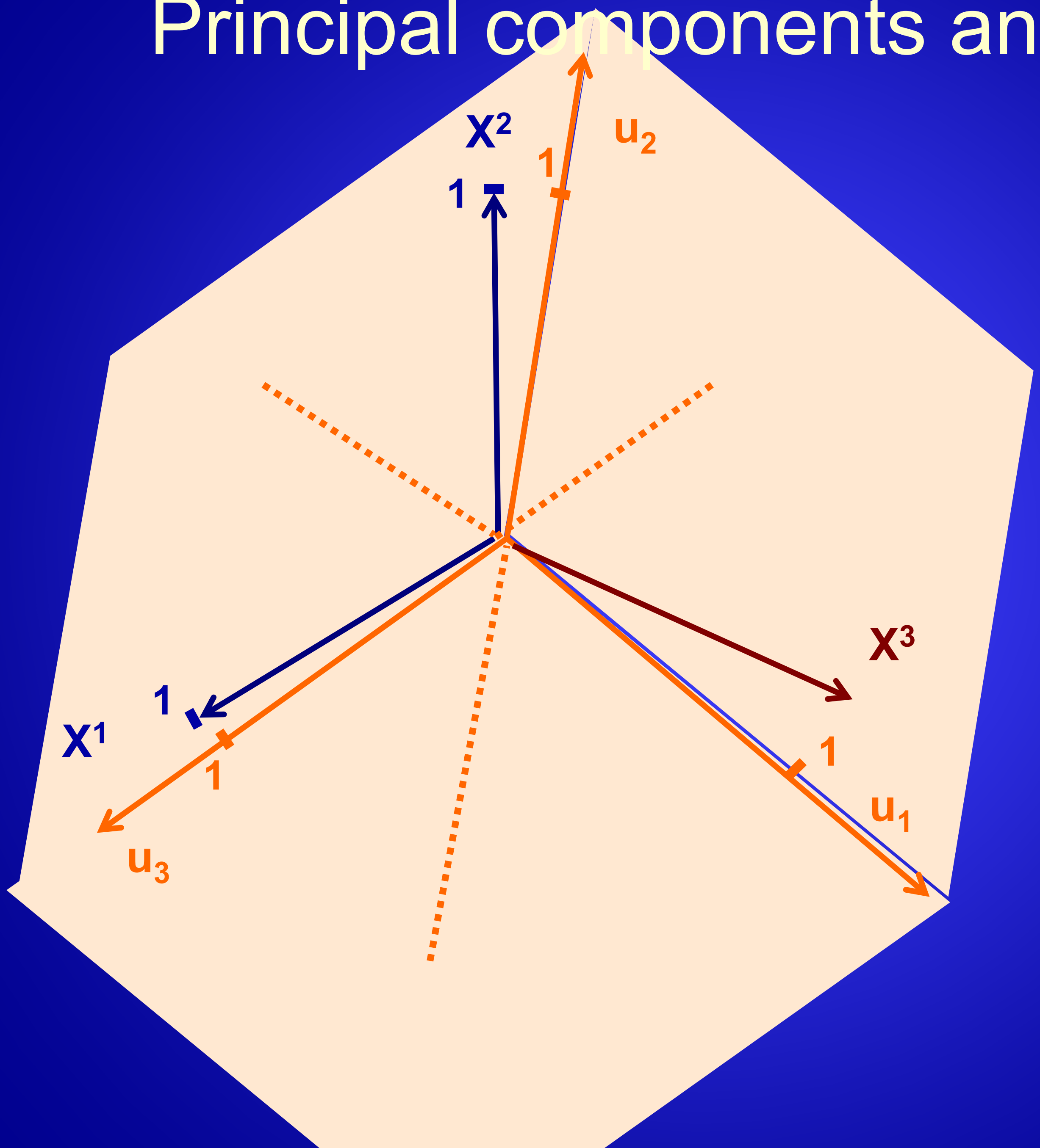
Principal components analysis



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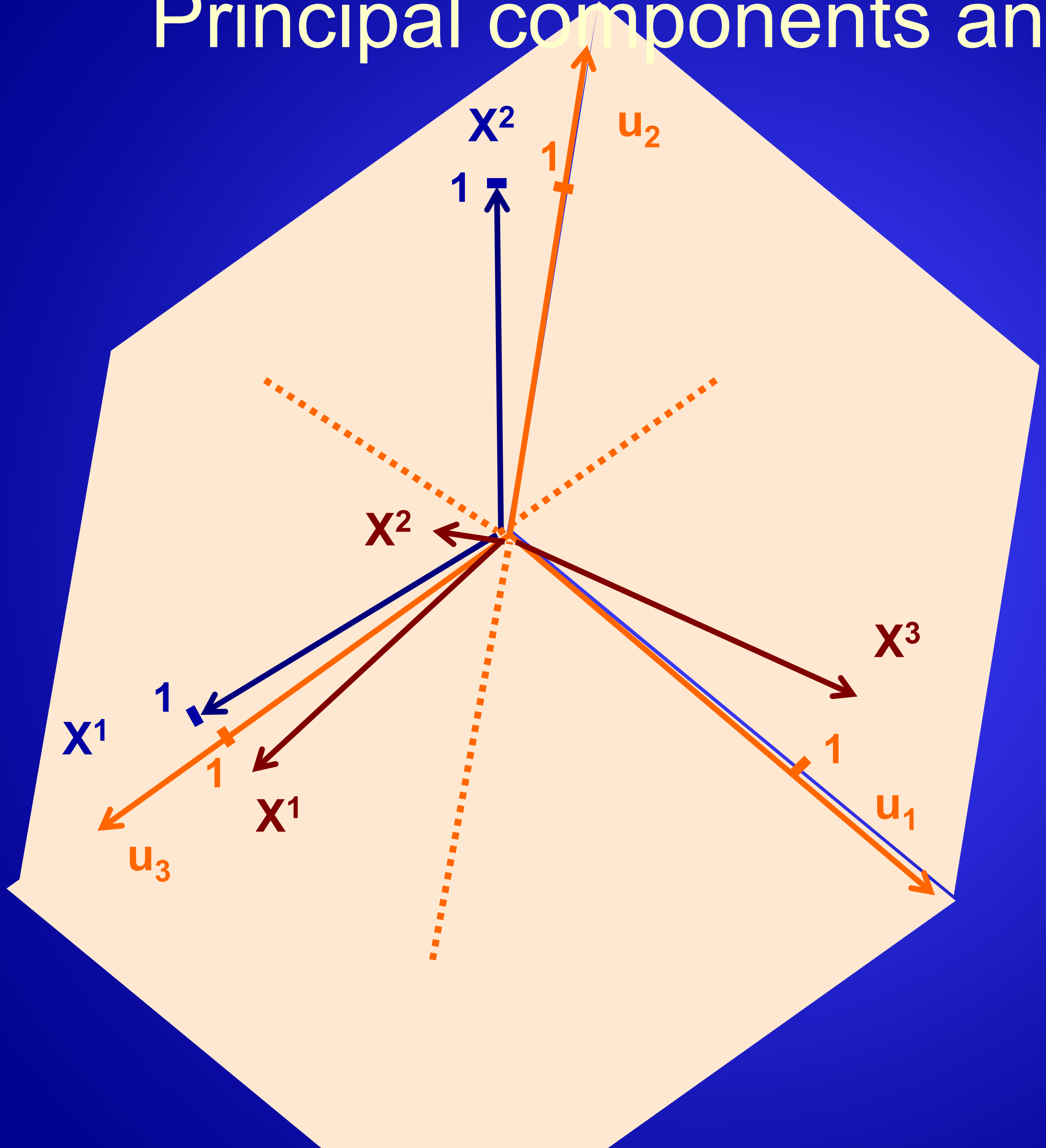
Principal components analysis



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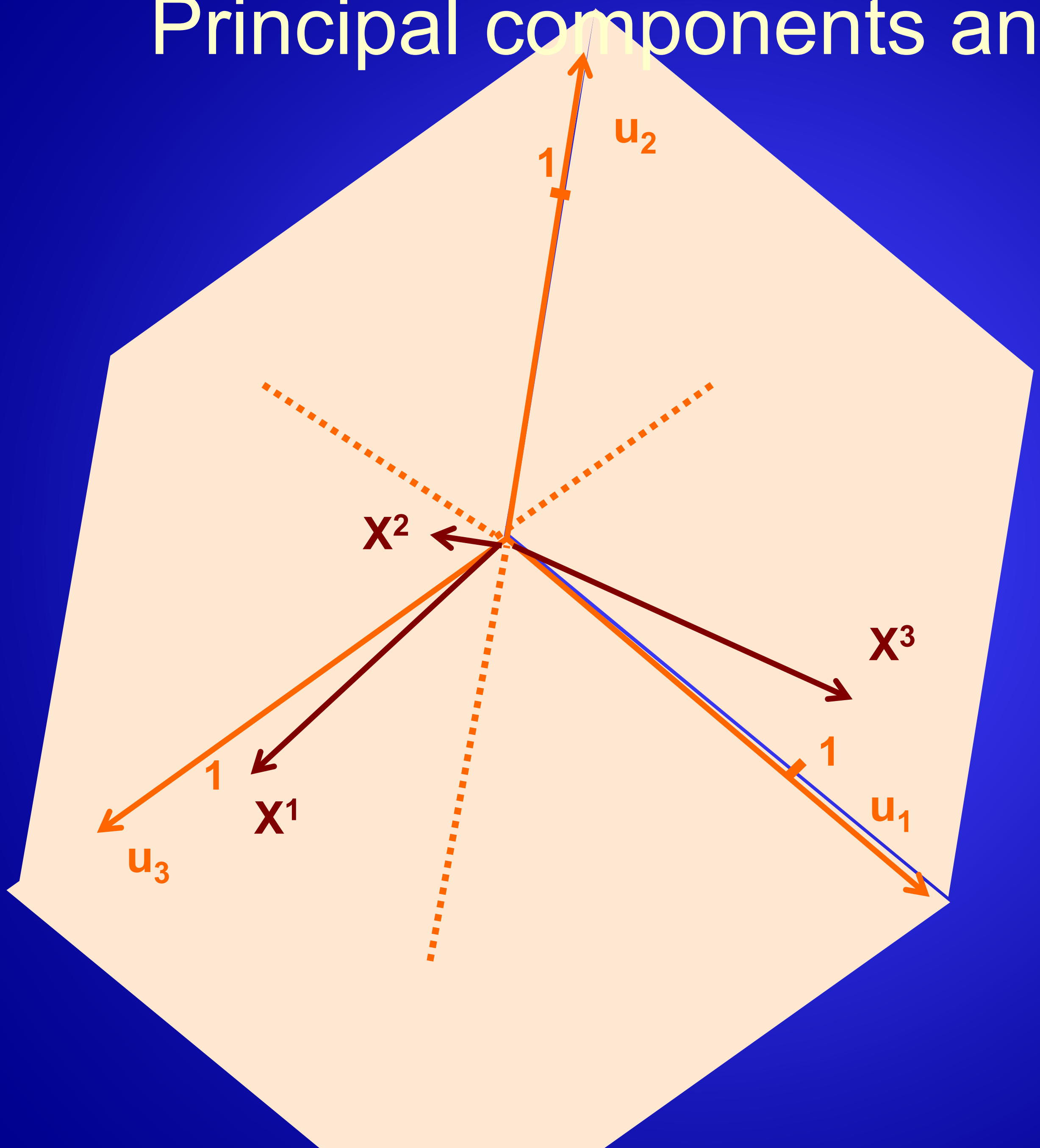
Principal components analysis



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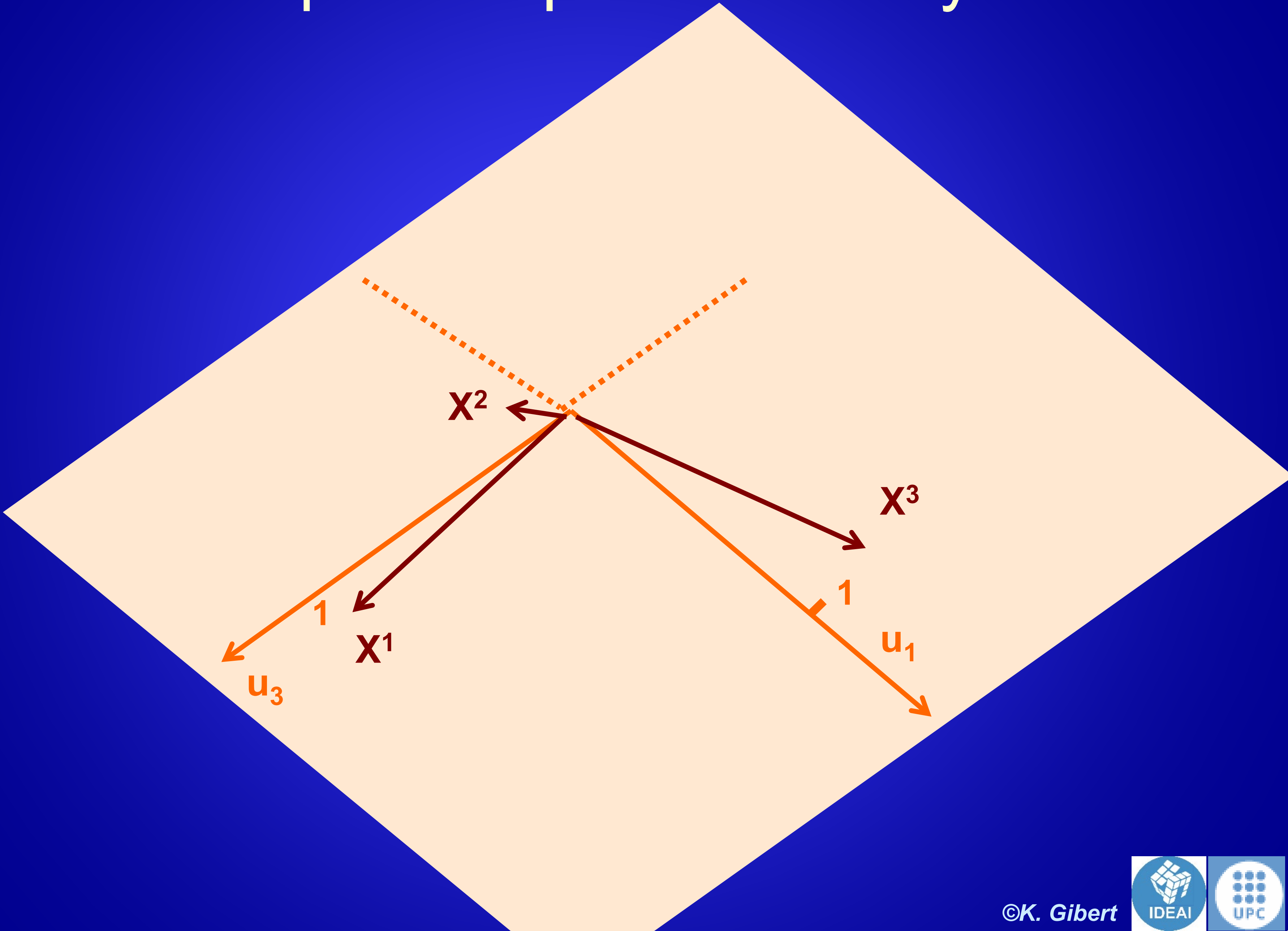
Principal components analysis



