

Statistical Learning and Data Mining

CS 363D/ SSC 358

Lecture: Data

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Adapted From: Pang-Ning Tan, Steinbach, Kumar

What is Data

- Collection of data objects and their attributes
- An attribute is a property or characteristic of an object
 - Examples: eye color of a person, temperature, etc.
- A collection of attributes describe an object

Attributes

Objects

Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

What is Data

- Collection of data objects and their attributes
- An attribute is a property or characteristic of an object
 - Examples: eye color of a person, temperature, etc.
 - Attribute is also known as variable, field, characteristic, or feature
- A collection of attributes describe an object
 - Object is also known as record, point, case, sample, entity, or instance

Attributes

Objects

Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
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3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Attribute Values

- Attribute values are numbers or symbols assigned to an attribute

Attribute: Income

Attribute Value: 125K
(real number)

Attributes

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7	Yes	Divorced	220K	No
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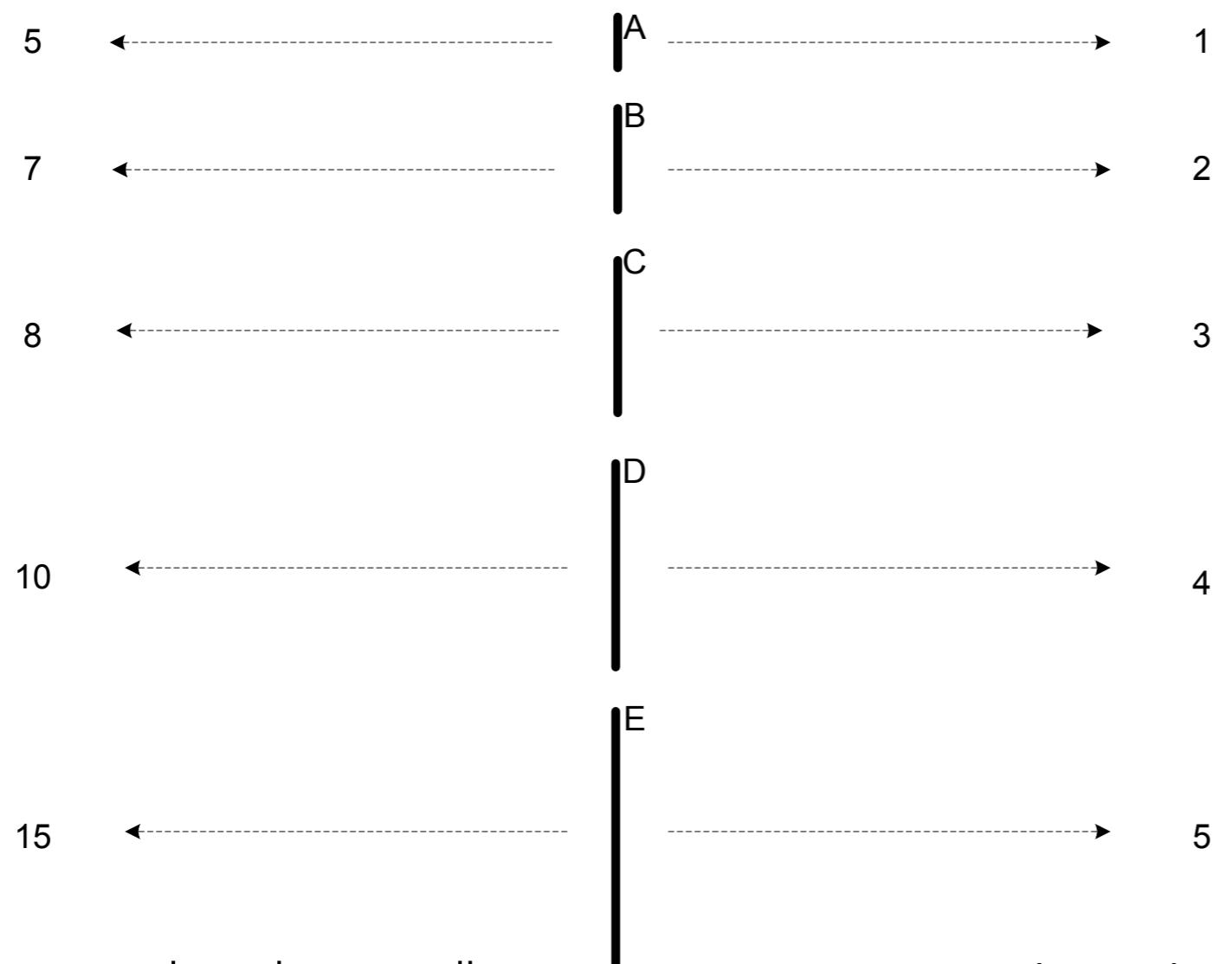
Objects

Attribute Values

- Attribute values are numbers or symbols assigned to an attribute
- Distinction between attributes and attribute values
 - Same attribute can be mapped to different attribute values
 - ◆ Example: height can be measured in feet or meters
 - Different attributes can be mapped to the same set of values
 - ◆ Example: Attribute values for ID and age are integers
 - ◆ But properties of attribute values can be different
 - ID has no limit but age has a maximum and minimum value

Example: Properties of Attributes and their Values

- The way you measure an attribute is somewhat may not match the attributes properties.



