A Mean Field Stoner–Wohlfarth Hysteresis Model

DAVID L. ATHERTON, SENIOR MEMBER, IEEE, AND J. R. BEATTIE

***Abstract*—A model of magnetic hysteresis is proposed. It is based on the Stoner–Wohlfarth coherent rotation model, but with a macroscopic mean field interaction, similar to Weiss's molecular field theory, added to represent interaction between domains. Rotation of domain magnetization vectors gives both reversible and irreversible changes in bulk magnetization. A qualitative correlation is obtained between hysteresis curves calculated using this theoretical model and experimentally measured hysteresis curves of a ferrite permanent magnet.**

INTRODUCTION

THE phenomenon of magnetic hysteresis is very complex, and it is not yet fully understood despite the many attempts which have been made to develop a mathematical model of hysteresis effects in ferromagnetic and ferrimagnetic materials. Many different approaches have been undertaken. For example, Stoner and Wohlfarth [1], [2, pp. 333–341] postulated a theory based on the rotation of the magnetic moments of single-domain particles with respect to their easy axes. Consideration of these rotations results in both reversible and irreversible changes in magnetization. However, a shortcoming of this model is that no interaction between domains is considered.

Jiles and Atherton [3] have developed a theory in which pinning sites (i.e., inclusions, voids, crystal boundaries, and lattice defects) inhibit domain wall motion. This theory is most helpful for describing the behavior of materials in which domain motion is the primary hysteresis mechanism.

Another competing mathematical description is the Preisach model [4], [5], which assumes a ferromagnet to consist of many small magnetic domains, each with its own characteristic hysteresis loop. These loops are square, but the positive and negative transition points of the loop are different for each domain. The interaction between domains is accounted for by offsetting the switching points of the domains by fixed amounts. This makes the interaction static and independent of magnetization. All the various types of hysteresis curves, including minor hysteresis loops, can readily be generated by considering a distribution of these domains. The model is primarily of interest in magnetic recording applications where the particles on the recording medium have squarish hysteresis loops and are spread out in a thin layer, thus minimizing their interaction.

Many other models of hysteresis exist, but micromagnetics approaches [6]–[9] generally require too much computing power to be useful for calculating hysteresis curves for bulk ferromagnetic materials, while most mathematical curve fitting approaches are too empirical [10]–[12] and have little or no physical basis underlying them. In addition to these time-independent models, there are interesting new differential equation rate-dependent models by Hodgdon [13] and Chua-type models by Saito *et al.* [14].

The Stoner–Wohlfarth model appears to be the most promising time-independent model on two grounds: first, it easily takes into account anisotropies, and second, it is inherently a three-dimensional model. This is in contrast with the Jiles–Atherton theory and the Preisach model, both of which can be extended to three dimensions only with considerable effort [15], [16].

STONER-WOHLFARTH THEORY

In the Stoner–Wohlfarth model, it is assumed that a magnetic material consists of a collection of small particles, each with anisotropy due to either stress, crystal structure, or particle shape. Each particle is uniformly magnetized to saturation, giving a single magnetic domain with moment m which is free to rotate in any direction. No interaction, either due to quantum exchange forces or to magnetic dipole–dipole forces, is considered. Each particle rotates to the orientation which results in a minimum energy. Fig. 1 defines the variables used. In small fields, the particle magnetization is only slightly perturbed from the easy axis, and there are two energy minima, designated by m_1 and m_2 in Fig. 1. As the magnetic field increases, the positions of these energy minima change. Initially, these changes are fully reversible. However, once a certain critical field H_c is exceeded, one of the energy minima becomes unstable, and the domain magnetization rotates suddenly to the other minimum, which is the global energy minimum. This constitutes an irreversible magnetization change. Reducing the field will not return the magnetic moment to its previous orientation.

The total energy of a single domain with saturation magnetic moment m_s is $E = -\mu_0 H m_s \cos(\psi - \theta) + k_u \sin^2\theta$ where k_u, the domain crystal anisotropy constant, is the product of the customary specific anisotropy con-

Manuscript received September 18, 1989; revised May 29, 1990. This work was supported by the Natural Sciences and Engineering Research Council of Canada.

