

# Lecture 3: Supervised Learning

## Machine Learning (BBWL)

Michael Mommert, University of St. Gallen

# Today's lecture

Supervised learning setup

Supervised learning concepts

Benchmarking and metrics

Linear models

Nearest Neighbor models

Tree-based models

# Supervised Learning setup

# Supervised Learning setup

## General goal for supervised problems:

Find a function ("task") that relates input data ( $\mathbf{x}$ ) to output data ( $\mathbf{y}$ ) with hyperparameters ( $\theta$ )

such that:  $f(\mathbf{x}; \theta) = \mathbf{y}$

A **hyperparameter** is a model parameter that the model not learns.

# Supervised Learning setup

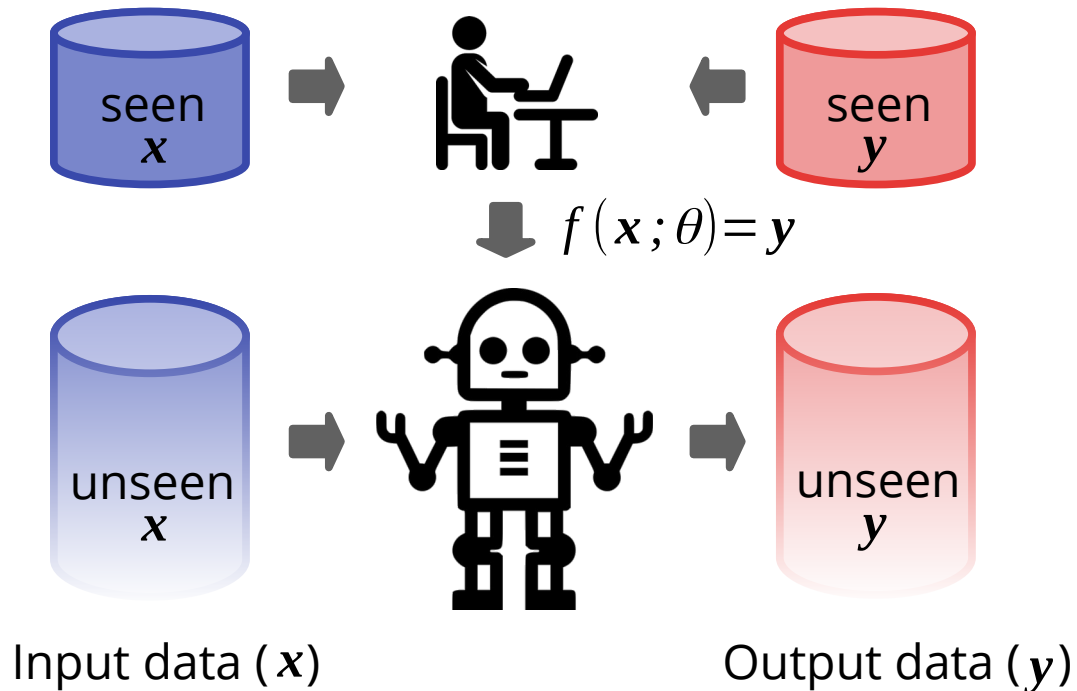
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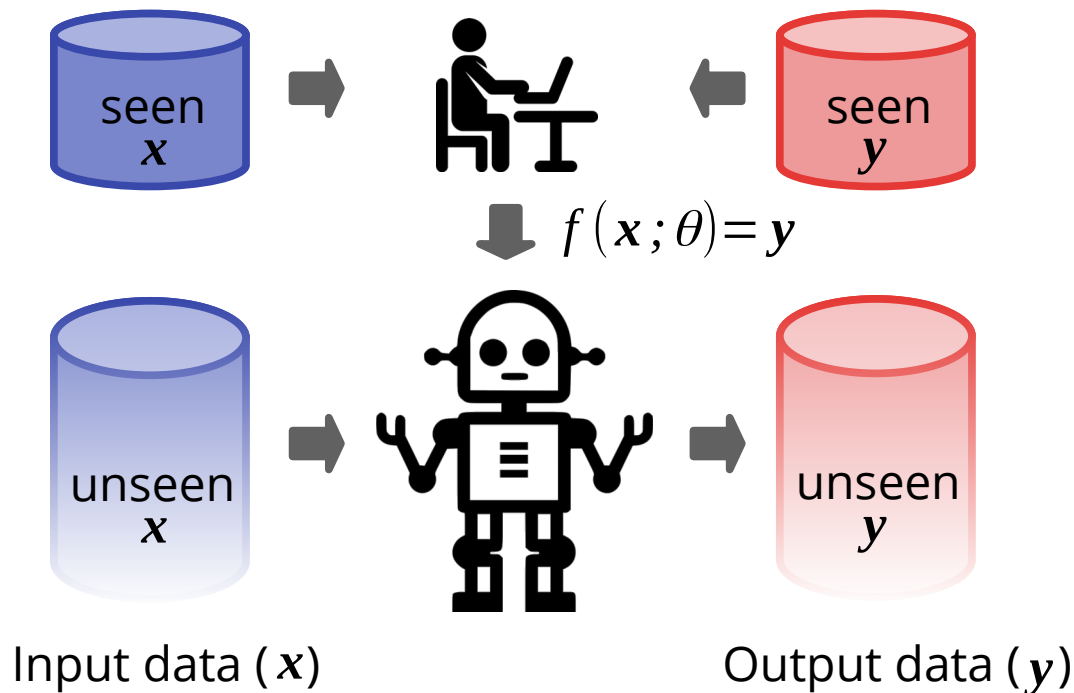
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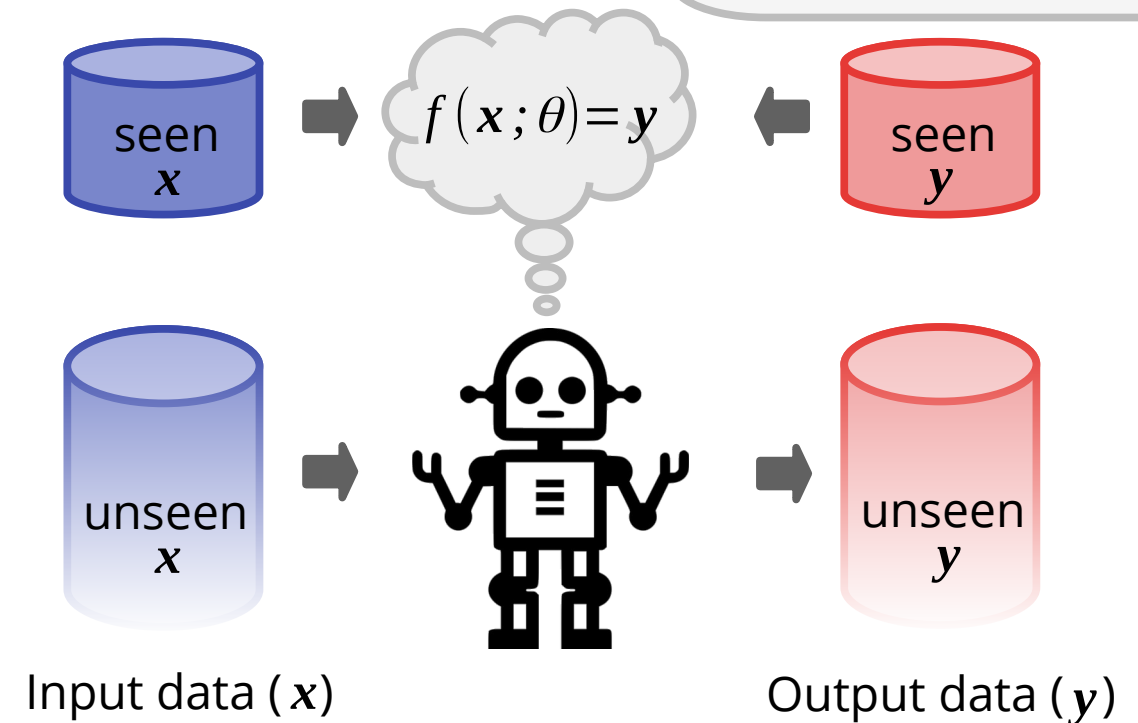
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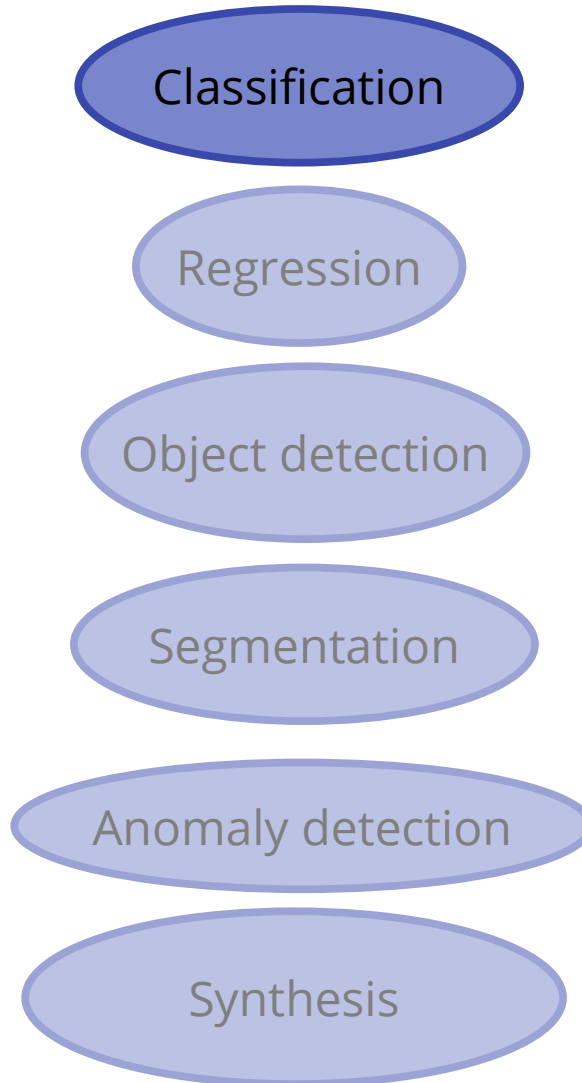
## Traditional (Rule-based) Approach:



## Machine-Learning Approach:



# What tasks can ML learn?



# What tasks can ML learn?

## (Multi-class) Classification:

Mapping input features to discrete classes of a single label

*Example:*

label	<b>Color</b>
classes {	red
	green
	blue

Classification

Regression

Object detection

Segmentation

Anomaly detection

Synthesis



# What tasks can ML learn?

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## Binary classification:

Mapping input features to a binary label

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label	<b>Status</b>
two classes {	on
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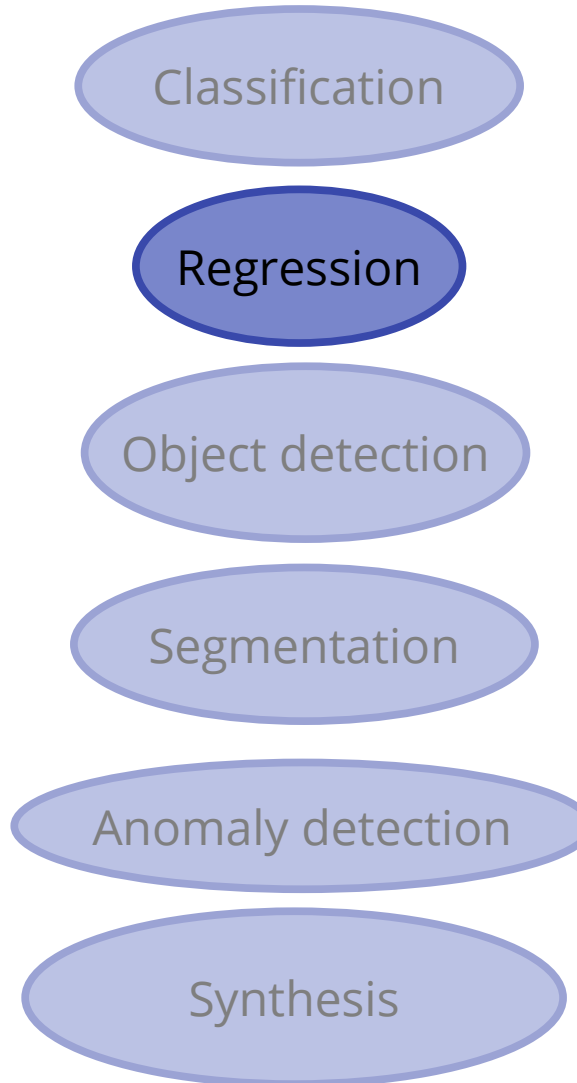
## Multi-label classification:

Mapping input features to discrete classes of multiple labels

*Example:*

labels	<b>Color</b>	<b>Sort</b>	<b>Quality</b>
classes {	red	A	good
	green	B	medium
	blue	C	bad
	...	...	...

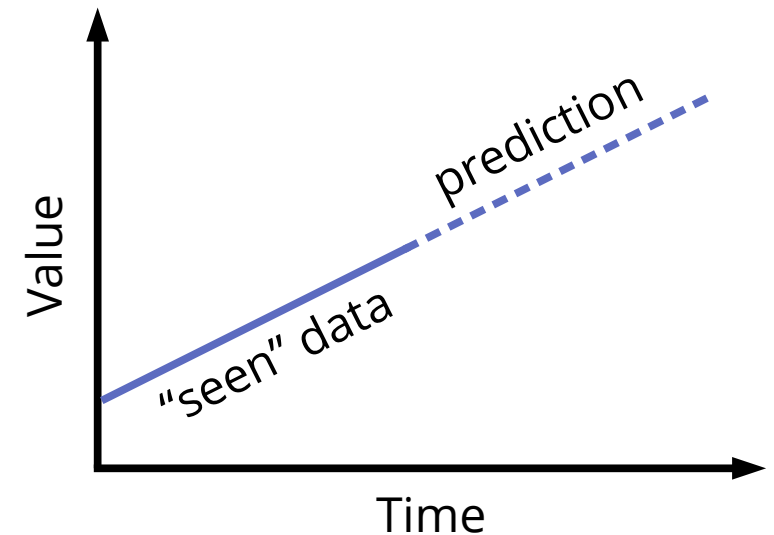
# What tasks can ML learn?



## **Regression:**

Mapping input features to continuous variable

*Example:*



# What tasks can ML learn?

## Object detection:

Approximately localize features in image data with bounding boxes

*Example:*



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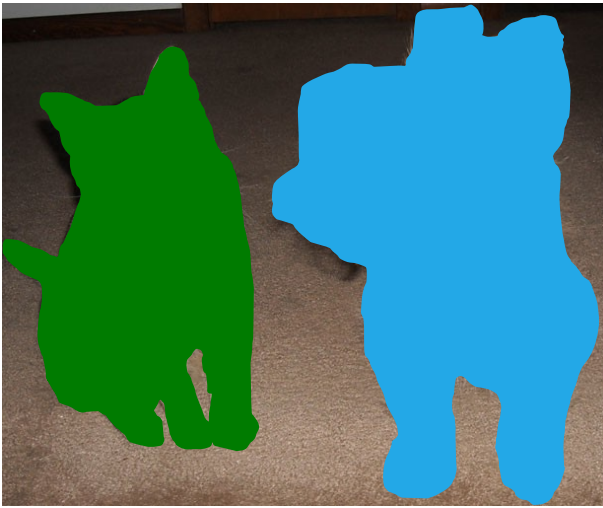
Synthesis

# What tasks can ML learn?

## Semantic segmentation:

Assign class label to each pixel of an image based on what it is showing

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## Instance segmentation:

Assign class label to each pixel of an image based on what it is showing and discriminate different instances of the class

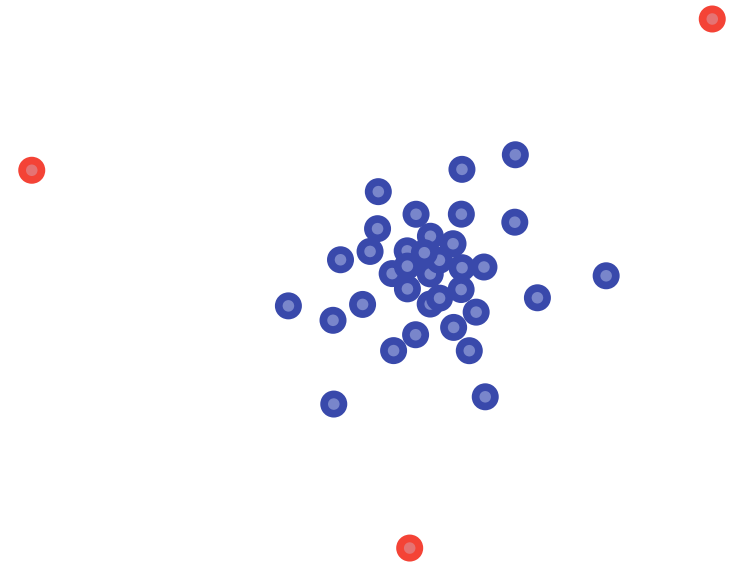
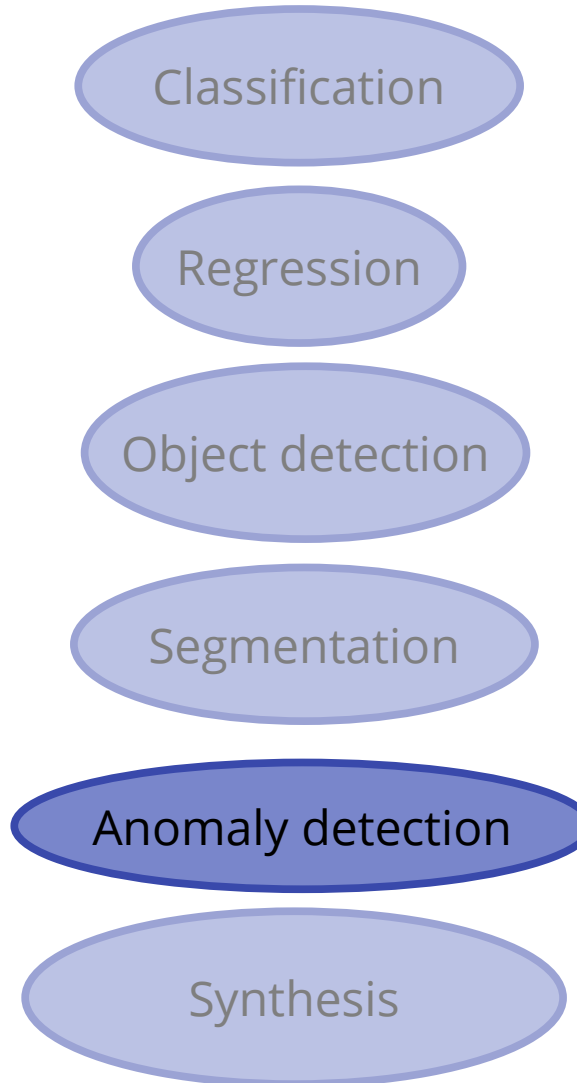
*Example:*



# What tasks can ML learn?

## Anomaly detection:

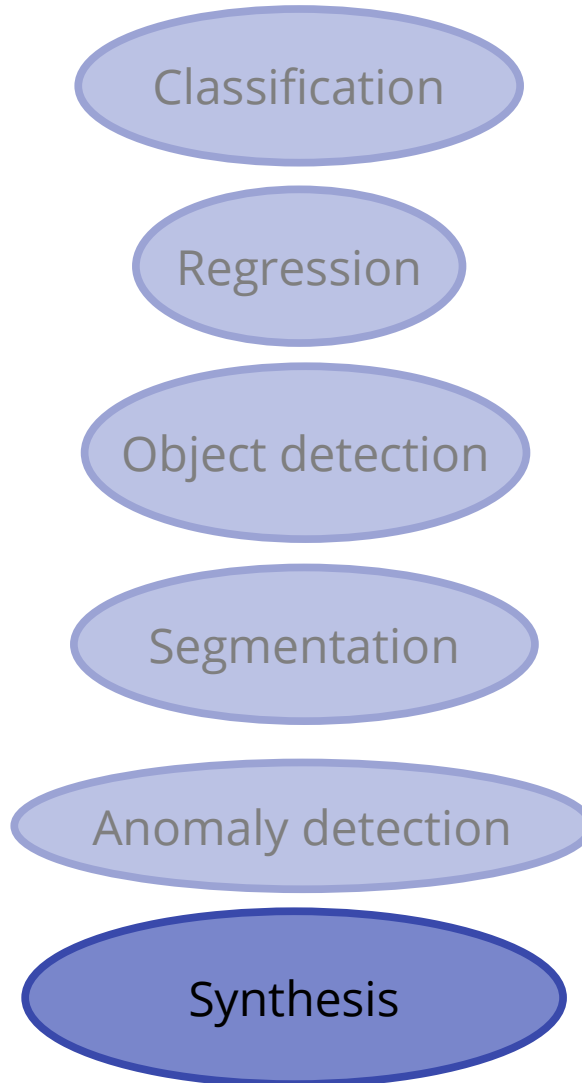
Identify anomalous or unusual data points within a data set



# What tasks can ML learn?

## Synthesis:

Generate new data points based on a learned distribution



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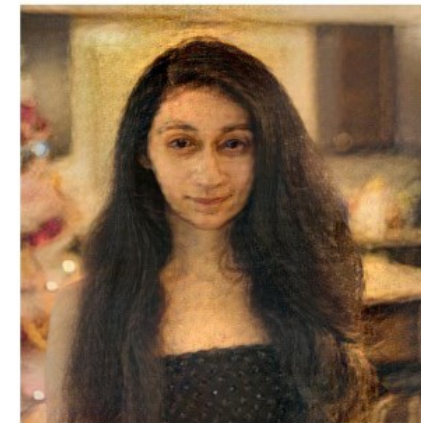
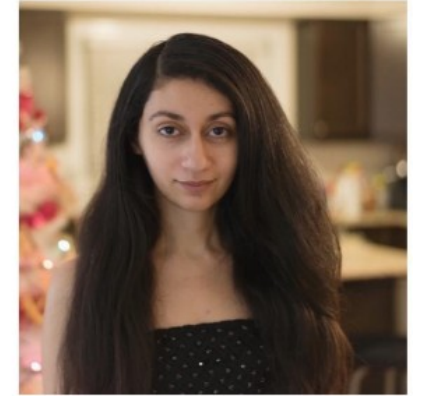
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Style Transfer (Gatys et al. 2016)



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StyleGAN2 (Karras et al. 2020)

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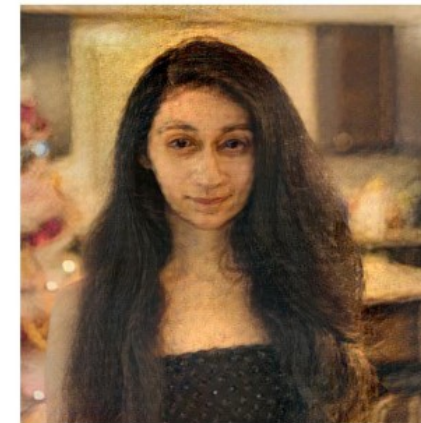
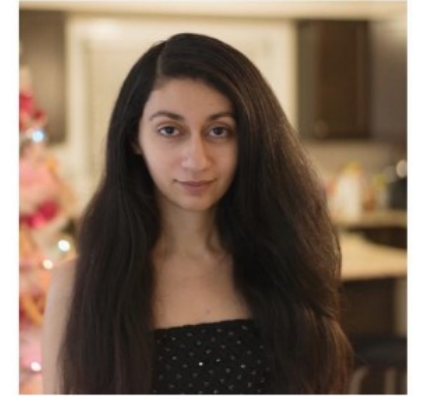
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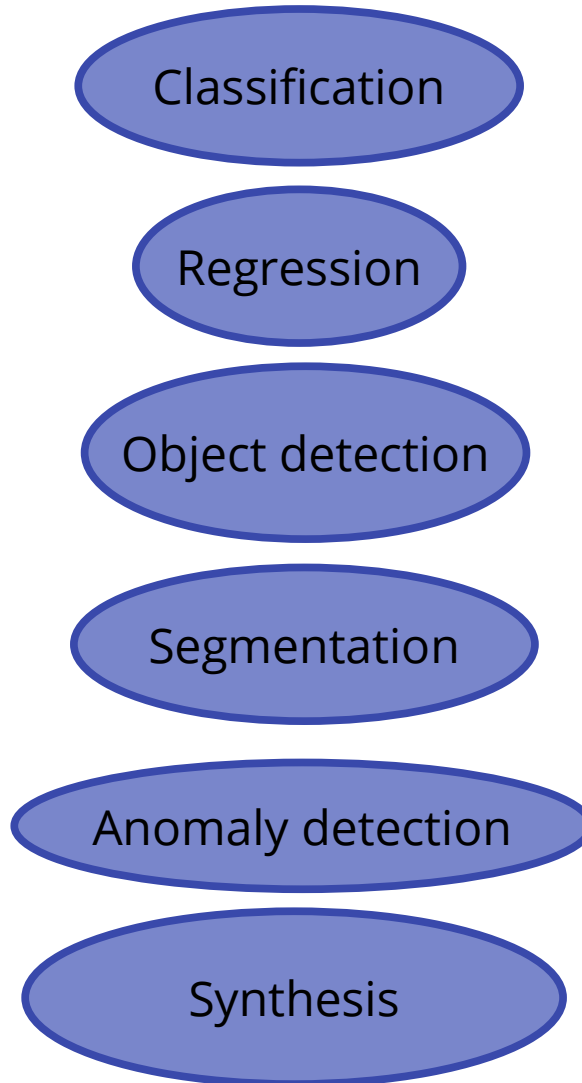
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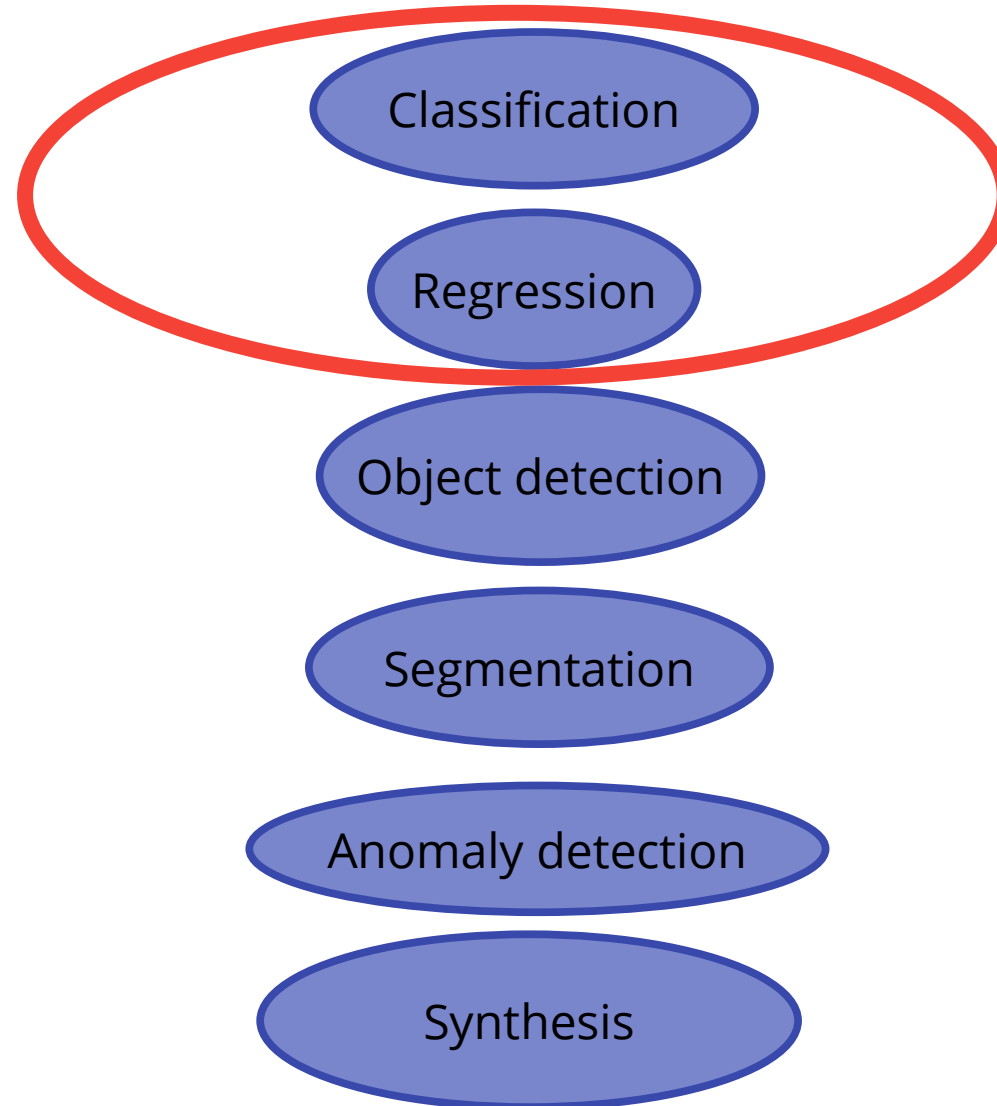


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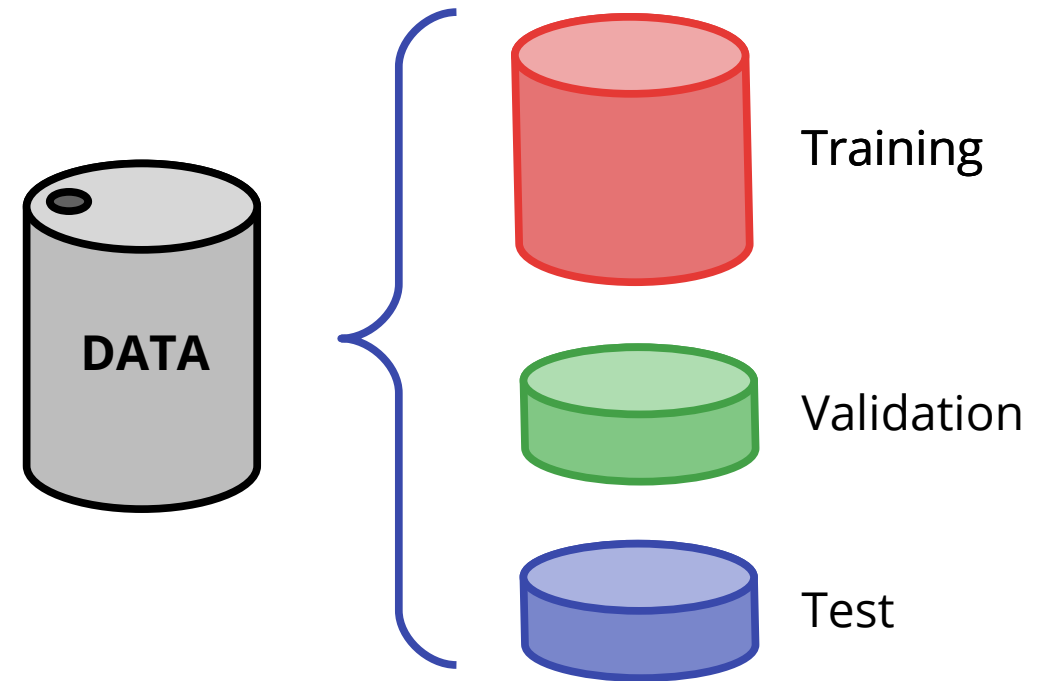
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## Supervised learning concepts



# Independent and identically distributed (iid) data

**iid** is a core concept of ML. When running an ML model on previously unseen data, we implicitly assume that the unseen (new) data and the already seen (training) data are **iid**, i.e., the individual samples in both data sets are *produced by the same data generation process*.