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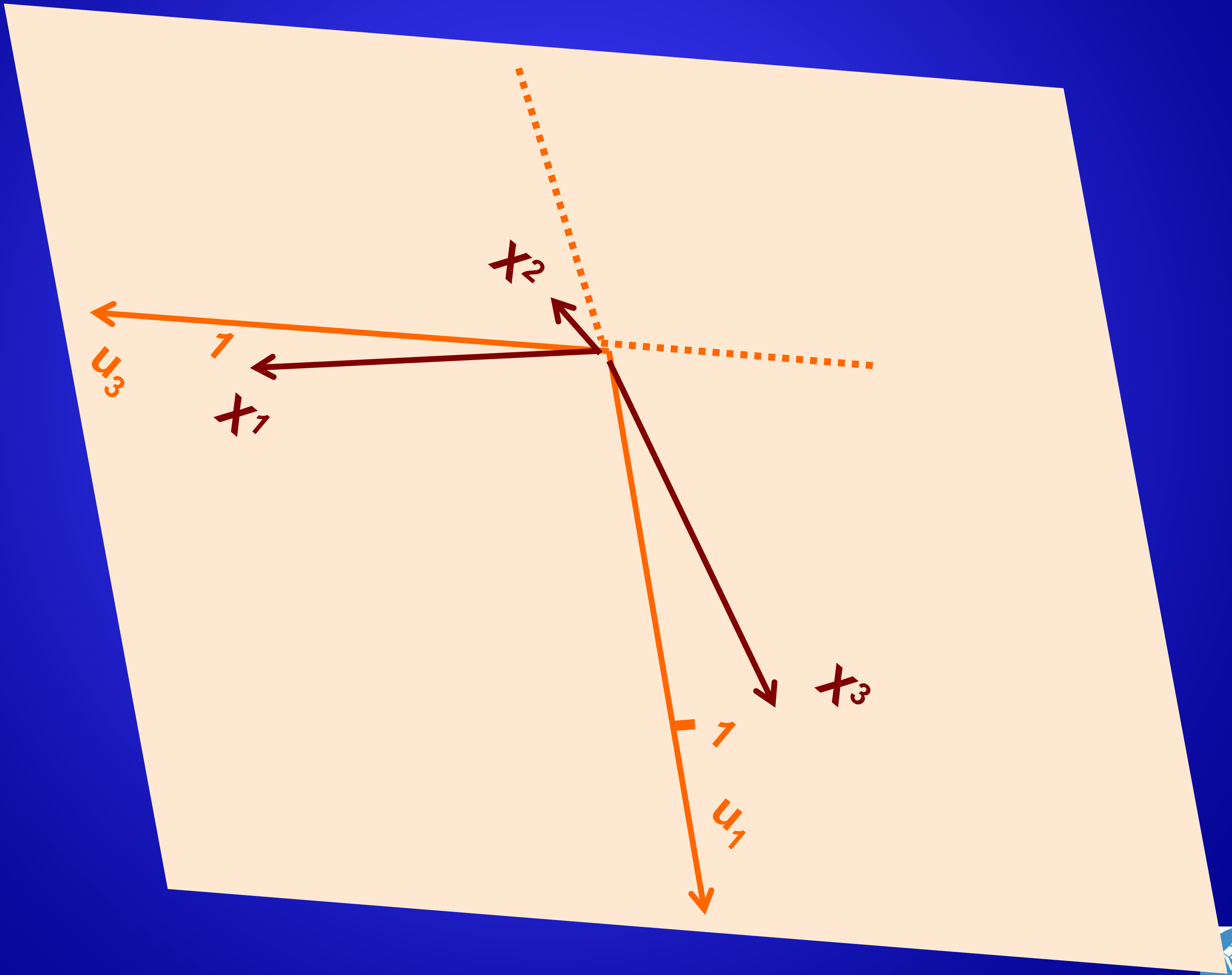
Principal components analysis



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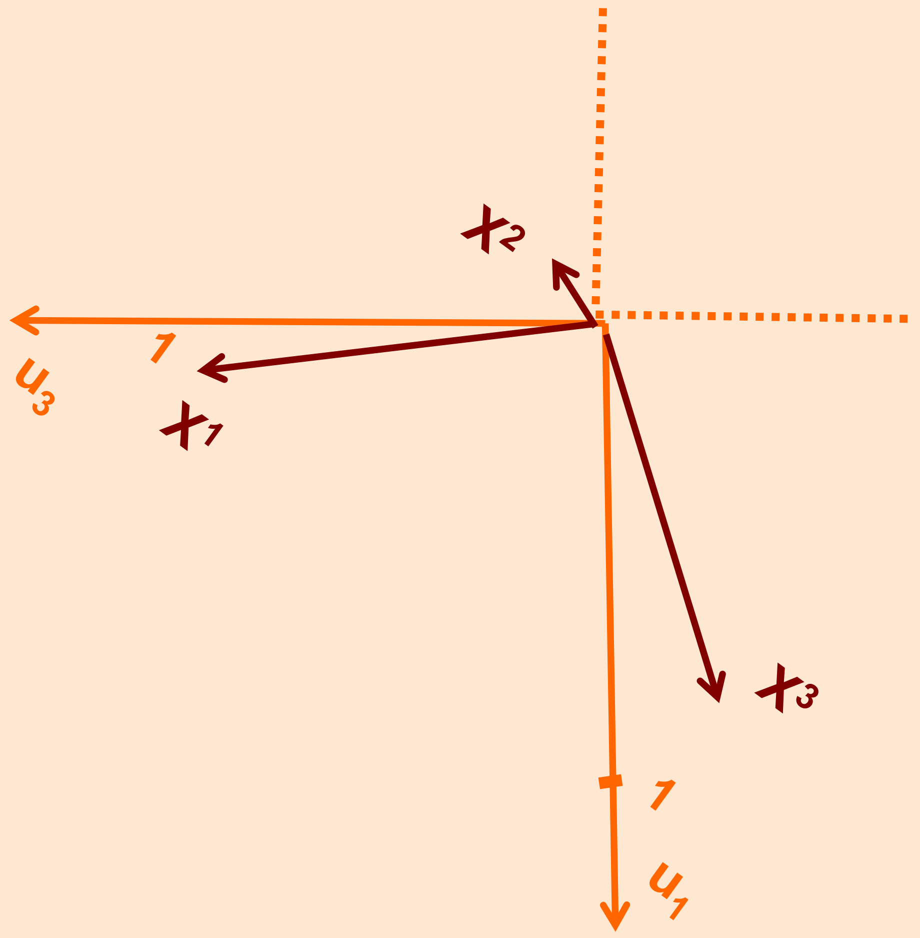
Principal components analysis



Map of projected variables

Angles linked with Association

Small angles : correlation

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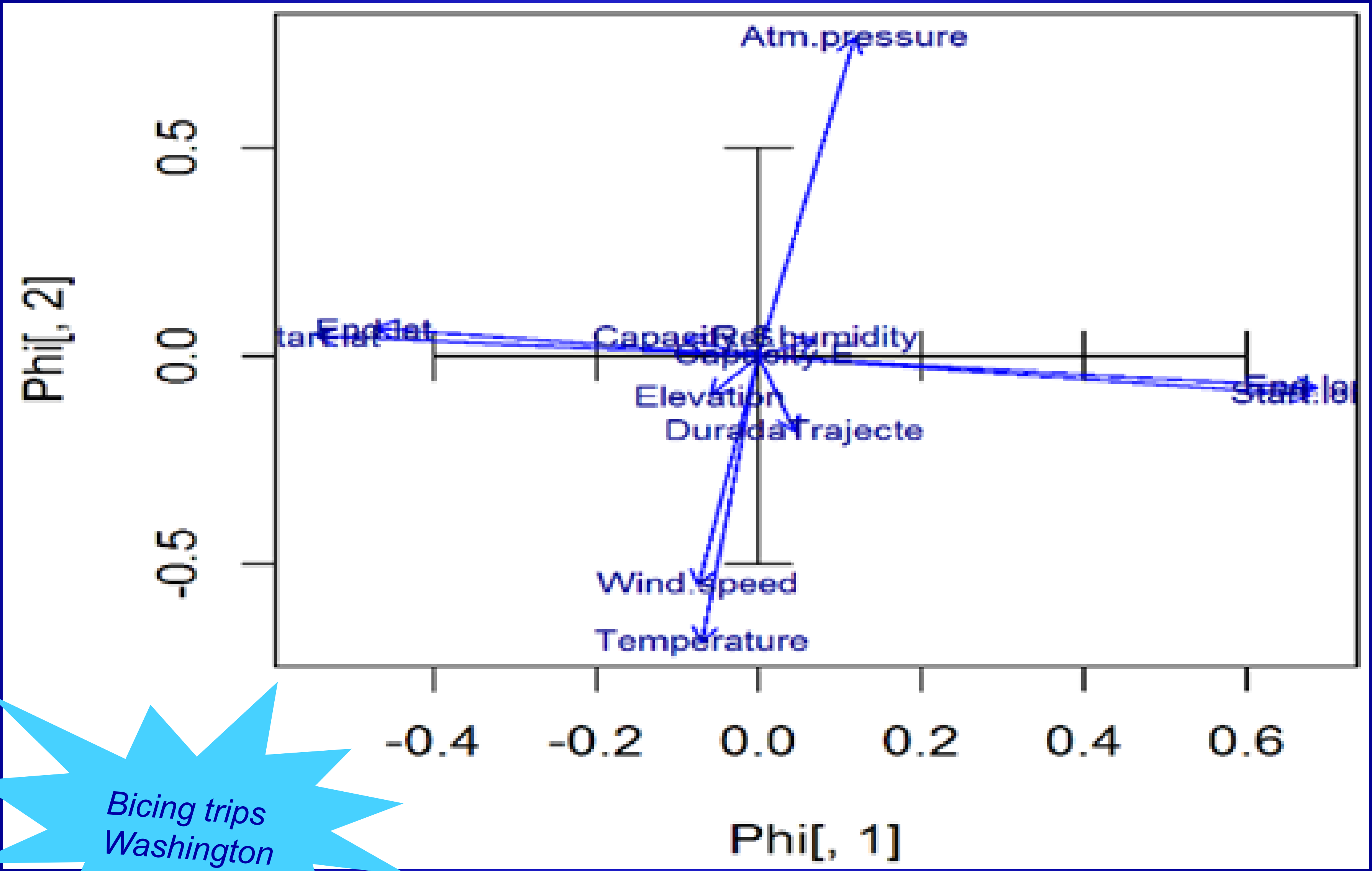
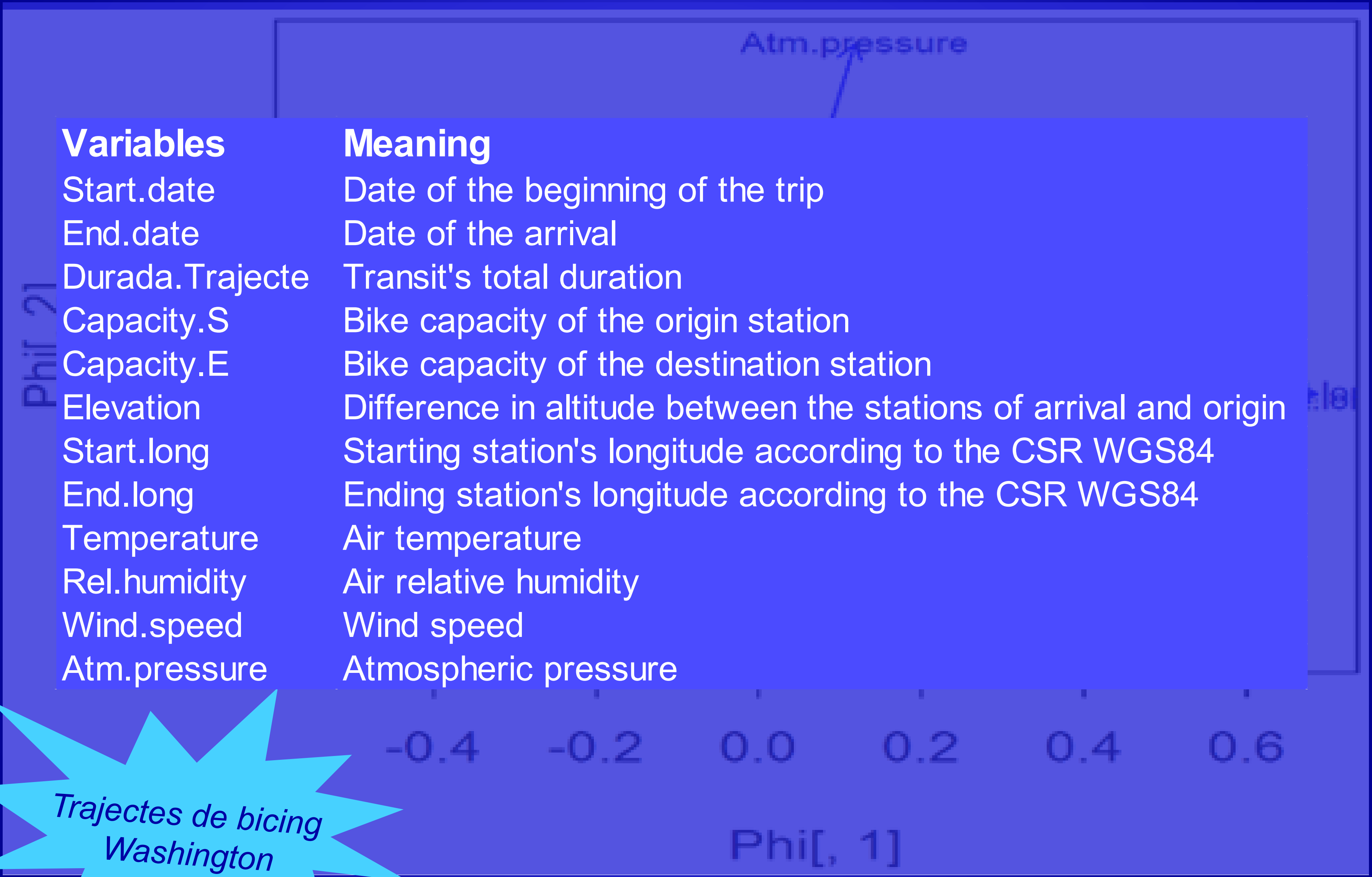
Principal components analysis