The authors are with the Department of Physics, Queen's University, Kingston, Ont., Canada K7L 3N6.

IEEE Log Number 9038058.

0018-9464/90/1100-3059$01.00

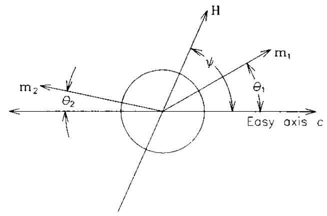

Fig. 1. Variables for a Stoner-Wohlfarth particle.

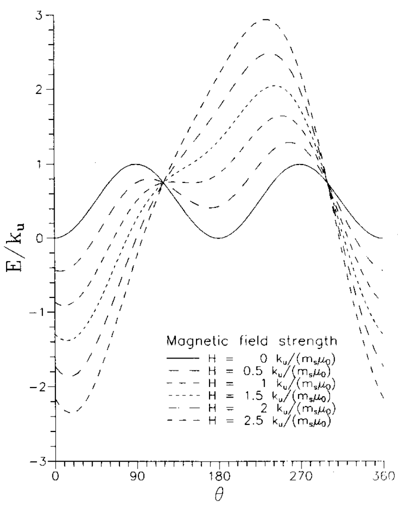

Fig. 2. Energy of a single Stoner-Wohlfarth particle as a function of θ, the angle between m and the easy axis c. The angle ψ, between H and the easy axis c, was 30° for this example.

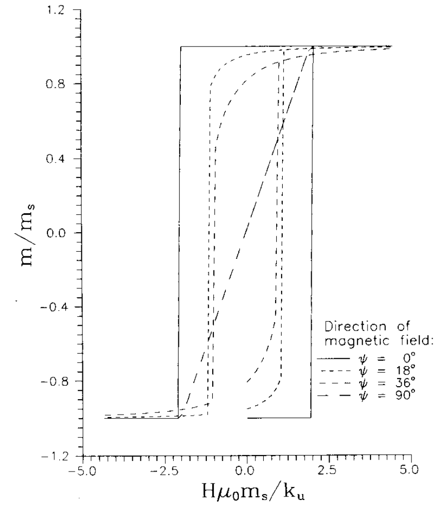

Fig. 3. Hysteresis curves for single Stoner-Wohlfarth particles with various orientations of the easy axis with respect to H.

stant K_u and the domain volume. The angular variables are defined in Fig. 1. Fig. 2 shows the energy of a Stoner-Wohlfarth domain in magnetic fields of various intensities. The positions of minimum energy are easily seen. These can be calculated for any particular (ψ, H) by finding the zeros of $\partial E(\theta, \psi, H)/\partial\theta$ for which $\partial^2 E(\theta, \psi, H)/\partial\theta^2 > 0$. The critical point at which irreversible domain rotation occurs is the point of minimum energy for which $\partial^2 E(\theta, \psi, H)/\partial\theta^2 = 0$.

Fig. 3 shows hysteresis loops for several individual Stoner-Wohlfarth domains with different angles between the easy axis and applied magnetic field. The magnetization shown is the component of the magnetization parallel to the magnetic field: $m_s \cos(\psi - \theta)$.

The hysteresis of a bulk material is modeled by a collection of these Stoner-Wohlfarth domains. The magnetization is the vector sum of the contributions of all of the constituent domains. M_H, the component of magnetization aligned with the field, is

$$M_H = \frac{1}{V}\int_0^{2\pi}\int_0^{\pi} m_s \cos[\psi - \theta(\psi, H)] \cdot \rho(\phi, \psi)\sin(\psi)\, d\psi\, d\phi \qquad (1)$$

where

$$\int_0^{2\pi}\int_0^{\pi} \rho(\phi, \psi)\sin(\psi)\, d\psi\, d\phi = 1. \qquad (2)$$

$\theta(\psi, H)$ is the angle of the Stoner-Wohlfarth domain at field H with angle ψ between c and H, V is the sample volume, and $\rho(\phi, \psi)$ is the easy axis distribution in spherical coordinates (ϕ, ψ).

