Introduction to Theoretical Computer Science, Fall 2024
Assignment 6 Solutions

Q1. (a) True. Every Turing machine semidecides exactly one language, which is L(M).
(b) False. If a Turing machines does not always halt, then it does not decides any language.

Q2. Since L is a recursive language, it is decided by some Turing machine $M = (K, \Sigma, \delta, s, \{y, n\})$. We can obtain a Turing machine that decides $\overline{L}$ by exchanging the role of y and n, so $\overline{L}$ is recursive.

Q3. (a) $A_w = \{w : D \text{ accepts } w\}$

(b) Yes. A_w can be decided by the the following Turing machine.

$M =$ on input w :
1. run D on w
2. if D accepts w
3. accept w
4. else
5. reject w

(c) Note that $A_w = L(D)$. Since D is an arbitrary DFA, it follows that any regular language is recursive.

Q4. Suppose Turing machine M decides EQ_{DFA}. We construct a Turing machine M' that decides A_L as follows.

$M' =$ on input "D" :
1. construct a DFA D_0 with $L(D_0) = L$
2. run M_{EQ} on "D" "D_0"
3. output the result

The reduction is $f(\text{"}D\text{"}) = \text{"}D\text{"}\,\text{"}D_0\text{"}$ where $L(D_0) = L$.