Introduction to Theoretical Computer Science, Fall 2024
Assignment 8 Solutions

Q1. Since B is recursive enumerable, there is a Turing machine M_B that semidecides B. We can construct the following Turing machine M_A.

M_A = on input x:
1. compute $f(x)$
2. run M_B on $f(x)$
3. accepts x if M_B accepts $f(x)$

M_A accepts x if and only if M_B accepts $f(x)$ if and only if $f(x) \in B$ if and only if $x \in A$. So M_A semidecides A.

Q2. Let f be a reduction from A to B. By definition, for any $x \in \Sigma^*$, $x \in A$ if and only if $f(x) \in B$. In other words, $x \in \overline{A}$ if and only if $f(x) \in \overline{B}$. So f is also a reduction to $\overline{A}$ to $\overline{B}$.

Q3. Since A is recursively enumerable, there is a Turing machine M_A that semidecides A. Let w be a string. Construct a Turing machine M_w as follows.

M_w = on input x:
1. run M_A on w
2. if M_A accepts w
3. accept x
4. else
5. looping

One can see that M_w halts on any input if and only if M_A accepts w. We can let x be an arbitrary sting, say e. Then $w \in A$ if and only if "M_w"$e \in H_{\mathrm{TM}}$.