Advance Harrow–Hassidim–Lloyd algorithm by Variational Quantum Circuit

The Harrow–Hassidim–Lloyd (HHL) algorithm is a quantum algorithm designed to solve systems of linear equations. It estimates the result of a scalar measurement on the solution vector to a given linear system of equations. One of its key features is the ability to provide an exponential speedup over classical algorithms for linear systems with low condition numbers. However, the HHL algorithm typically involves quantum circuits of higher depth due to the use of Quantum Phase Estimation (QPE). QPE is an important quantum computing subroutine that plays a crucial role in various quantum applications, but it can be challenging to implement QPEs on near-term quantum hardware.

To address this depth issue, researchers have explored an intriguing approach: replacing the QPE step in the HHL algorithm with a Variational Quantum Circuit (VQC). We use this research as the main reference for our work and build upon their work.

- *Learning Quantum Phase Estimation by Variational Quantum Circuit, Chen-Yu Liu and Chu-Hsuan Abraham Lin and Kuan-Cheng Chen, 2023, arXiv [quant-ph]*
  https://doi.org/10.48550/arXiv.2311.04690

Objectives

• Reduce circuit depth of HHL algorithm implementations
• Improve accuracy of HHL implementations (optimization for increased efficiency)

Project Links:

- https://github.com/HSUYUCHAO/2024-Qhack

Method

Given a Hermitian matrix A and a unit vector b, the goal is to prepare a quantum state corresponding to the vector that solves the linear system Ax = b. Specifically, we want to compute expectation values of observables related to the solution vector. This can be done using the conventional HHL algorithm. We use the following code-base and paper as a reference to our implementation of the HHL algorithm.

- *Master Semester Project: On Quantum Algorithms for Solving Linear Systems of Equations, Adrien Vandenbroucque, 2019*
  https://github.com/Adirlou/epfl_master_semester_project/blob/master/README.md

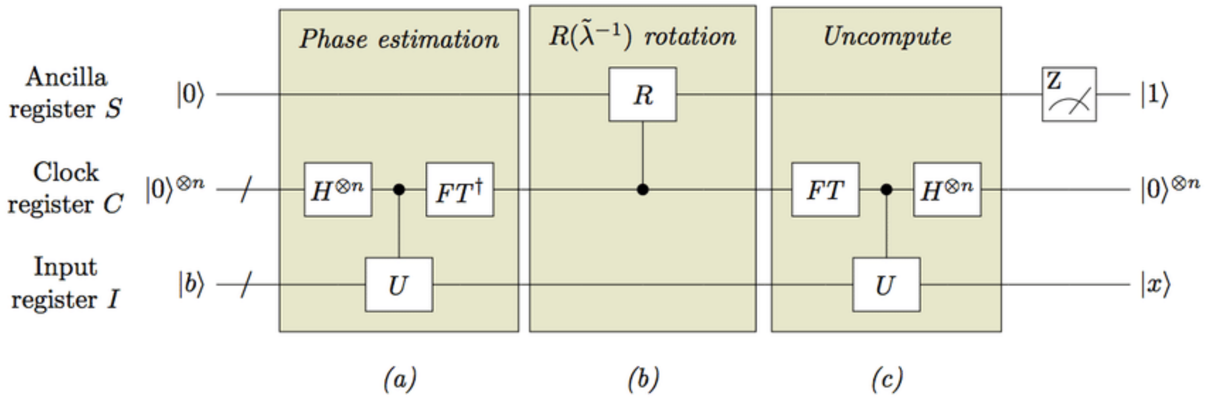

Figure 1: HHL Algorithm

In the HHL algorithm, a QPE circuit is typically employed to estimate the ground state energy of a quantum system. However, the QPE circuit requires a large number of qubits and circuit depth, which can be impractical for near-term quantum computers. To address this challenge, we propose using a variational quantum circuit (VQC) to estimate the solution vector of the HHL algorithm.

