Introduction to Theoretical Computer Science, Fall 2024
Quiz 2 Solutions

Q1. The following CFG generates A.

$$S \to AB$$
$$B \to B0|B1|e$$
$$A \to 0A0|1A1|\#B$$

Q2. We give a reduction from ALL_{PDA} to EQ_{PDA}. Let P be a PDA. Let P^* be a PDA with $L(P^*) = \Sigma^*$. P^* can be easily constructed. Then we have that "P" $\in \text{ALL}_{PDA}$ if and only if "P" "P^*" $\in \text{EQ}_{PDA}$. This completes the reduction.

Q3. Suppose that A is recursively enumerable. Let M_A be the TM that semidecides A. We shall use M_A to construct a TM M_H that halts on H_{TM}. Let x be an arbitrary string. Note that either $x \in H_{TM}$ or $x \in \overline{H_{TM}}$. Or equivalently, either $0x \in A$ or $1x \in A$. Therefore, M_A halts on exactly one of $0x$ and $1x$. We can run M_A on $0x$ and $1x$ in parallel, and see on which input it halts. If M_A halts on $0x$, then $x \in H_{TM}$. If M_A halts on $1x$, then $x \notin H_{TM}$.