Some numbers ordered according
to length of lines

Length of lines

Types of Attributes

- There are different types of attributes
 - Nominal
 - ◆ Examples: ID numbers, eye color, zip codes
 - Ordinal
 - ◆ Examples: rankings (e.g., taste of potato chips on a scale from 1-10), grades, height in {tall, medium, short}
 - Interval
 - ◆ Examples: calendar dates, temperatures in Celsius or Fahrenheit.
 - Ratio
 - ◆ Examples: temperature in Kelvin, length, time, counts

Types of Attributes

- There are different types of attributes
 - Nominal
 - ◆ Examples: ID **Just Names**, gender, zip codes
 - Ordinal
 - ◆ Examples: rankings (e.g., taste of potato chips on a scale from 1-10), grades, height in {tall, medium, short}
 - Interval
 - ◆ Examples: calendar dates, temperatures in Celsius or Fahrenheit.
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 - Ordinal
 - ◆ Examples: ratings of potato chips on a scale from 1-10), gender, size, {small, medium, short}
 - Interval
 - ◆ Examples: calendar dates, temperatures in Celsius or Fahrenheit.
 - Ratio
 - ◆ Examples: temperature in Kelvin, length, time, counts

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Differences are meaningful

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Differences, Ratios are meaningful

Properties of Attribute Values

- The type of an attribute depends on which of the following properties it possesses:
 - Distinctness: $= \neq$
 - Order: $< >$
 - Addition: $+$ $-$
 - Multiplication: $*$ $/$
 - Nominal attribute: distinctness
 - Ordinal attribute: distinctness & order
 - Interval attribute: distinctness, order & addition
 - Ratio attribute: all 4 properties

Examples

Attribute Type	Description	Examples	Operations
Nominal	The values of a nominal attribute are just different names, i.e., nominal attributes provide only enough information to distinguish one object from another. ($=, \neq$)	zip codes, employee ID numbers, eye color, sex: $\{male, female\}$	mode, entropy, contingency correlation, χ^2 test
Ordinal	The values of an ordinal attribute provide enough information to order objects. ($<, >$)	hardness of minerals, $\{good, better, best\}$, grades, street numbers	median, percentiles, rank correlation, run tests, sign tests
Interval	For interval attributes, the differences between values are meaningful, i.e., a unit of measurement exists. $(+, -)$	calendar dates, temperature in Celsius or Fahrenheit	mean, standard deviation, Pearson's correlation, t and F tests
Ratio	For ratio variables, both differences and ratios are meaningful. $(*, /)$	temperature in Kelvin, monetary quantities, counts, age, mass, length, electrical current	geometric mean, harmonic mean, percent variation

Discrete and Continuous Attributes

● Discrete Attribute

- Has only a finite or countably infinite set of values
- Examples: zip codes, counts, or the set of words in a collection of documents
- Often represented as integer variables.
- Note: binary attributes are a special case of discrete attributes

● Continuous Attribute

- Has real numbers as attribute values
- Examples: temperature, height, or weight.
- Practically, real values can only be measured and represented using a finite number of digits.
- Continuous attributes are typically represented as floating-point variables.

Types of Data Sets

● Record

- Data Matrix
- Document Data
- Transaction Data

● Graph

- World Wide Web
- Molecular Structures

● Ordered

- Spatial Data
- Temporal Data
- Sequential Data
- Genetic Sequence Data

Record Data

- Data that consists of a collection of records, each of which consists of a fixed set of attributes

<i>Tid</i>	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
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Data Matrix

- If data objects have the same fixed set of numeric attributes, then the data objects can be thought of as points in a multi-dimensional space, where each dimension represents a distinct attribute

Special Case of Record Data where all

- Special case of record data where all attribute values are numeric

Value of each attribute is represented by one column, one for each attribute

Projection of x Load	Projection of y load	Distance	Load	Thickness
10.23	5.27	15.22	2.7	1.2
12.65	6.25	16.22	2.2	1.1

Data Matrix

- If data objects have the same fixed set of numeric attributes, then the data objects can be thought of as points in a multi-dimensional space, where each dimension represents a distinct attribute
- Such data set can be represented by an m by n matrix, where there are m rows, one for each object, and n columns, one for each attribute

Projection of x Load	Projection of y load	Distance	Load	Thickness
10.23	5.27	15.22	2.7	1.2
12.65	6.25	16.22	2.2	1.1

Document Data

- Each document becomes a 'term' vector,
 - each term is a component (attribute) of the vector,
 - the value of each component is the number of times the corresponding term occurs in the document.

	team	coach	play	ball	score	game	win	lost	timeout	season
Document 1	3	0	5	0	2	6	0	2	0	2
Document 2	0	7	0	2	1	0	0	3	0	0
Document 3	0	1	0	0	1	2	2	0	3	0