The VQC consists of a parameterized quantum circuit followed by a measurement of the circuit's output state. The parameters of the VQC are optimized to minimize a cost function, which is typically the expectation value of a Hamiltonian or an observable of interest.
Compared to the QPE circuit, the VQC requires significantly fewer qubits and circuit depth. This is because the VQC does not require the precise estimation of the phase of the ground state wave function, but only needs to find a state that has a large overlap with the ground state.

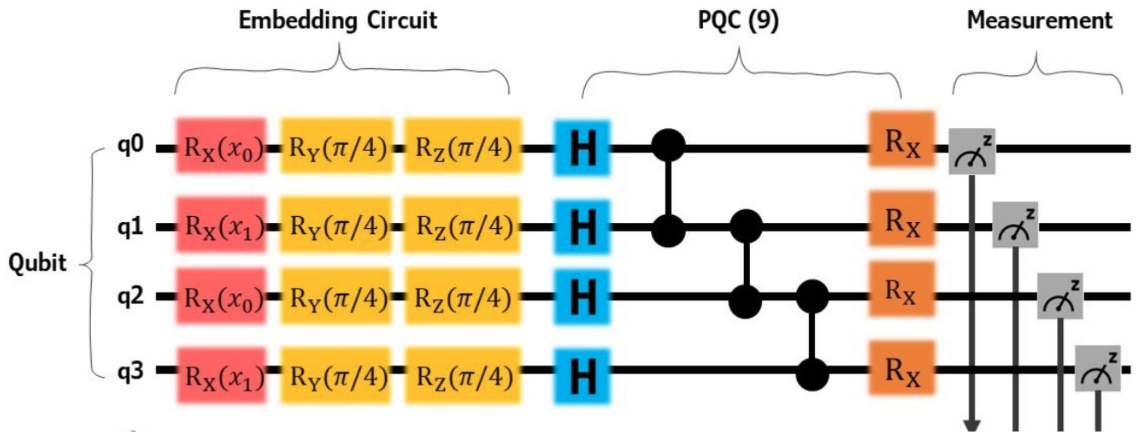

Figure 2: Variational Quantum Circuit(VQC)

Moreover, the VQC can be implemented on near-term quantum computers with limited resources. This makes it a promising approach for studying quantum systems and solving optimization problems on quantum computers. In our work, we have implemented the VQC-based HHL algorithm on a quantum simulator and demonstrated its effectiveness in estimating the ground state energy.

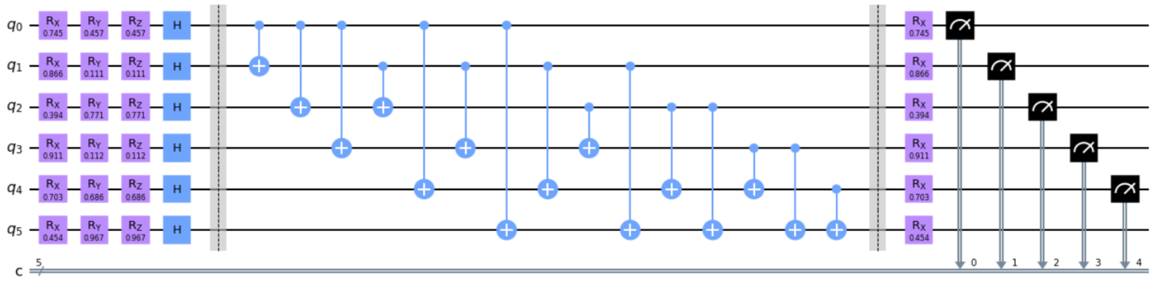

Figure 3: VQC implementation (circuit depth=14)

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Our results show that the VQC-based HHL algorithm can achieve high accuracy in estimating the ground state energy, while maintaining a small circuit depth(only 14). This makes it a viable alternative to the QPE-based HHL algorithm for near-term quantum computers.