This does not imply that the seen and unseen data sets are identical!

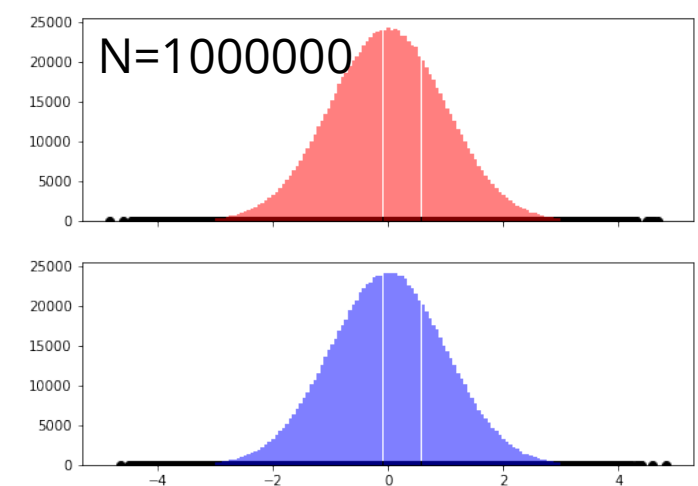
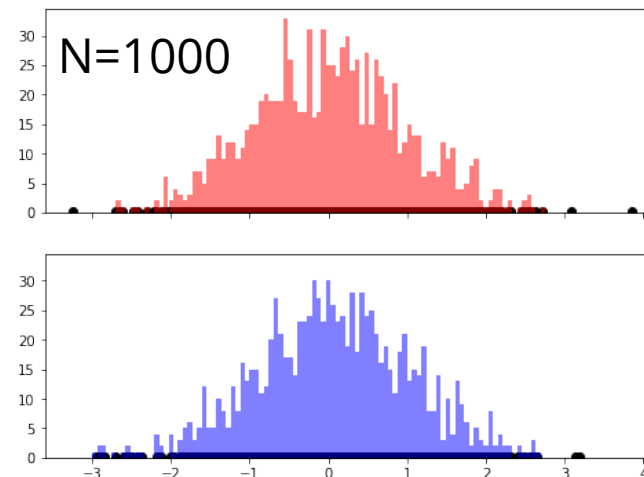
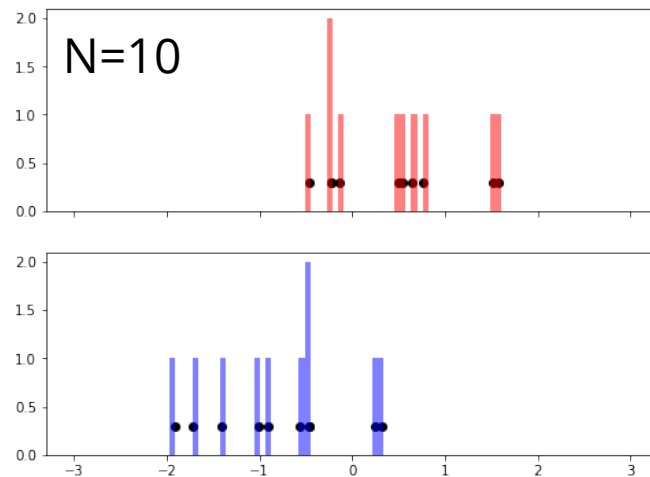
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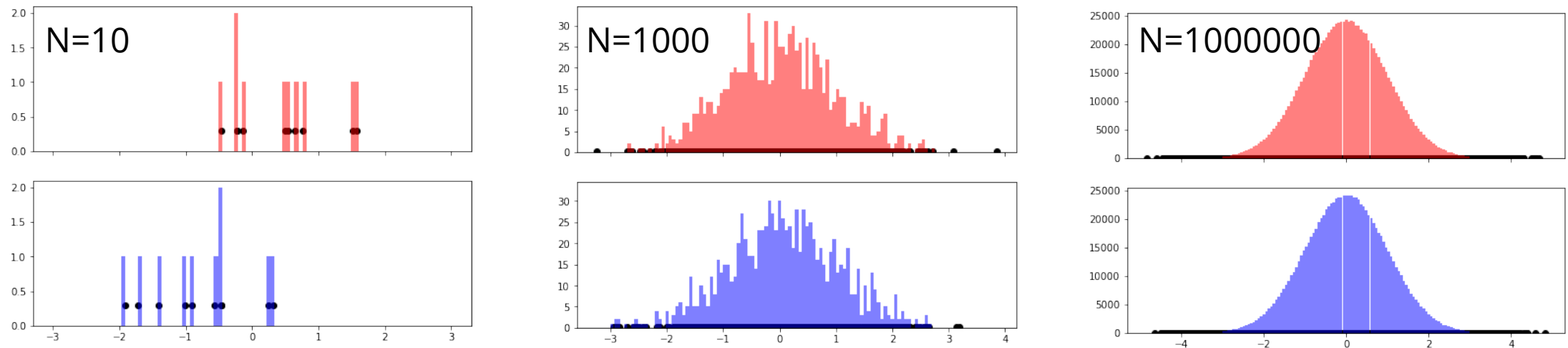


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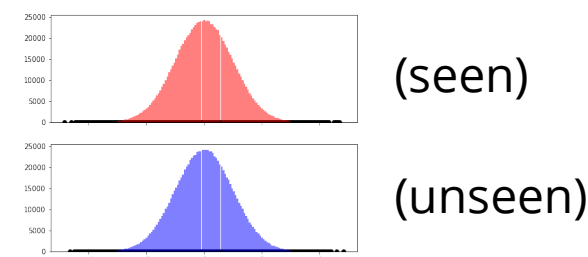
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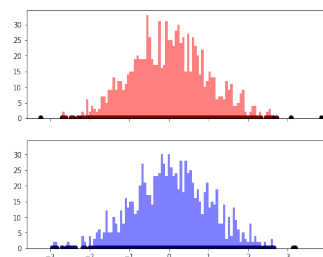
Lesson here: For small sample sizes, data sets that are iid may still differ significantly.

ML models are trained on existing data sets (seen data) and will be evaluated/applied to a new, previously unseen data set. Since **real data sets have a limited extent** (size), these distributions will look different, despite their iid nature.

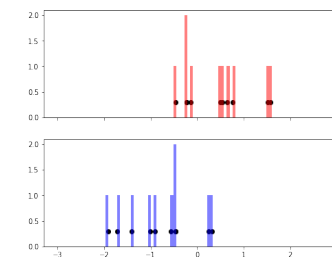
Ideally, the two distributions should be very similar:



But in real life, they tend to look more like this



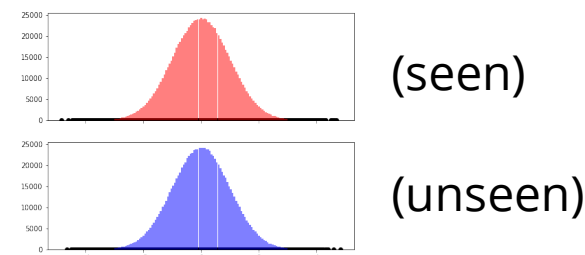
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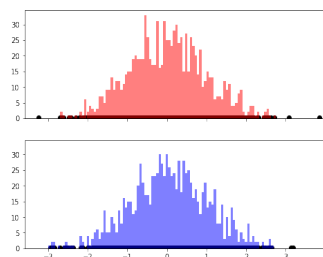


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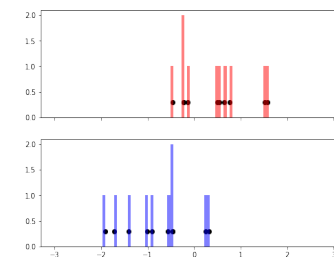
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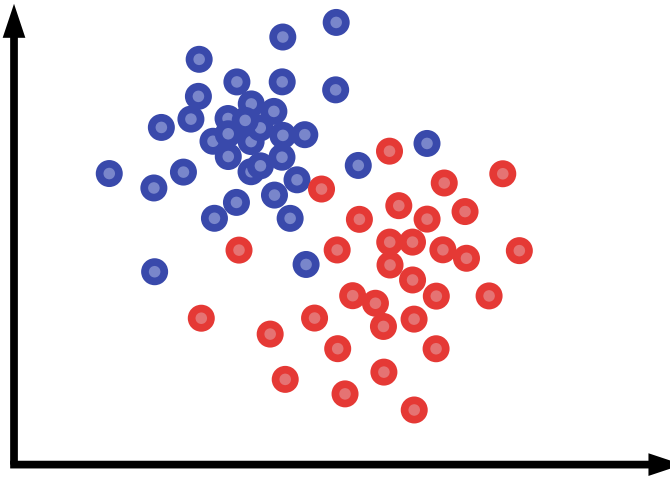
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Successful training on one data set does not imply good performance on unseen data  
 → the model has to **generalize** well by preventing **overfitting**

# Generalization, regularization, overfitting and underfitting

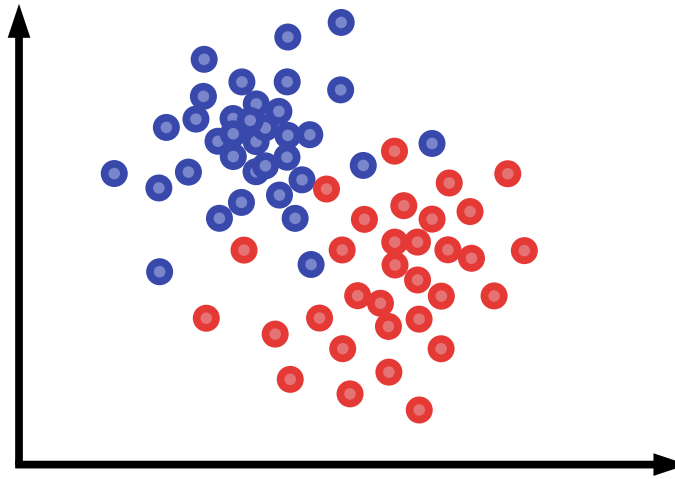
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What would be a good decision boundary between the two classes?

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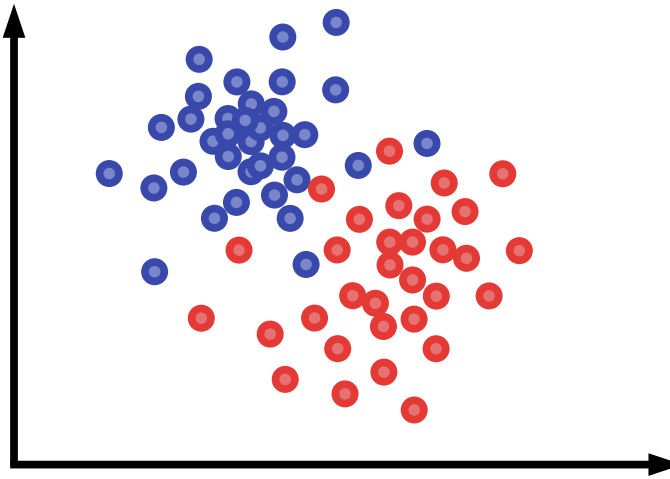


What would be a good decision boundary between the two classes?

The **decision boundary** separates the different classes as learned by the trained model.

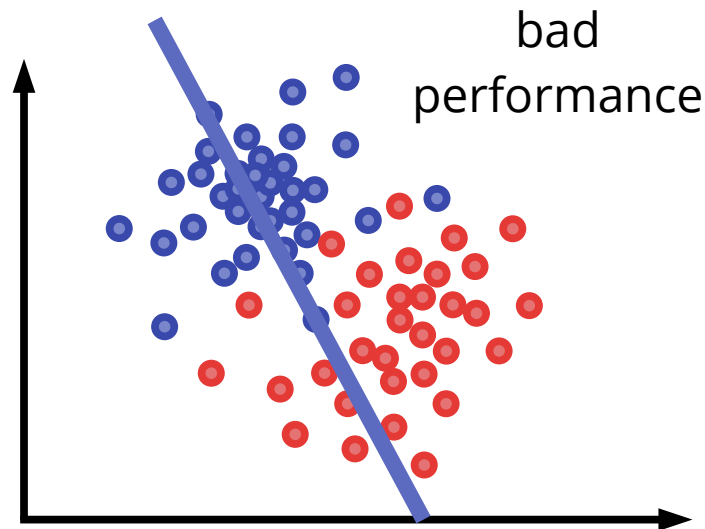
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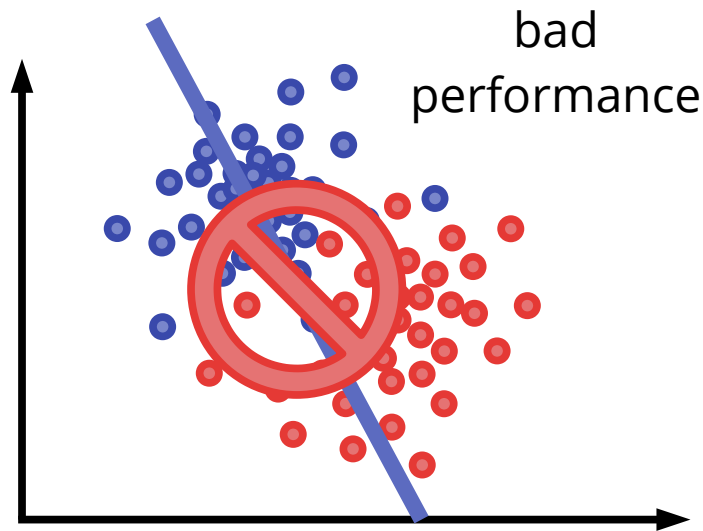


This would not be a good decision boundary as it barely allows to distinguish the two classes.

This model clearly **underfits** the data and leads to **poor performance**.

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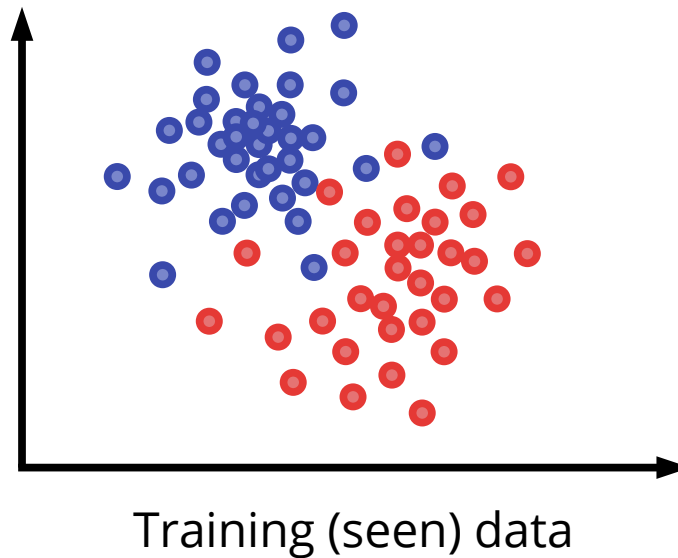


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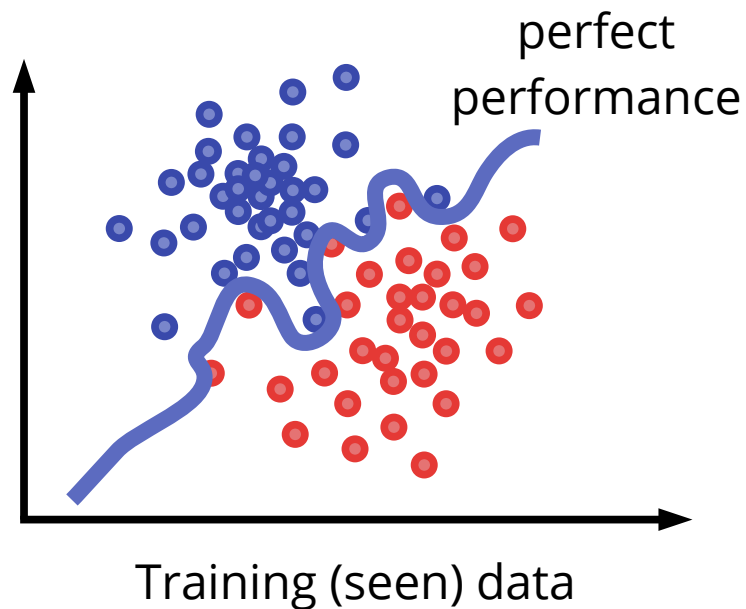
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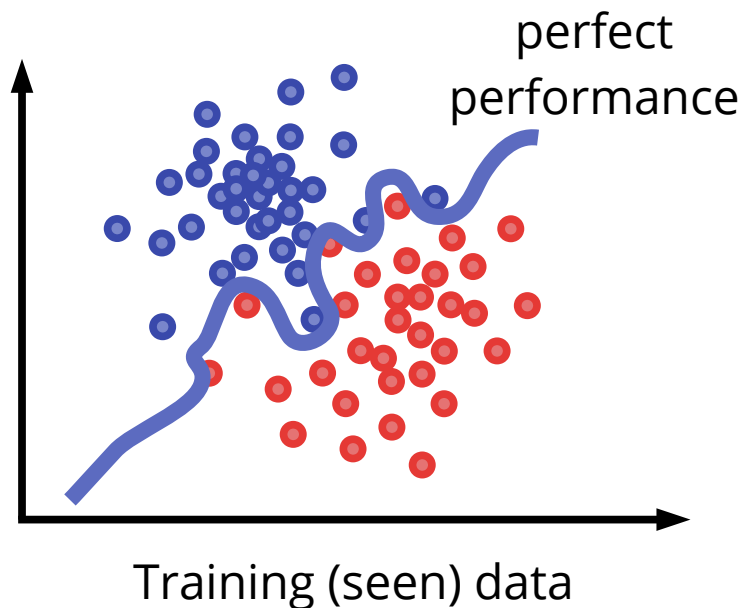
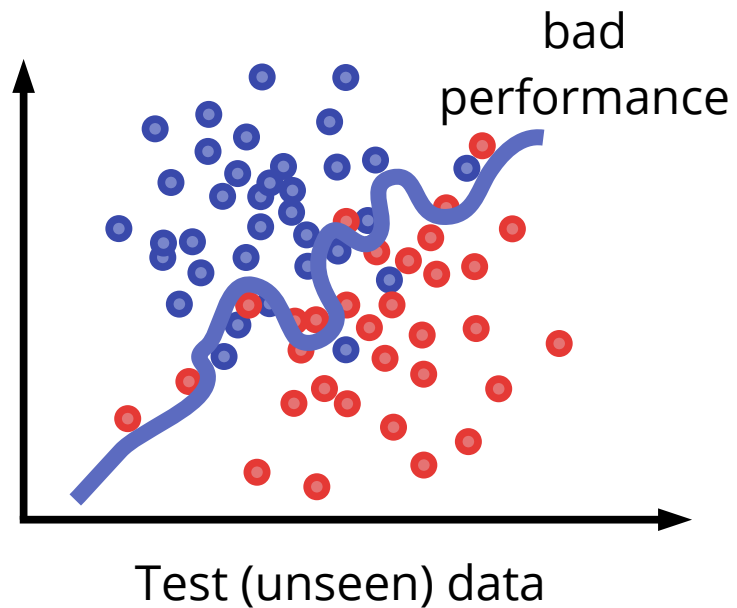
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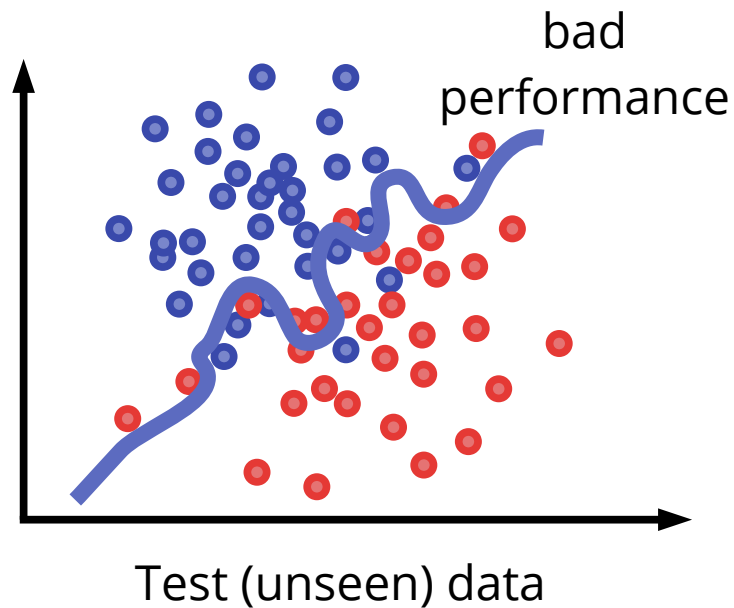


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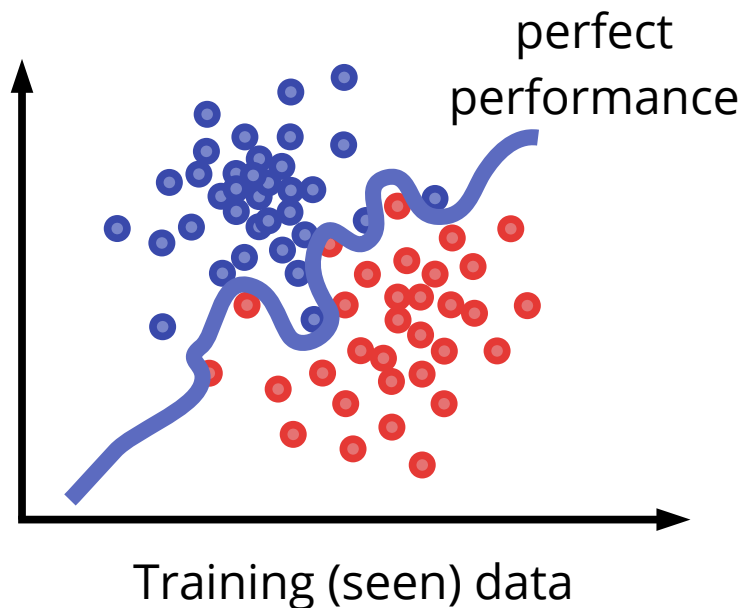
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This model is **overfitting**: it memorizes the structure of the training (seen) data and as a result **generalizes badly** on the overall data distribution.

We can improve its performance through **regularization** methods.