Bicing trips Washington

©K. Gibert

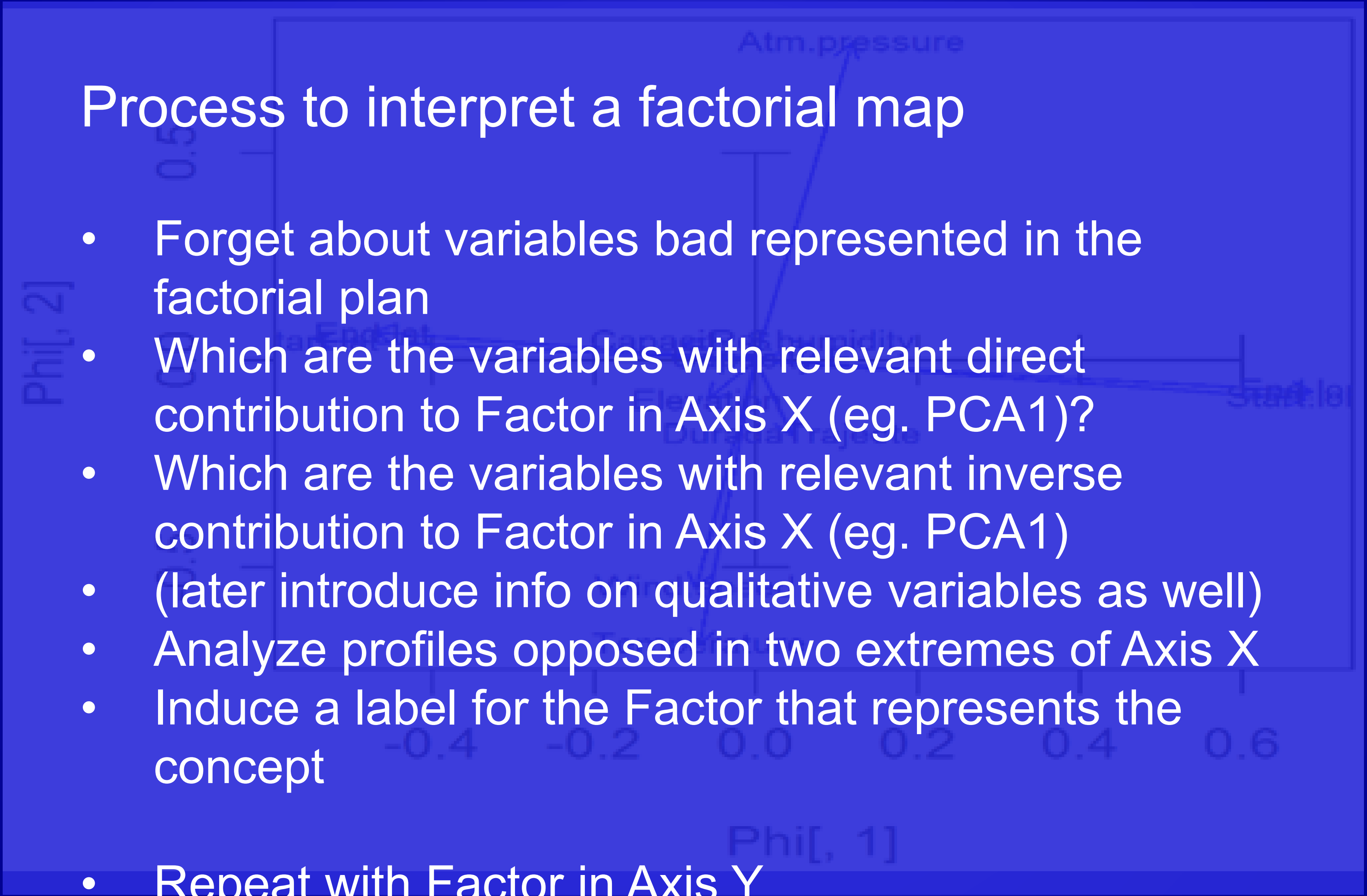
Principal components analysis



Variables	Meaning
Start.date	Date of the beginning of the trip
End.date	Date of the arrival
Durada.Trajecte	Transit's total duration
Capacity.S	Bike capacity of the origin station
Capacity.E	Bike capacity of the destination station
Elevation	Difference in altitude between the stations of arrival and origin
Start.long	Starting station's longitude according to the CSR WGS84
End.long	Ending station's longitude according to the CSR WGS84
Temperature	Air temperature
Rel.humidity	Air relative humidity
Wind.speed	Wind speed
Atm.pressure	Atmospheric pressure

Trajectes de bicing Washington

Principal components analysis

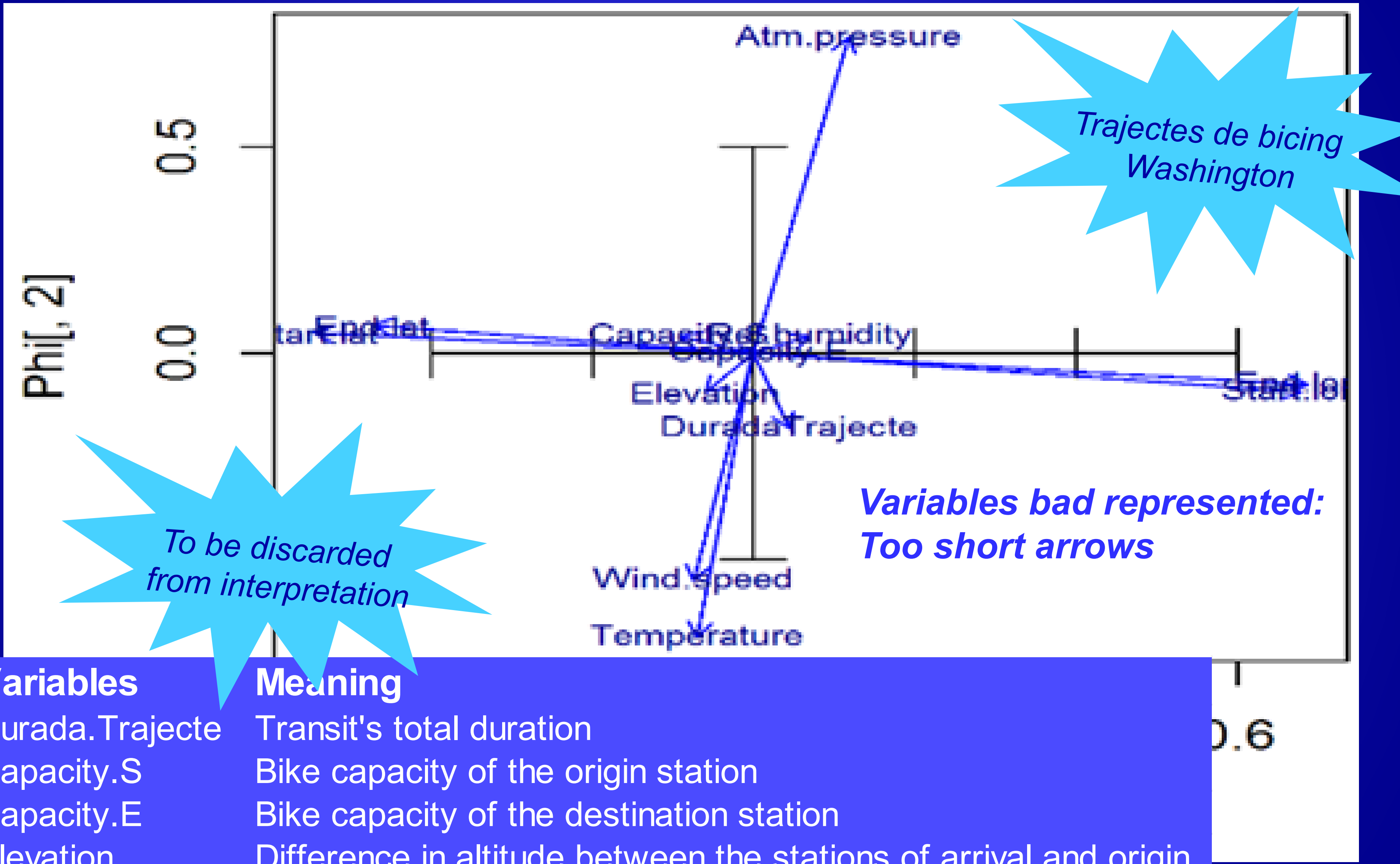


Process to interpret a factorial map

- Forget about variables bad represented in the factorial plan
- Which are the variables with relevant direct contribution to Factor in Axis X (eg. PCA1)?
- Which are the variables with relevant inverse contribution to Factor in Axis X (eg. PCA1)
- (later introduce info on qualitative variables as well)
- Analyze profiles opposed in two extremes of Axis X
- Induce a label for the Factor that represents the concept

- Repeat with Factor in Axis Y

Principal components analysis



Trajectes de bicing Washington

To be discarded from interpretation

Variables bad represented: Too short arrows

Variables	**Meaning**
Durada.Trajecte	Transit's total duration
Capacity.S	Bike capacity of the origin station
Capacity.E	Bike capacity of the destination station
Elevation	Difference in altitude between the stations of arrival and origin
Rel.humidity	Air relative humidity

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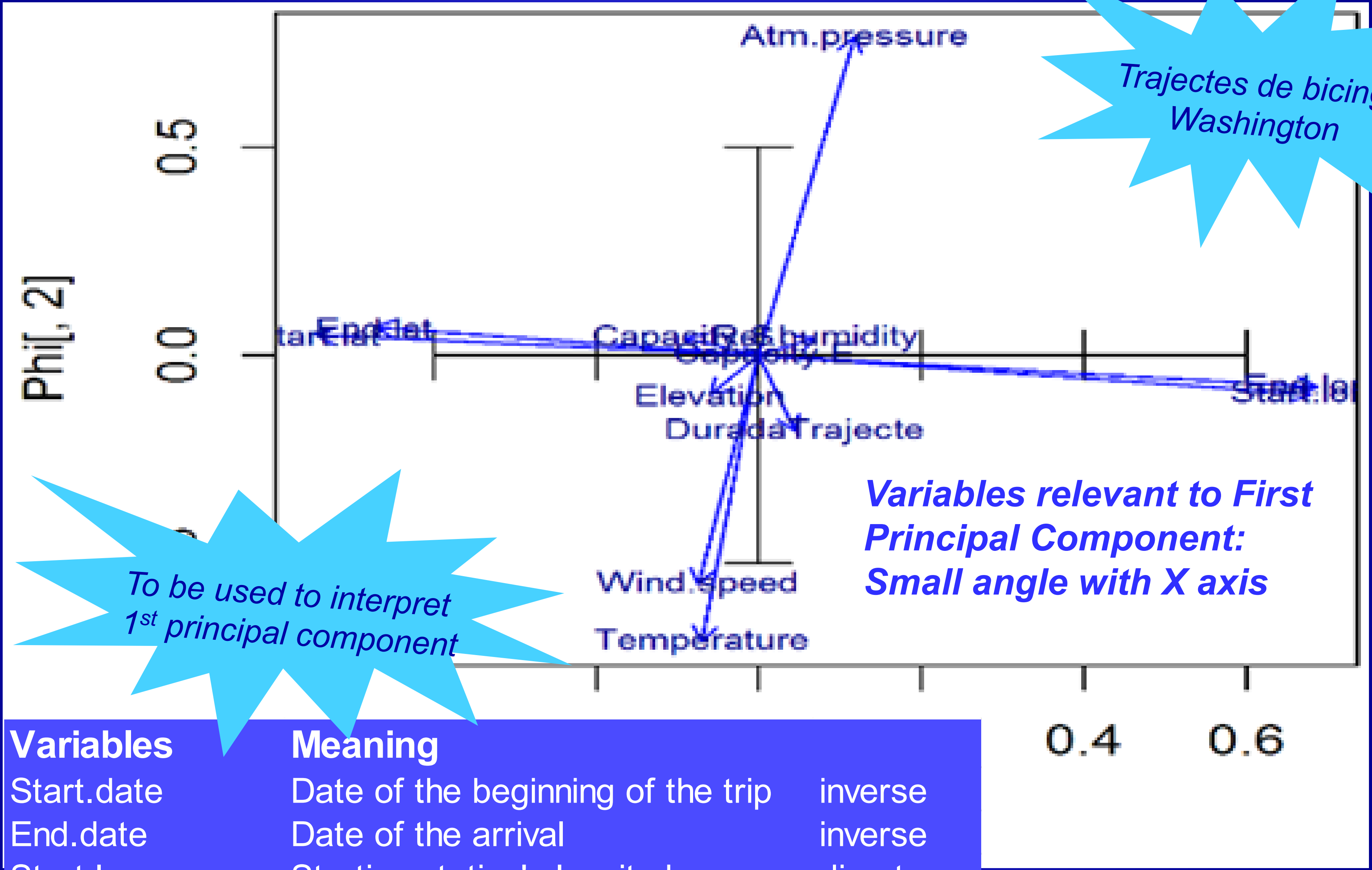
Principal components analysis



Trajectes de bicing Washington

Variables relevant to First Principal Component: Small angle with X axis

To be used to interpret 1st principal component

Variables	Meaning	
Start.date	Date of the beginning of the trip	inverse
End.date	Date of the arrival	inverse
Start.long	Starting station's longitude	direct
End.long	Ending station's longitude	direct

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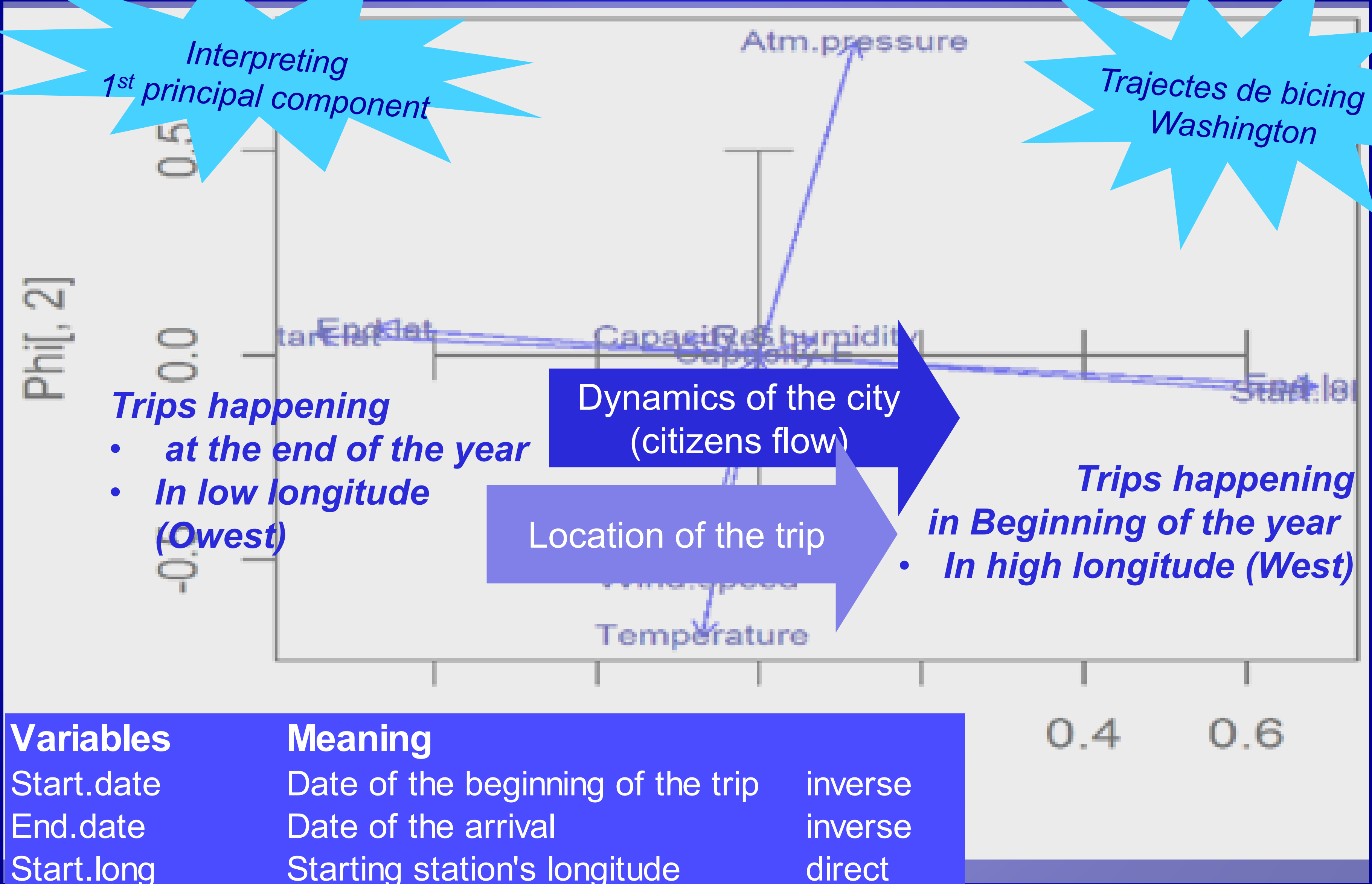
Principal components analysis



