Low Depth HHL with VQC

The Harrow–Hassidim–Lloyd (HHL) algorithm is a quantum algorithm designed for solving systems of linear equations. It estimates the result of a scalar measurement on the solution vector to a given linear system of equations. One of its key features is the ability to provide an exponential speedup over classical algorithms for linear systems with low condition numbers. However, the HHL algorithm typically involves quantum circuits of higher depth due to the use of Quantum Phase Estimation (QPE). QPE is a important quantum computing subroutine that play a crucial role in various quantum application, but it can be challenging to implement QPEs on near-term quantum hardware.

To address this depth issue, researchers have explored an intriguing approach: replacing the QPE step in the HHL algorithm with a Variational Quantum Circuit (VQC). We use this research as the main reference for our work, and build upon their work.

- Learning Quantum Phase Estimation by Variational Quantum Circuit, Chen-Yu Liu and Chu-Hsuan Abraham Lin and Kuan-Cheng Chen, 2023, arXiv [quant-ph] https://doi.org/10.48550/arXiv.2311.04690

Objective

- Reduce circuit depth of HHL algorithm implementations
- Improve accuracy of HHL implementations (optimization for increased efficiency)
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Method

Given a Hermitian matrix A and a unit vector b, the goal is to prepare a quantum state corresponding to the vector that solves the linear system Ax = b. Specifically, we want to compute expectation values of observables related to the solution vector. This can be done using the conventional HHL algorithm.

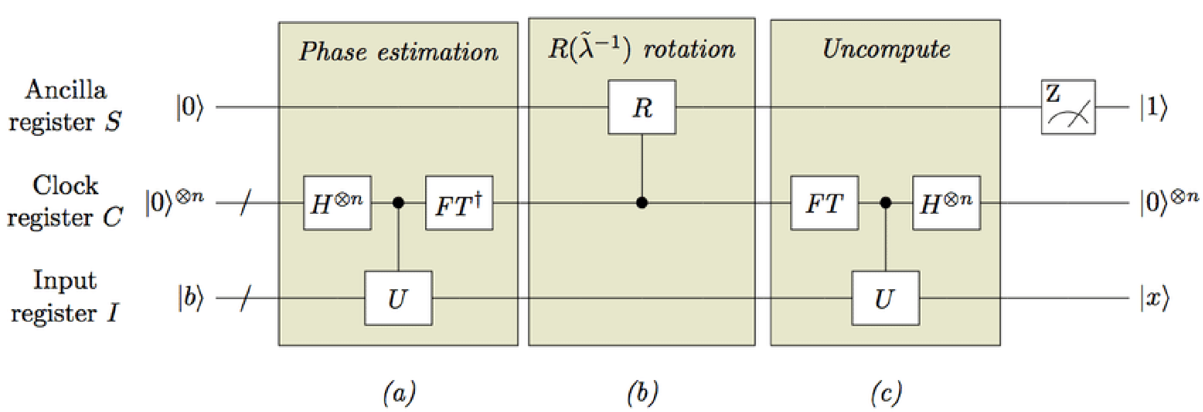
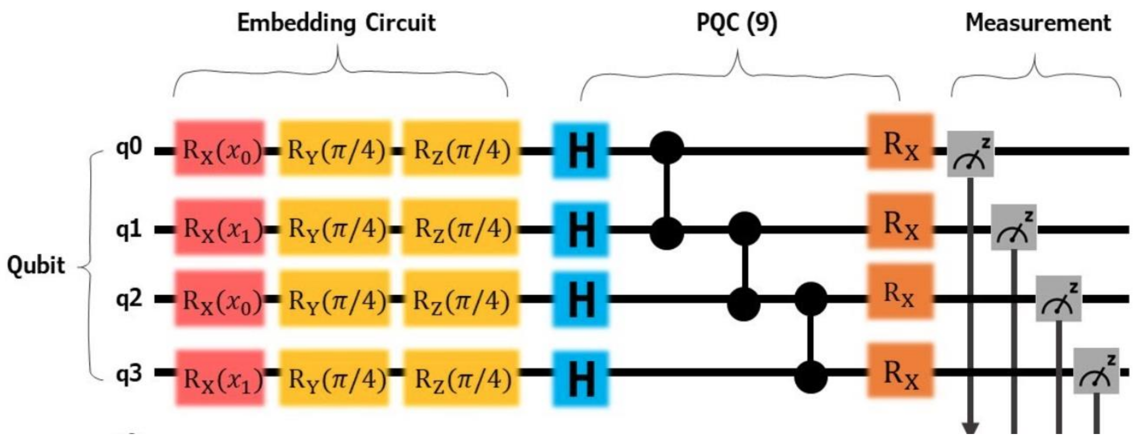
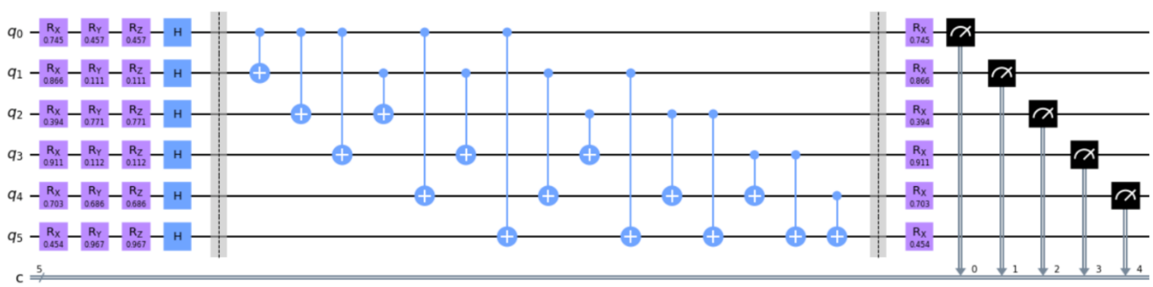


Figure 1: HHL Algorithm

In the HHL algorithm, instead of using QPE, we plan to employ a VQC to estimate the solution vector. The VQC reduces the circuit depth significantly while maintaining precision in correlation energies. To achieve shallow-depth implementations, we plan to experiment with different VQC parameters and combine it with the HHL algorithm. Furthermore, we also plan to deviate from the naïve brute force approach of Controlled Rotation which requires exponential amount of controlled gates unless optimized manually to the equation.





Hardware Demonstrations

We plan to validate this approach on real quantum hardware. And therefore, we would need powerups to run these in AWS.