ElMag - Formelsammlung

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2 Idiotenseite

2.1 SI-Vorsätze

Symbol	Name	Wert	Binär	Symbol	Name	Wert
da	Deka	10^1		d	Dezi	10^{-1}
h	Hekto	10^2		c	Centi	10^{-2}
k	Kilo	10^3	$2^{10} = 1024$	m	Mili	10^{-3}
M	Mega	10^6	2^{20}	y, μ	Mikro	10^{-6}
G	Giga	10^9	2^{30}	n	Nano	10^{-9}
T	Tera	10^{12}	2^{40}	p	Piko	10^{-12}
P	Peta	10^{15}	2^{50}	f	Femto	10^{-15}

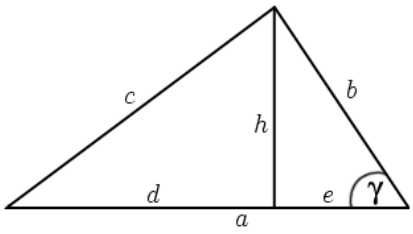
2.2 Dreiecksformeln

Cosinussatz

$$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$$

Sinussatz

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} = 2r = \frac{u}{\pi}$$



Pythagoras beim Sinus

$$\sin^2(b) + \cos^2(b) = 1 \qquad \tan(b) = \frac{\sin(b)}{\cos(b)}$$

$$\sin\beta = \frac{b}{a} = \frac{\text{Gegenkathete}}{\text{Hypotenuse}} \qquad \tan\beta = \frac{c}{b} = \frac{\text{Gegenkathete}}{\text{Ankathete}}$$
$$\cos\beta = \frac{c}{a} = \frac{\text{Ankathete}}{\text{Hypotenuse}} \qquad \cot\beta = \frac{c}{b} = \frac{\text{Ankathete}}{\text{Gegenkathete}}$$

2.3 Funktionswerte für Winkelargumente

deg	rad	sin	cos	tan
0	0	0	1	0
30	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

deg	rad	sin	cos
90	$\frac{\pi}{2}$	1	0
120	$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$
135	$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$
150	$\frac{5\pi}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$

deg	rad	sin	cos
180	π	0	-1
210	$\frac{7\pi}{6}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
225	$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$
240	$\frac{4\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$

deg	rad	sin	cos
270	$\frac{3\pi}{2}$	-1	0
300	$\frac{5\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
315	$\frac{7\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
330	$\frac{11\pi}{6}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$

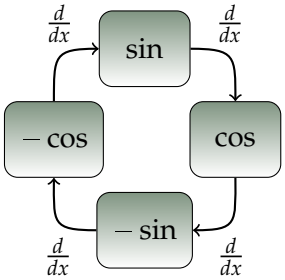
2.4 Periodizität

$$\cos(a + k \cdot 2\pi) = \cos(a) \qquad \sin(a + k \cdot 2\pi) = \sin(a) \qquad (k \in \mathbb{Z})$$

2.5 Quadrantenbeziehungen

$$\sin(-a) = -\sin(a) \qquad \cos(-a) = \cos(a)$$
$$\sin(\pi - a) = \sin(a) \qquad \cos(\pi - a) = -\cos(a)$$
$$\sin(\pi + a) = -\sin(a) \qquad \cos(\pi + a) = -\cos(a)$$
$$\sin\left(\tfrac{\pi}{2} - a\right) = \sin\left(\tfrac{\pi}{2} + a\right) = \cos(a) \qquad \cos\left(\tfrac{\pi}{2} - a\right) = -\cos\left(\tfrac{\pi}{2} + a\right) = \sin(a)$$

2.6 Ableitungen



2.7 Additionstheoreme

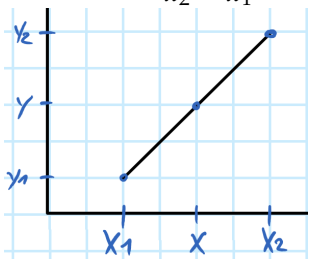
$$\sin(a \pm b) = \sin(a) \cdot \cos(b) \pm \cos(a) \cdot \sin(b)$$
$$\cos(a \pm b) = \cos(a) \cdot \cos(b) \mp \sin(a) \cdot \sin(b)$$
$$\tan(a \pm b) = \frac{\tan(a) \pm \tan(b)}{1 \mp \tan(a) \cdot \tan(b)}$$