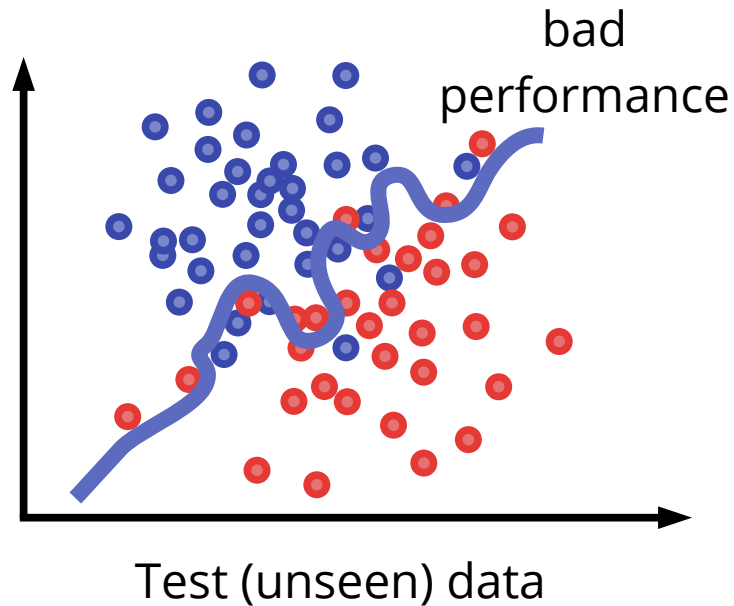
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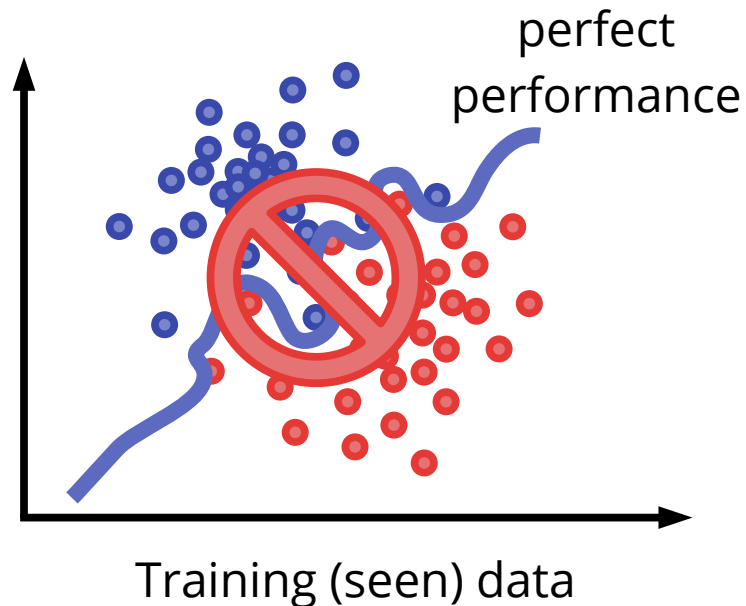
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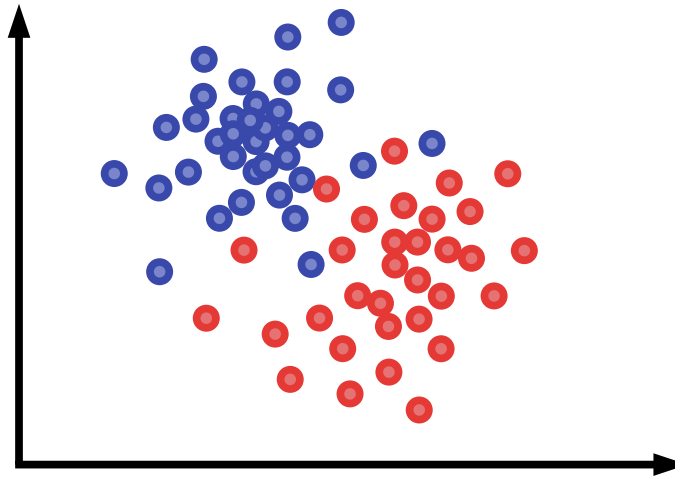
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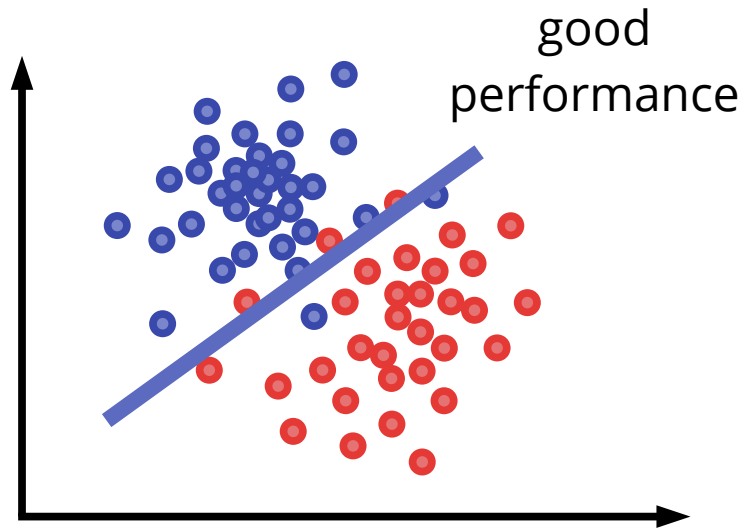
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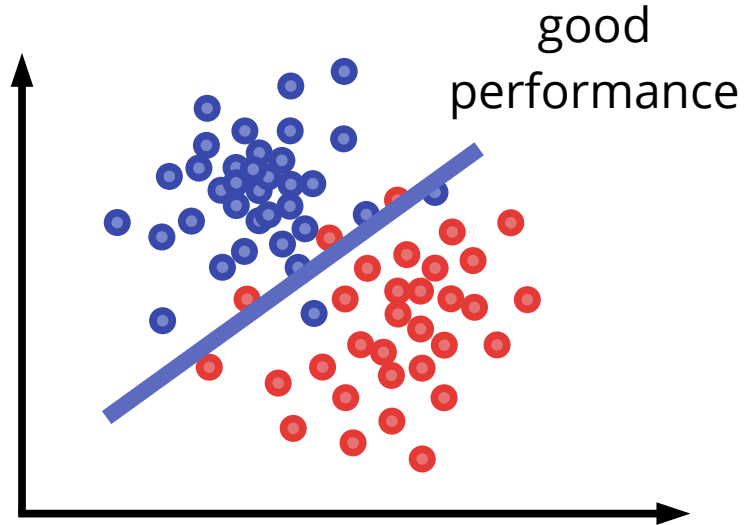
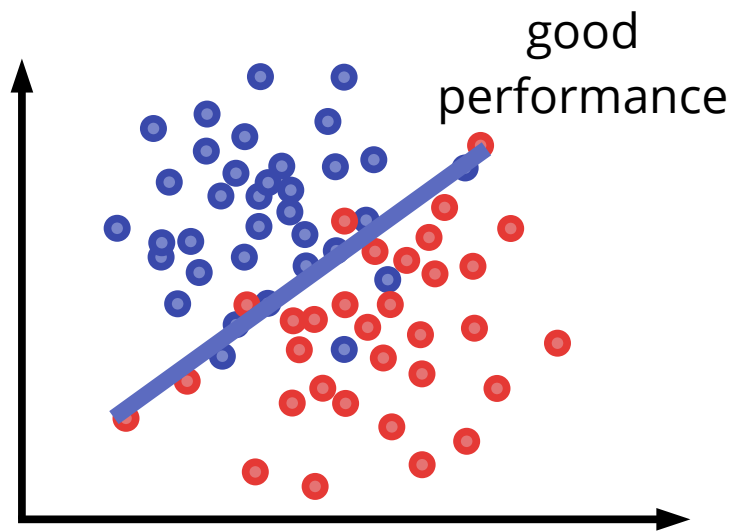
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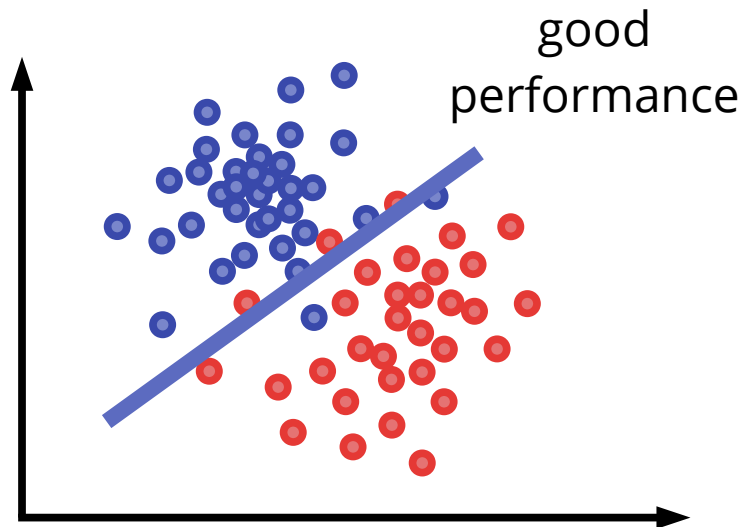
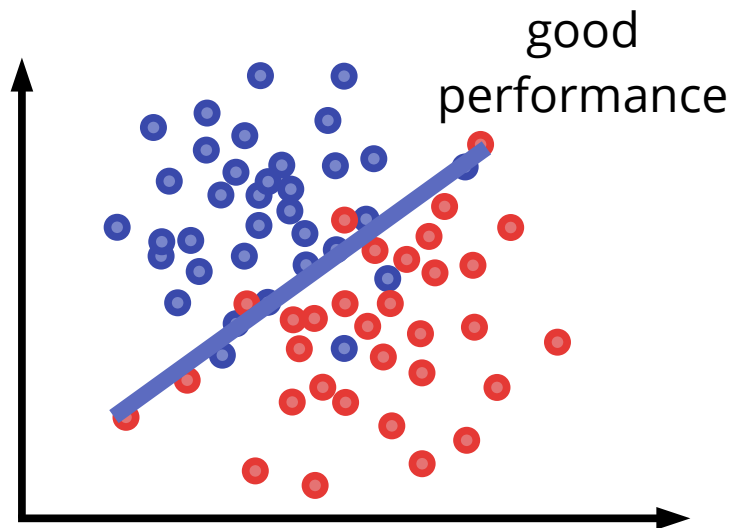
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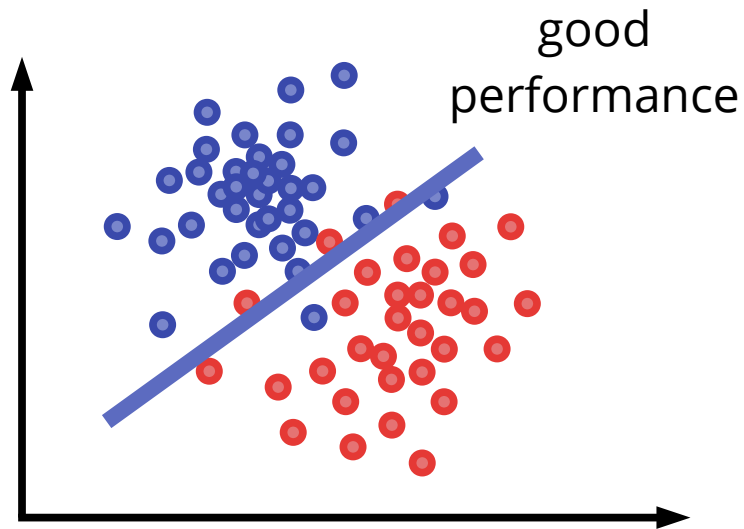
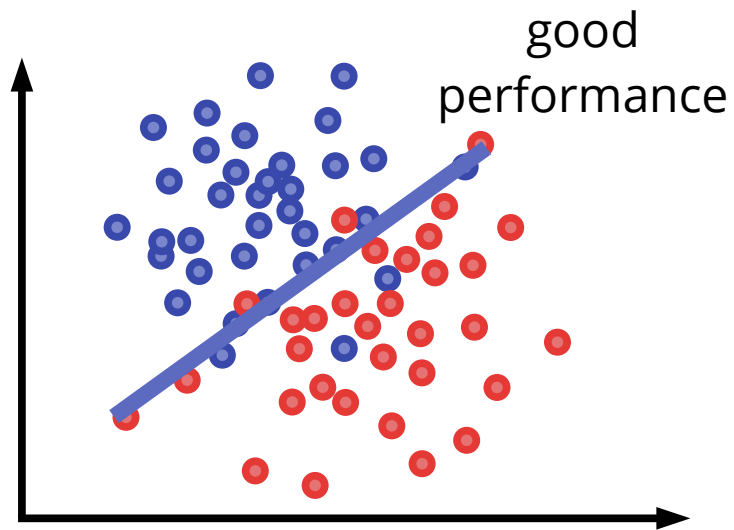
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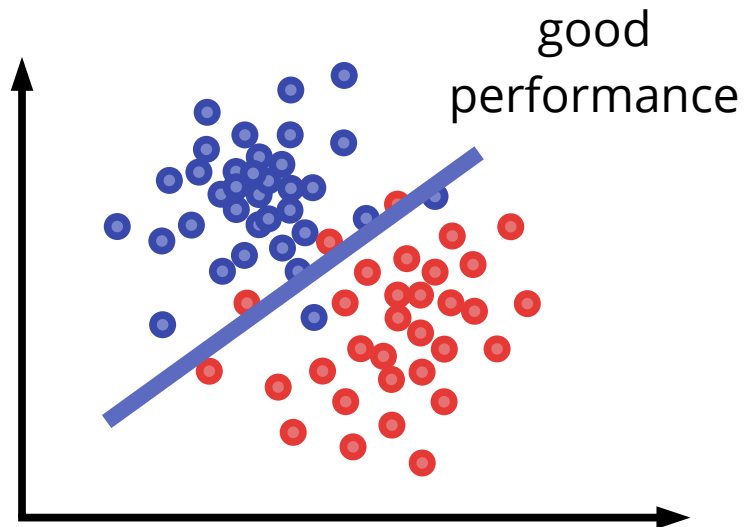
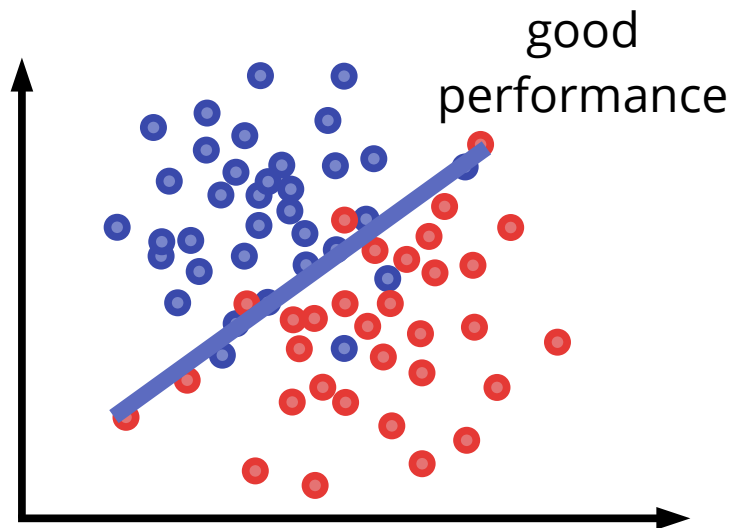
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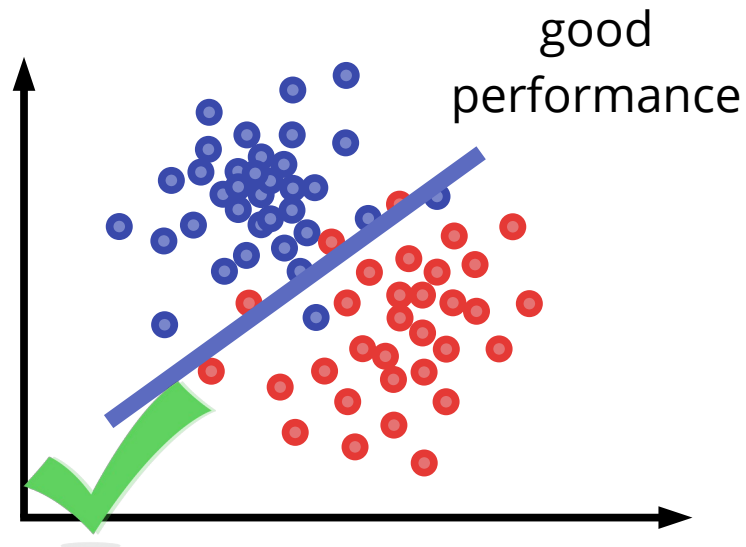
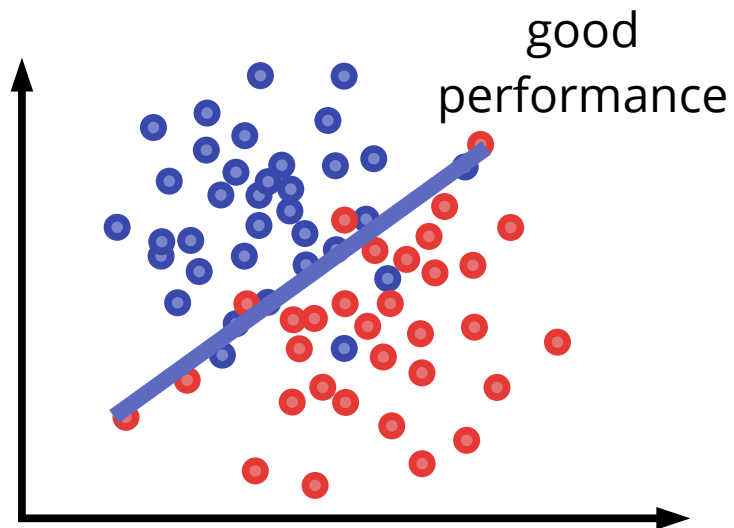
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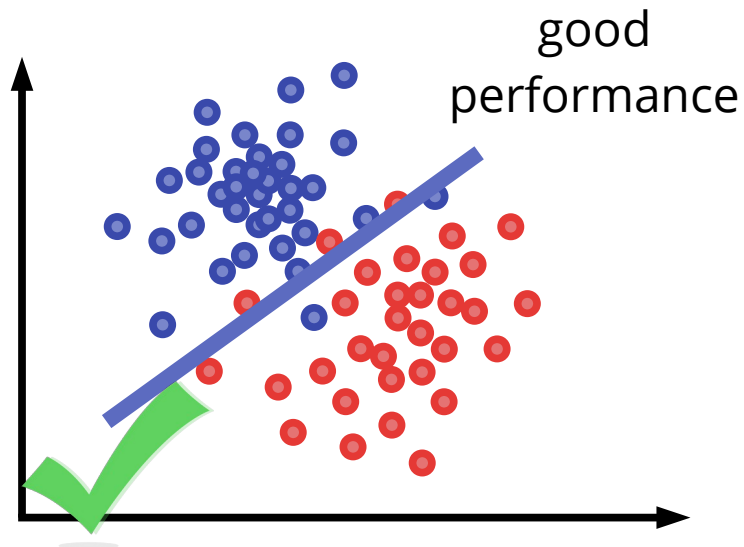
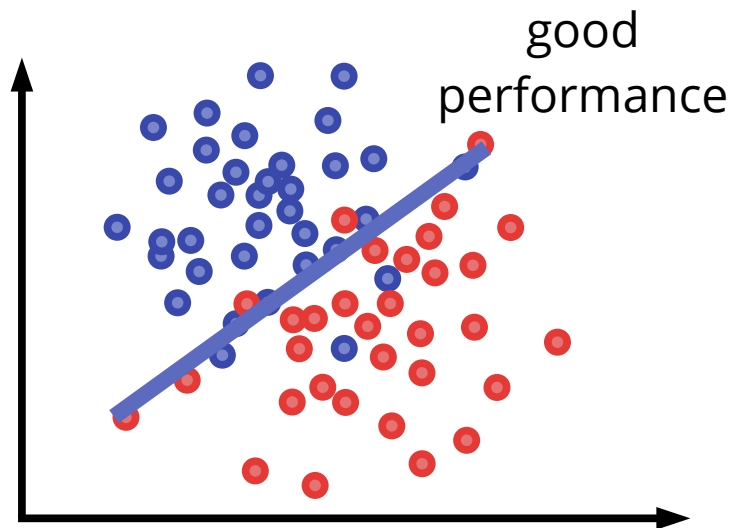
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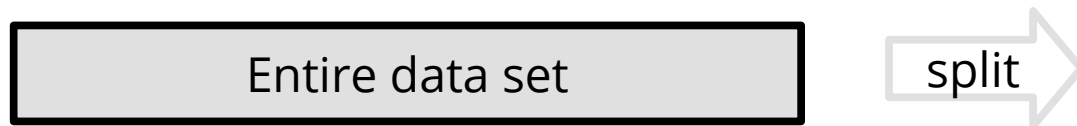
Entire data set

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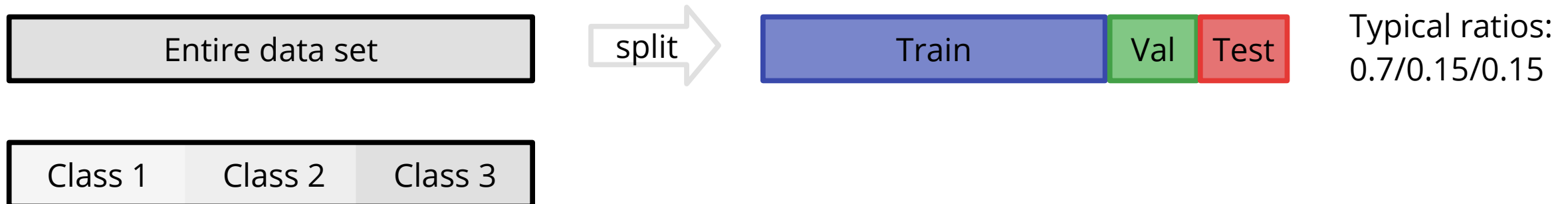


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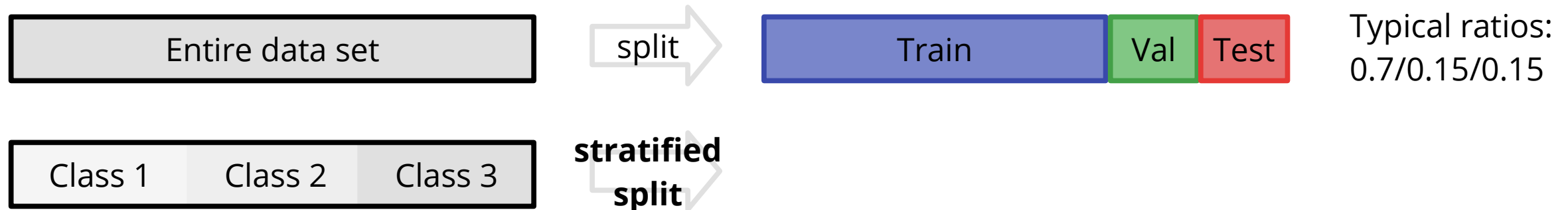


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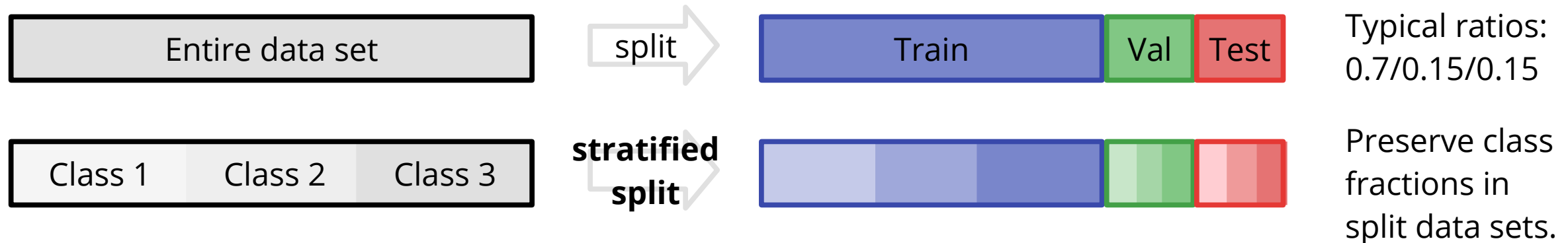


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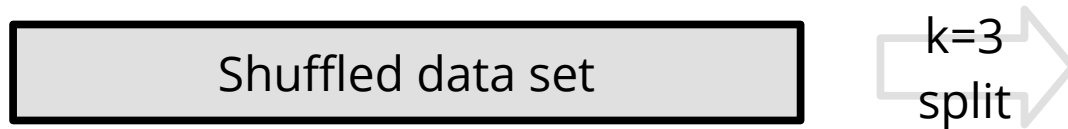
Shuffled data set



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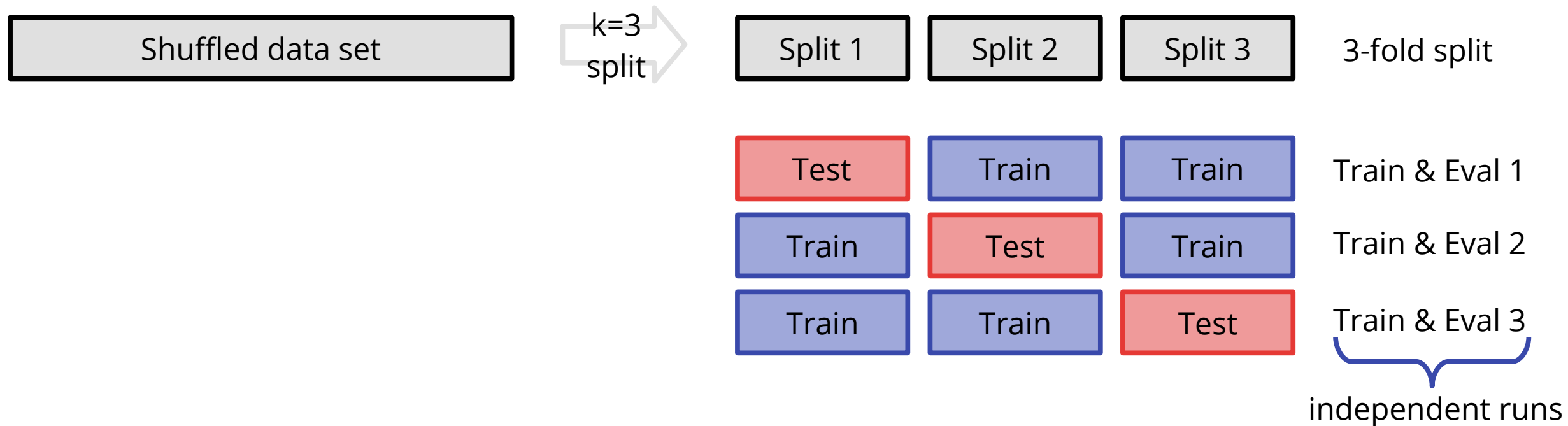
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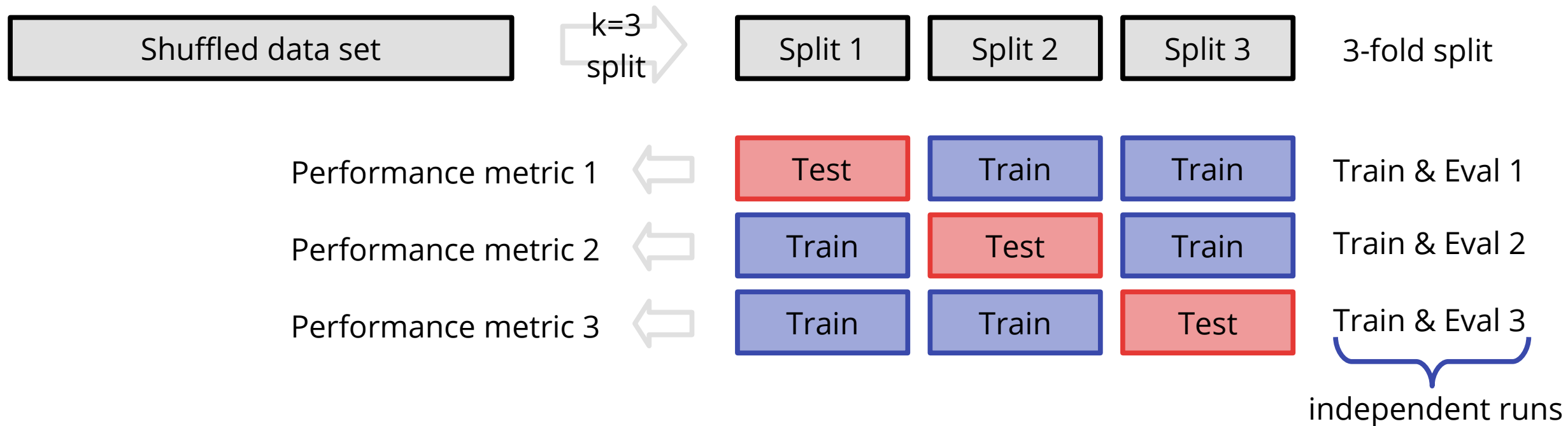
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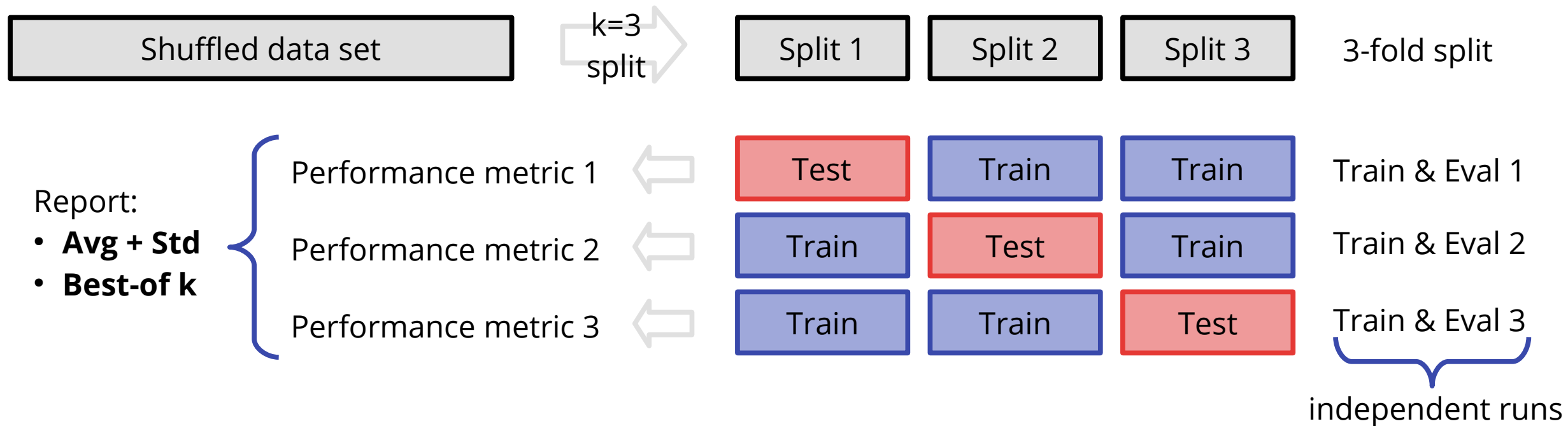
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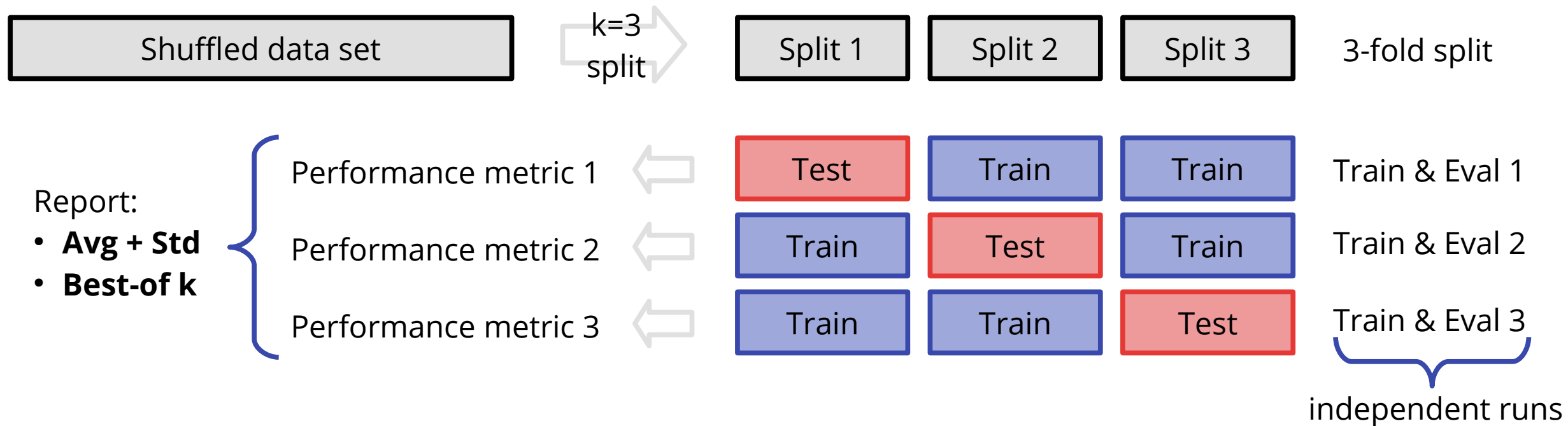
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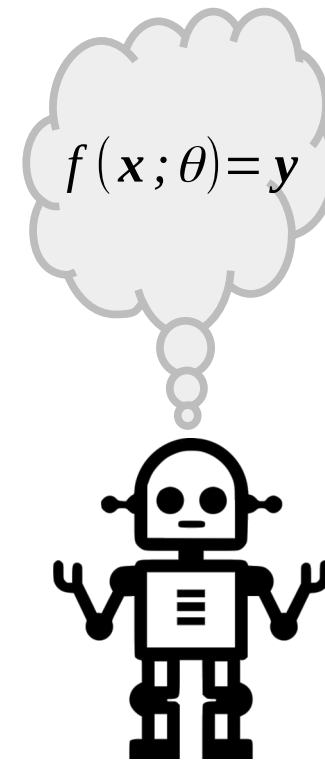
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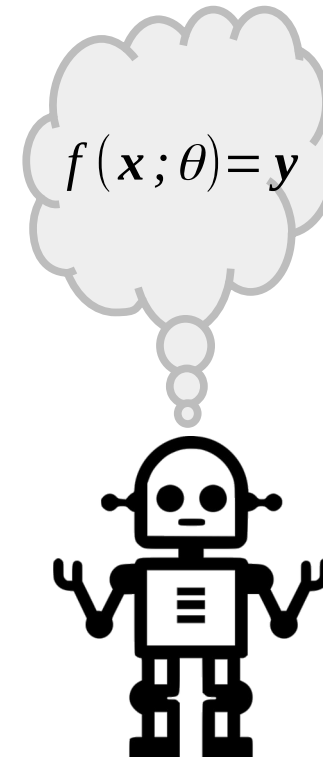
Note: Keep in mind that cross-validation will not improve your model performance; it will simply give you a more reliable estimate of its performance.

