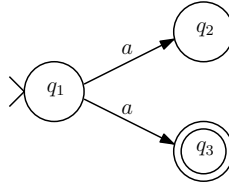


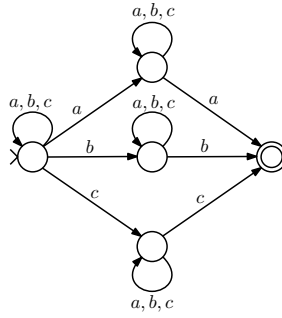
# Introduction to Theoretical Computer Science, Fall 2024

## Assignment 2 Solutions

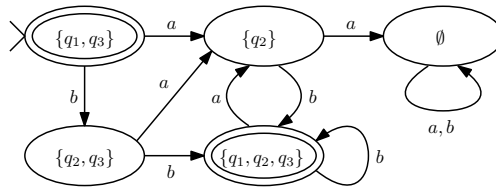
Q1. No. Let  $M$  denotes the following NFA over  $\Sigma = \{a\}$ , then  $L(M) \cap L(M') = \{a\} \neq \emptyset$ .



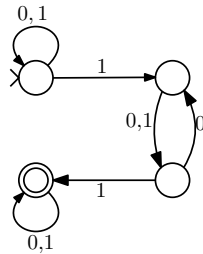
Q2. The NFA guesses at the last symbol of  $w$ .



Q3. The DFA is as follows.



Q4. Whenever a 1 is read, it may or may not be the first 1 in the pair. The NFA makes a guess.



Q5. (a) Let  $A = \{(p, q) : \delta(p, a) = q \text{ for some } a \in \Sigma\}$ . We construct a NFA as  $M' = (K, \Sigma, \Delta, s, F)$ , where  $\Delta = \{(p, a, q) : (p, q) \in A, a \in \Sigma\}$ .

Basically, if the DFA  $M$  can go from state  $p$  to state  $q$  when it reads some symbol in  $\Sigma$ , then the NFA  $M'$  can go from state  $p$  to state  $q$  when it reads any symbol in  $\Sigma$ . One can see that

if some string  $u$  is accepted by  $M$ , then any string with the same length as  $u$  will be accepted by  $M'$ , and vice versa.

(b) One can observe that  $D = B \cap C$ . Since both  $B$  and  $C$  are regular, by the closure properties,  $D$  is regular.