With symmetry in ϕ, i.e., about the H axis, $(\phi, 0)$:

$$M_H = \frac{2\pi}{V}\int_0^{\pi} m_s \cos[\psi - \theta(\psi, H)]\,\rho(\psi)\sin(\psi)\, d\psi \qquad (3)$$

and all other components of M are zero; thus, $M = M_H$. Alternatively, the vector magnetization can be computed by components in more complex cases.

Typically, the easy axes of the domains are taken to be uniformly distributed in all directions. However, for modeling a material which has a macroscopic anisotropy, a nonuniform distribution of easy axis orientations is necessary.

The Stoner-Wohlfarth theory has several potential advantages: a single mechanism generates both reversible and irreversible changes in magnetization, it is naturally three dimensional, and it easily accounts for anisotropy. Furthermore, it is computationally simple enough so that it can now be readily solved on a PC AT. However, it also has potential flaws: the interaction between domains is ignored, each particle must consist of a single, uniformly magnetized domain, and pinning effects are ignored. For those types of materials which have a small grain size, the assumption that all particles are single domain in reasonable. However, in many ferromagnetic and ferrimagnetic materials, the interaction between domains is important, and it cannot be neglected. The addition of a macroscopic mean field interaction offers a means of correcting this shortcoming.

Mean Field Theory

One of the earliest explanations of ferromagnetism is due to Weiss [2, pp. 119–121], who postulated the existence of a molecular field γM which caused the molecular magnetic moments to align. The total effective field experienced by a molecular moment is therefore $H_{\text{eff}} = H_{\text{external}} + \gamma M$. Quantum mechanics has since shown that the interaction between atomic magnetic moments is due to the exchange force. However, at a larger scale, that of individual domains, Jiles and Atherton [3] postulated a mean field interaction $H_m = \alpha M$, which offers a means of approximating the actual interactions. Micromagnetic models [6], [7] compute these interactions more precisely, but these require very large amounts of computing time to solve. A mean field interaction provides a compromise between the accuracy and the speed of the model.

Mean Field Stoner-Wohlfarth Model

Particle interaction is incorporated into the Stoner-Wohlfarth model by the addition of a mean field term. The magnetic field experienced by a particle becomes $H_{\text{eff}} = H + \alpha M$. This modifies the energy of a particle E.

In the symmetric case, H and M are collinear and the components may be treated as scalars. The magnetizatiohn of a bulk material then becomes

$$M = \frac{2\pi}{V} \int_0^{\pi} m_s \cos\left[\psi - \theta(\psi, H + \alpha M)\right] \cdot \rho(\psi) \sin(\psi)\, d_c. \tag{4}$$

This is an implicit equation for M, and it cannot be solved analytically. Instead, numerical methods must be used.

A Pascal program was written to compute hysteresis curves using the Stoner-Wohlfarth model with this mean field interaction. The calculation of a hysteresis loop consists of five parts.

1) For each domain, calculation of the critical field $H_c(\psi)$ and the corresponding critical angle $\theta_c(\psi)$ at which one of the energy minima disappears and the domain magnetization rotates abruptly to the global energy minimum. This need be done only once, during the initialization process. This procedure is described in detail in [1] and [2, pp. 333–341].

2) For each domain, determination of the new minimum energy position $\theta(\psi, H)$ given a previous position θ_{old}. Brent's algorithm [18] is used to find the root of the equation $\partial E(\theta, \psi, H)/\partial\theta = 0$.

3) Approximation of the integration of m over all of the domains using the trapezoidal rule. A more sophisticated integration algorithm could be used in the future.

4) Solution of the implicit equation for M, giving the magnetization after accounting for the mean field effect. Brent's algorithm is used to find the root M of (4).