# General supervised learning pipeline



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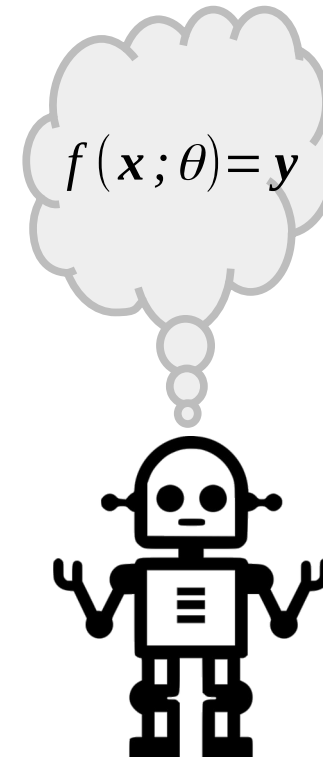
1) Feature engineering: raw data → features





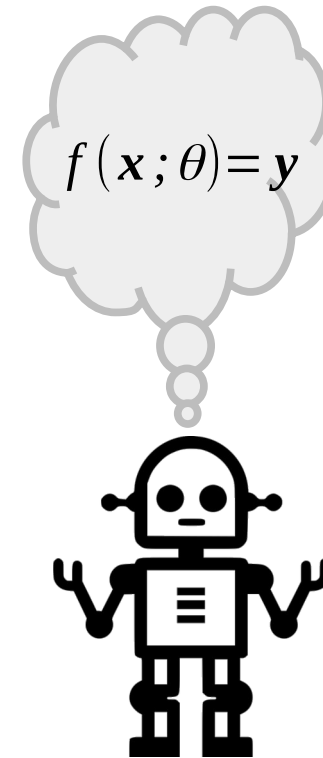
# General supervised learning pipeline

- 1) Feature engineering: raw data → features
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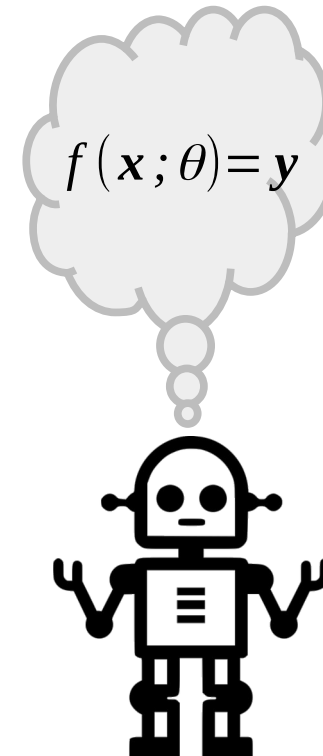
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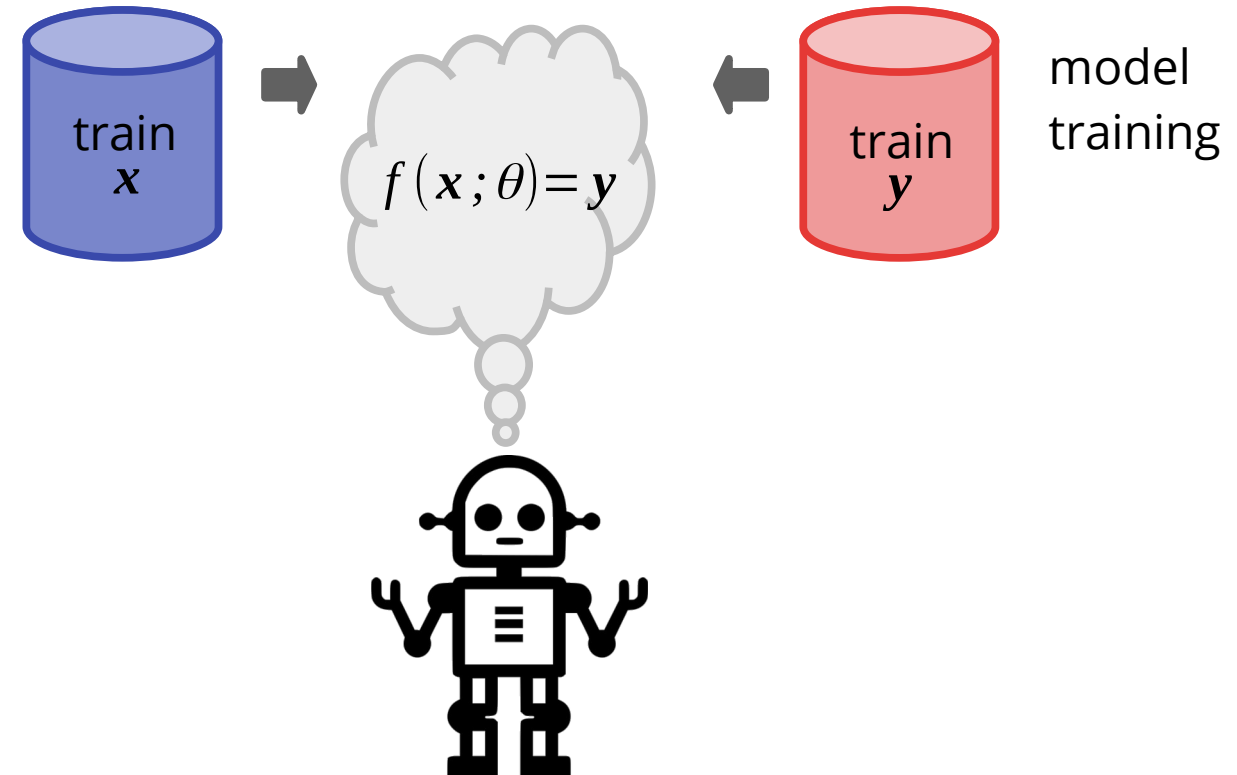
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- 4) Define hyperparameters



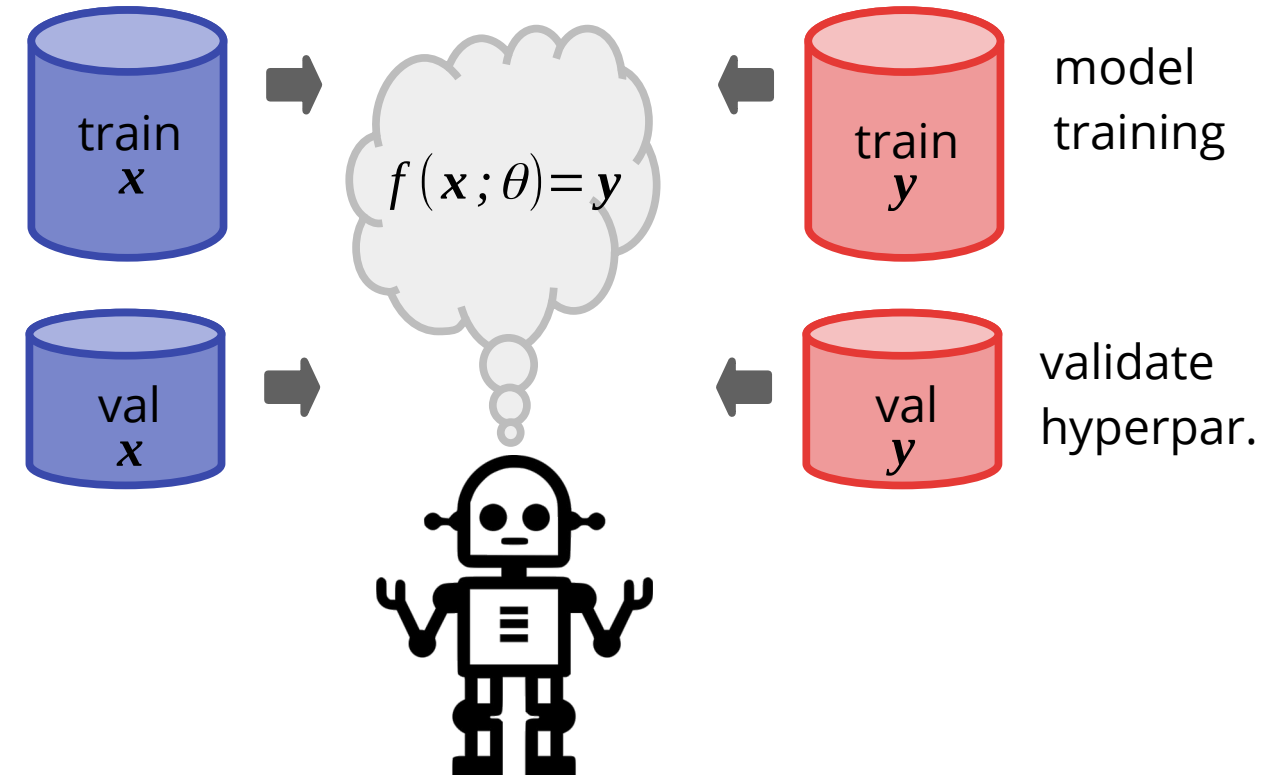
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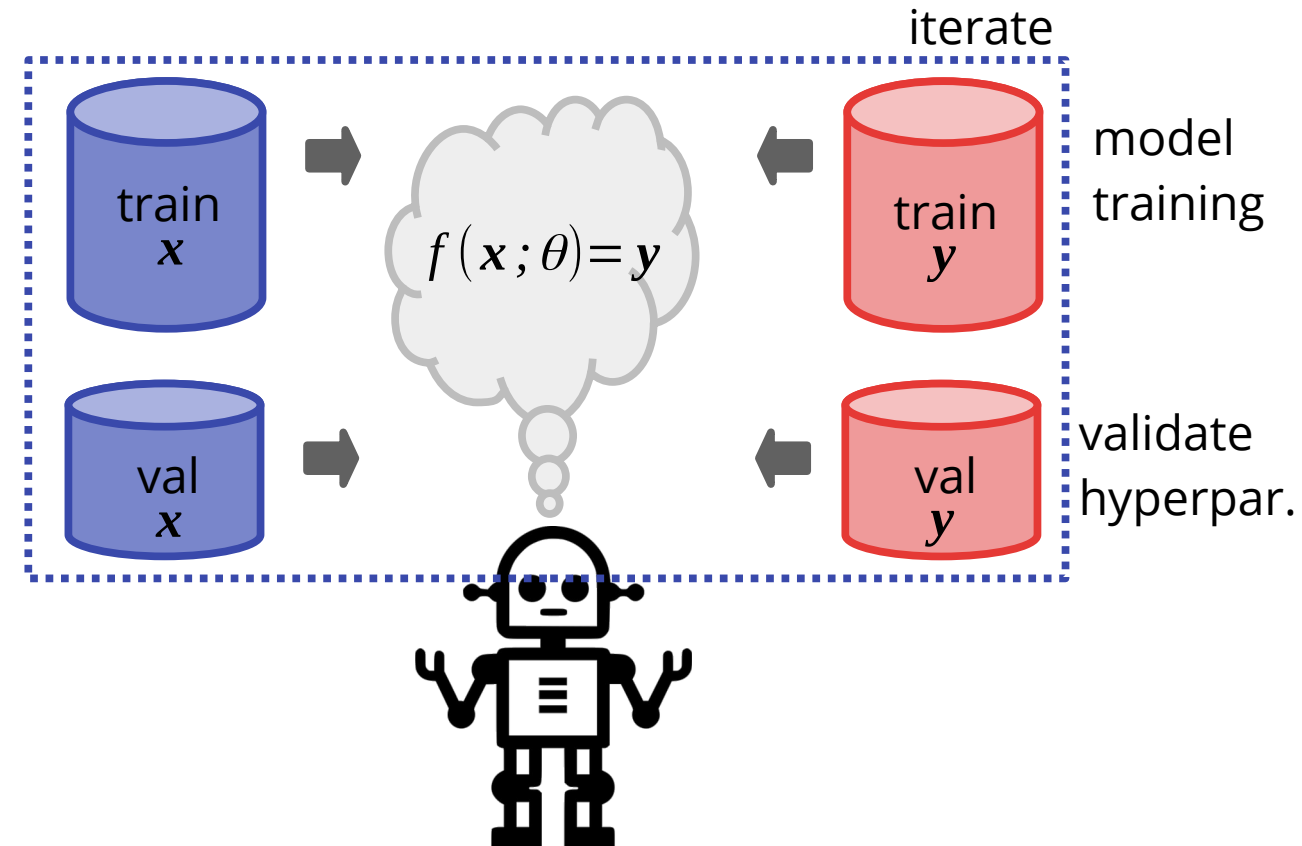
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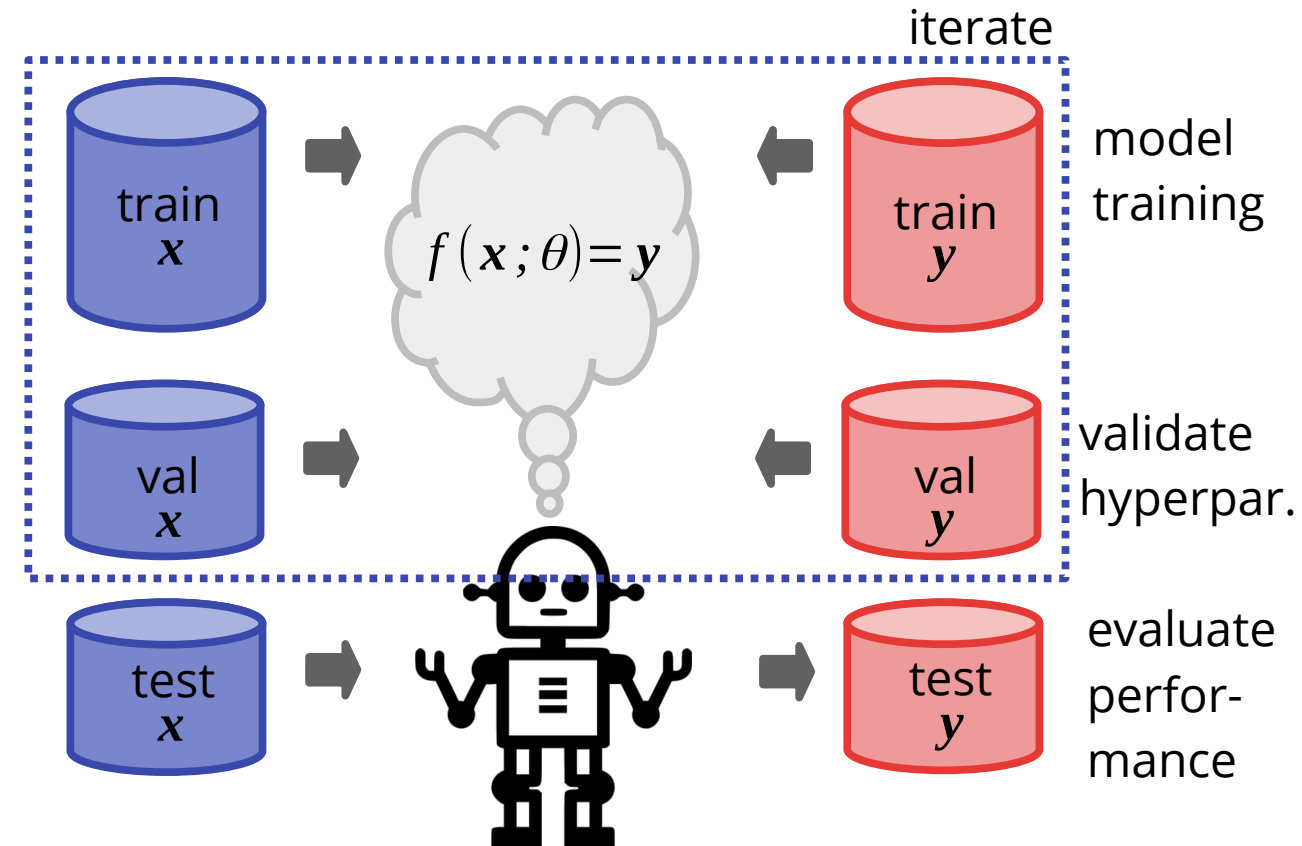
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- 7) Repeat 4) to 6) until performance on validation data maximized



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- 7) Repeat 4) to 6) until performance on validation data maximized
- 8) Evaluate trained model on test data → report test data performance



# **Benchmarking and metrics**





# How do we measure the performance of our model?

**Benchmarking** refers to the process of quantitatively assessing your ML model's performance.

Performance is measured based on pre-defined metrics; a **metric** can be thought of as a measure for how well an ML model performs on a specific task and data set.

*Examples:*

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Based on...

- speed
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## Medical diagnosis

What is most important?

- correctness of diagnosis
- minimizing failures
- patient's comfort
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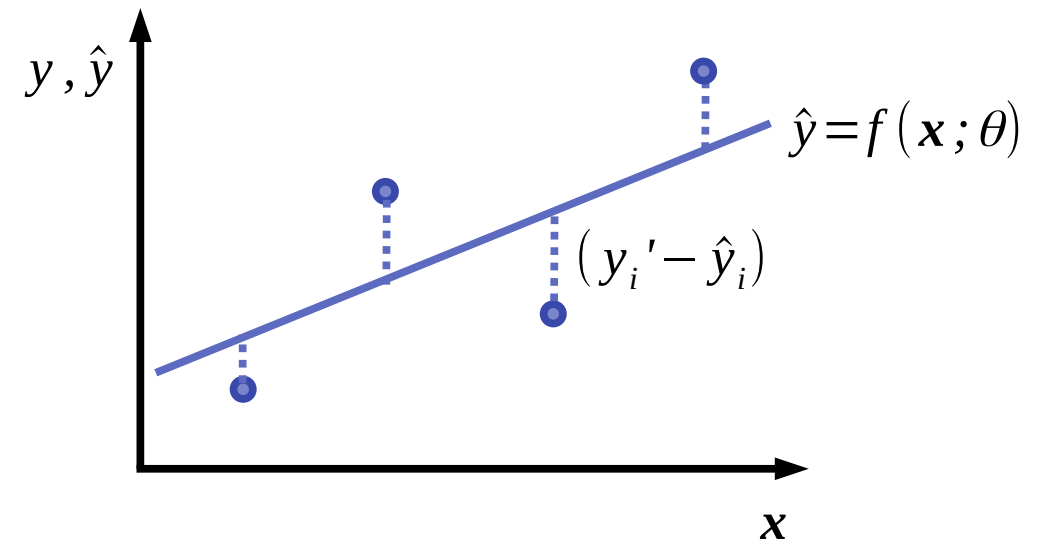
*Examples:*

	Which athlete is best?	Which company is successful?	Medical diagnosis
	Based on...	Contributing factors:	What is most important?
metrics	<ul style="list-style-type: none"><li>• speed</li><li>• strength</li><li>• number of victories</li><li>• income</li></ul>	<ul style="list-style-type: none"><li>• revenue</li><li>• overall value</li><li>• number of employees</li><li>• annual CO2 emissions</li></ul>	<ul style="list-style-type: none"><li>• correctness of diagnosis</li><li>• minimizing failures</li><li>• patient's comfort</li><li>• cost</li></ul>

# Different ML tasks require different metrics

## Regression task:

Input data:  $\mathbf{x}_i, i \in \{1 \dots N\}$   
Target ground-truth:  $y_i'$   
Target prediction:  $\hat{y}_i = f(\mathbf{x}_i; \theta)$



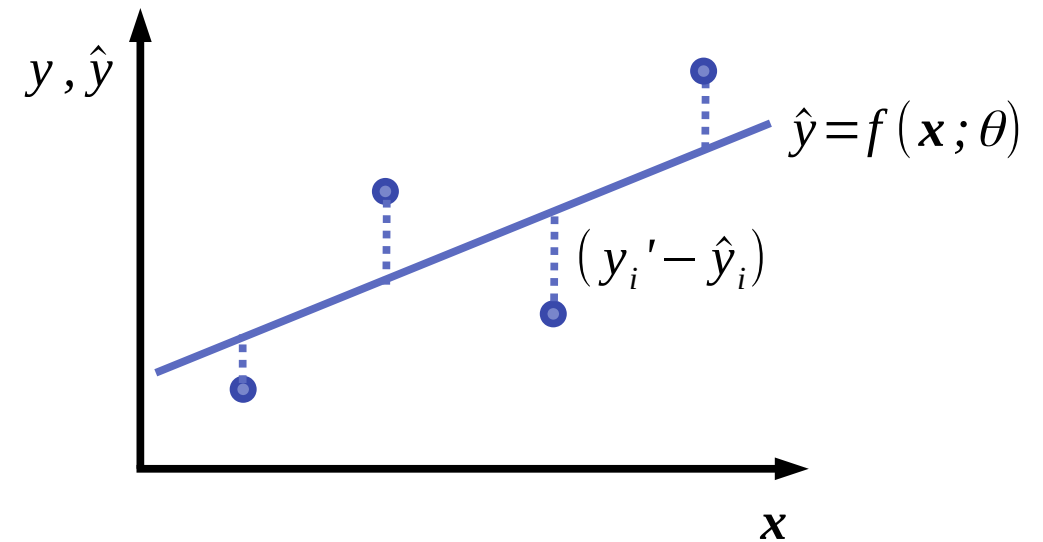
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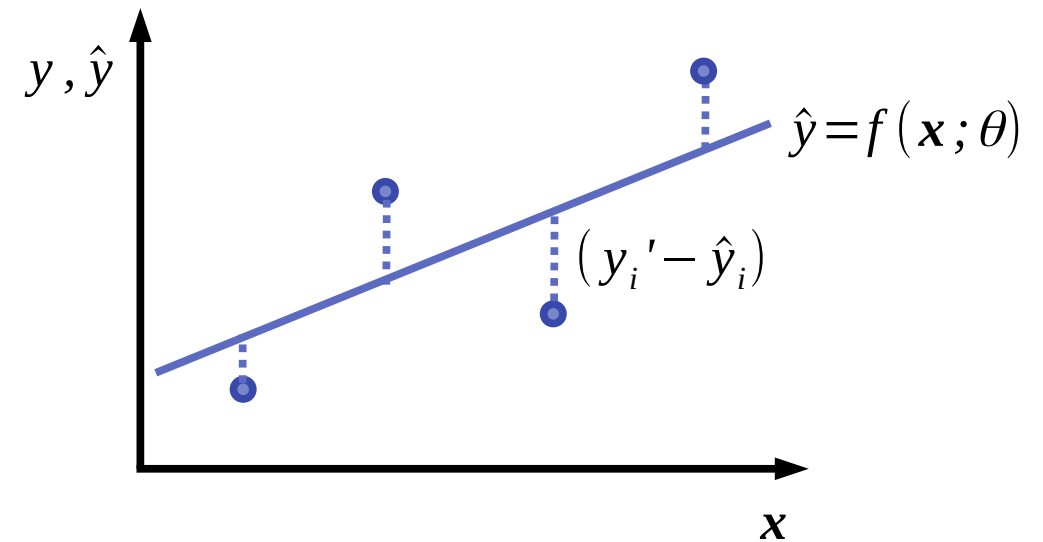
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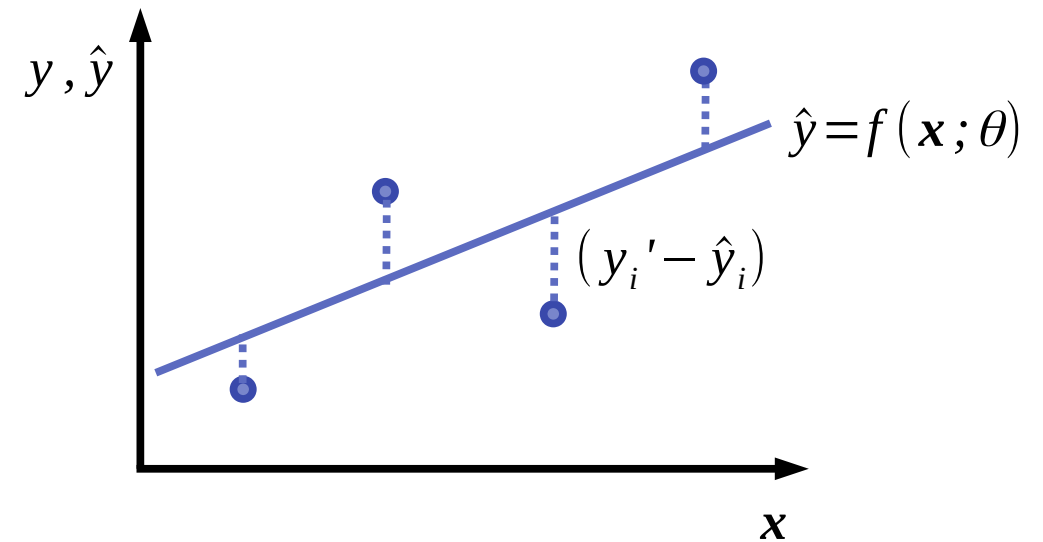
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Intuition: by how much deviates your model prediction from the ground-truth on average.

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Prediction	positive	negative
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Ground-Truth	True positive	False positive
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*What is the overall fraction of correct (positive and negative) predictions?*

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# Different ML tasks require different metrics

## (Binary) Classification:

**Accuracy** =  $(TP + TN) / (TN + TP + FP + FN)$

Requires somewhat balanced classes

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Less susceptible to imbalance.

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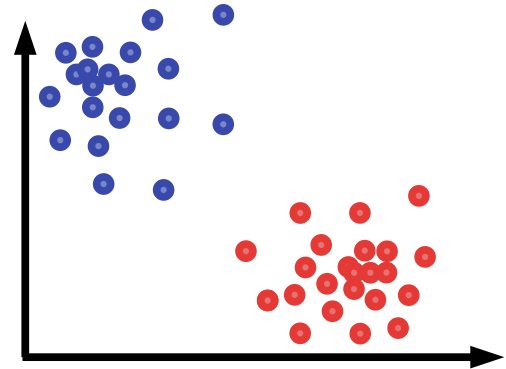
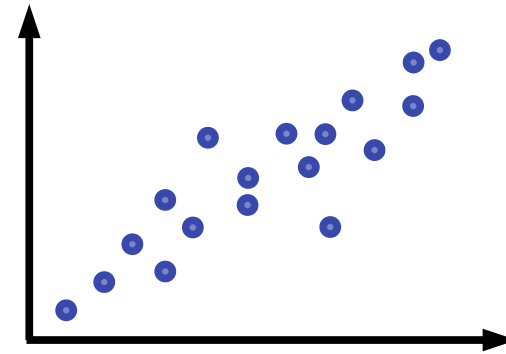
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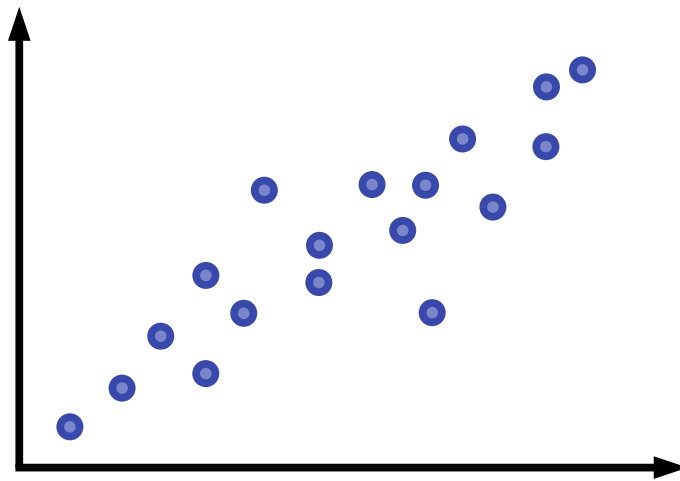
Low precision means that we issue some false alarms – this is something we can deal with.

Low recall means that we miss some asteroids that are about to impact, which is not exactly what we want. Therefore, **recall** is the really

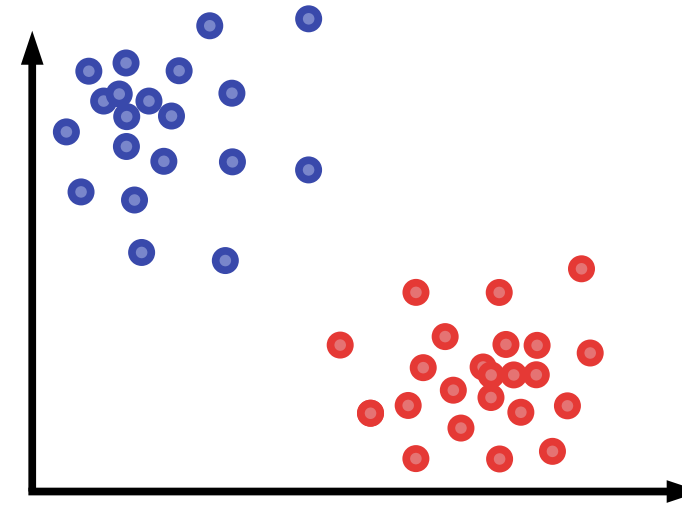
## Linear models



Linear models assume linearity in the underlying data. They are rather simple but convey many of the concepts utilized in other, more complex models.



