Functions and Intro to Linear Algebra Practice Questions

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Functions

Basics of functions

1. How can we write AJR's argument into mathematical functions?

   "We exploit differences in European mortality rates to estimate the effect of institutions on economic performance. Europeans adopted very different colonization policies in different colonies, with different associated institutions. In places where Europeans faced high mortality rates, they could not settle and were more likely to set up extractive institutions. These institutions persisted to the present." (Acemoglu, Johnson, and Robinson. 2001. "The Colonial Origins of Comparative Development: An Empirical Investigation." *American Economic Review*)

Nonlinear functions

Simplify

1. x^0
2. $x^{-2} \times x^3$
3. $y^7 y^6 y^5 y^4$
4. $(b \cdot b \cdot b) \times c^{-3}$
5. $\left(\frac{1}{27b^3}\right)^{1/3}$
6. $\sqrt{x} \times \sqrt[5]{x}$
7. $\ln(5) \cdot \ln(45)$
8. $\ln(x^5 y^3) - \ln \frac{x^3}{y}$
9. Simplify into one term $\ln(3x) - 2\ln(x+2)$

Linear functions

Rewrite the following by taking the log of both sides. Is the result a linear function?

1. $y = \alpha + x_1^{\beta_1} + \beta_2 x_2 + \beta_3 x_3$
2. $y = \alpha \times x_1^{\beta_1} \times x_2^{\beta_2} \times x_3^{\beta_3}$
3. $y = \alpha \times x_1^{\beta_1} \times \frac{x_2^{\beta_2}}{x_3^{\beta_3}}$

Logarithms

Explain how one unit change in the right-hand side variable (L and $\ln(L)$, respectively) leads to changes in the left-hand side variable (U) for each equation.

- $U = 4 + 2L$
- $U = 4 + 2\ln(L)$

Inverse Function

When a function is defined as $f(x) : A \to B$, the inverse function is $f^{-1}(x) : B \to A$. For example, if $f(x) = 2x+3$, then its inverse is $f^{-1}(x) = \frac{x-3}{2}$. Find the inverse of the following functions. $f(x) = 8x - 5$ $f(x) = 2/(x-4)$ $f(x) = \sqrt{(x+3)}$

Finding equilibrium

1. Find equilibrium P^*, Q^*

$$\begin{aligned} 1.Q_1 &= 51 - 3P_1 \\ Q_2 &= 6P_2 - 10 \end{aligned} \qquad \begin{aligned} 2.Q_1 &= 30 - 2P_1 \\ Q_2 &= -6 + 5P_2 \end{aligned}$$

2. We have a more advanced model for national income as follows:

$$\begin{aligned} Y &= C + I + G \\ C &= 7 + 0.4(Y - T) \\ T &= 15 + 0.3Y \end{aligned}$$

where T and t represents taxation amount and its rates, respectively. How many endogenous variables do we have? What are Y^*, T^*, and C^*?

Notation

Find solutions to the following questions

1. When $A = \{A, I, J, K\}, B = \{C, D, J, Q\}$, and $C = \{B, C, E, I\}$, find $A/B \cap C$.
   $A/B \cap C =?$
2. $C = \{2^1, 2^3, 2^4\},\ c_i \in C.\ \prod_{i=1}^{n} c_i =?$
3. When $x \in \{3, 4, 5, 6\}$, and $f(x) = x + 3$,
   $\max_x f(x) =?$
   $\arg\max_x f(x) =?$
4. $x \in \mathbb{R}, f(x) = 19 - (x - 2)^2$.
   $\max_x f(x) =?$
   $\arg\max_x f(x) =?$

Simplify the following:

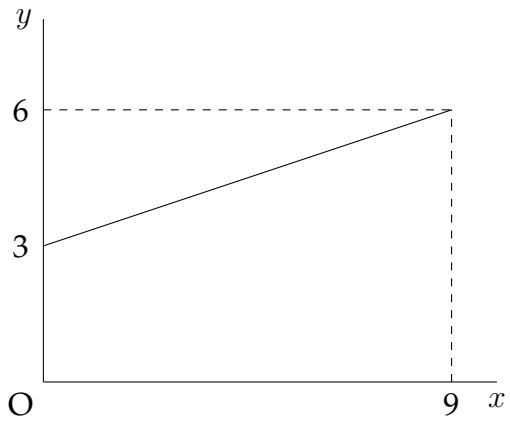
$$\log(\prod_{i=1}^{n} x_i)$$

Show that in general

$$\sum_{i=1}^{m}\prod_{j=1}^{n} x_i y_j \neq \prod_{j=1}^{n}\sum_{i=1}^{m} x_i y_j$$

Linear equations on graphs

Find the intercept and slope of the following graph.



Intro to Linear Algebra I

Basic Vector Algebra

Define $s = 3, t = 1, u = [2, 4, 8], v = [9, 7, 5]$.

1. Calculate $\mathbf{u} \cdot \mathbf{v}$
2. Calculate $\mathbf{u} \cdot \mathbf{u}'$
3. Calculate $(s + t)(\mathbf{u} + \mathbf{v})$.