To achieve shallow-depth implementations, we plan to experiment with different VQC parameters and combine it with the HHL algorithm. Furthermore, we also plan to deviate from the naive brute force approach of Controlled Rotation, which requires an exponential amount of controlled gates unless optimized manually to the equation.

Our work

Implementation

Based on the references, we first implemented the HHL algorithm as shown below.

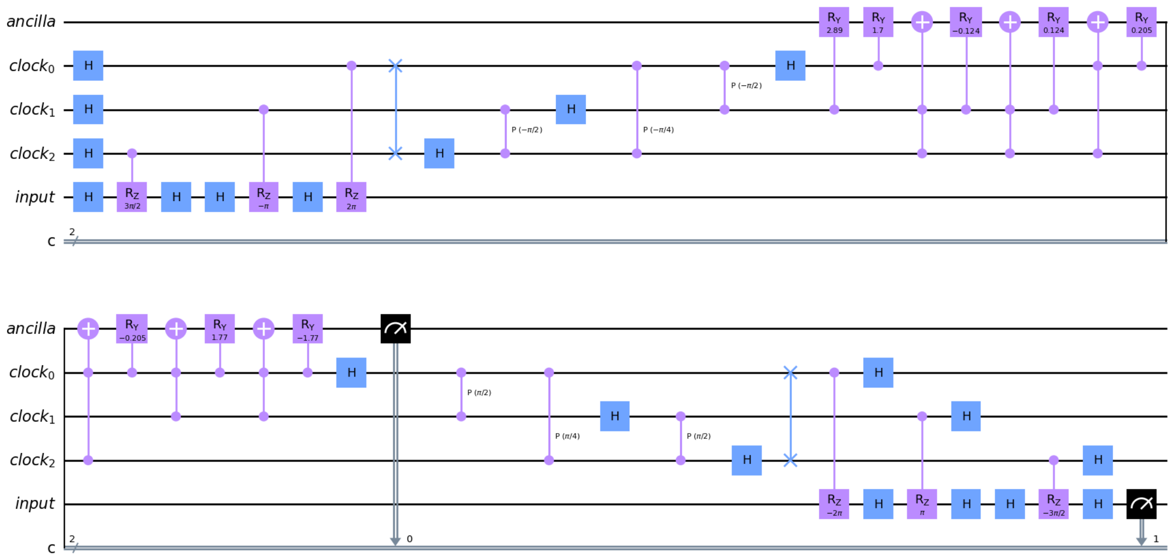

Figure 4: HHL implementation

The implementation employs 5 qubits, comprising ancilla and input qubits, resulting in a circuit depth of 40. The limited clock register limits the circuit's accuracy to a maximum of 1/(2^8). Nevertheless, implementing the circuit on a 7-qubit register would increase its circuit depth, due to the increased number of gates the QPE would demand.

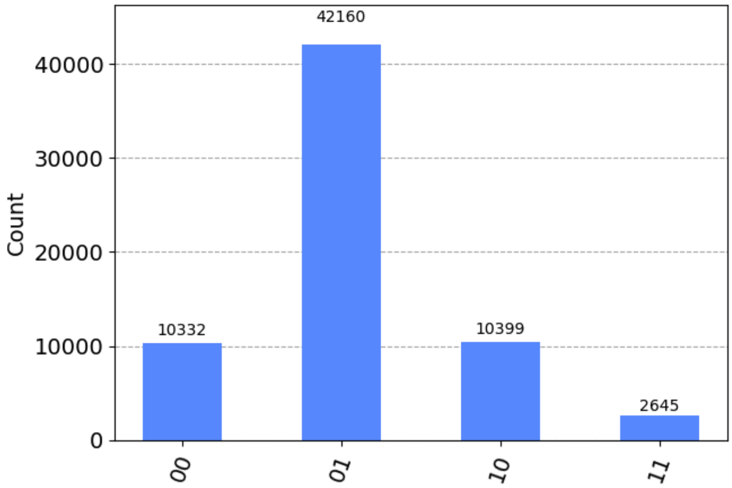

Figure 5: HHL implementation Simulation Results

On the other implementation we coupled our VQC implementation with the HHL by removing the QPE and inverse QPE functions and replacing them with the VQC and reverse VQC implementations.