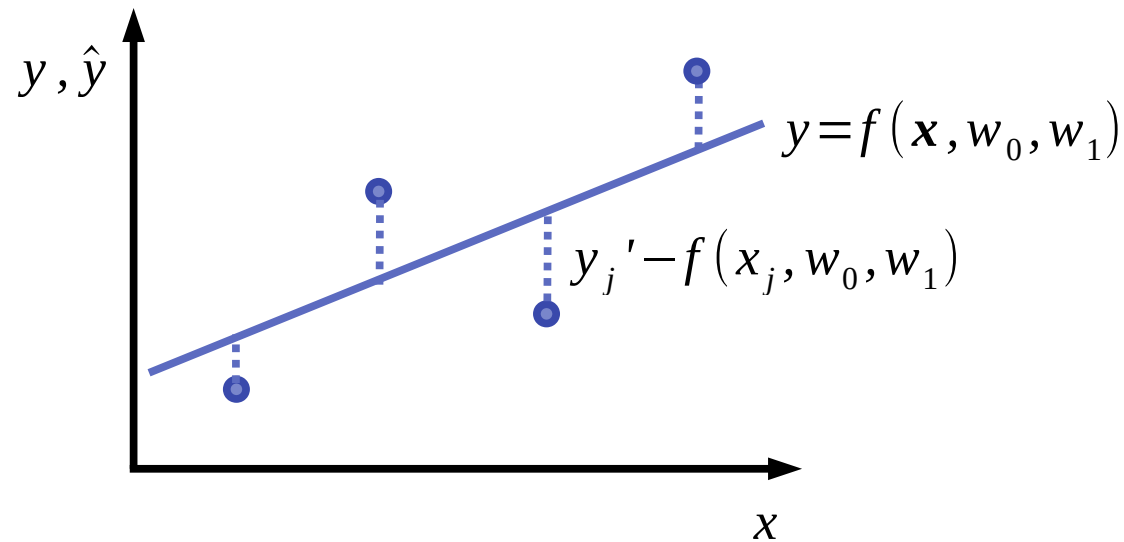
**Linear regression**



**Linear classification**

# Linear regression (univariate)

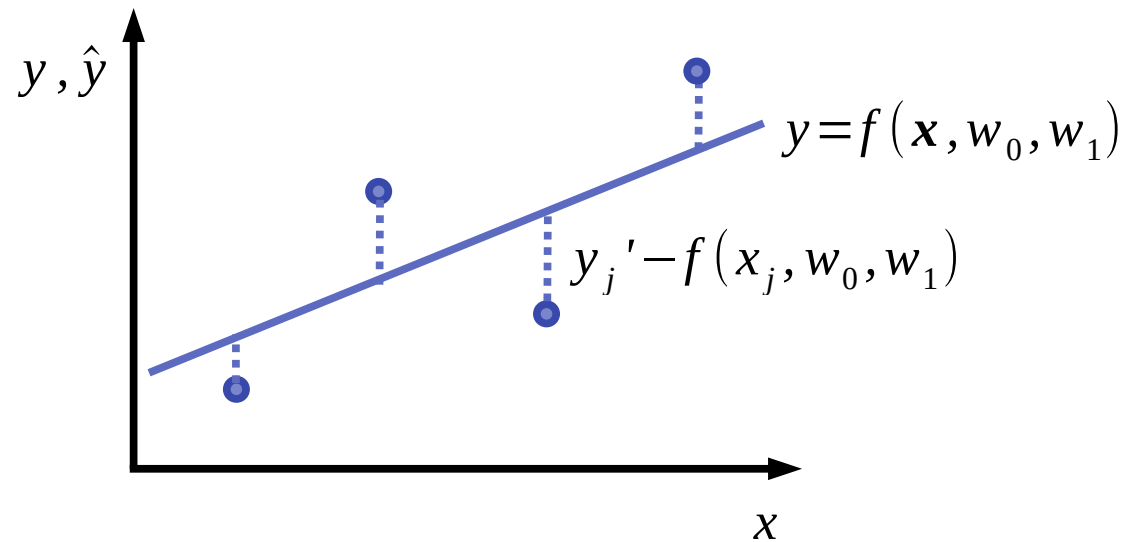
Find weights  $w_0$  and  $w_1$  so that the linear function  $f(x) = w_1 x + w_0$  with input  $x$  and output  $y$  best fits the data containing ground-truth values  $y'$ .





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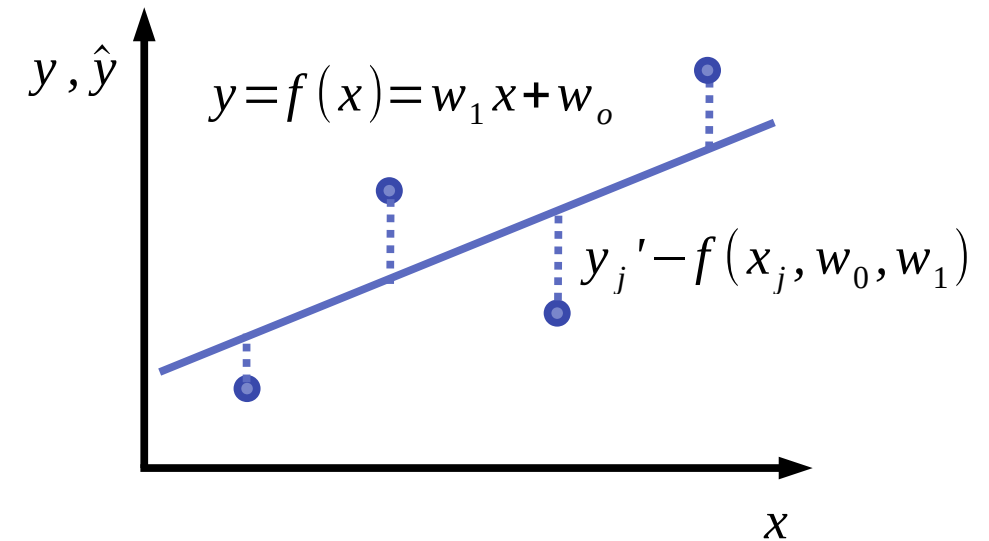


How can we learn  $w_0$  and  $w_1$  from data?

# Linear regression (univariate) – Least squares fitting

**Idea:** minimize squared errors of prediction with respect to ground truth for each data point:

$$\text{for data point } j: [y_j' - f(x_j, w_0, w_1)]^2$$

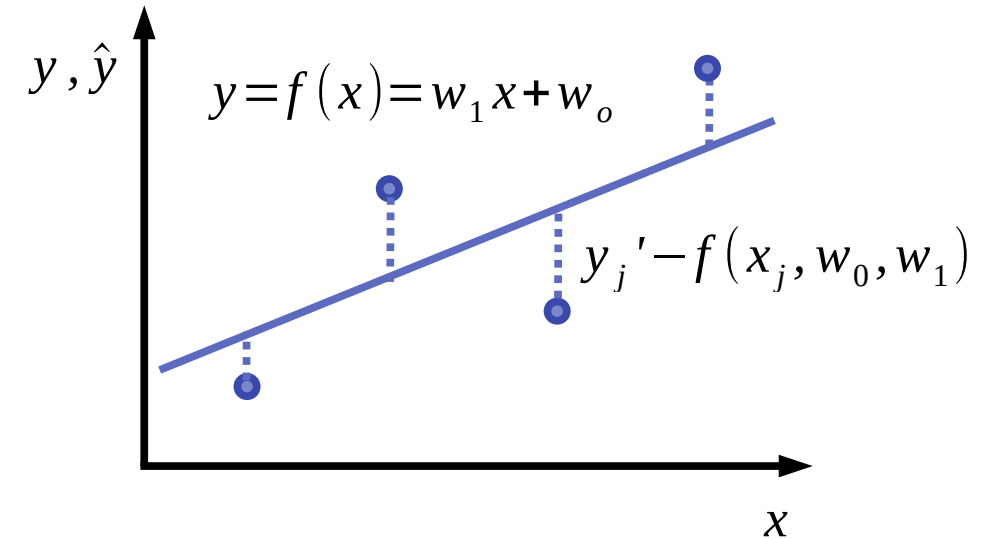


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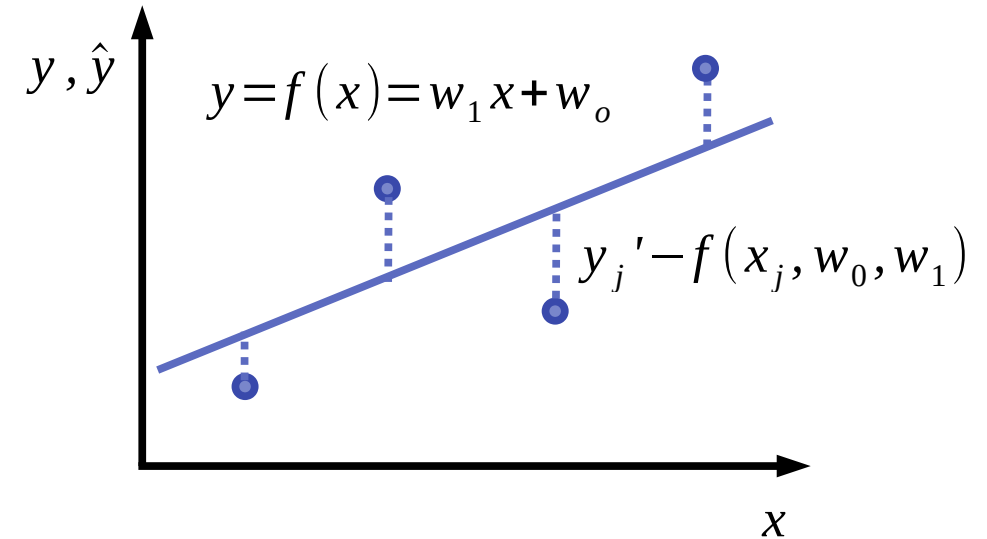
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$$L = \sum_j^N (y_j' - f(x_j, w_0, w_1))^2 = \sum_j^N (y_j' - (w_1 x_j + w_0))^2$$



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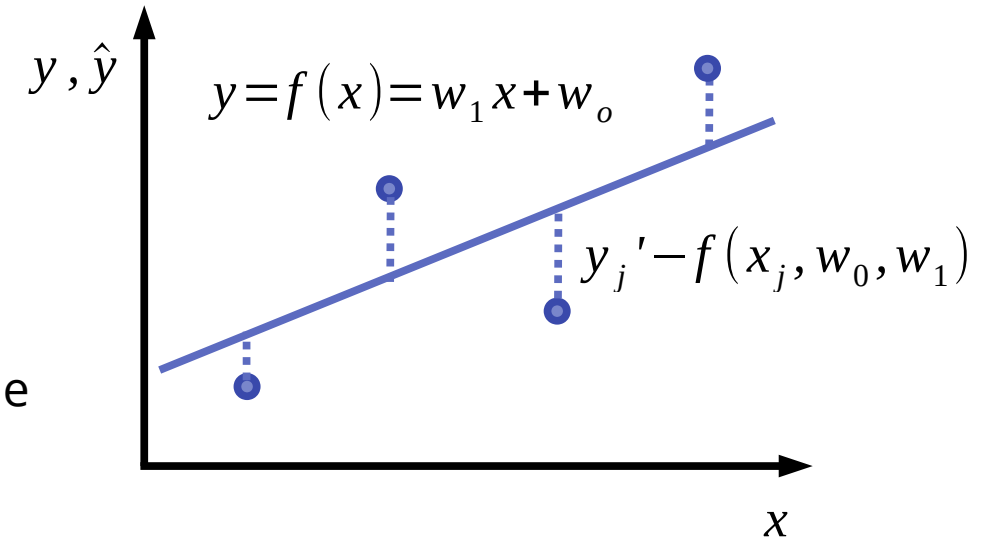
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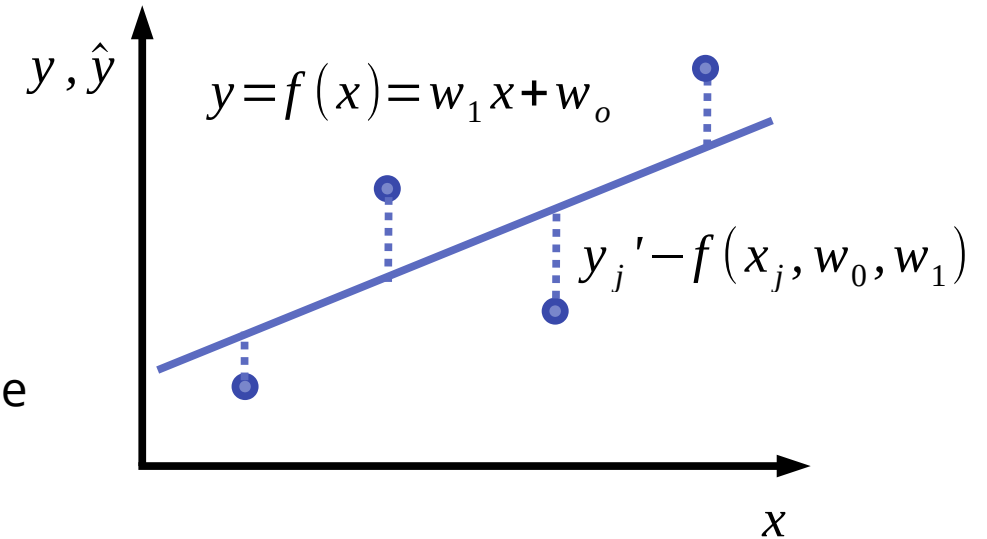
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$$\frac{\partial L}{\partial w_0} = 0 \quad \frac{\partial L}{\partial w_1} = 0$$



# Linear regression (univariate) – Least squares fitting

**Idea:** minimize squared errors of prediction with respect to ground truth for each data point:

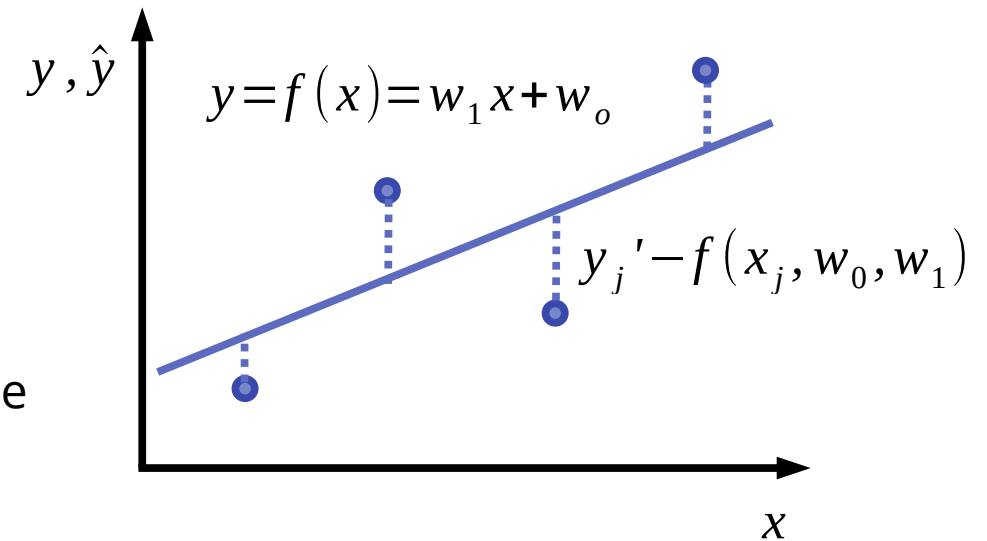
$$\text{for data point } j: [y_j' - f(x_j, w_0, w_1)]^2$$

We define a **Loss** (or Objective) function that is the sum of the squared errors over all data points:

$$L = \sum_j^N (y_j' - f(x_j, w_0, w_1))^2 = \sum_j^N (y_j' - (w_1 x_j + w_0))^2$$

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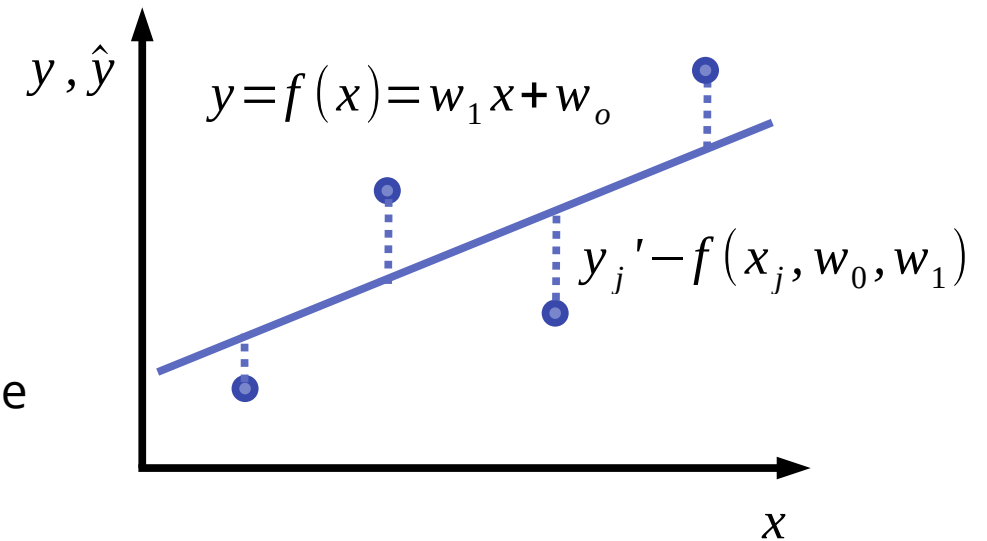
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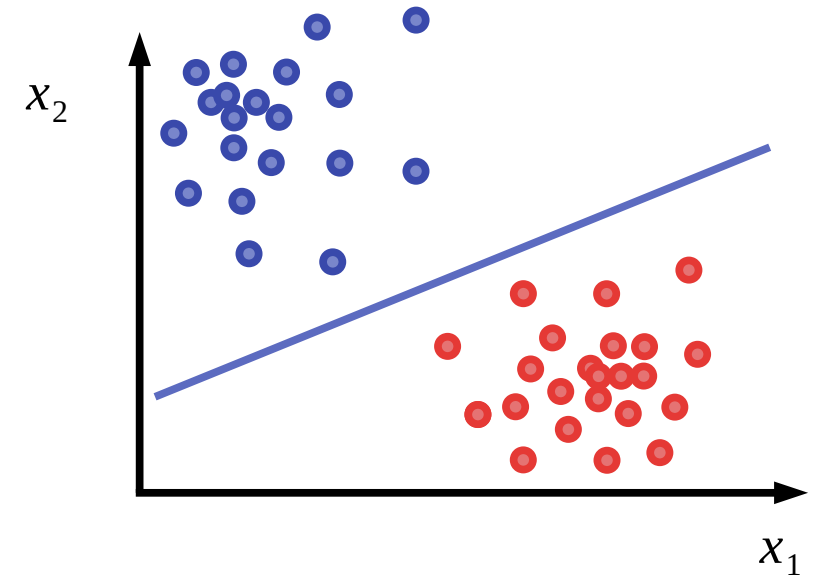
Least-squares + linear model function: the resulting minimum of the Loss function is **global**, i.e., the solution is by default the best-possible solution!





# Linear classifier (two-dimensional case)

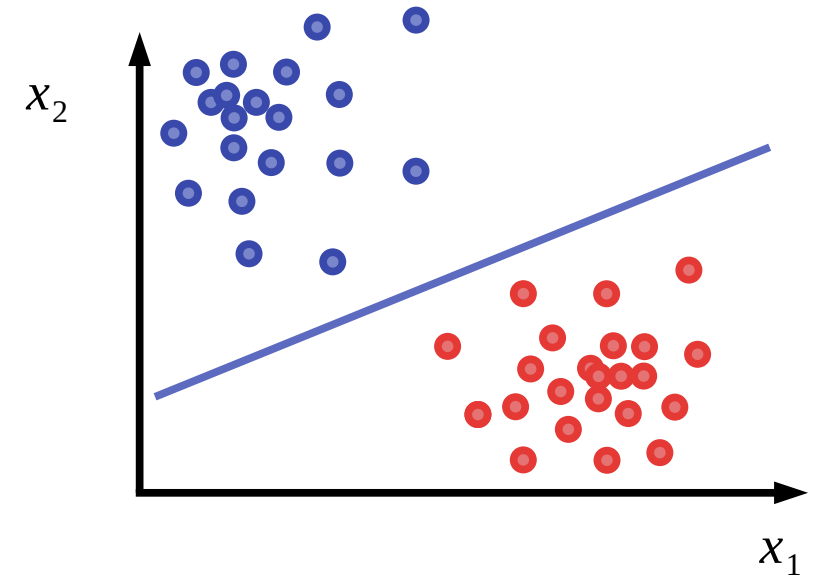
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Define decision boundary  $f(\mathbf{x}, \mathbf{w}) = \mathbf{w} \cdot \mathbf{x} = w_0 + w_1 x_1 + w_2 x_2$   
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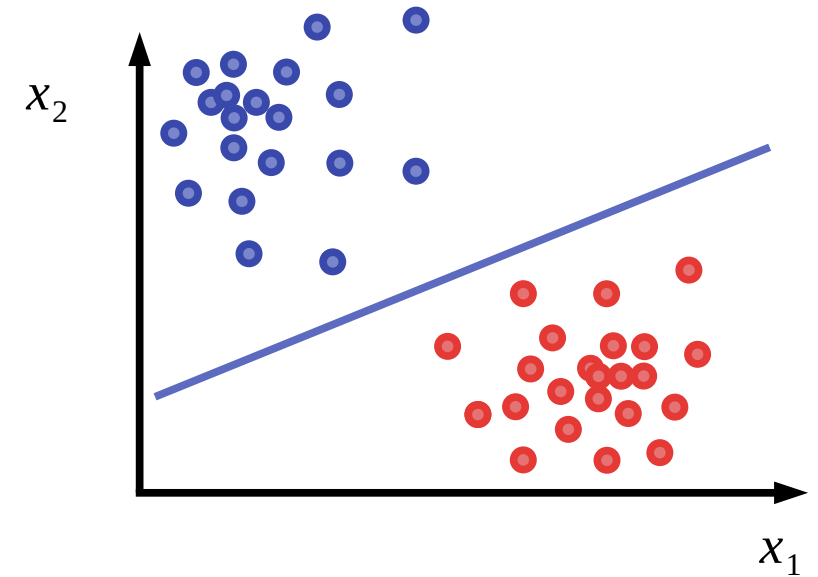


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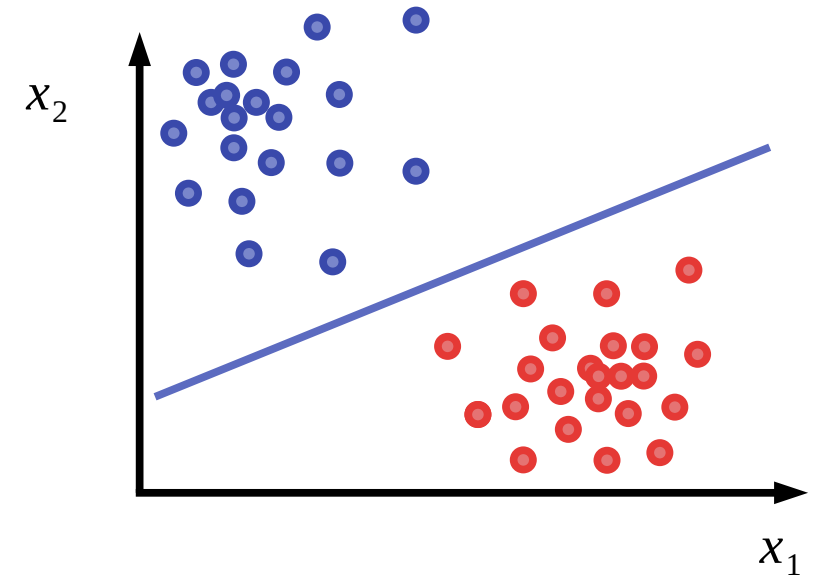
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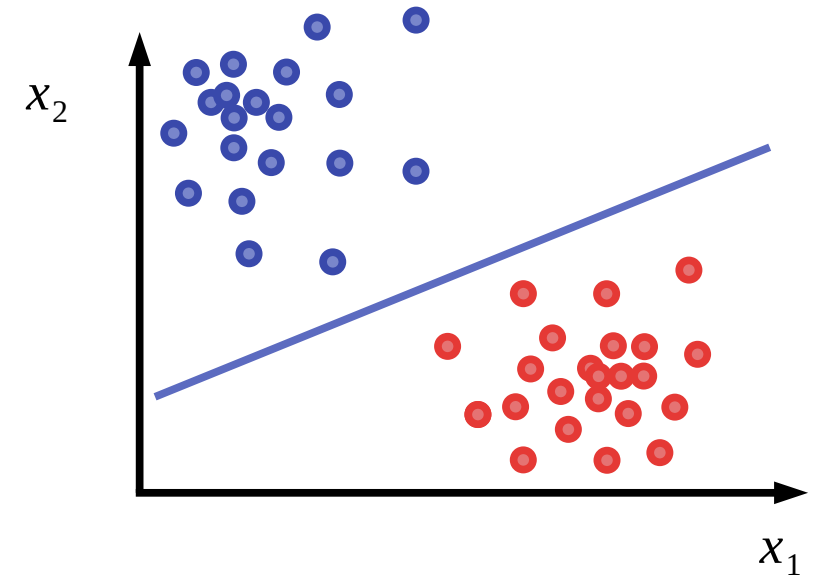
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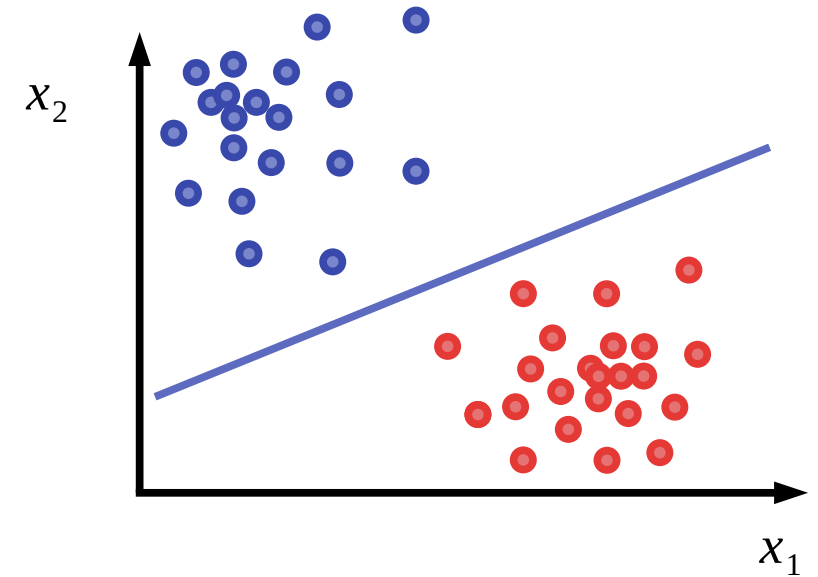
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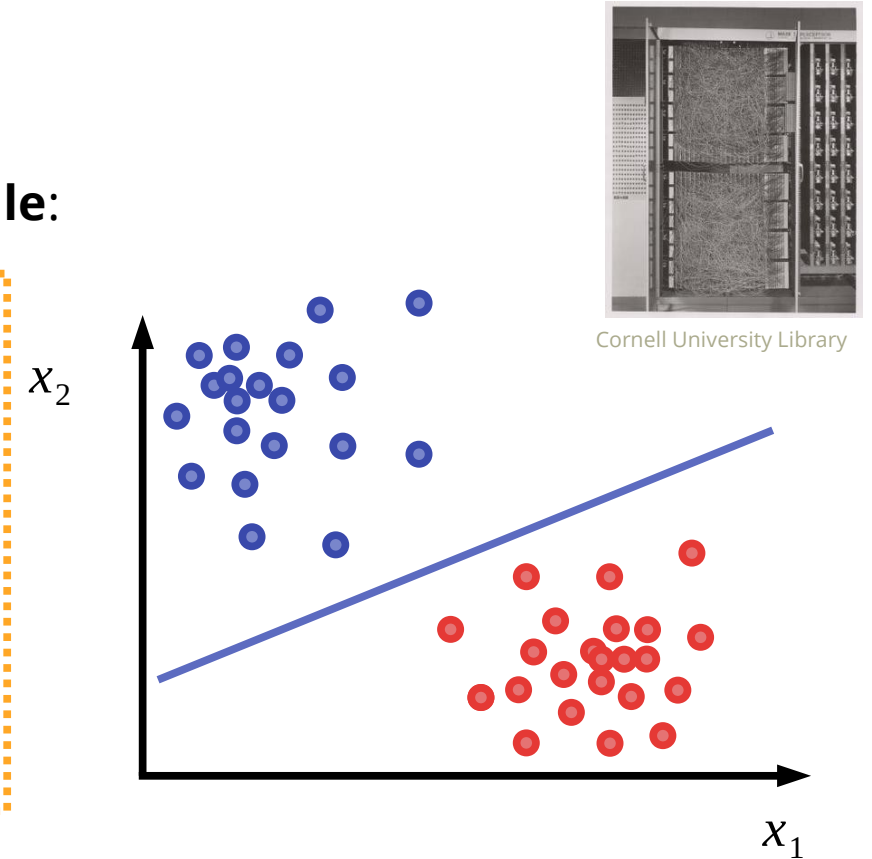
How can we learn  $\mathbf{w}$  from data?

# Linear classifier (two-dimensional case) – Perceptron learning rule

We define the following algorithm as the **Perceptron learning rule**:

We consider each data point, consisting of  $\mathbf{x}$  and ground-truth label  $y'$  and check whether the prediction from  $\bar{f}(\mathbf{x}, \mathbf{w})$  is correct, or not. If...

- $\bar{f}(\mathbf{x}, \mathbf{w}) = y$ , then do nothing.
- $\bar{f}(\mathbf{x}, \mathbf{w}) = 0$  but  $y' = 1$ , then increase  $w_i$  if  $x_i \geq 0$ , or vice versa.
- $\bar{f}(\mathbf{x}, \mathbf{w}) = 1$  but  $y' = 0$ , then decrease  $w_i$  if  $x_i \geq 0$ , or vice versa.



Weights are adjusted by a step size that is called the **learning rate**. By iteratively running this algorithm over your training data multiple times, the weights can be learned so that the model performs properly.