Interpreting 1st principal component

Trajectes de bicing Washington

Trips happening

- ***at the end of the year***
- ***In low longitude (Owest)***

Dynamics of the city (citizens flow)

Location of the trip

Trips happening

in Beginning of the year

- ***In high longitude (West)***

Variables	**Meaning**	
Start.date	Date of the beginning of the trip	inverse
End.date	Date of the arrival	inverse
Start.long	Starting station's longitude	direct
End.long	Ending station's longitude	direct

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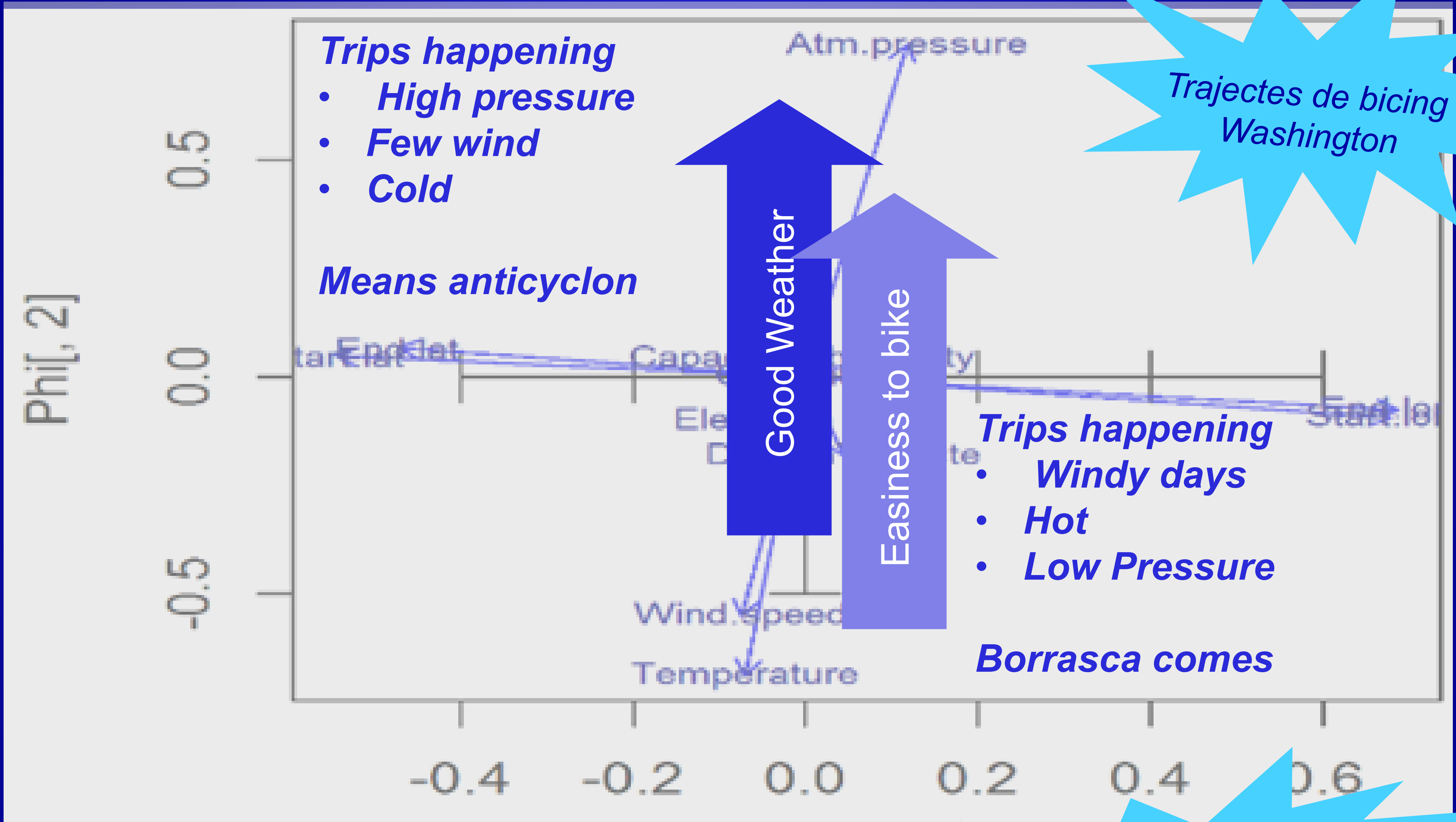
Principal components analysis



Trips happening

- ***High pressure***
- ***Few wind***
- ***Cold***

Means anticyclon

Trips happening

- ***Windy days***
- ***Hot***
- ***Low Pressure***

Borrasca comes

Trajectes de bicing Washington

Variables	**Meaning**	
Temperature	Air temperature	invers
Rel.humidity	Air relative humidity	invers
Atm.pressure	Atmospheric pressure	direct

Interpreting 2nd principal component

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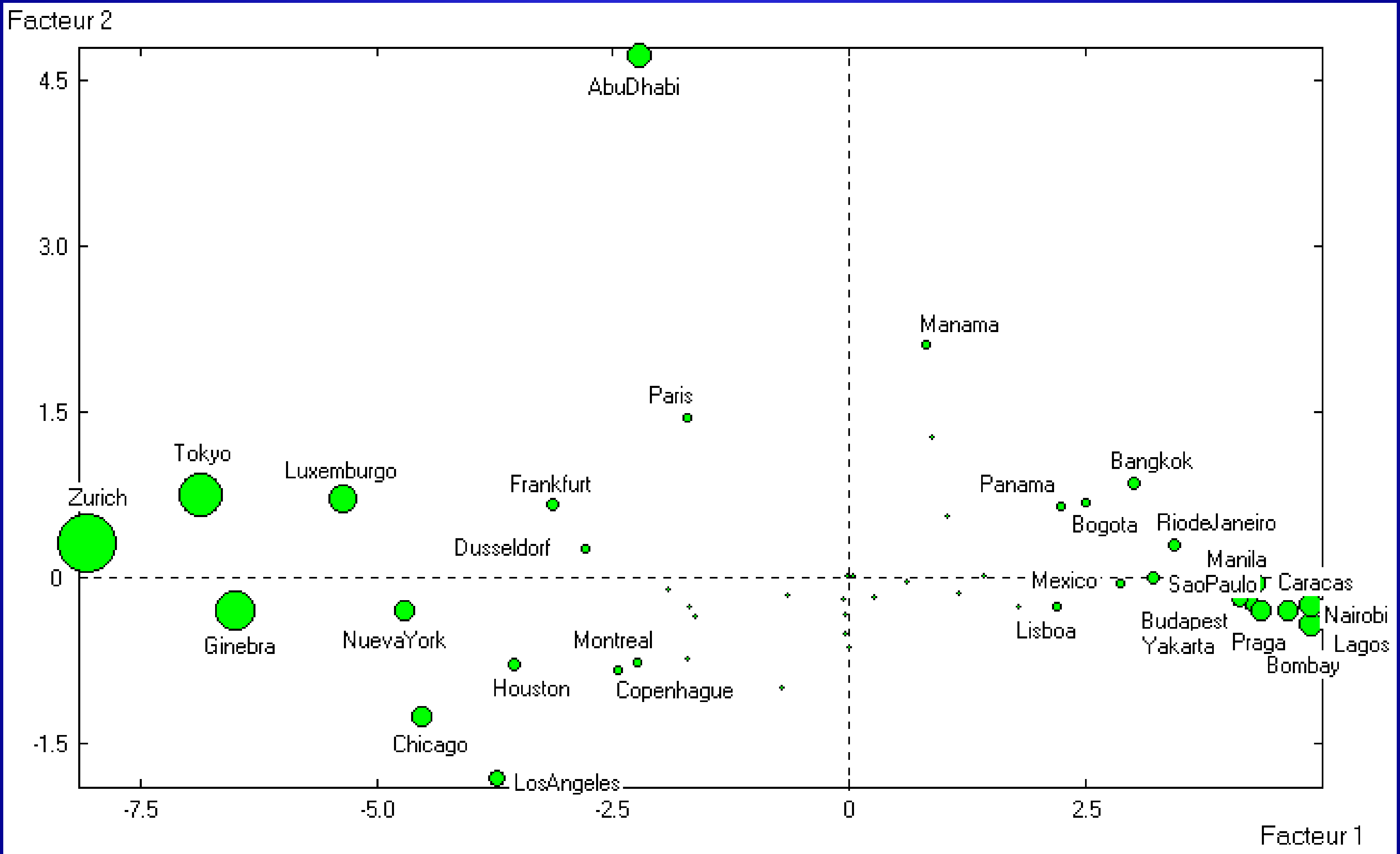
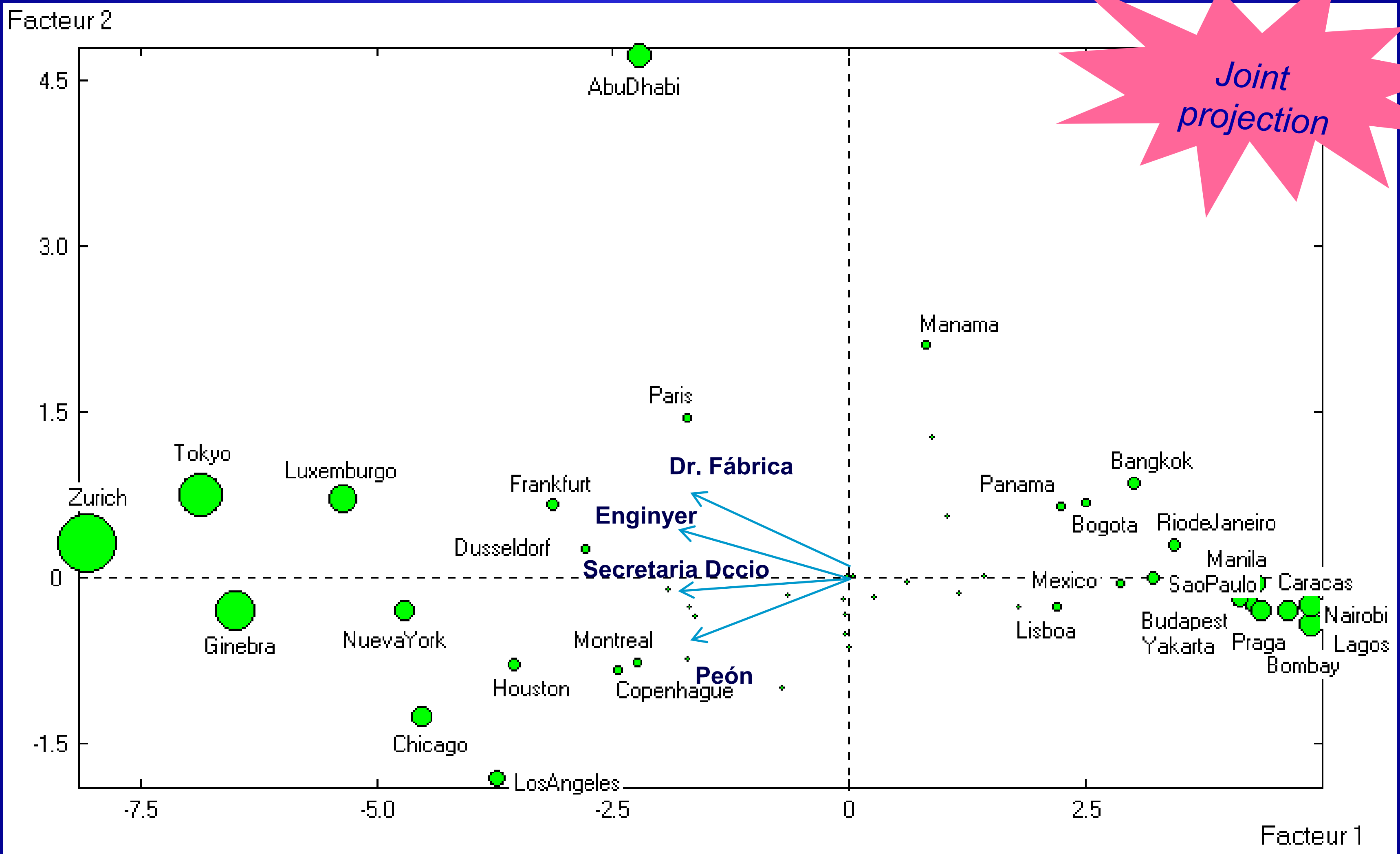
Visualisation of international cities according their salaries. USB 1994.



Visualisation of international cities according their salaries. USB 1994.