2.8 Doppel- und Halbwinkel

$$\sin(2a) = 2\sin(a)\cos(a)$$
$$\cos(2a) = \cos^2(a) - \sin^2(a) = 2\cos^2(a) - 1 = 1 - 2\sin^2(a)$$
$$\cos^2\left(\frac{a}{2}\right) = \frac{1+\cos(a)}{2} \qquad \sin^2\left(\frac{a}{2}\right) = \frac{1-\cos(a)}{2}$$

2.9 Geradengleichung Interpolieren

$$y(x) = y_1 + \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$



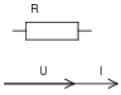
2.10 Grad <-> Rad

$$\alpha_{rad} = \alpha_{grad} \cdot \frac{\pi}{180}$$
$$\alpha_{grad} = \alpha_{rad} \cdot \frac{180}{\pi}$$

2.11 Grundelemente

Ohmscher Widerstand R

u und i können sprunghaft ändern



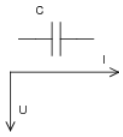
$$u(t) = R \cdot i(t)$$
$$i(t) = \frac{u(t)}{R}$$
$$\underline{Z}_R = R$$

nicht linear:

$$R_=(u) = \frac{U}{I(u)}, r_D = \frac{\mathrm{d}U}{\mathrm{d}I}|_{U_0}$$
$$P = I^2 \cdot R = \frac{U^2}{R}$$

Kapazitität C

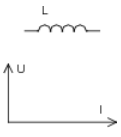
u kann nicht sprunghaft ändern



$$u(t) = \frac{1}{C}\int_0^t i(\tau)d\tau + u(0)$$
$$i(t) = C\frac{du(t)}{dt}$$
$$\underline{Z}_C = \frac{1}{j\omega C} = -\frac{j}{\omega C}$$
$$X_C = -\frac{1}{\omega C} \quad B_C = \omega C$$
$$Q_C = -U^2 \cdot \omega C = -\frac{I^2}{\omega C}$$
$$W_C = \frac{1}{2}CU_C^2$$

Induktivität L

i kann nicht sprunghaft ändern



$$u(t) = L\frac{di(t)}{dt}$$
$$i(t) = \frac{1}{L}\int_0^t u(\tau)d\tau + i(0)$$
$$\underline{Z}_L = j\omega L$$
$$X_L = \omega L \quad B_L = -\frac{1}{\omega L}$$
$$Q_L = I^2 \cdot \omega L = \frac{U^2}{\omega L}$$
$$W_L = \frac{1}{2}LI_L^2$$

2.12 Begriffe der Impedanz und Admittanz

Scheinwiderstand		$Z = \frac{U_{eff}}{I_{eff}}$	$= \sqrt{R^2 + X^2}$	Ohm
Komplexer Widerstand	Impedanz	$\underline{Z} = R + jX = Z \cdot e^{j\varphi}$	$= \frac{\underline{U}}{\underline{I}} = \frac{\underline{U} \cdot \underline{U}^*}{\underline{S}^*} == \frac{U^2}{\underline{S}^*} = \frac{\underline{S}}{I^2}$	Ohm
Komplexer Leitwert	Admittanz	$\underline{Y} = G + jB = \frac{1}{\underline{Z}} = \frac{1}{Z}e^{-j\varphi}$	$= \frac{\underline{I}}{\underline{U}}$	Siemens
Wirkwiderstand	Resistanz	$R = \mathrm{Re}(\underline{Z})$	$= Z \cdot cos(\varphi)$	Ohm
Wirkleitwert	Konduktanz	$G = \mathrm{Re}(\underline{Y})$	$\neq \frac{1}{R}$	Siemens
Blindwiderstand	Reaktanz	$X = \mathrm{Im}(\underline{Z})$	$= Z \cdot sin(\varphi)$	Ohm
Blindleitwert	Suszeptanz	$B = \mathrm{Im}(\underline{Y})$	$\neq \frac{1}{X}$	Siemens
Phasenverschiebung		$\varphi = \varphi_u - \varphi_i = \arctan\left(\frac{\mathrm{Im}(\underline{Z})}{\mathrm{Re}(\underline{Z})}\right)$		Radiant