# Linear models – pros and cons



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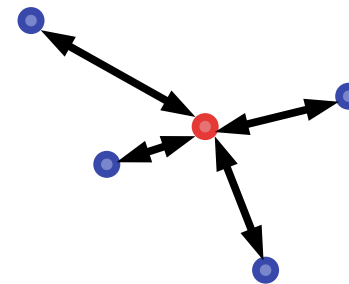
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## Nearest-Neighbor models





# Nearest neighbor models

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# Nearest neighbor models

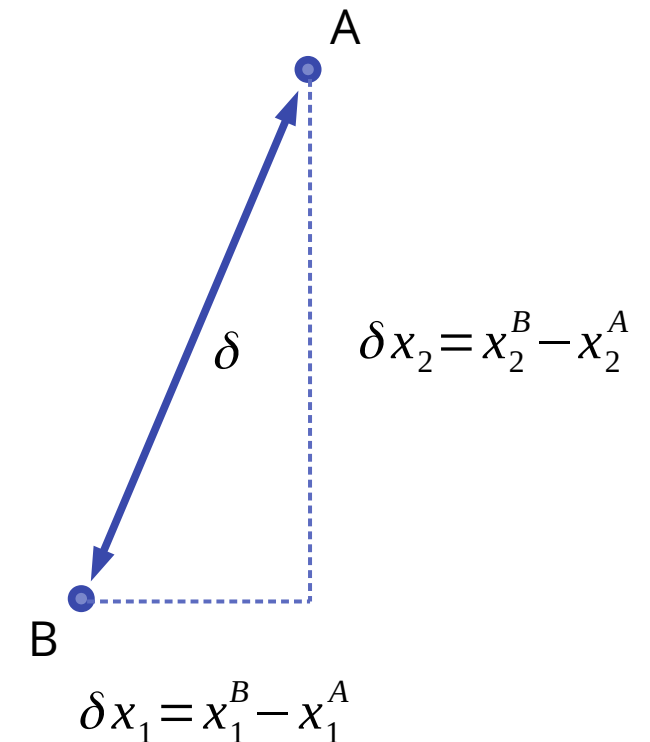
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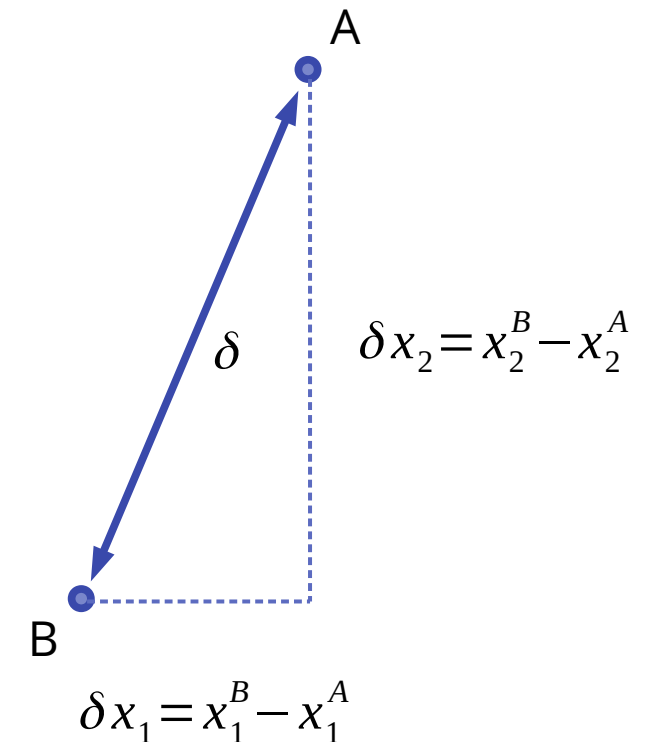


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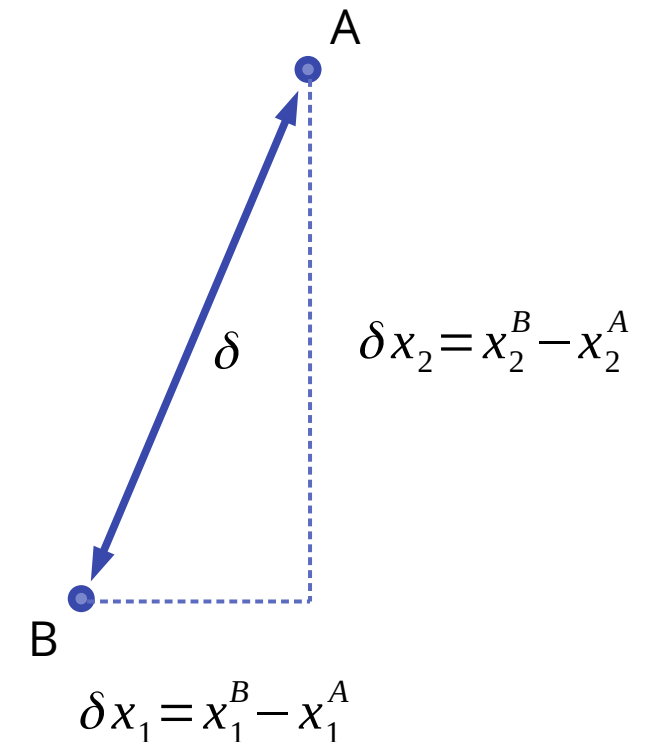
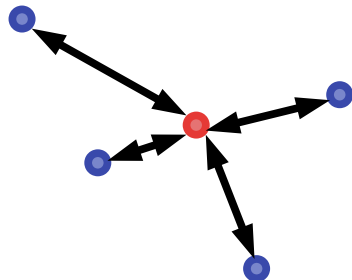
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Nearest neighbor methods utilize distances between datapoints for **classification** and **regression** tasks.



# ***k*-nearest neighbor classification**

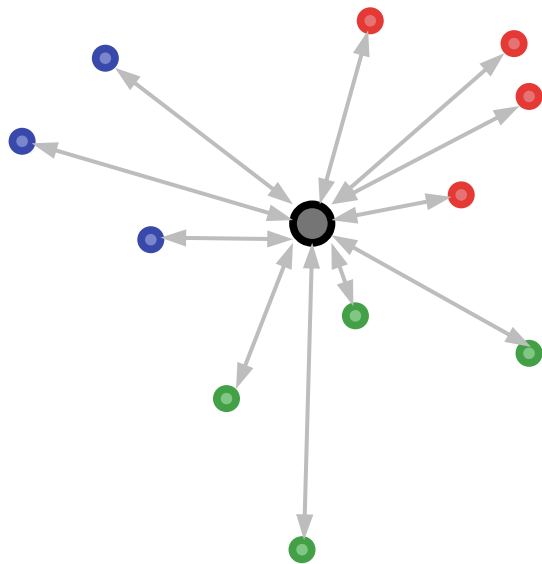
*k*-nearest neighbor (knn) classifiers predict class affiliation of an unseen data point based on **majority voting** of its ***k* nearest neighbors** in a seen data set with ground-truth labels.

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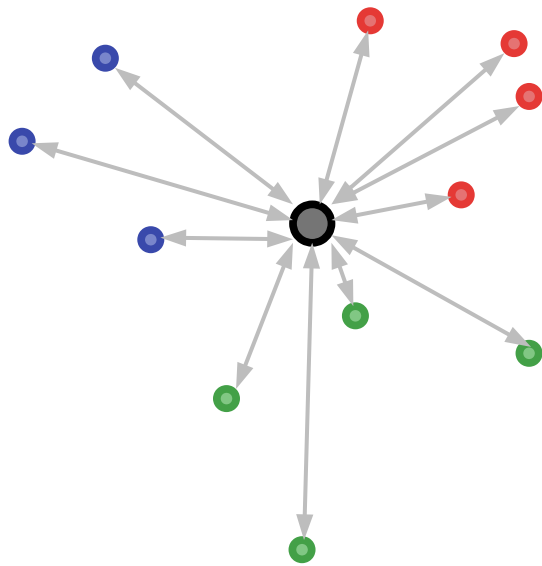


compute distances

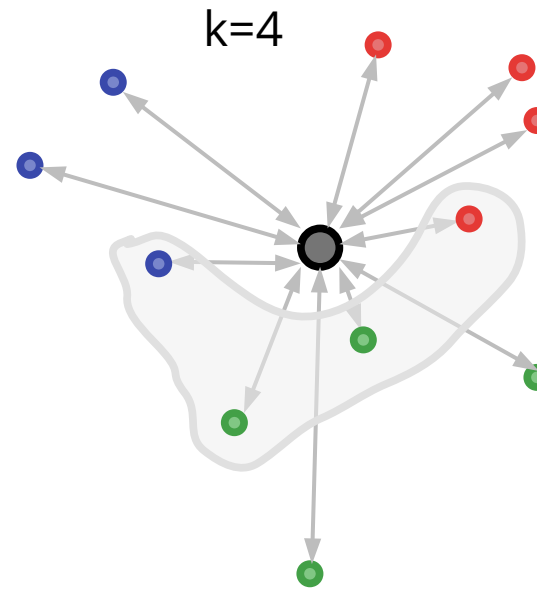
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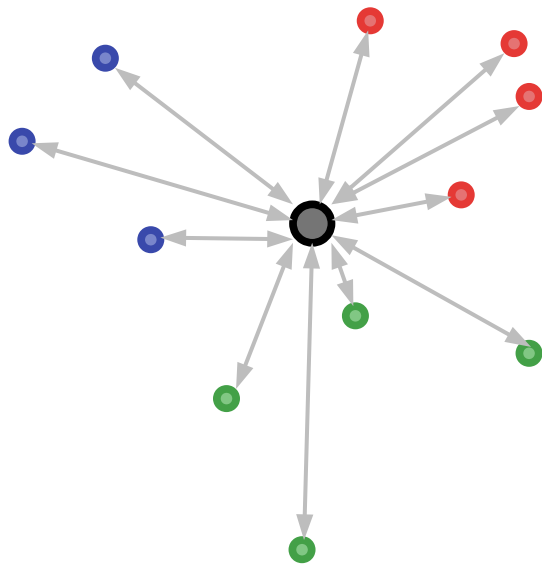
identify *k* nearest neighbors



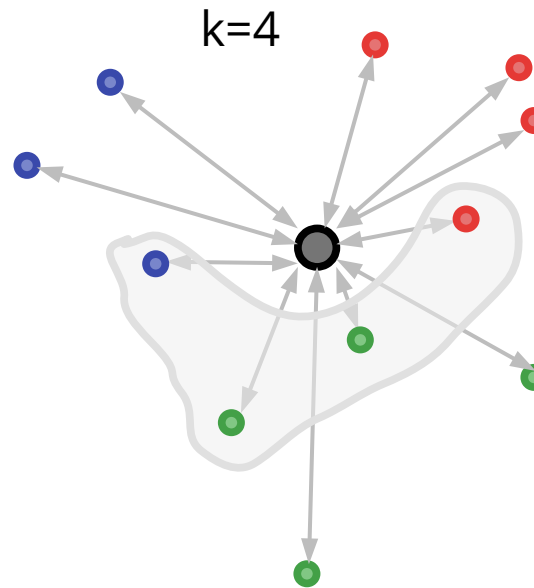
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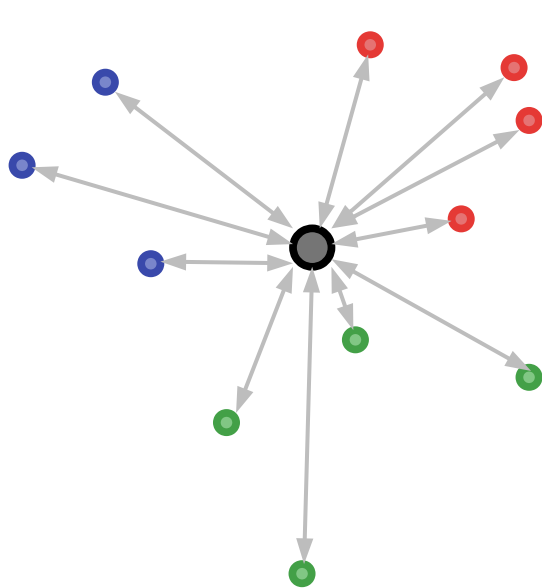
k=4:



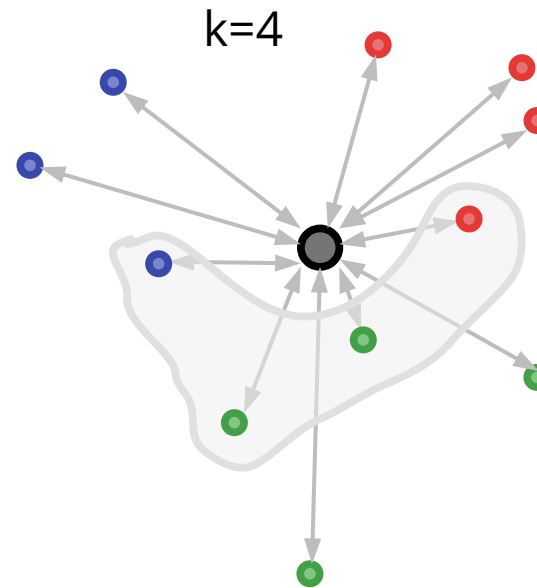
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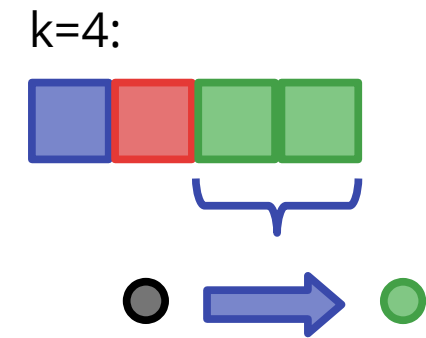
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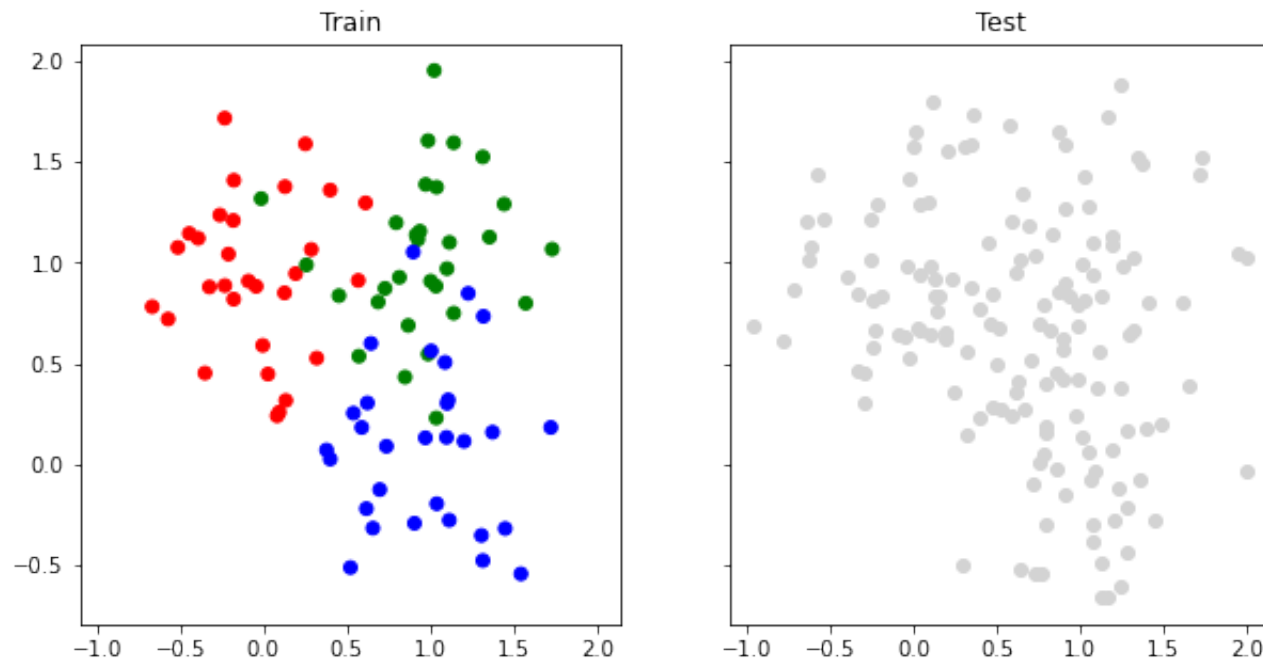


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class assignment

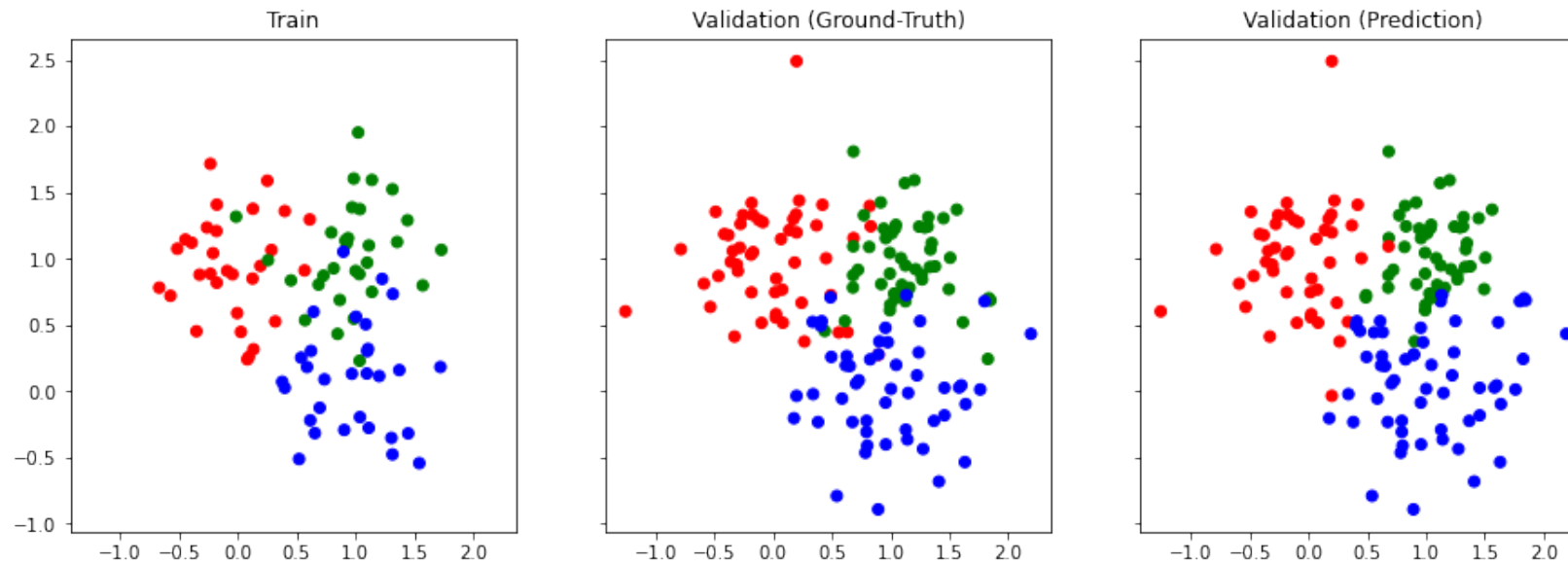
# k-nearest neighbor classification - Example



3 overlapping clusters

How well can knn classify  
our test data set?

# k-nearest neighbor classification - Example

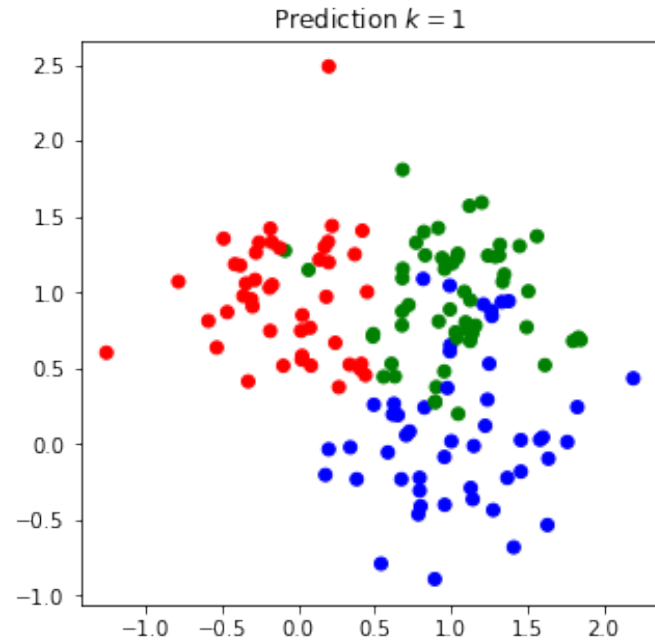


$k=5$ : accuracy=0.867

Hyperparameter  $k$  has an impact on how well the model generalizes to unseen data: perform a **hyperparameter search**!

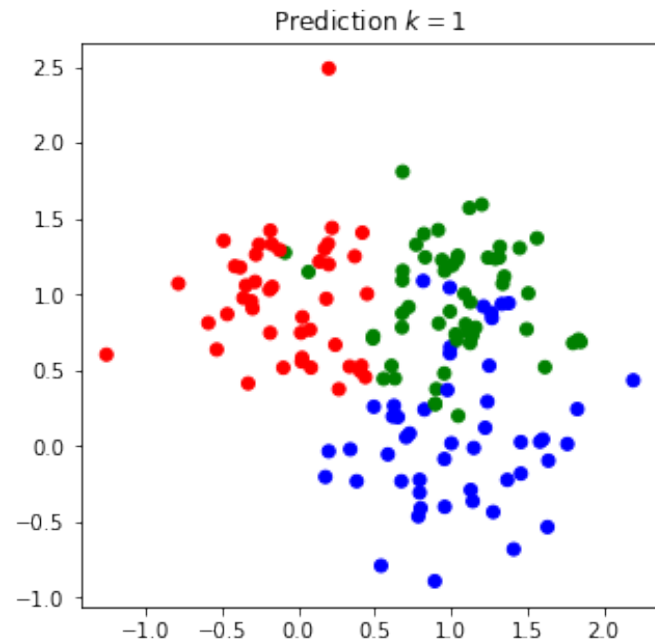
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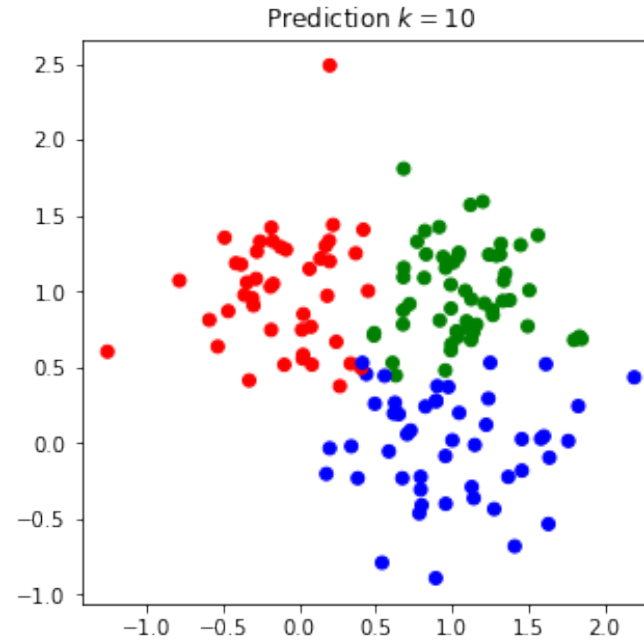


$k=1$   
accuracy<sub>val</sub>=0.800

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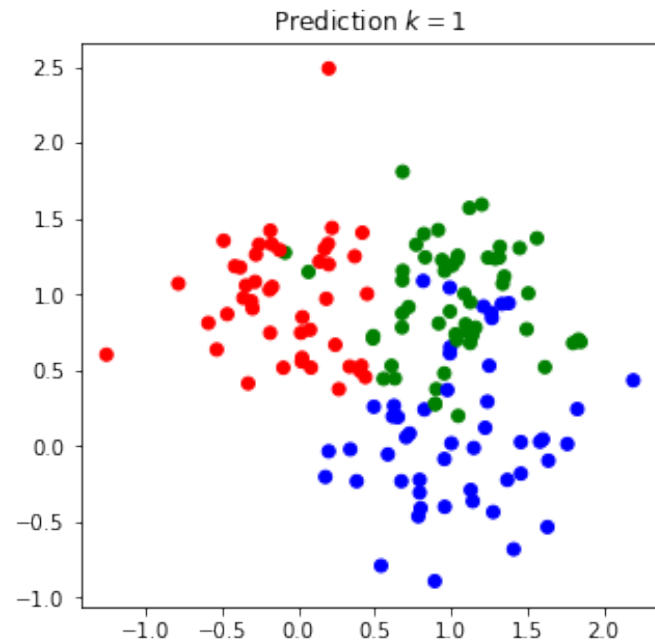


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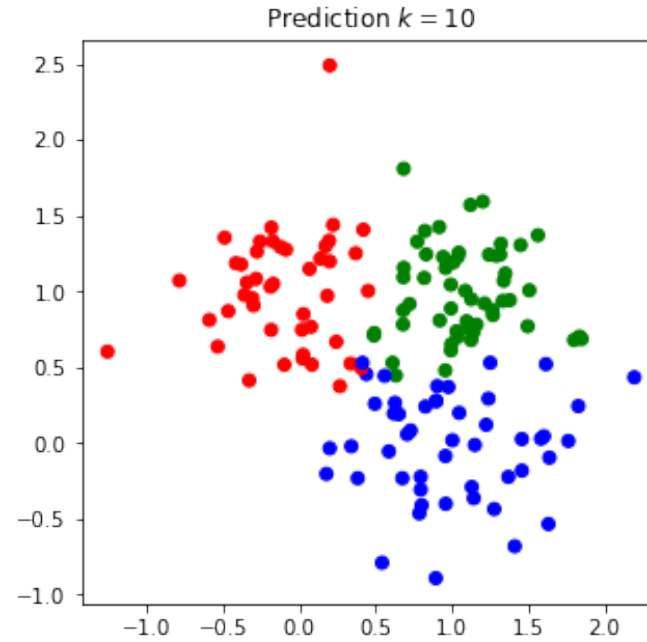


$k=10$   
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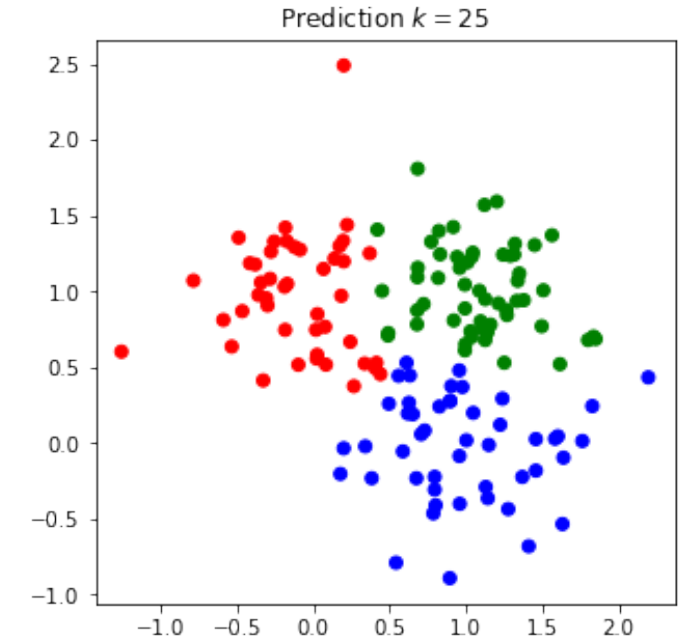
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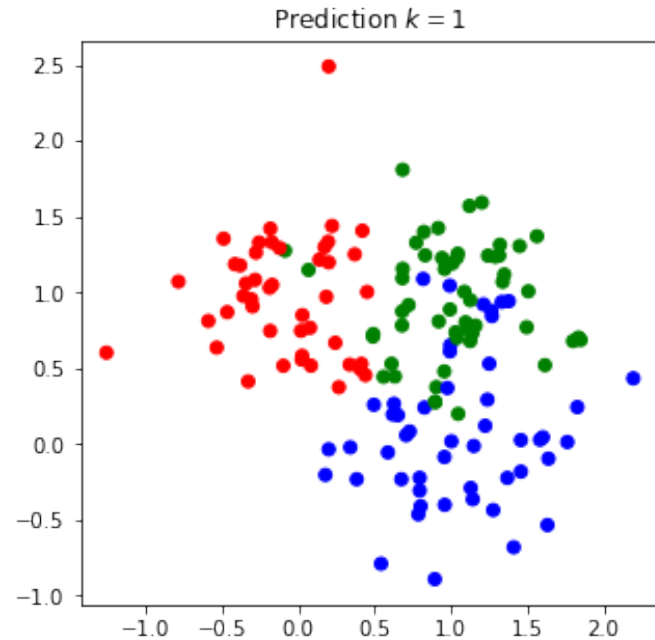
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$k=25$   
accuracy<sub>val</sub>=0.873

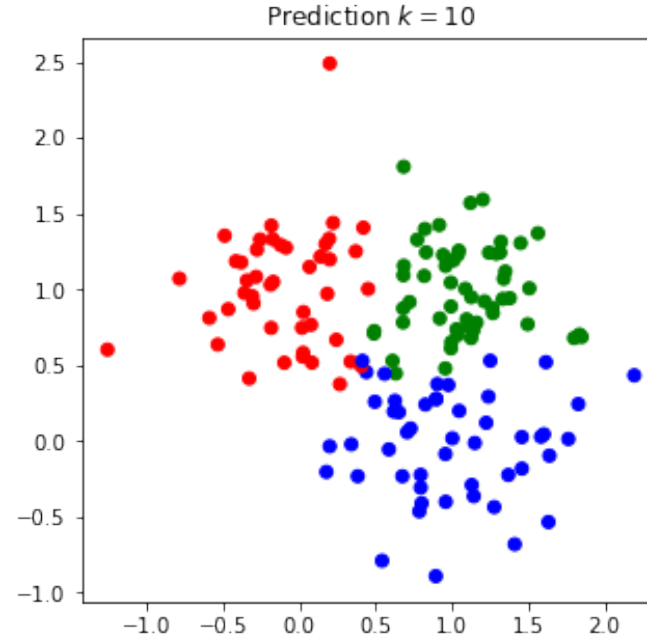


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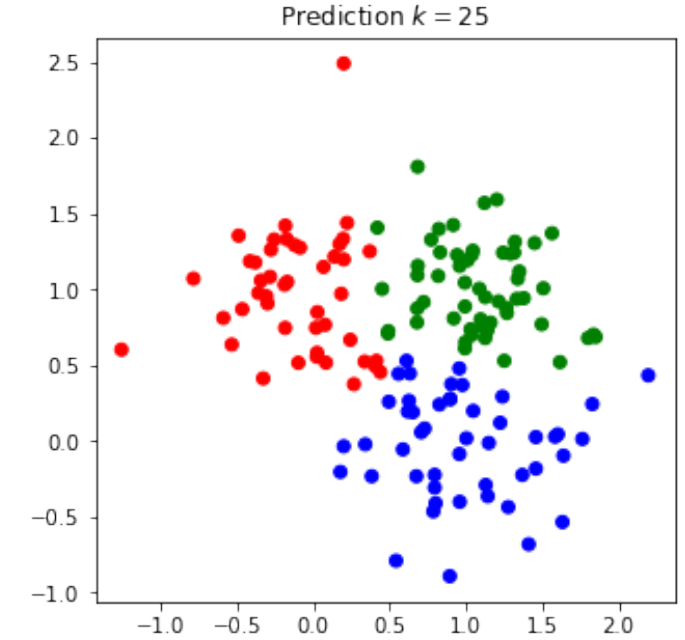
$k=1$   
accuracy<sub>val</sub>=0.800

**Overfitting!**



$k=10$   
accuracy<sub>val</sub>=0.893  
accuracy<sub>test</sub>=0.880

**Best performance**



$k=25$   
accuracy<sub>val</sub>=0.873

**Underfitting!**

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Number of data points should grow exponentially with data dimensionality.