Joint projection

Course DM: Multivariate Visualisation. T. Aluja

©K. Gibert

Factorial Methods

- Principal Components Analysis
  - Output: K factors rotating original X variables
  - Factors: Linear combinations of original variables

Several uses:

- As an associative data mining method to analyze relationships among variables

  Project variables and modalities and find associations

- As a preprocessing method for elicitation of latent variables

  Project active and illustrative variables/individuals on first/second factorial plane and interpret factors (find latent variables)

- As a preprocessing method for multidimensionality reduction

  Select more informative factors κ<<p *(accumulate 80% inertia)*

  Reduce data matrix to selected factors

  Alternative, keep variables mainly contributing to selected factors (smaller angles with factorial axis)

Factorial Methods

- Given <X,M,D>

Diagonalize Correlations matrix X'DX

Get r eigen values λ_α and sort decreasingly

$\{\lambda_\alpha\}_{\alpha=1:r}$ $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq_{.....} \geq \lambda_r$

Corresponding eigenvectors $u_\alpha = (u_{\alpha 1} .... u_{\alpha p})$

for M= $\mathbb{I}_p$ *:* $u^*_\alpha = u_\alpha$ *; for* M≠ $\mathbb{I}_p$ *:* $u^*_\alpha = M^{-½} u_\alpha$

$\{u^*_\alpha\}_{\alpha=1:r}$ *orthonormal base for individuals*

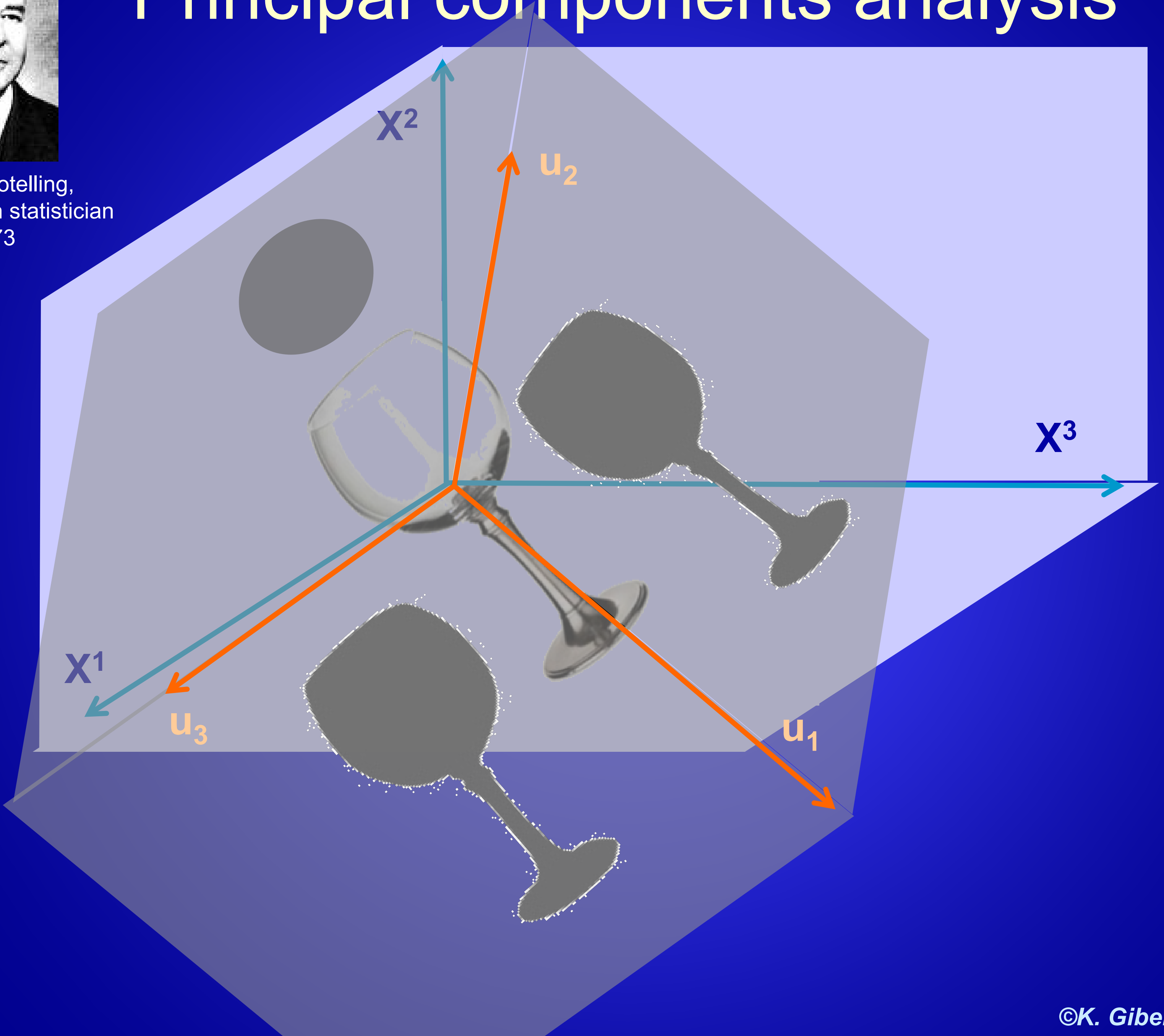
u^*_α are the principal factors of X : good rotation directions

$U^*=([u^*_1]\ [u^*_2] ..... [u^*_r])$ is the basis for the projection space

$U^*_k=([u^*_1]\ [u^*_2] ..... [u^*k])$ is the basis for projecting in first k dimensions(k<r)

Principal components analysis

Harold Hotelling,
American statistician
1895-1973



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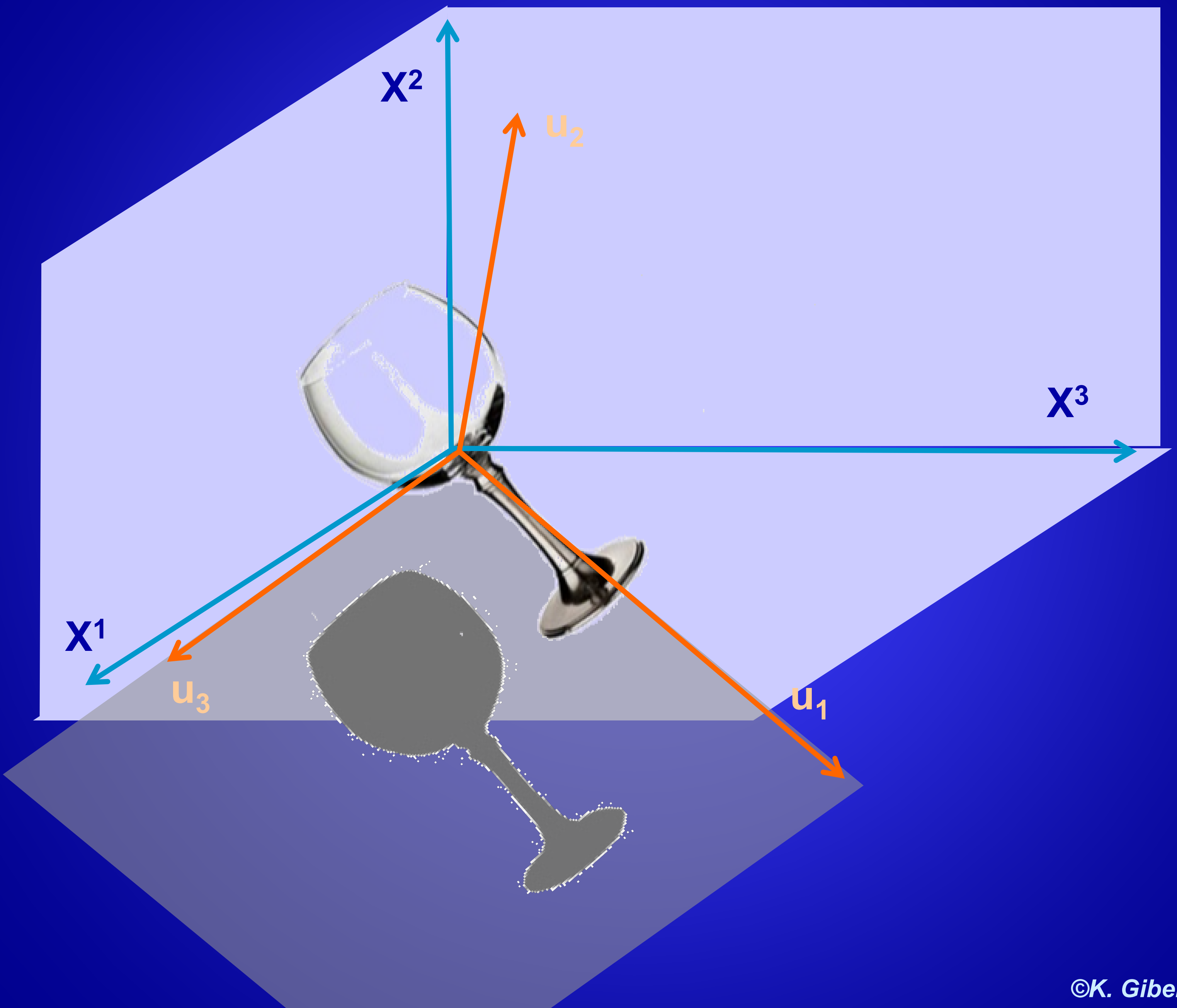
Principal components analysis



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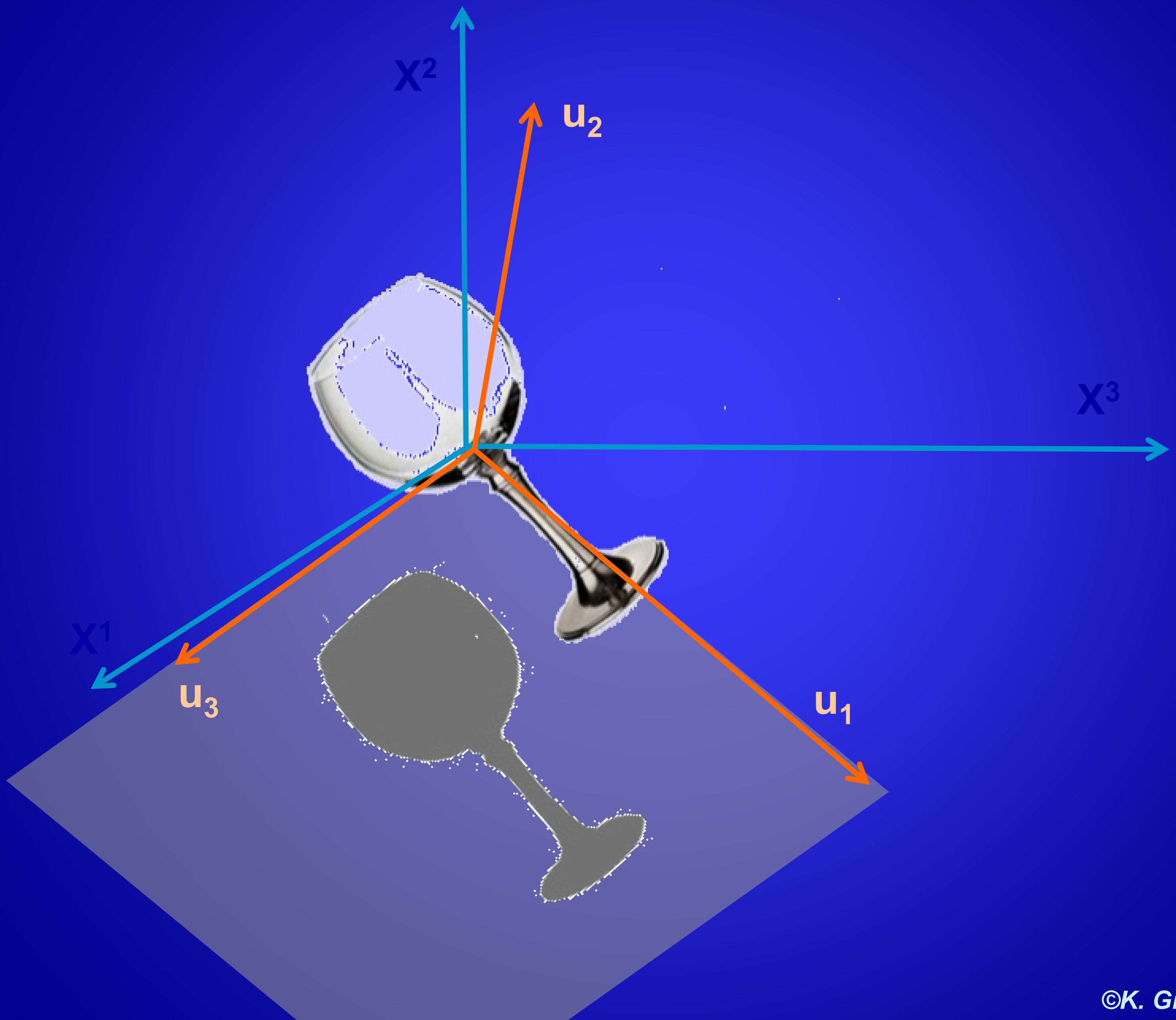
Principal components analysis



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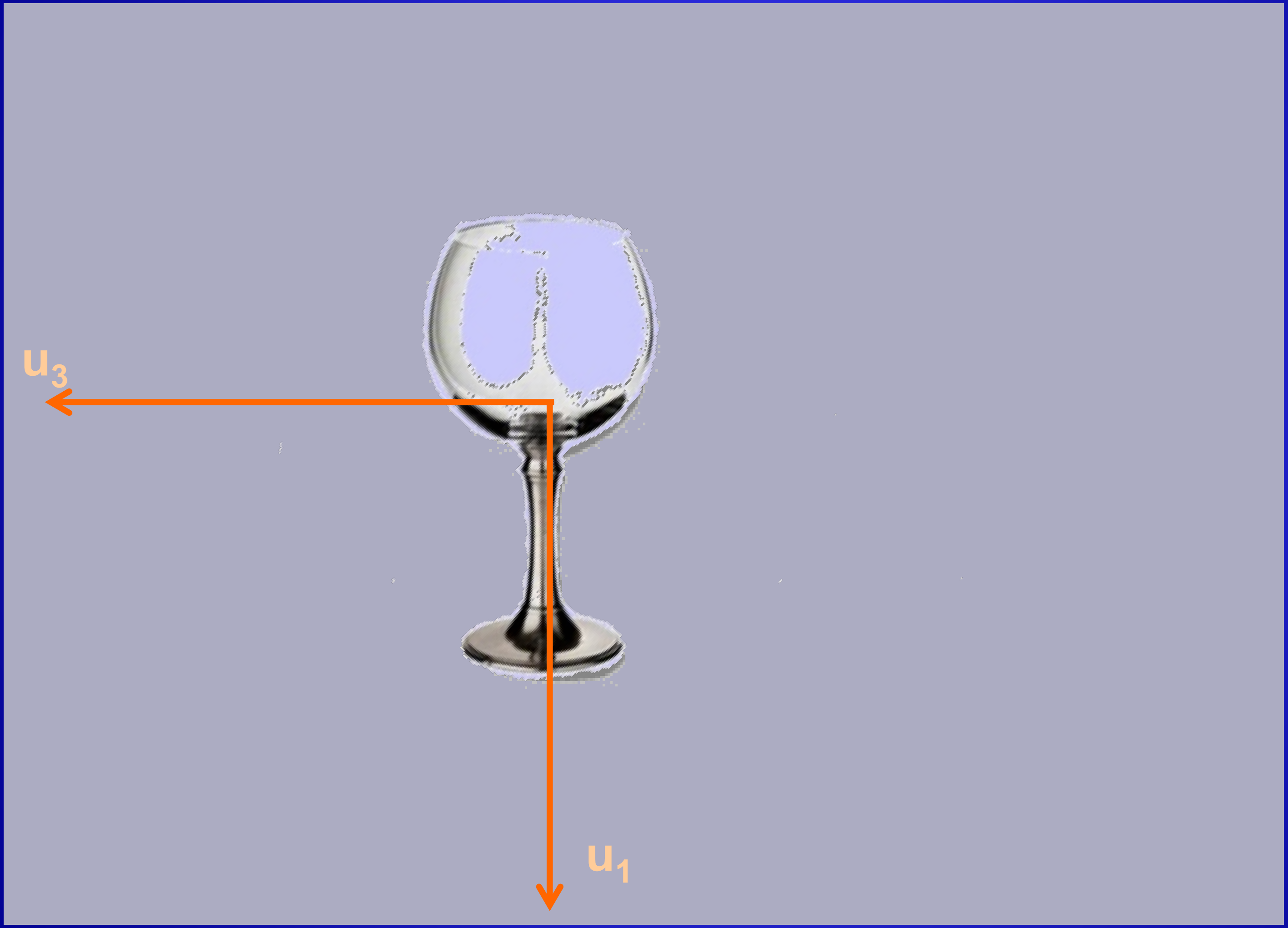
Principal components analysis