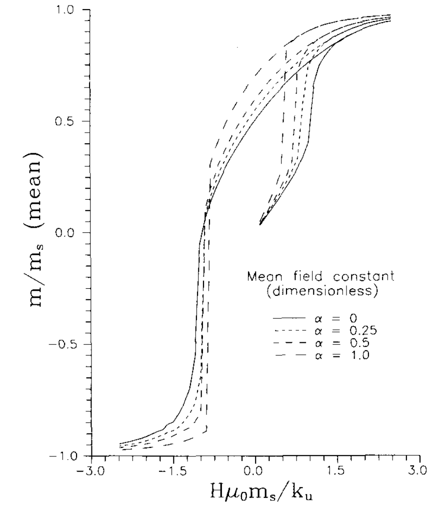

Fig. 4. Hysteresis curves for the mean field Stoner–Wohlfarth model with the particle easy axes uniformly distributed. The range of dimensionless mean field parameter α (not directly related to the Weiss molecular field parameter γ) is larger than normally used for Jiles–Atherton mean field pinning models.

5) Generation of a hysteresis curve by stepping through the values of H. After each small step in H, the magnetization is recomputed using steps 2)–4) above. The previous values of θ for each particle are remembered and used in computing the next value of θ for that particle.

Results

The effects of adding a range of mean field interactions are shown in Fig. 4. A uniform distribution of particle easy axes is used here. Increasing the dimensionless mean field interaction parameter α results in a latching effect, giving a steeper curve near the coercive point. However, although the range of interaction parameters used is much greater than typically used for Jiles–Atherton models, none of these curves compares well to Fig. 5, which shows an experimentally determined hysteresis curve for an Indiana General Indox 5 hard ferrite material. The easy axis of the ferrite sample was aligned in the same direction as the magnetic field.

It is known that in Indox 5, the domains are preferentially aligned. This indicates that a Stoner–Wohlfarth model with a nonuniform particle distribution should provide a more accurate model. As a first approximation, a linear variation of the particle orientation density with the angle ψ was selected, as shown in Fig. 6. The theoretical curve resulting from this nonuniform distribution is shown in Fig. 7, and is much closer to the experimental curve of Fig. 5.

Figs. 8 and 9 show the total and reversible permeabilities of the upper branch of the hysteresis loops for the theory and experiment, respectively. The reversible permeability is simply the slope of a small minor hysteresis loop (a reversal or recoil) made along the major hys-

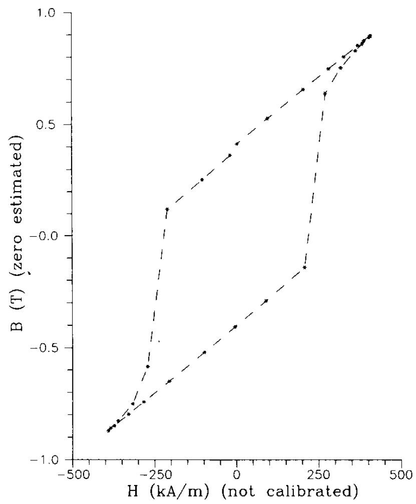

Fig. 5. Experimental major hysteresis loop for Indox 5 Ferrite. The B scale zero was estimated from symmetry because the sample could not be demagnetized before zeroing the integrating fluxmeter. There is a small uncertainty in the H scale occurring in the measurement of the sample's demagnetizing factor.

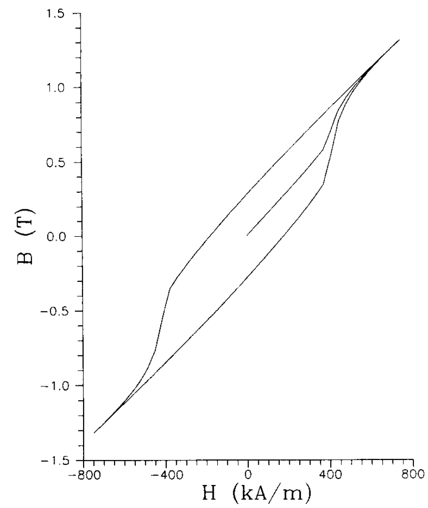

Fig. 7. Hysteresis curves for the mean field Stoner–Wohlfarth model with the particle easy axes distributed as shown in Fig. 6. $K_u = 1.65 \times 10^5$ J/m^3, $\alpha = 10^{-3}$ (dimensionless).