1. Practice: calculate vector norms

$$\begin{bmatrix} 3 \\ 7 \end{bmatrix} \qquad \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix} \qquad \begin{bmatrix} 5 \\ -2 \\ 6 \end{bmatrix}$$

2. Which vector is longer, $\begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$ or $\begin{bmatrix} 5 \\ -2 \\ 6 \end{bmatrix}$?

Matrix calculations

1. Given the following matrices, perform the calculations below. (Some of the calculations cannot be performed.)

$$A = \begin{bmatrix} 5 & 1 & 2 \\ 6 & 2 & 3 \end{bmatrix}, B = \begin{bmatrix} 3 & 4 & 5 \\ -2 & -3 & 6 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 \\ -5 & 3 \\ -3 & 1 \end{bmatrix}, D = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}, E = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

   a) $A + B$

   b) $A - C$

   c) $A + 5B$

   d) $3A$

   e) $B^T - C$

   f) BA

   g) AC

   h) DB

   i) DE

2. Given the following matrices and their dimensions, calculate the dimensions after matrix multiplication. $\underset{3\times2}{A}, \underset{2\times4}{B}, \underset{5\times2}{C}, \underset{4\times3}{D}$

   a) AB

   b) BD

   c) BC

   d) DA

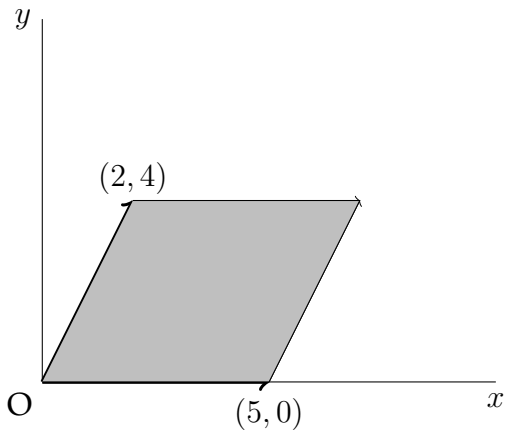
   e) CA^T

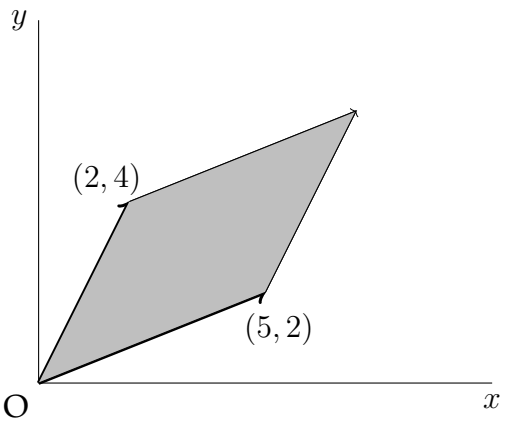
f) $B^T C^T$

g) $D^T B^T$

Determinant and Inverse of a matrix

1. Calculate the size of the shaded area.





Now consider the following two matrices: $\begin{bmatrix} 5 & 2 \\ 0 & 4 \end{bmatrix}$ and $\begin{bmatrix} 5 & 2 \\ 2 & 4 \end{bmatrix}$. Calculate determinants of each matrix. Are they similar to what you calculated above?

2. Show that when $A = \begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}$, $\quad A \cdot A^{-1} = A^{-1} \cdot A = I$

Linear systems of equations

1. Let's flip this page and go to the Finding Equilibrium question on page 3. We have two sets of equations in Q1. Can you rewrite them in matrix form?

2. Can you find solutions to those equations?

3. Rewrite the following linear systems of equations into matrix form. (No need to solve them.)

$$\begin{aligned} x + y + 2z &= 2 \\ 3x - 2y + z &= 1 \\ y - z &= 3 \end{aligned} \qquad \begin{aligned} 2x + 3y - z &= -8 \\ x + 2y - z &= 2 \\ -x - 4y + z &= -6 \end{aligned} \qquad \begin{aligned} x - y + 2z &= 2 \\ 4x + y - 2z &= 10 \\ x + 3y + z &= 0 \end{aligned}$$

4. For the following expression on the relationship between political blame and regional political variables, simplify it in matrix algebra form.

$$\begin{aligned} Y_i = {} & \beta_0 + \beta_1 CHANGELIV + \beta_2 BLAMECOMM + \beta_3 INCOME \\ & + \beta_4 FARMER + \beta_5 OWNER + \beta_6 BLUESTATE \\ & + \beta_7 WHITESTATE + \beta_8 FORMMCOMM + \beta_9 AGE \\ & + \beta_{10} SQAGE + \beta_{11} SEX + \beta_{12} SIZEPLACE \\ & + \beta_{13} EDUC + \beta_{14} FINHS + \beta_{15} ED * HS \\ & + \beta_{16} RELIG + \beta_{17} NATION + E_i, \text{ for } i\text{=1 to } n \end{aligned}$$