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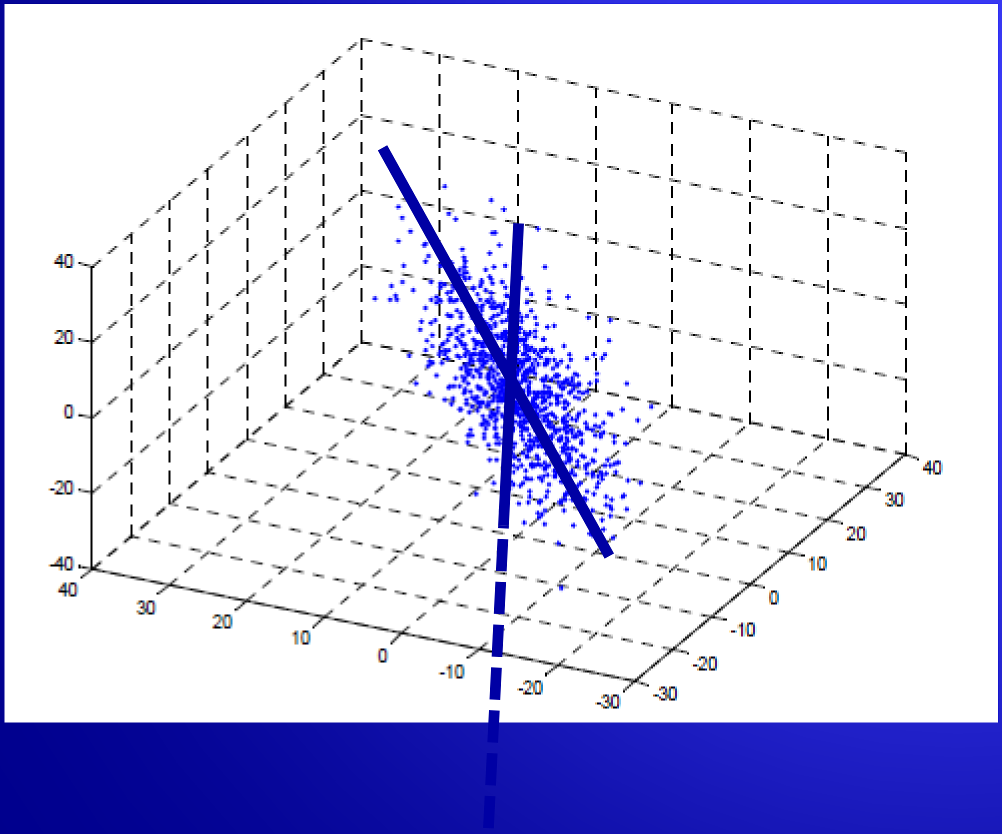
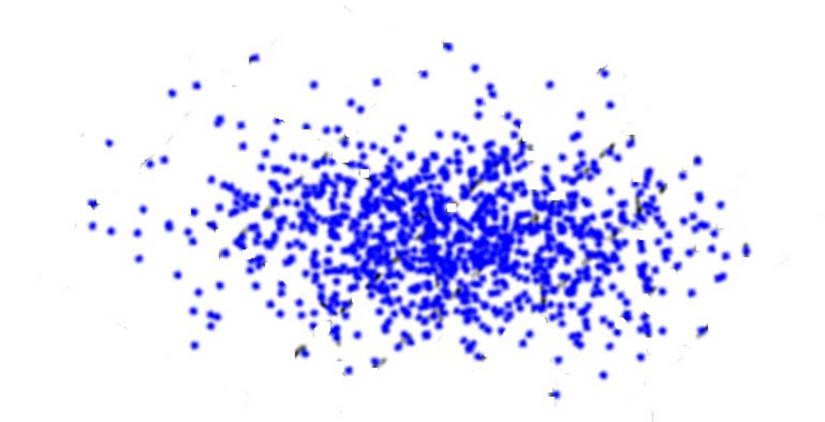
Principal components analysis

- Find the most informative projection planes of data cloud *(factorial planes)*





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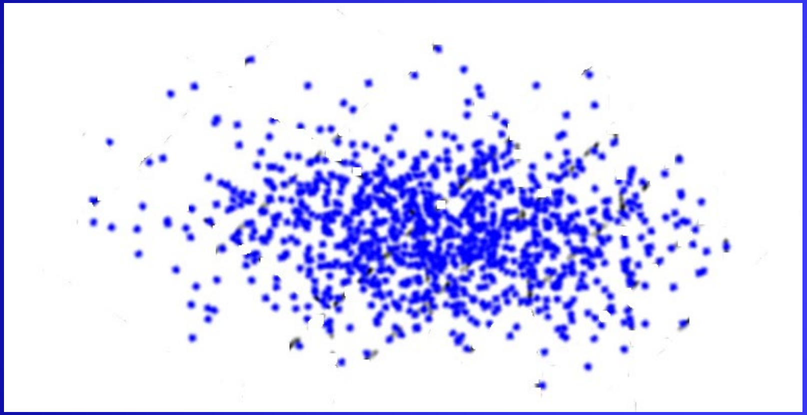
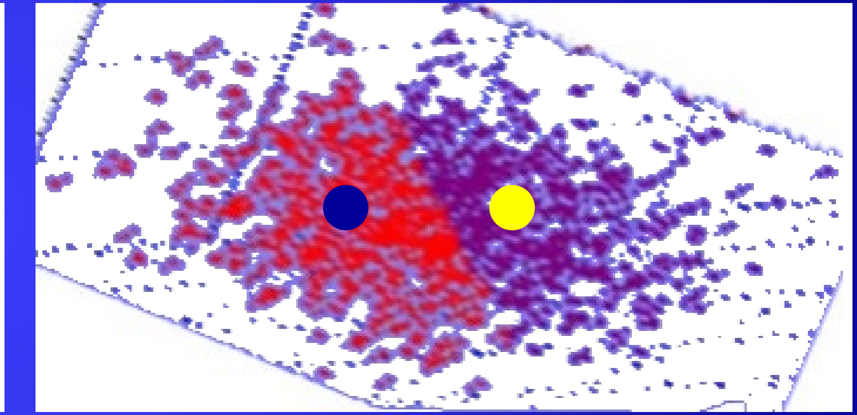
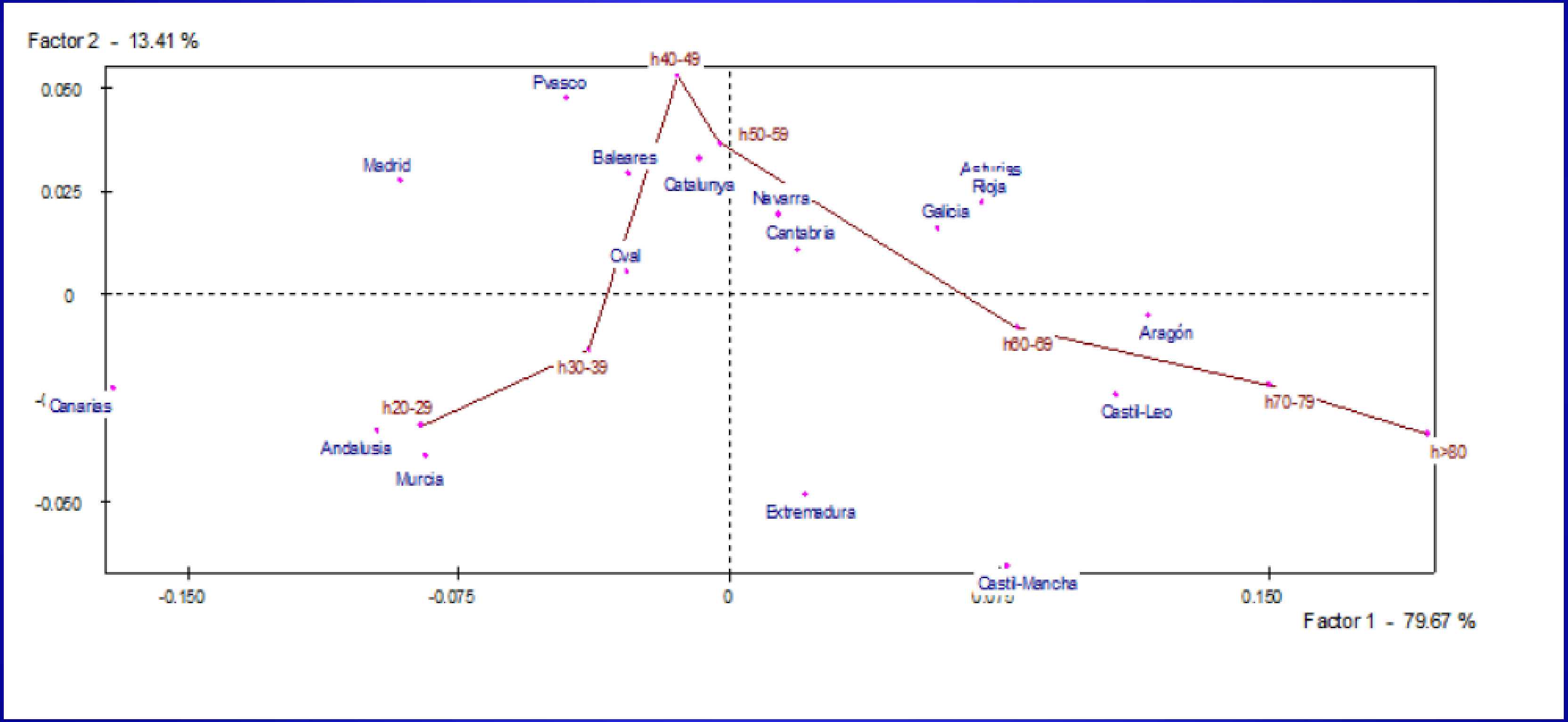
Principal components analysis

- Introduce qualitative information (projecting modalities)






Factor 2 - 13.41 %
0.050
0.025
0
-0.050
-0.150
-0.075
0
0.075
0.150
Factor 1 - 79.67 %
h40-49
Pvasco
h50-59
Madrid
Balears
Catalunya
Navarra
Asturias
Rioja
Galicia
Cantabria
Cval
Aragón
h60-69
h30-39
Canarias
h20-29
Castil-Leo
h70-79
Andalusia
h>80
Murcia
Extremadura
Castil-Mancha

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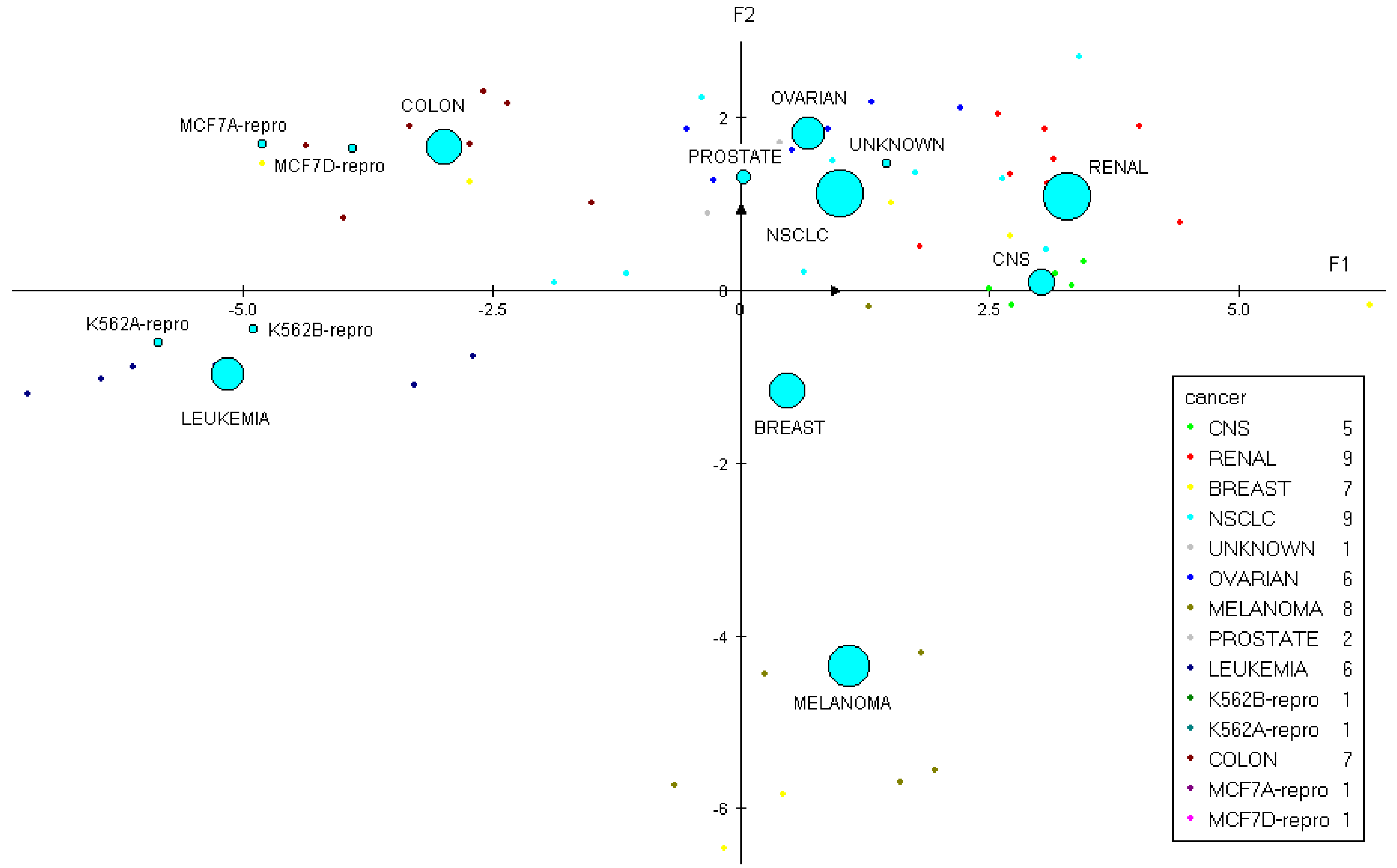
Microarray data: 64 cancers 6830 gen cromotografy