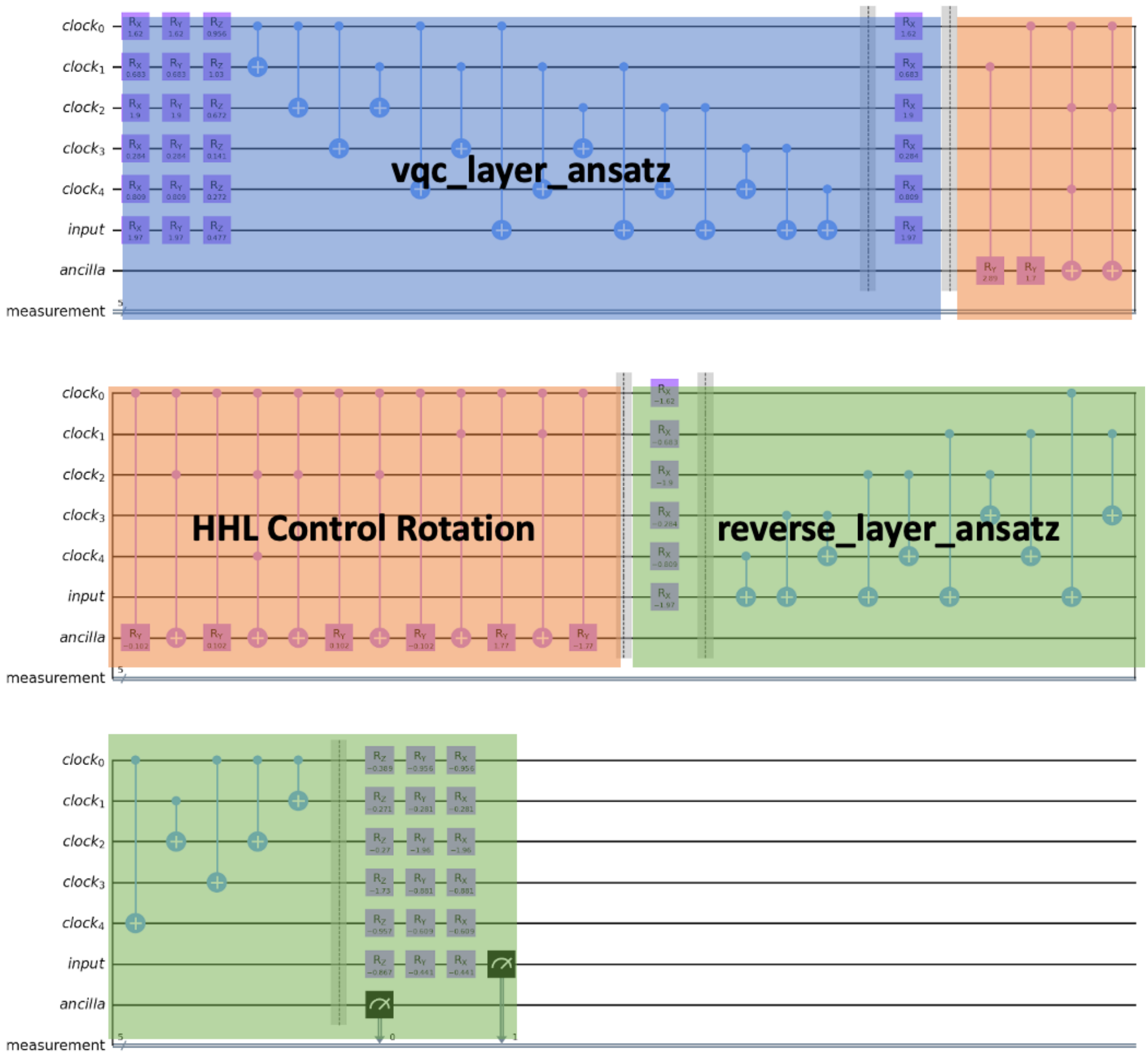

Figure 6: Advance HHL implementation

This implementation reduces the circuit depth from 40 in the previous implementation with half the qubits in the clock register to 43 with 7 qubits. **This is a significant improvemen**t.

Certainly, achieving a comparable circuit depth for a 5-qubit (clock register) implementation to a 3-qubit (clock register) implementation would result in significant improvement in terms of accuracy. In essence, if we want a highly accurate implementation with similar depth, increasing the number of qubits in the Advanced HHL algorithm can enhance accuracy without increasing the circuit depth. Alternatively, implementing both at the same accuracy level with an equal number of qubits would still yield a significant reduction in circuit depth.

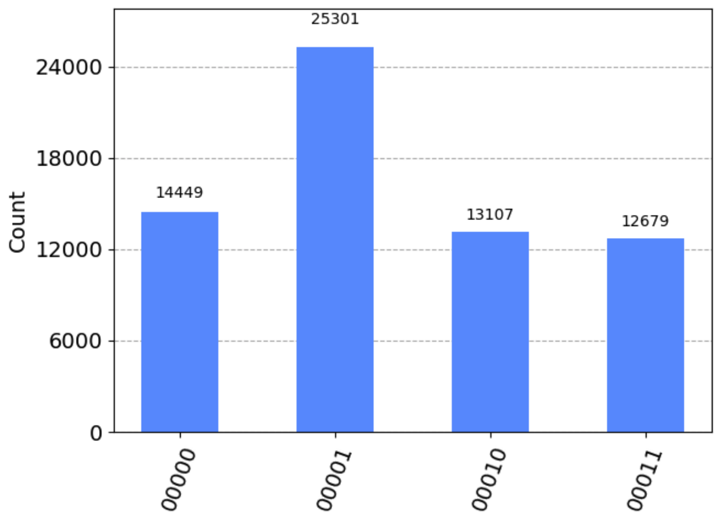

Figure 7: Advance HHL Simulation Results

Conclusion

By replacing the quantum phase estimation part of the HHL algorithm with a variational quantum circuit (VQC), we significantly reduced the circuit depth, which has profound implications for the future application of the HHL algorithm. This optimization not only reduces the demand for quantum resources and improves the robustness of the algorithm on noisy quantum devices but also enhances the scalability and efficiency of the algorithm. It opens new possibilities for solving more complex linear equations and accelerating the application of quantum computing across various industries.

Future work

Expected Outcomes

- A more efficient implementation of the HHL algorithm with reduced circuit depth
- Improved accuracy of HHL implementations for solving linear systems of equations
- A better understanding of the relationship between quantum computing and linear algebra

Significance

The HHL algorithm is a powerful tool for solving linear systems of equations. However, its high circuit depth makes it challenging to implement on near-term quantum hardware. Our work aims to address this issue by reducing the circuit depth of the HHL algorithm using a VQC instead of QPE. This will make the HHL algorithm more practical for use on near-term quantum hardware and open up new possibilities for quantum computing applications.

Hardware Demonstrations:

In the realm of quantum computing, validating theoretical concepts and algorithms on real hardware is a crucial step toward practical applications. To assess the feasibility and performance of our algorithm, we plan to conduct hardware demonstrations on real quantum devices.

By running our quantum algorithm on real hardware, we can obtain valuable insights into its behavior and performance. We can measure the execution time, evaluate the error rates, and analyze the impact of noise and decoherence on the algorithm's output. This information is essential for optimizing our algorithm and assessing its potential for practical applications.