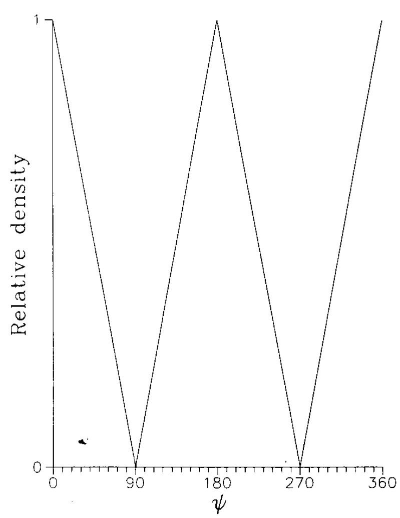

Fig. 6. A nonuniform distribution of the particle easy axes for the mean field Stoner–Wohlfarth model.

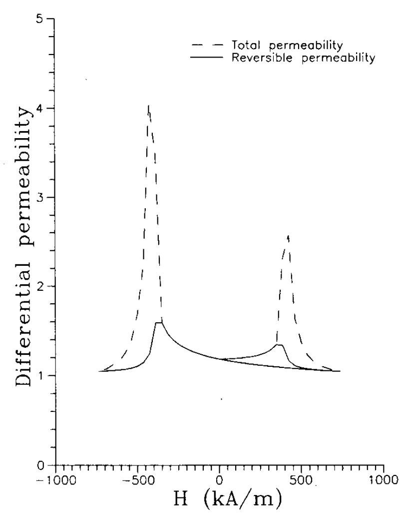

Fig. 8. Differential permeabilities for the mean field Stoner–Wohlfarth model with parameters as for Fig. 7.

teresis loop [18]. Although these graphs are not identical, they are very similar in character. This suggests that the model is qualitatively substantially correct, but requires some further small adjustments, in particular, to the easy axes distribution, in order to obtain a better quantitative correlation.

A final point of interest concerns the behavior when a diminishing amplitude ac field is applied. With zero bias field, this tends to demagnetize the majority of ferromagnets [2, pp. 20–21], while, with a nonzero bias, an anhysteretic point is obtained.

The Stoner–Wohlfarth model precludes demagnetization. When in the demagnetized state, for each value of ψ, there are two identical minimum energy orientations, both of which will be occupied. However, once $H_c(\psi)$ has been exceeded, all these domains will be aligned in the same direction, and they will remain synchronized after H is lowered below $H_c(\psi)$. These domain magnetizations will no longer cancel each other, and it then becomes impossible to return the domain ensemble to the initial demagnetized state, although nondemagnetized $(M, H) = (0, 0)$ states are still attainable using asymmetric hysteresis loops.

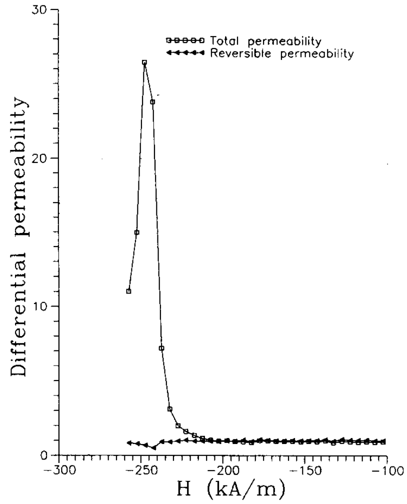

Fig. 9. Measured differential permeabilities for the Indox 5 ferrite.

This very behavior was observed in the Indox 5 ferrite sample. Once magnetized, it was not possible to return the sample to the initial demagnetized state in the normal way. Instead, the sample must be raised above the Curie point and then cooled [19]. This corresponds to returning the domains to the original starting positions in the Stoner–Wohlfarth model.

Conclusion

The mean field Stoner–Wohlfarth model offers a method by which the Stoner–Wohlfarth model can be extended to include the interaction between domains. Although the macroscopic mean field interaction used is not strictly correct, it is sufficient for the purposes of providing a workable model of hysteresis.

Furthermore, this model exhibits behavior which is similar to that observed in a sample of Indox 5 ferrite permanent magnet material. This suggests that in a hard ferrite magnetic material such as the Indox 5 studied, domain rotation may be the dominant mechanism.