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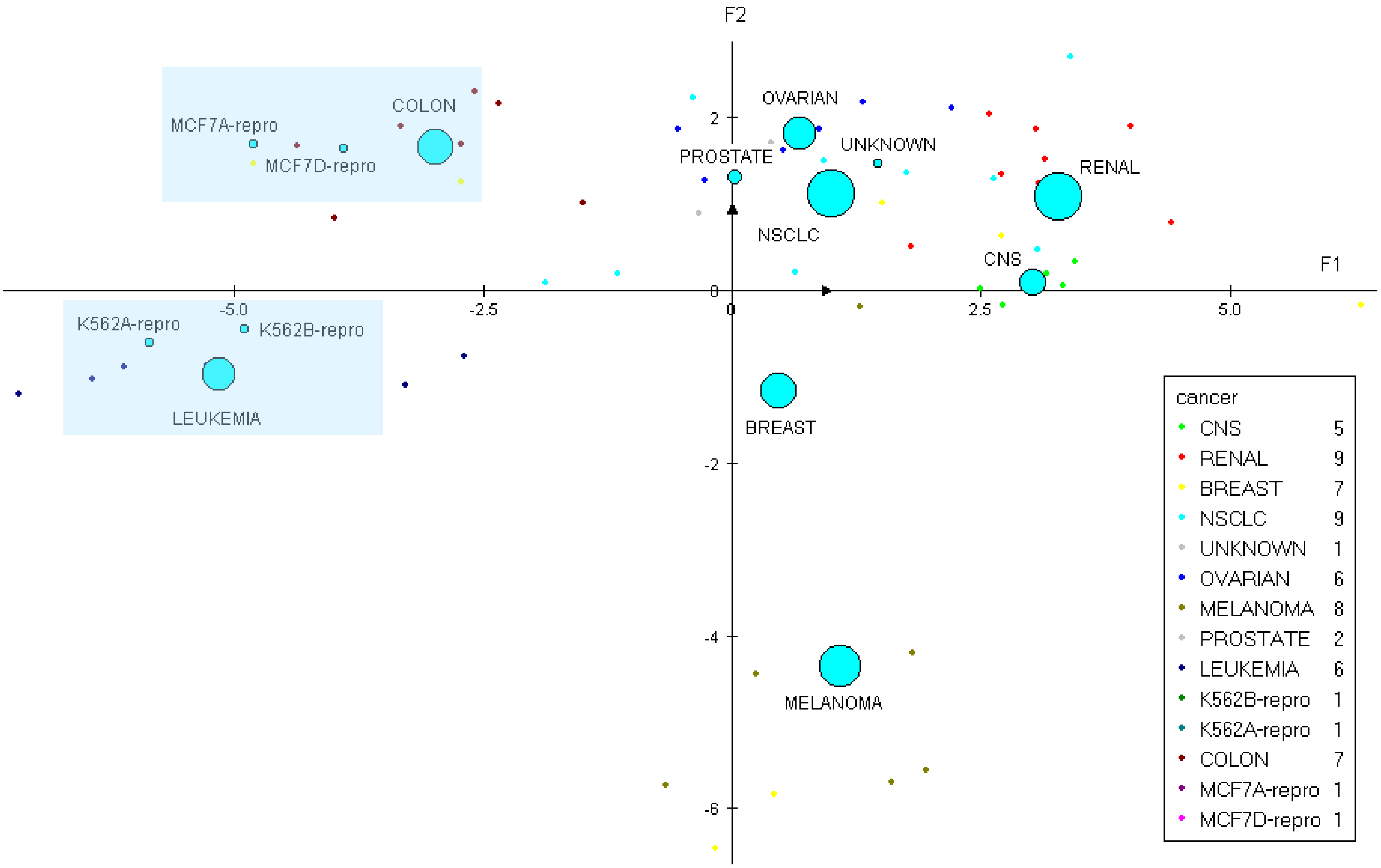
Microarray data: 64 cancers 6830 gen cromotografy



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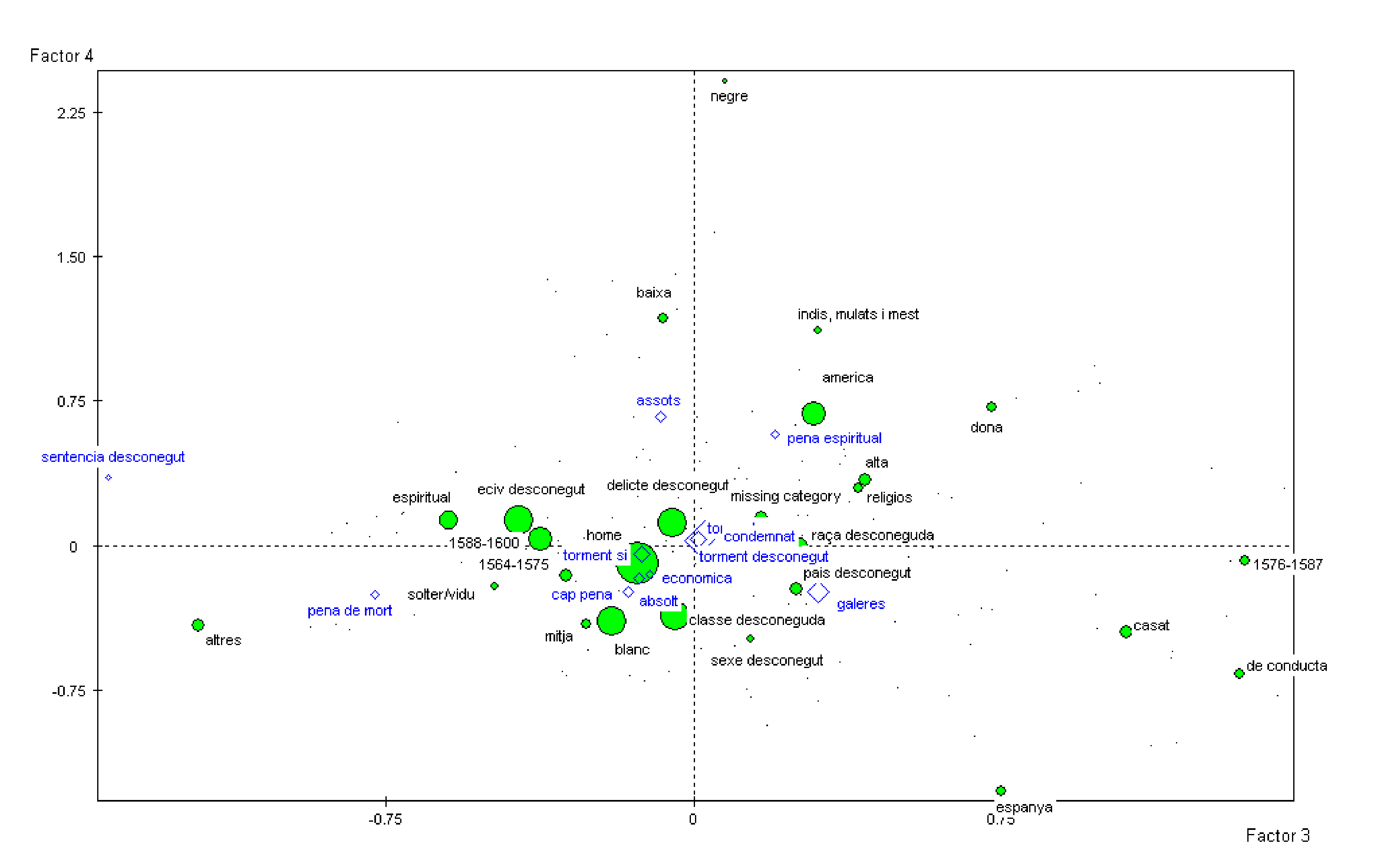
Spanish inquisition 1567-1600

sentences & crimes



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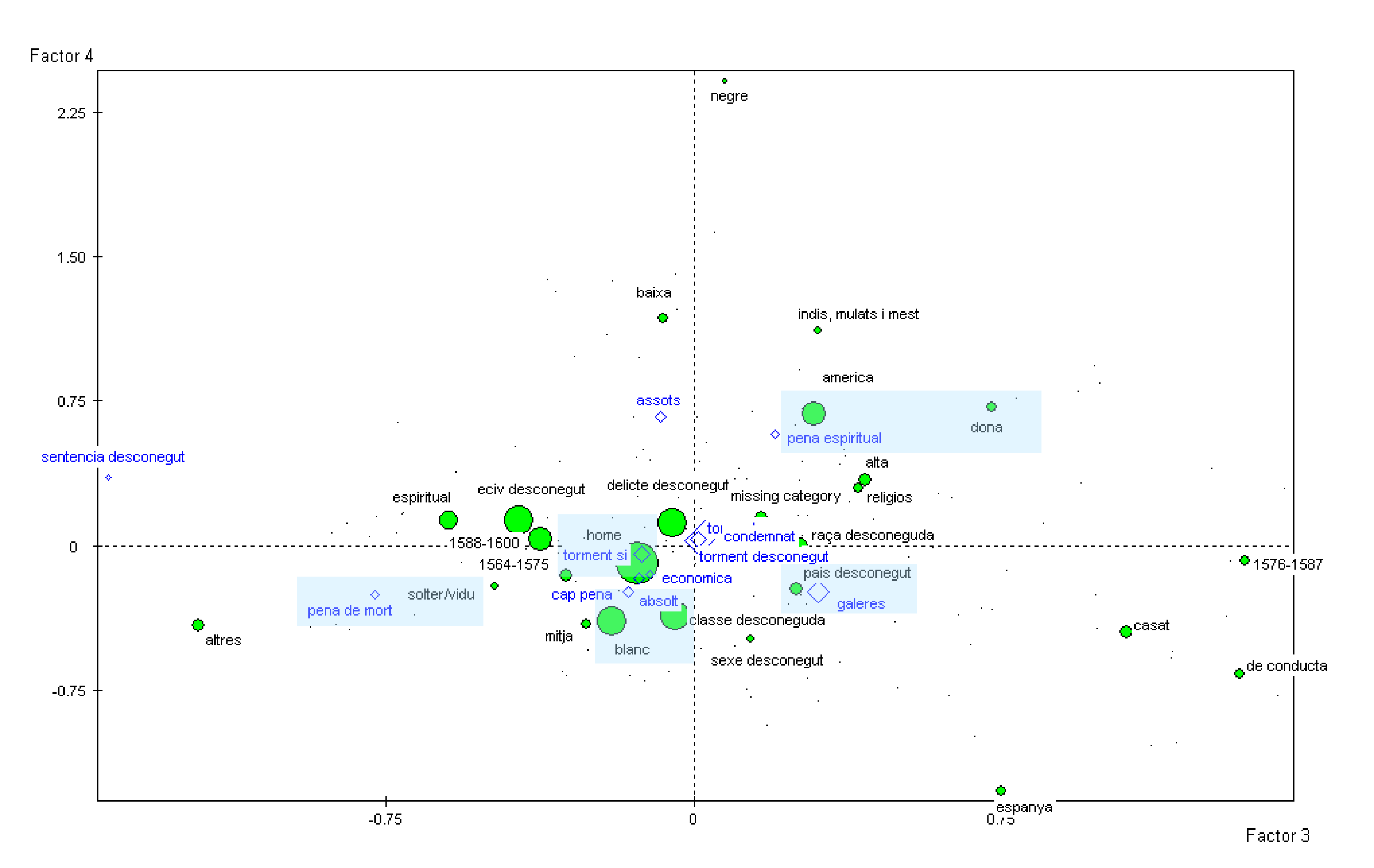
Spanish inquisition 1567-1600

sentences & crimes



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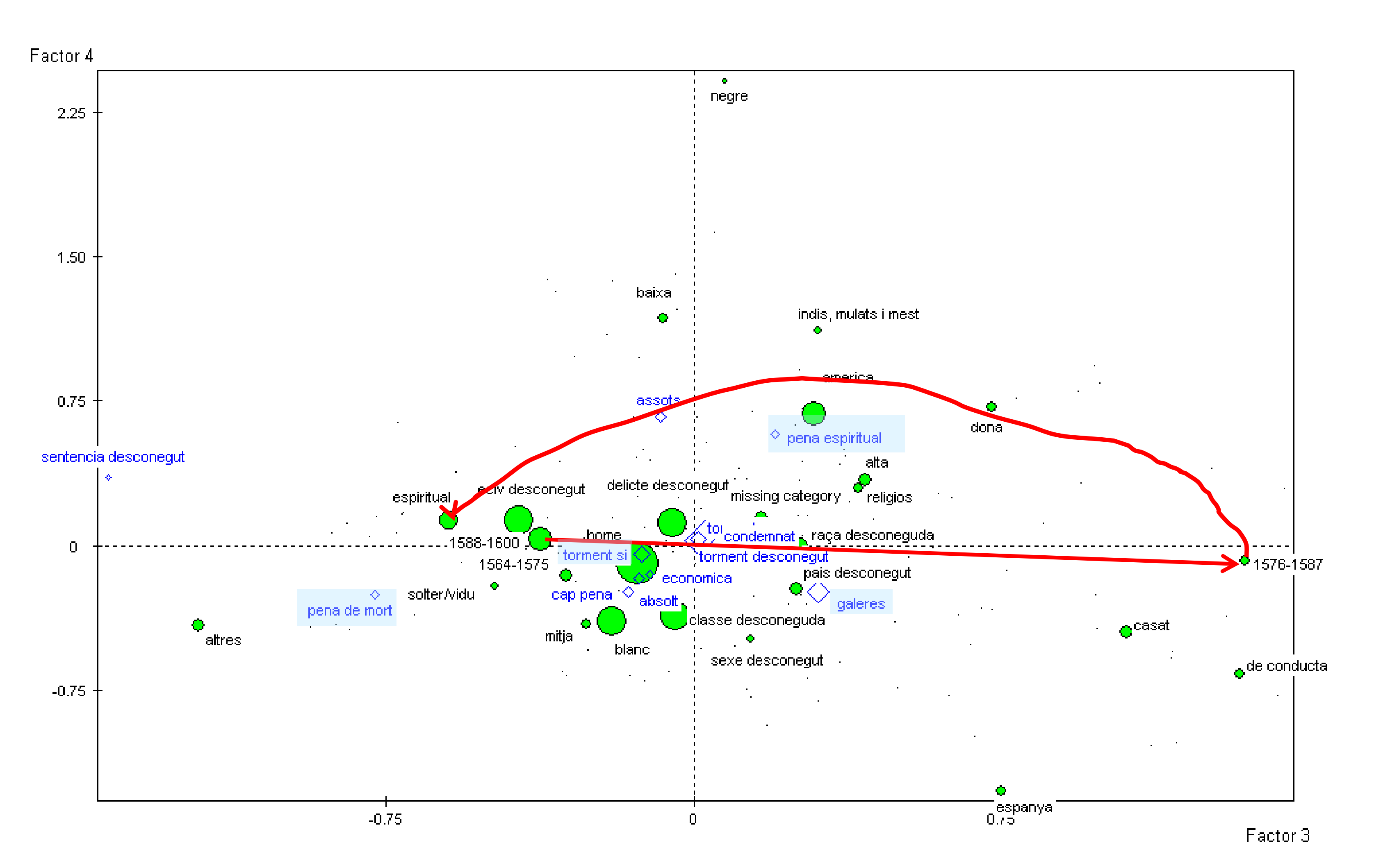
Spanish inquisition 1564-1600