If parameter space is insufficiently sampled, the model does not have enough data points for training properly.

# Bonus Slide: The Curse of Dimensionality

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Datasets we deal with typically have a limited size,  $N$ . The number of features in our dataset,  $d$ , defines the dimensionality of the feature space; the volume of the feature space,  $V$ , grows exponentially with  $d$ .

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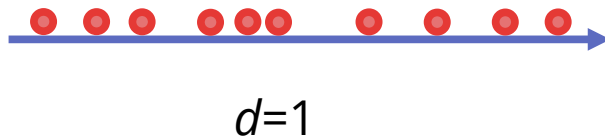
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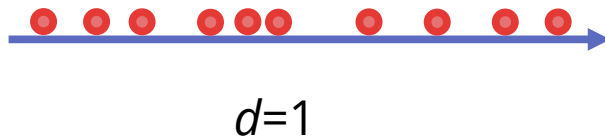


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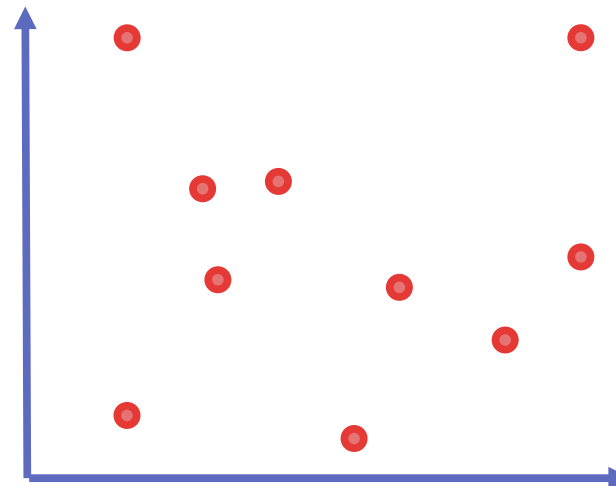
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$d=1$

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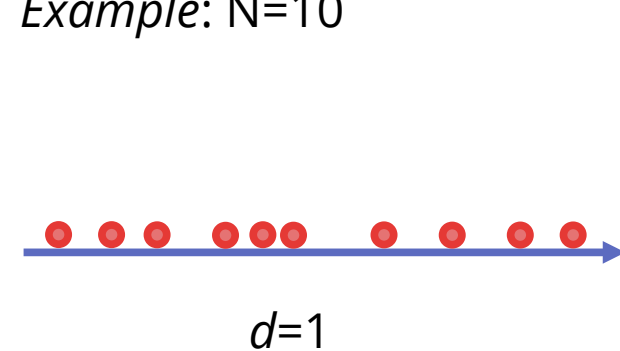
$d=2$

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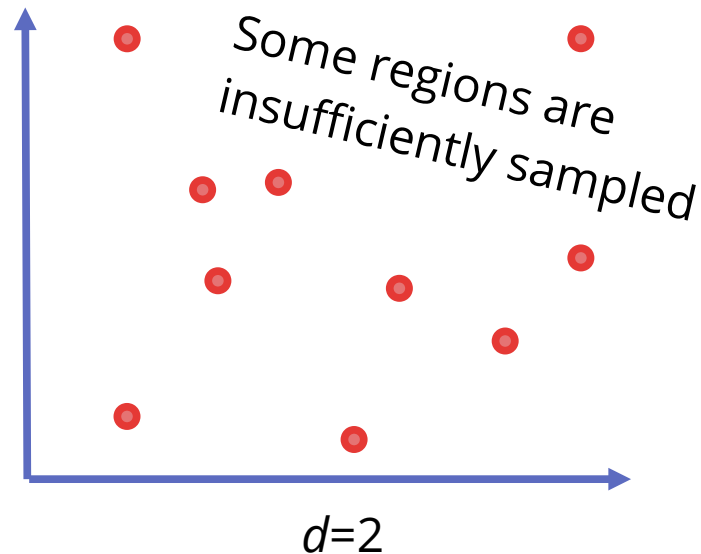
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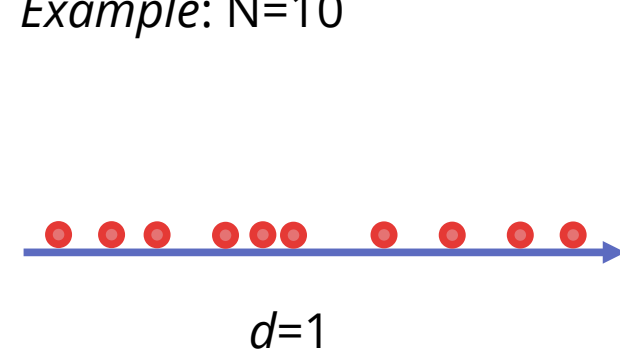


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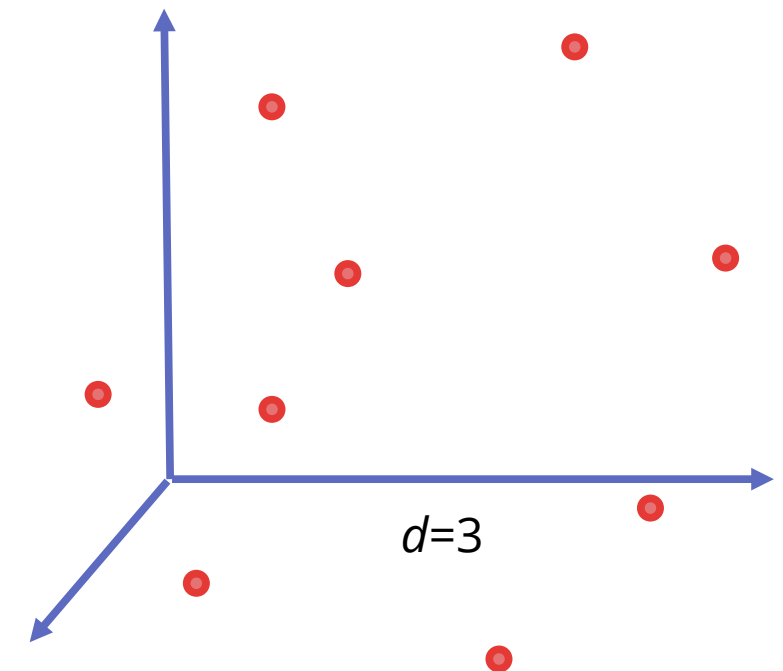
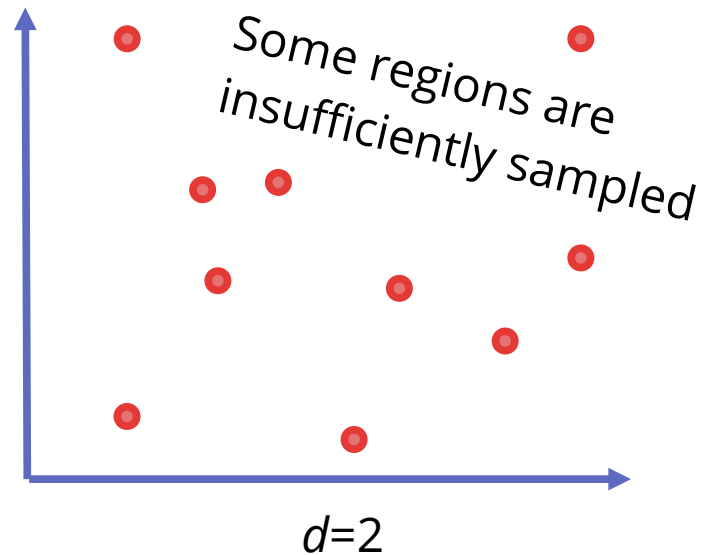
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Curse of Dimensionality: for large  $d$  (high-dimensional input data),  $N$  may be too small to sample  $V$ .

Example:  $N=10$



Feature space is well-sampled

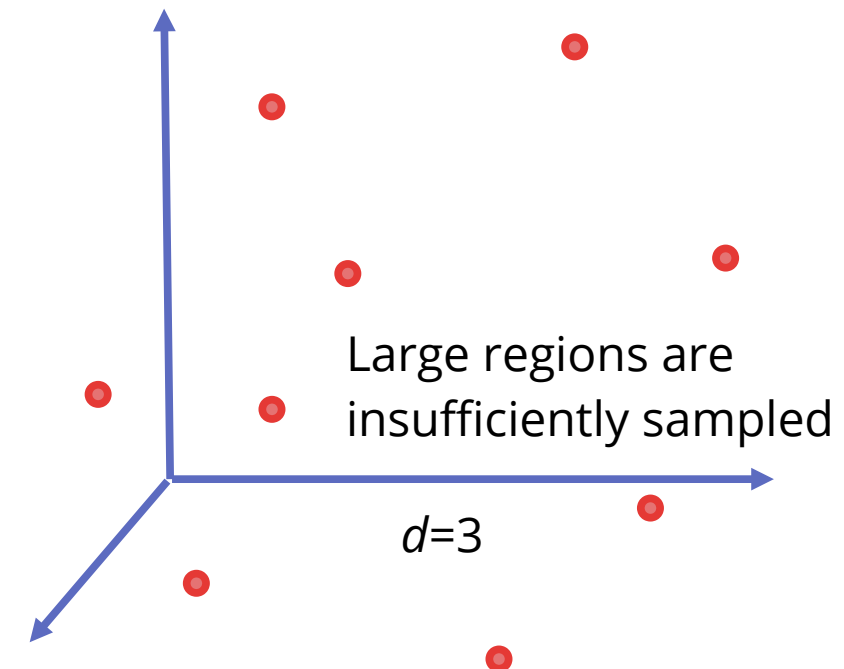
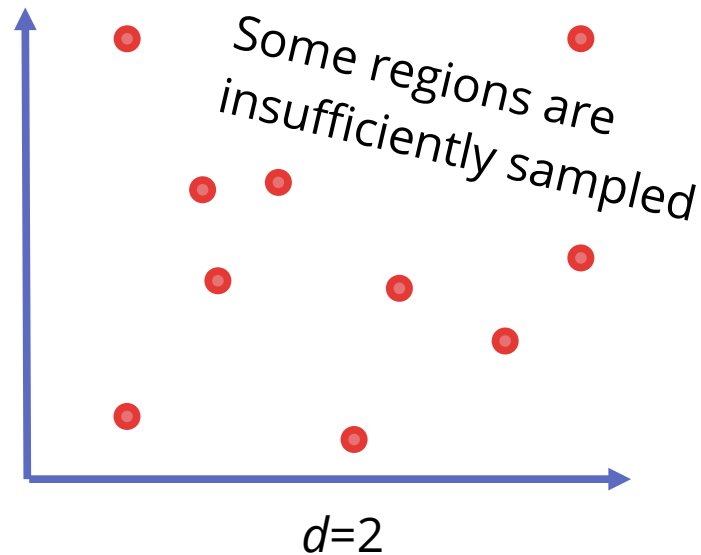
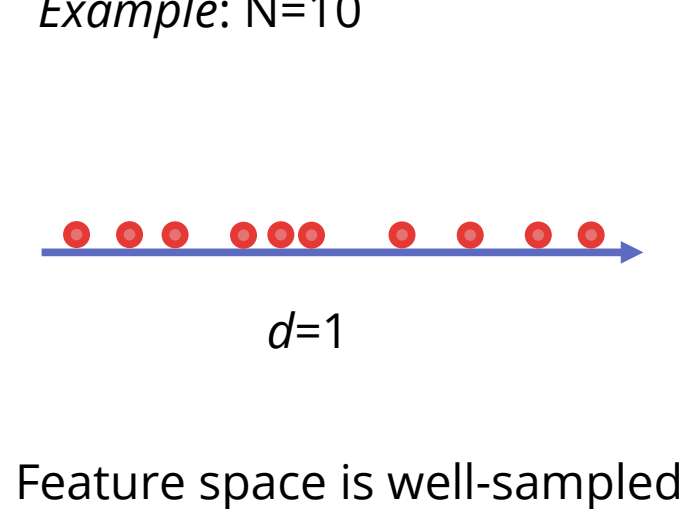


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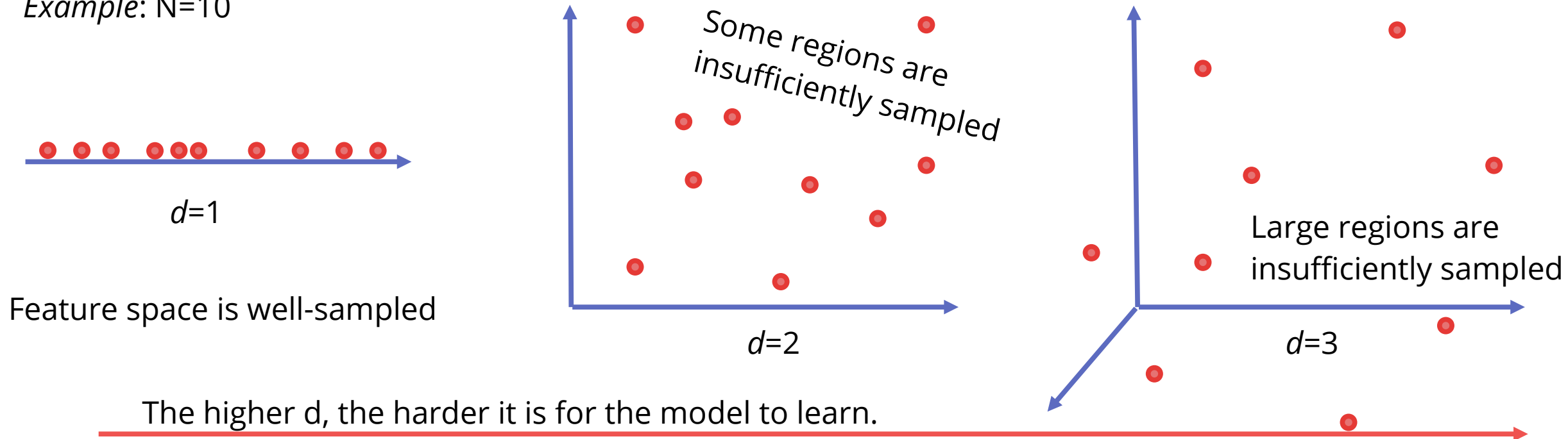


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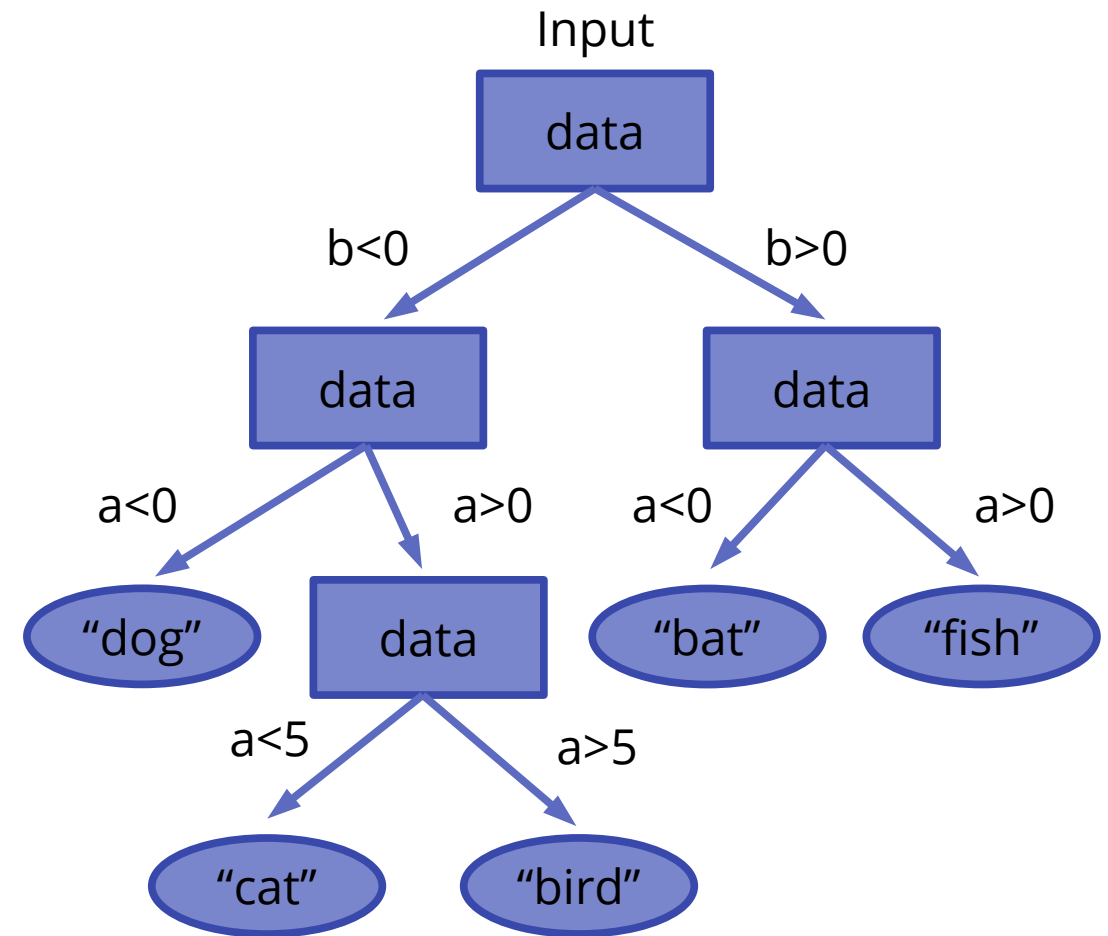
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## Tree-based models (a high-level introduction)



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Vectorial input  
"Root" a=1, b=-2

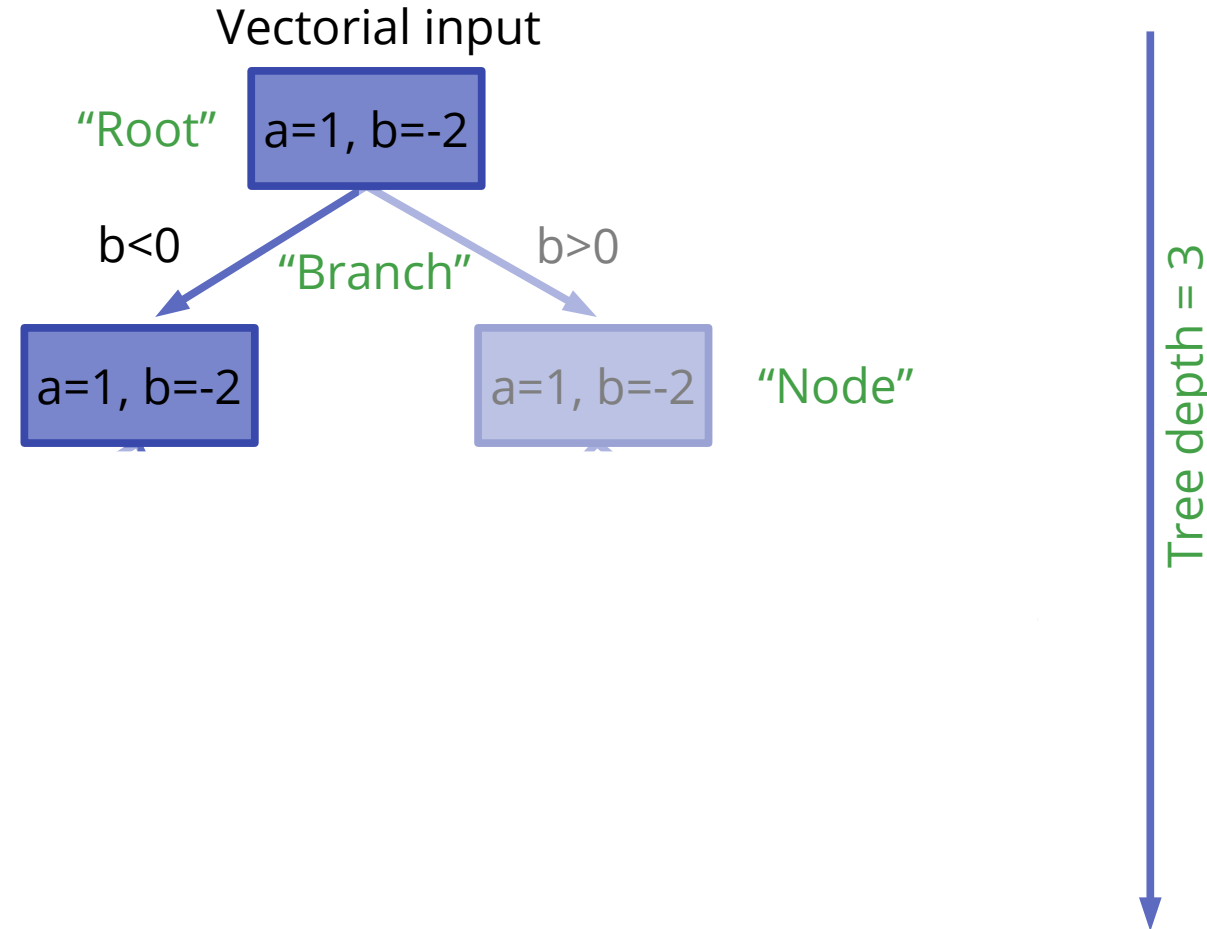
Tree depth = 3

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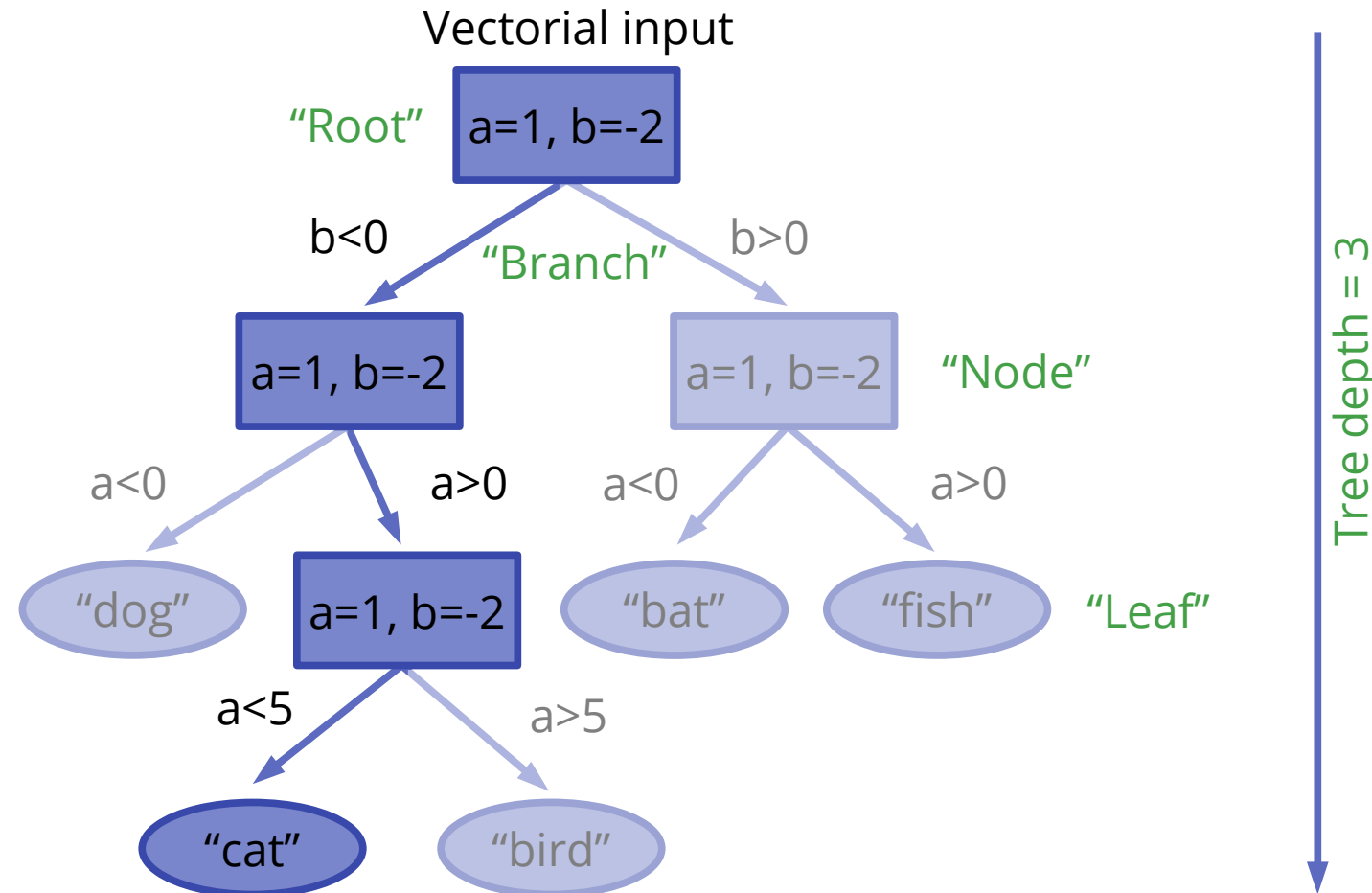


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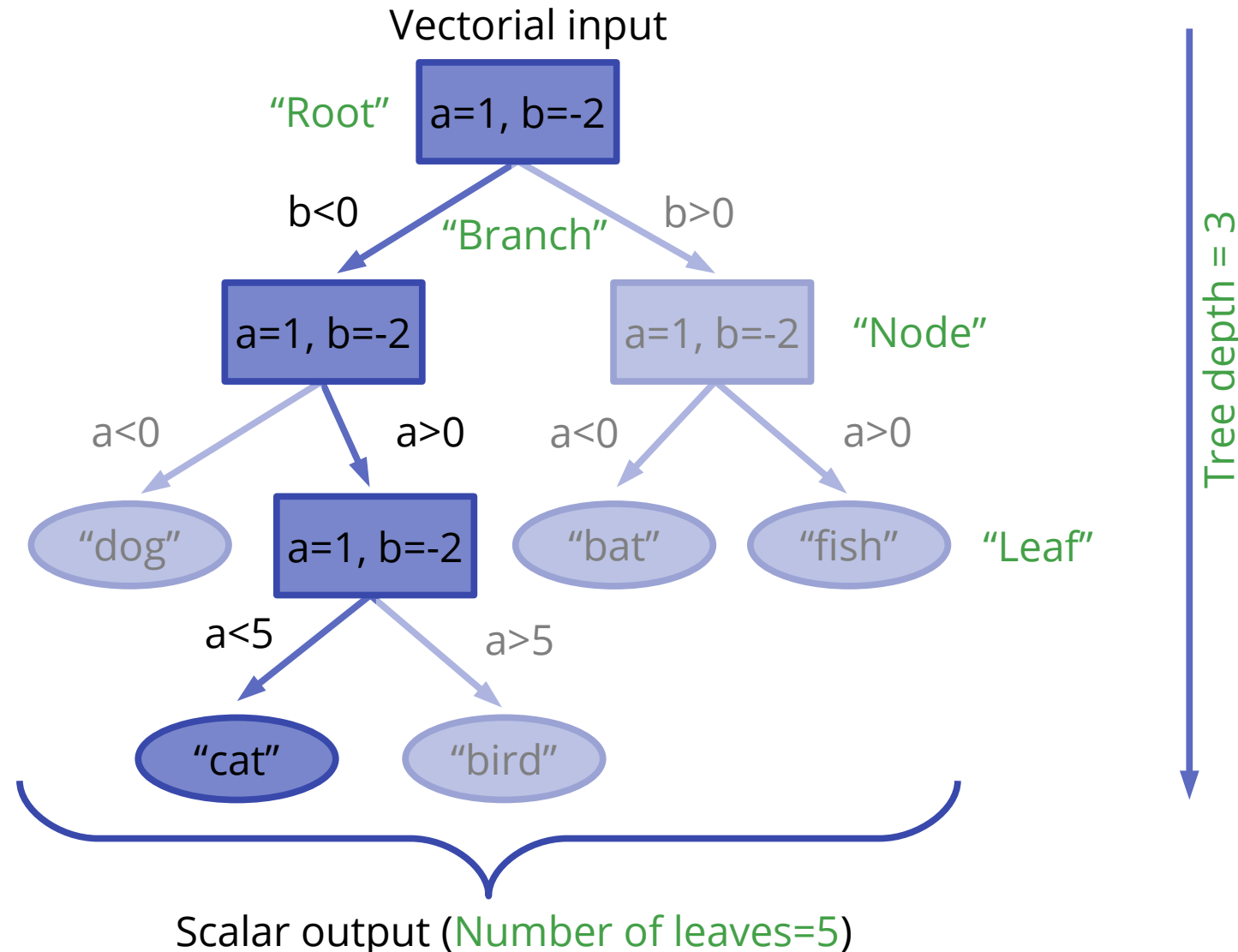