References

[1] E. C. Stoner and E. P. Wohlfarth, "A mechanism of magnetic hysteresis in heterogeneous alloys," *Phil. Trans. Roy. Soc.*, vol. 240A, pp. 599–642, May 4, 1948.

[2] B. D. Cullity, *Introduction to Magnetic Materials*. Reading, MA: Addison-Wesley, 1972.

[3] D. C. Jiles and D. L. Atherton, "Theory of ferromagnetic hysteresis," *J. Magn. Magn. Mater.*, vol. 61, pp. 48–60, 1986.

[4] I. D. Mayergoyz, "Vector Preisach hysteresis models," *J. Appl. Phys.*, vol. 63, pp. 2995–3000, 1988.

[5] R. M. DelVecchio, "Analytical representation of arbitrary hysteresis paths in magnetic materials," Westinghouse R&D Cen., Pittsburgh, PA, Res. Rep. 78-ID5-MTEST-R1.

[6] W. Williams and D. J. Dunlop, "Three dimensional micromagnetic modelling of ferromagnetic domain structure," *Nature*, vol. 337, pp. 634–637, Feb. 16, 1989.

[7] J.-G. Zhu and H. N. Bertram, "Micromagnetic studies of thin metallic films," *J. Appl. Phys.*, vol. 63, pp. 3248–3253, Apr. 15, 1988.

[8] D. R. Fredkin and T. R. Koehler, "Numerical micromagnetics of small particles," *IEEE Trans. Magn.*, vol. 24, pp. 2362–2367, 1988.

[9] R. Klein and J. J. Niez, "A new method for finite element calculation of micromagnetic problems," *IEEE Trans. Magn.*, vol. 24, pp. 2371–2373, 1988.

[10] J. Rivas, J. M. Zamarro, E. Martin, and C. Pereira, "Simple approximation for magnetization curves and hysteresis loops," *IEEE Trans. Magn.*, vol. MAG-17, pp. 1498–1502, 1981.

[11] J. R. Koehler, "A computationally fast, two dimensional vector hysteresis model," *J. Appl. Phys.*, vol. 61, pp. 1568–1575, Feb. 15, 1987.

[12] G. F. Dionne, J. A. Weiss, and G. A. Allen, "Hysteresis loops modeled from coercivity, anisotropy, and microstructure," *J. Appl. Phys.*, vol. 61, pp. 3862–3864, Apr. 15, 1987.

[13] M. L. Hodgdon, "Mathematical theory and calculations of magnetic hysteresis curves," *IEEE Trans. Magn.*, vol. 24, pp. 3120–3122, 1988.

[14] Y. Saito, K. Fukushima, S. Hayano, and N. Tsuya, *IEEE Trans. Magn.*, vol. MAG-23, pp. 2237–2229, 1987.

[15] I. D. Mayergoyz, "Mathematical models of hysteresis," *IEEE Trans. Magn.*, vol. MAG-22, pp. 603–608, 1986.

[16] ——, "Dynamic Preisach models of hysteresis," *IEEE Trans. Magn.*, vol. 24, pp. 2928–2930, 1988.

[17] R. P. Brent, *Algorithms for Minimization without Derivatives*. Englewood Cliffs, NJ: Prentice-Hall, 1973.

[18] D. L. Atherton and M. Schönbächler, "Measurements of reversible magnetization component," *IEEE Trans. Magn.*, vol. 24, pp. 616–620, 1988.

[19] L. R. Moskowitz, *Permanent Magnet Design and Application Handbook*. Boston, MA: Cahners Books Int., 1976.

David L. Atherton (M'64–SM'83) received the B.A. and M.A. degrees from Cambridge University, Cambridge, U.K.

He came to Canada in 1959 and joined Ferranti Packard Electric in Toronto, becoming Head of Superconductivity Research. He moved to Queen's University in 1971 where he is Professor of Physics. He has published more than 130 papers principally on applied superconductivity and magnetics. His current research is concentrated on magnetic inspection techniques for pipelines, the remote field eddy current technique, ferromagnetic hysteresis theory, and the effects of stress on the magnetization of steel. He has acted as a specialist consultant on magnetics to many international companies.

J. R. Beattie, biography not available at the time of publication.