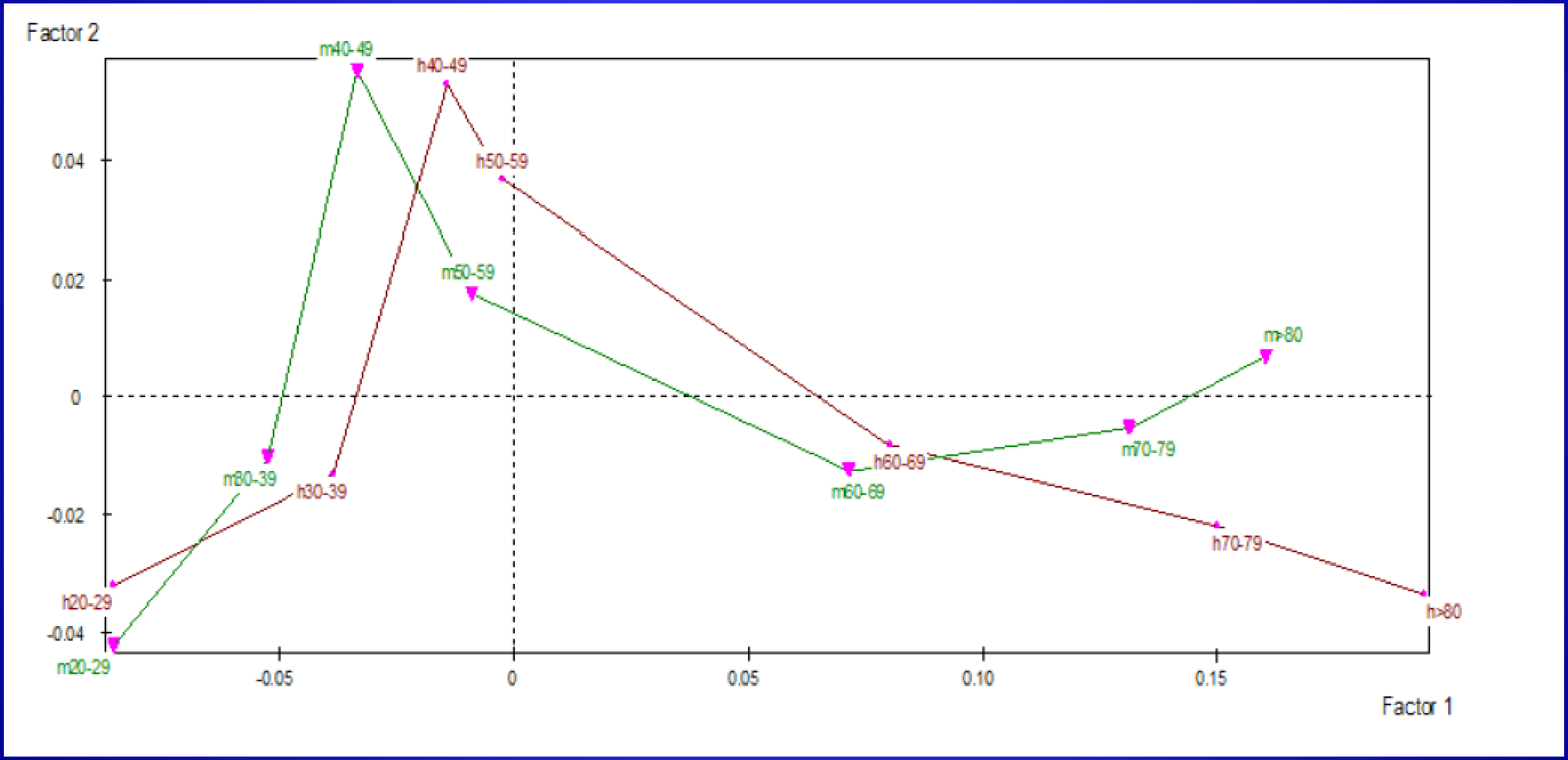
sentences & crimes



Projecting qualitative variables

Respect the following principles:

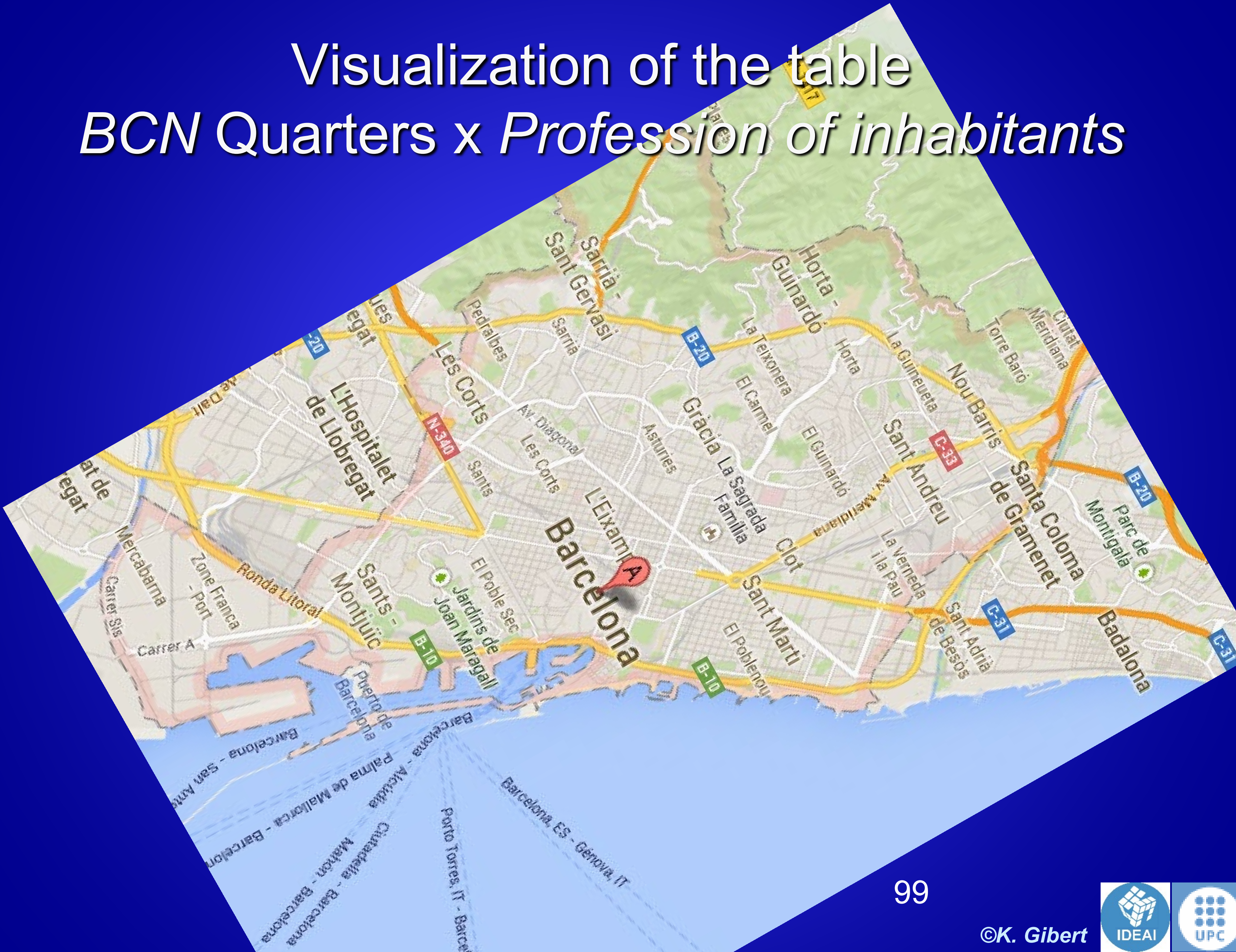
- Choose a diferent color for each qualitative variable
- Use the color of the variable for all centroids corresponding to the modalities of the variable
- Include a legent with the list of variables and associated color
- Ensure that legend to not hide any centroide in the factorial map
- For ordinal variables link modalities with rows in the right order and use the color of the variable for the arrows
- Manipulate the size of the font to guarantee the màximum visibility of the map


Factor 2
m40-49
h40-49
0.04
h50-59
0.02
m50-59
m>80
0
m70-79
h60-69
m60-69
m30-39
h30-39
-0.02
h70-79
h20-29
-0.04
h>80
m20-29
-0.05
0
0.05
0.10
0.15
Factor 1

Efecte guttmann

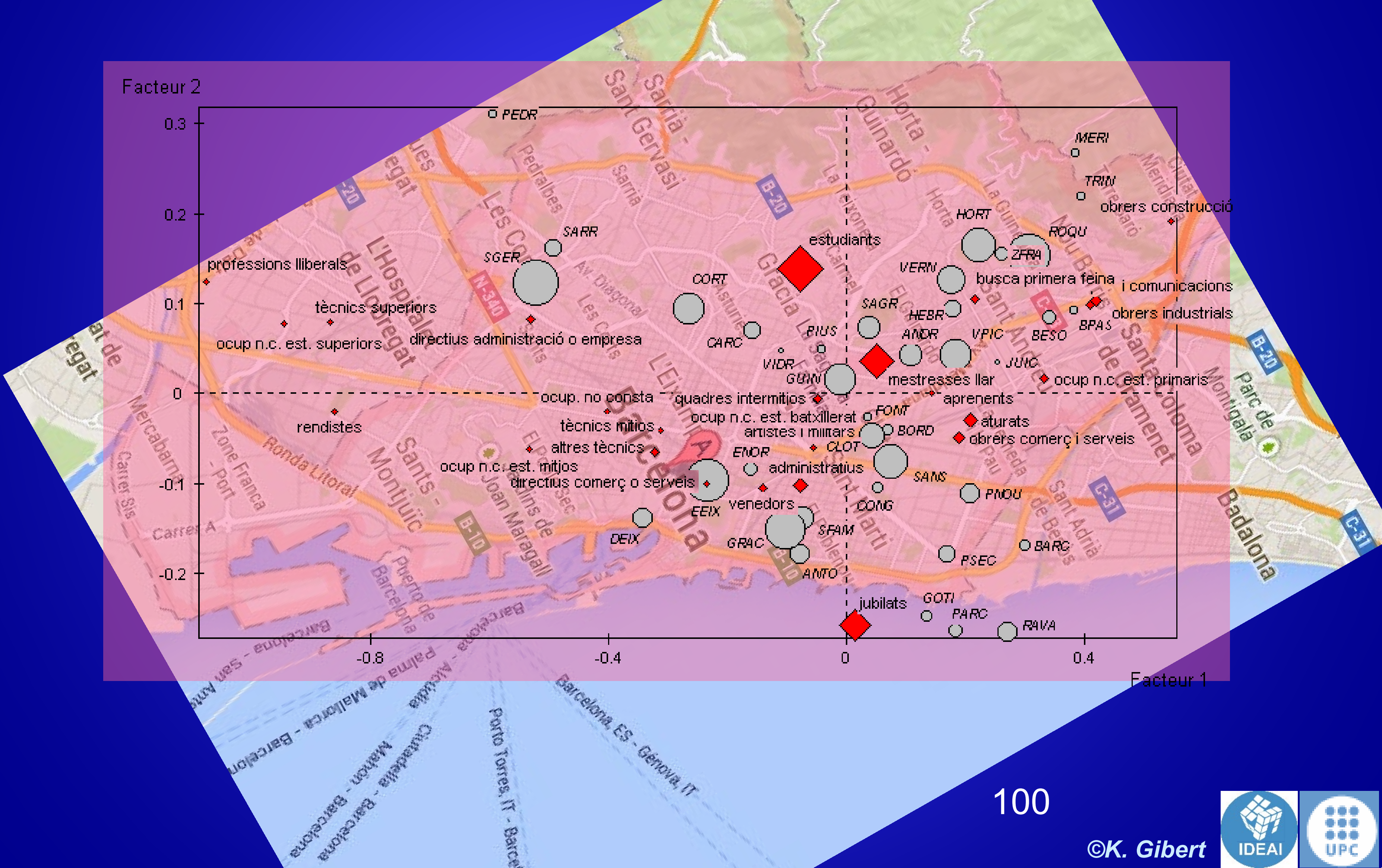
http://www.ugr.es/~gallardo/

Visualization of the table *BCN* Quarters x *Profession of inhabitants*



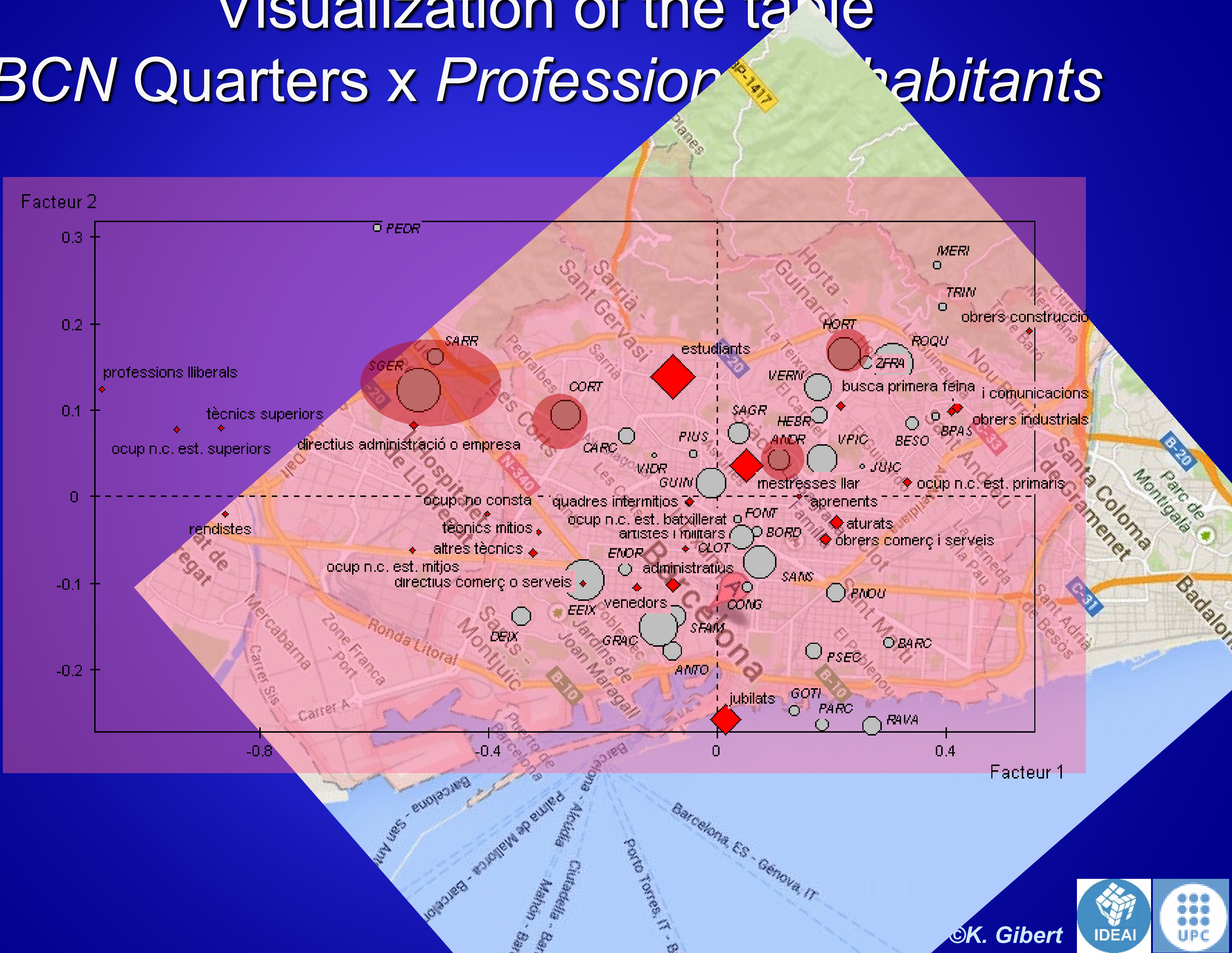
Visualization of the t

BCN Quarters x *Profes* *bitants*



Visualization of the table

BCN Quarters x *Professions inhabitants*



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