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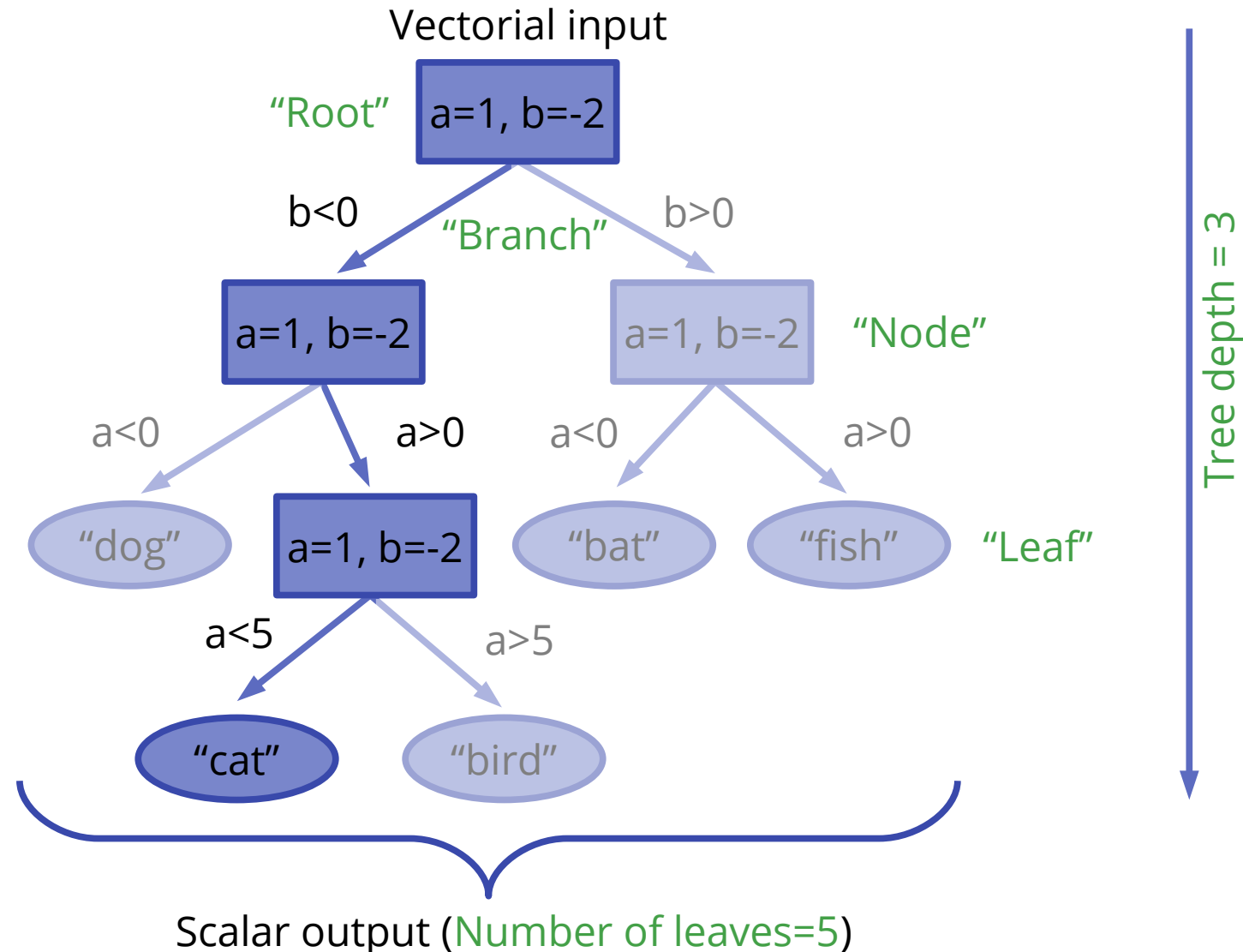
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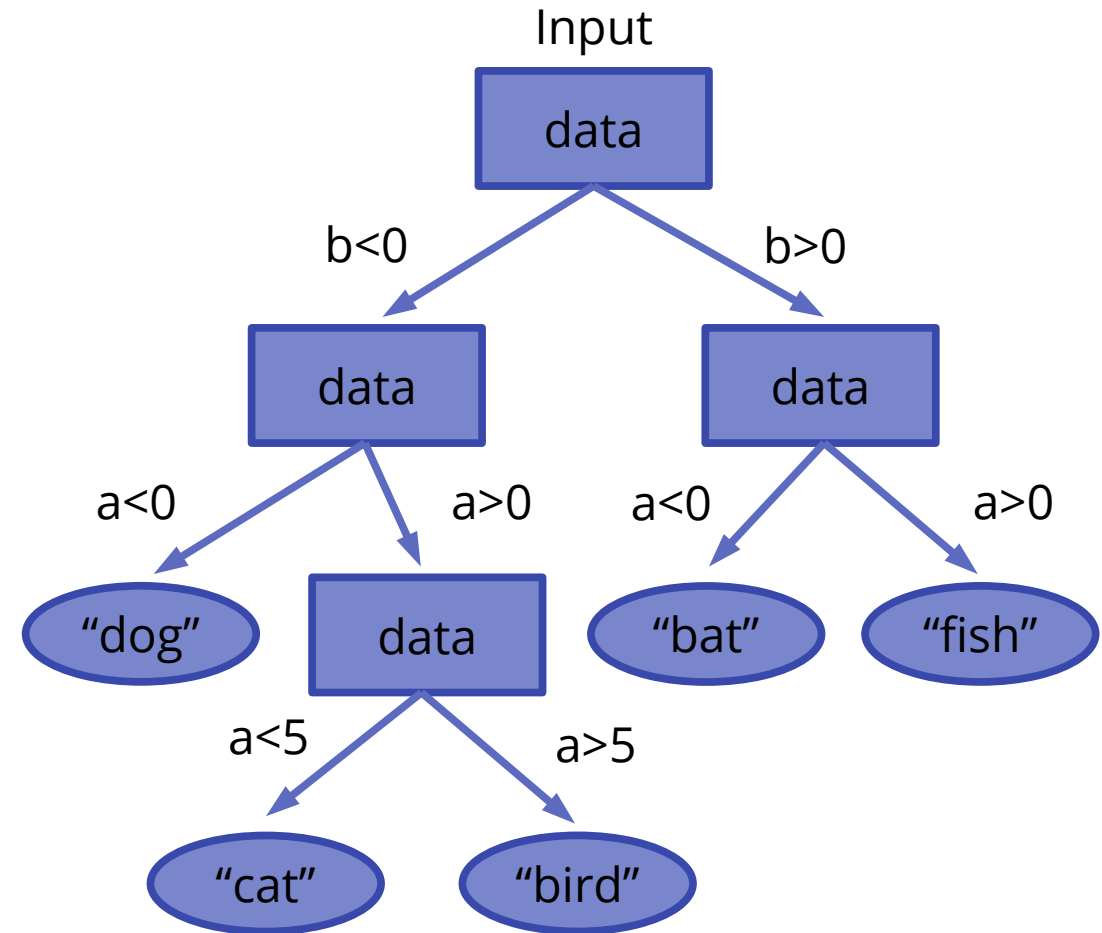
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How can the rules stored in the nodes be learned?

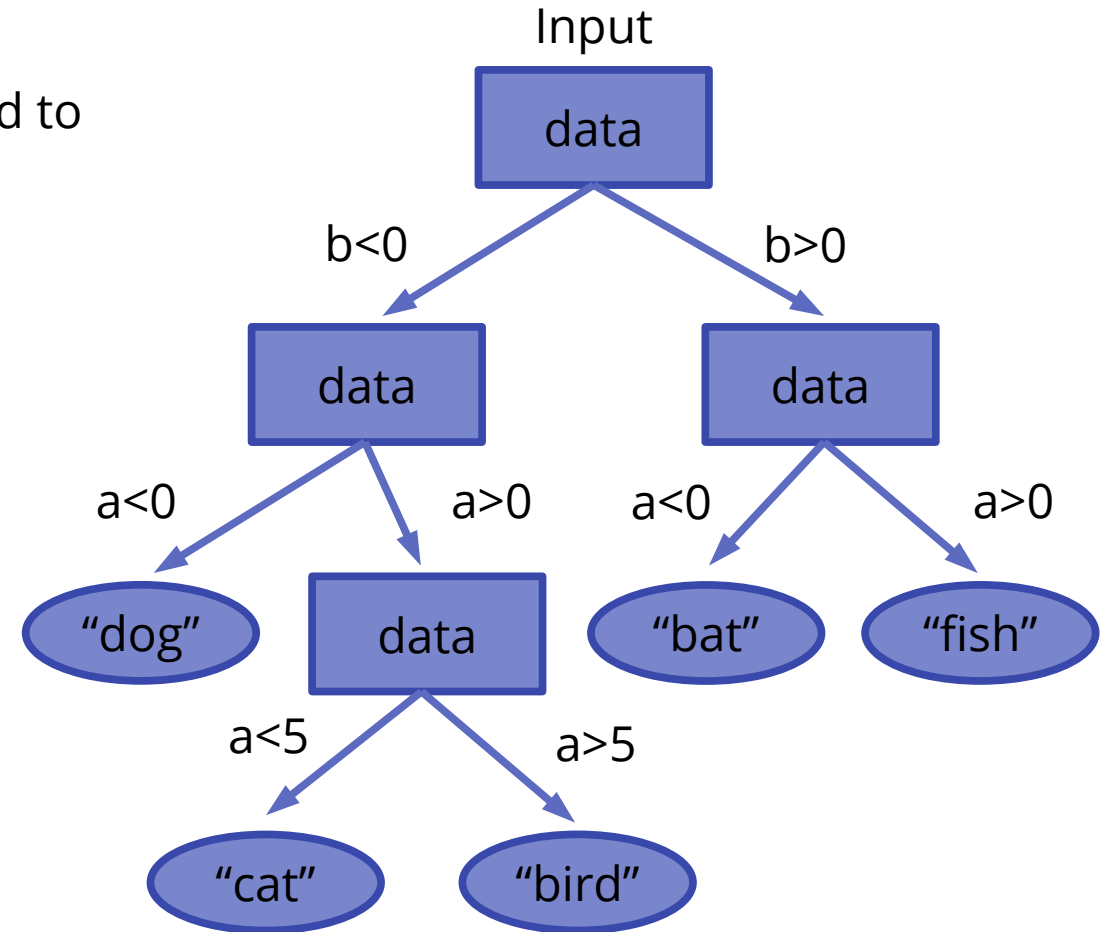


# Decision tree learning



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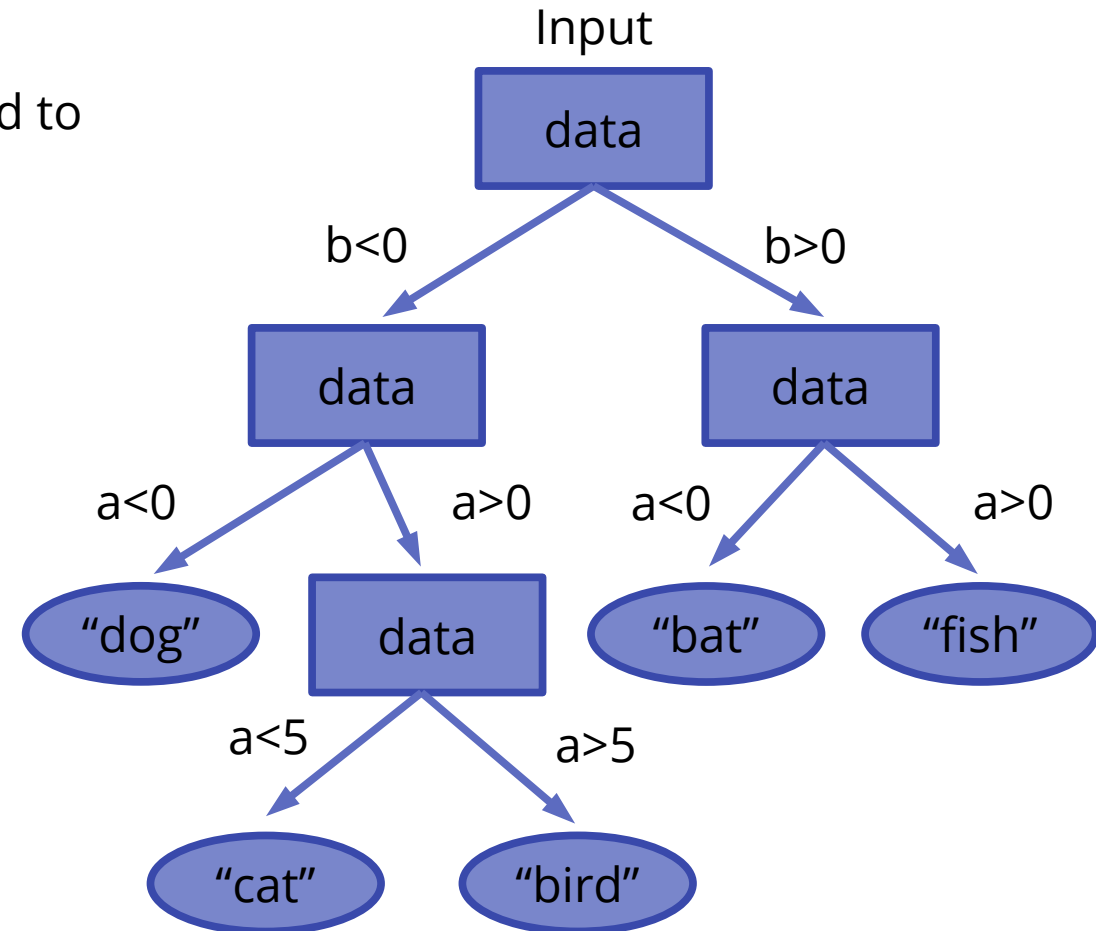
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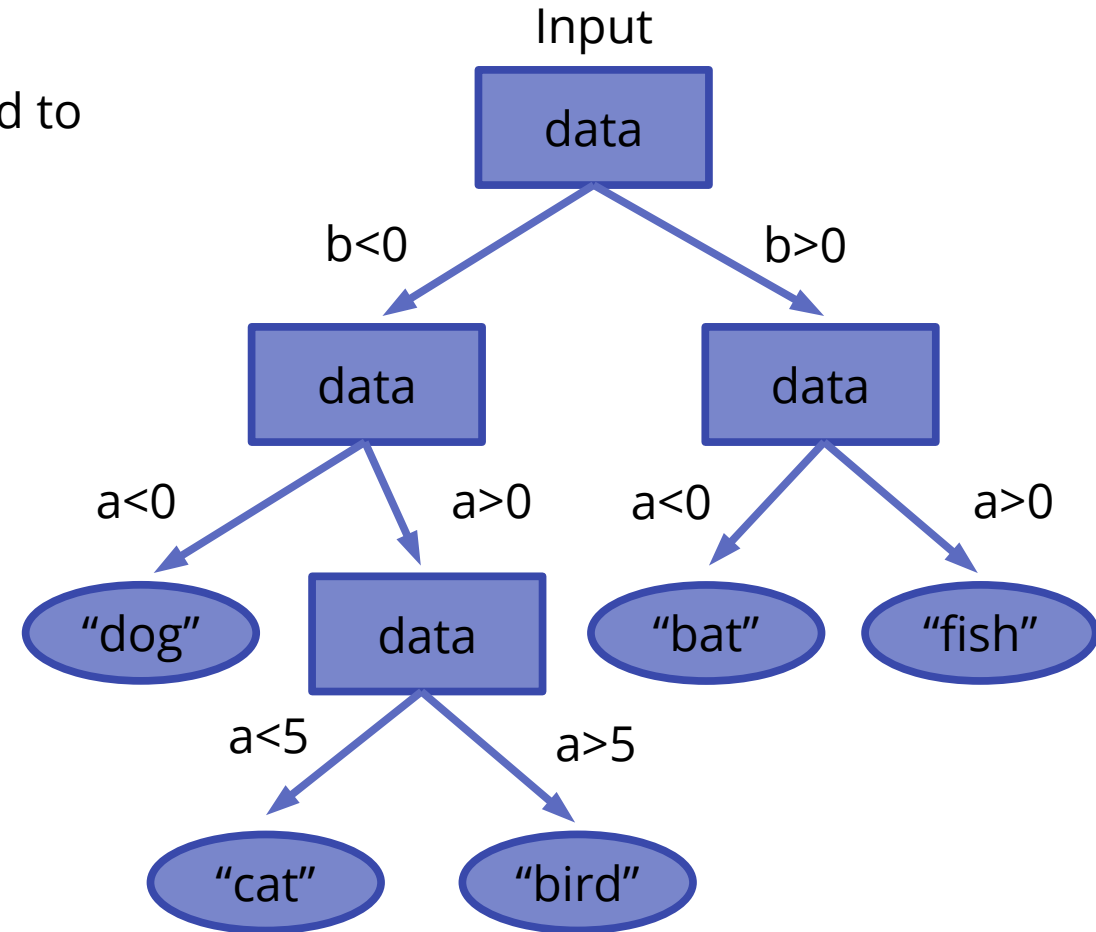




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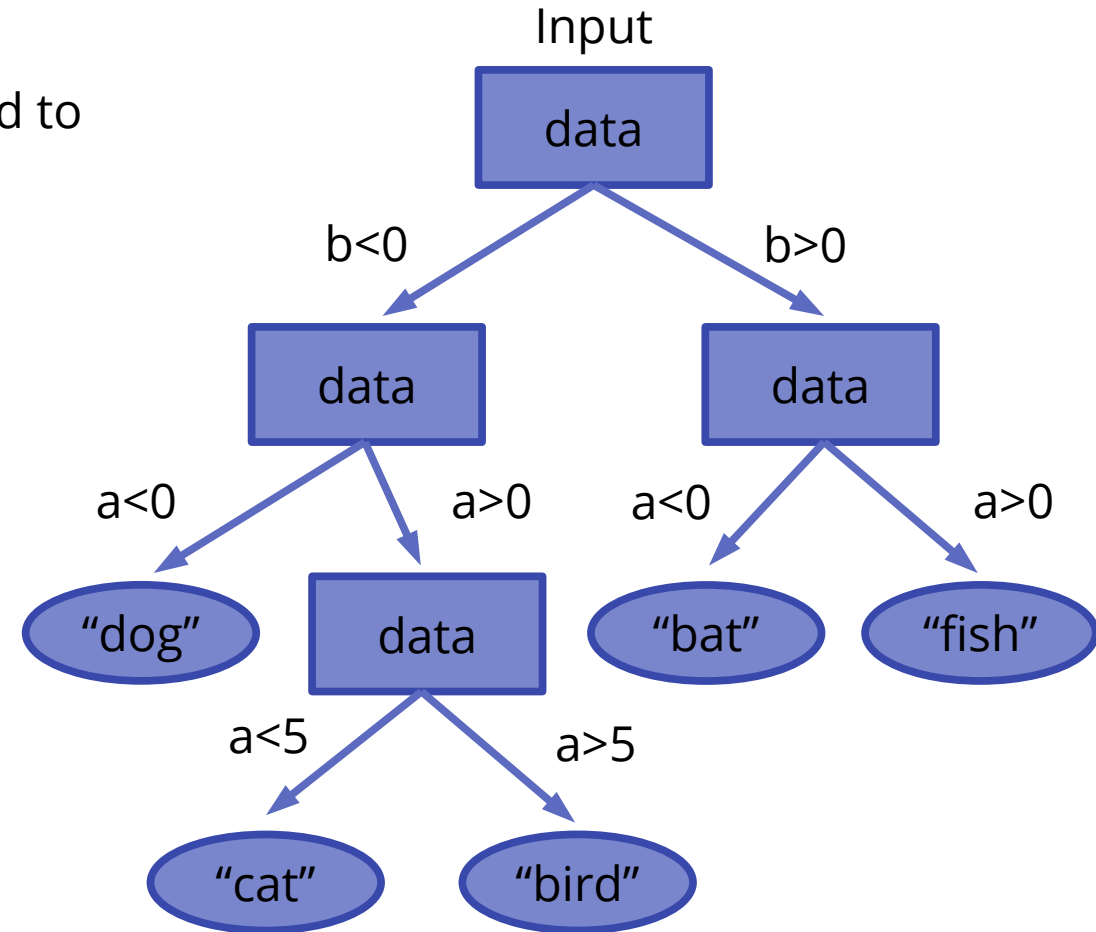
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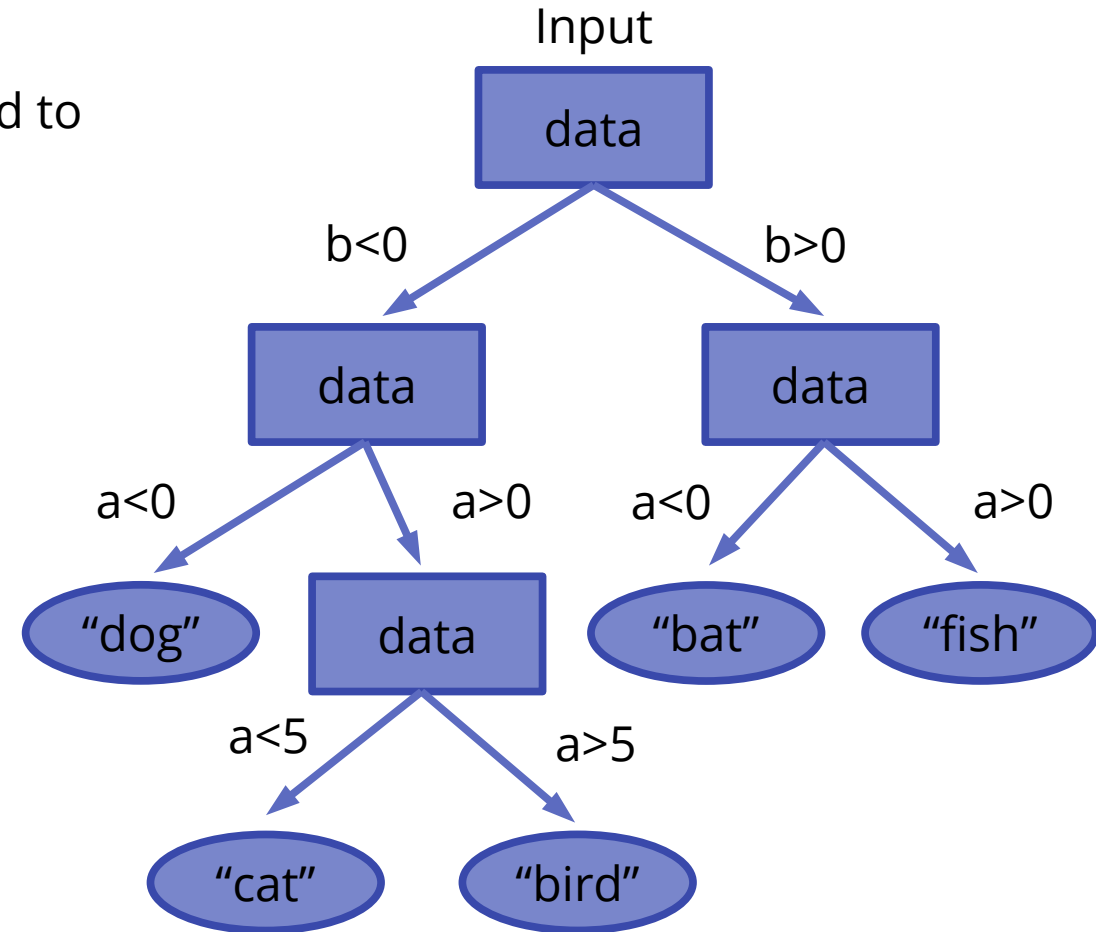
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## But what is the “most important feature”?

Generally, it means that feature that makes the most difference to the classification of a single sample.

There different implementation of this definition, e.g., utilizing **information entropy** or other useful measures.

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Trees in a **random forest** are shallower than other decision tree models. The trees therefore act as “**weak learners**” that perform badly by themselves. However, combining a large number of weak learners performs much better than individual trees. The intuition behind is that weak learners “on average” compensate for their individual shortcomings.

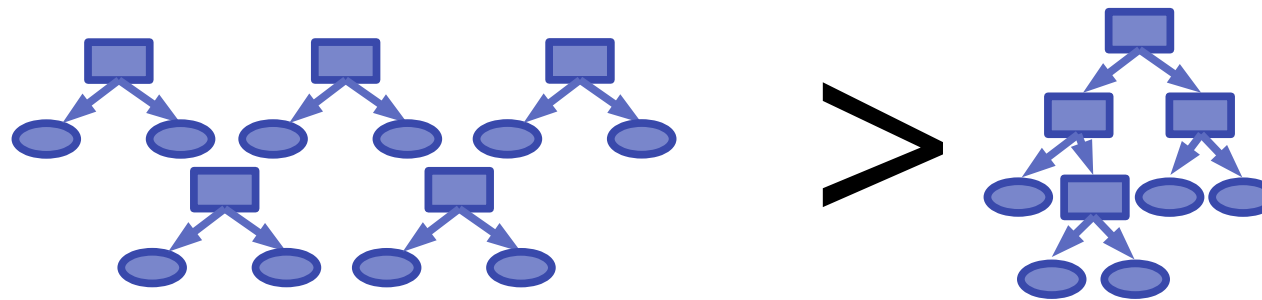


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# Gradient-boosted tree-based models

Gradient-boosted tree-based models are random forests (decision tree ensembles) that are built successively in such a way that *every newly created tree compensates for the shortcomings of the previous trees*.

The term **gradient-boosting** refers to the fact that new base learners (individual decision trees) are fitted to the model's pseudo-residuals, based on the gradient of the loss of the ensemble:

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$$f(\mathbf{x}) = \sum_m^M \beta_m h(\mathbf{x}, \theta_m)$$

Ensemble model with learning rate  $\beta_m$ , base learners  $h(\mathbf{x}, \theta_m)$  with parameters  $\theta_m$ .

$$\mathbf{r}_m^i = - \left[ \frac{\partial L(y^i, f(\mathbf{x}^i))}{\partial f(\mathbf{x}^i)} \right]_{f=f_{m-1}}$$

Pseudo-residuals to which the next base learner will be fitted; by default, the loss of the updated ensemble will be less or equal to the loss of the current ensemble.

# Gradient-boosted tree-based models

Gradient-boosted models are very successful in regression and classification tasks and still represent state-of-the-art in traditional ML.

If you have a classification or regression problem, it is always worth trying out gradient-boosted methods.

Common implementations:

- XGBoost
- LightGBM



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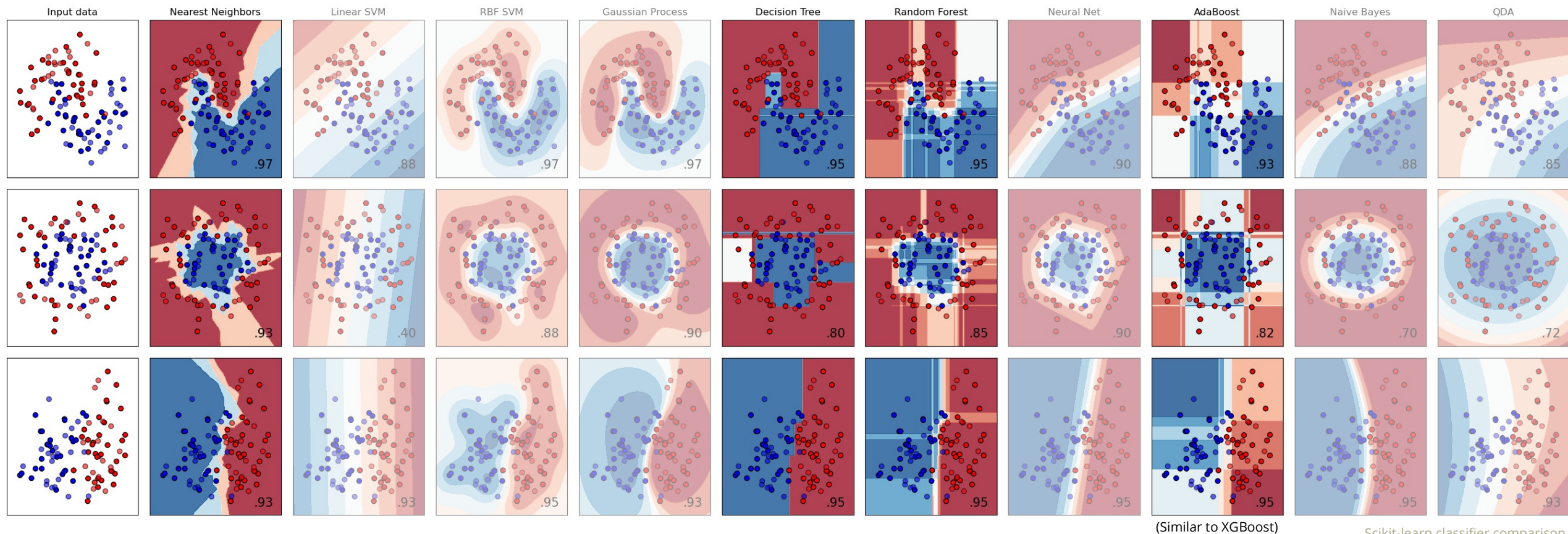
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## Cons:

- Decision boundaries and regression predictions may be discrete instead of continuous (see next slide)

# Supervised learning - summary



**That's all folks!**

## Today's lecture

### Supervised learning

Supervised learning setup

Supervised learning concepts

Benchmarking and metrics

Linear models

Nearest Neighbor models

Tree-based models

## Next lecture (20 March)

### Next Week:

Lab 1: Supervised learning

### Unsupervised learning

Unsupervised learning setup

Hierarchical clustering

k-means clustering

Expectation Maximization clustering

DBSCAN

Principal component analysis