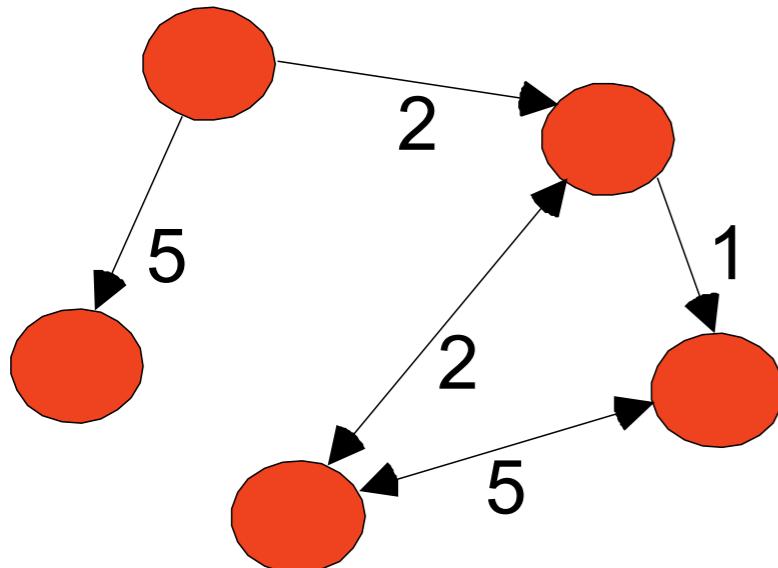
Transaction Data

- A special type of record data, where
 - each record (transaction) involves a set of items.
 - For example, consider a grocery store. The set of products purchased by a customer during one shopping trip constitute a transaction, while the individual products that were purchased are the items.

<i>TID</i>	<i>Items</i>
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Coke, Diaper, Milk

Graph Data I

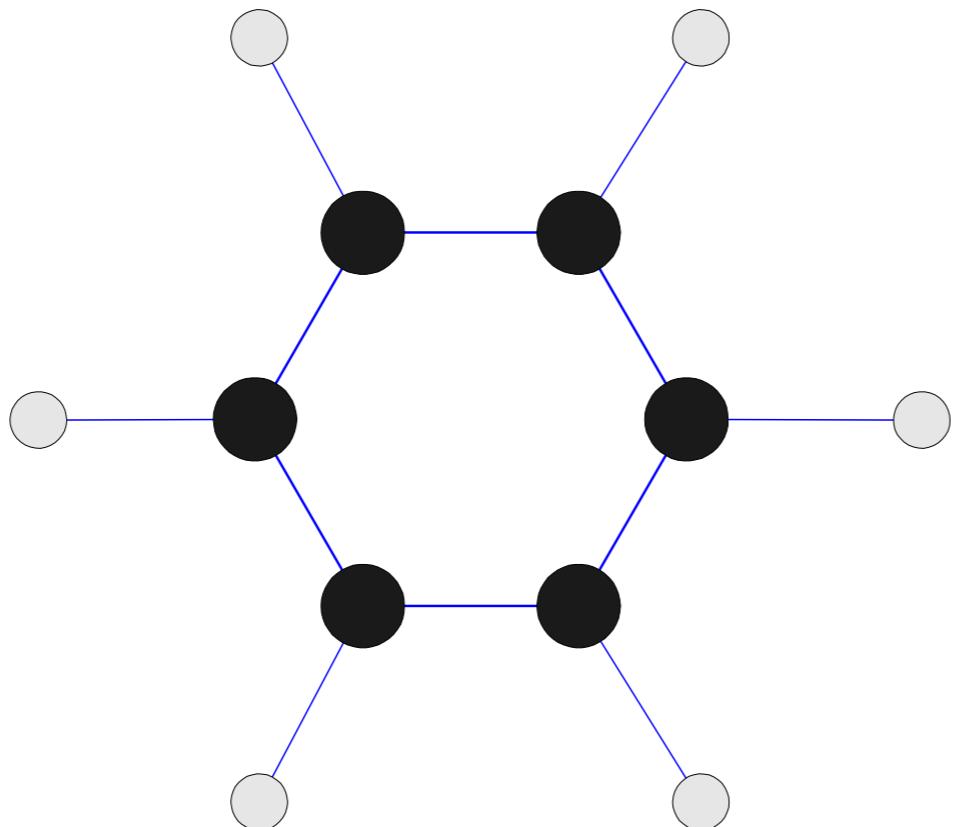
- Examples: Generic graph and HTML Links



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<a href="papers/papers.html#bbbb">  
Data Mining </a>  
<li>  
<a href="papers/papers.html#aaaa">  
Graph Partitioning </a>  
</li>  
<a href="papers/papers.html#aaaa">  
Parallel Solution of Sparse Linear System of Equations </a>  
</li>  
<a href="papers/papers.html#ffff">  
N-Body Computation and Dense Linear System Solvers
```

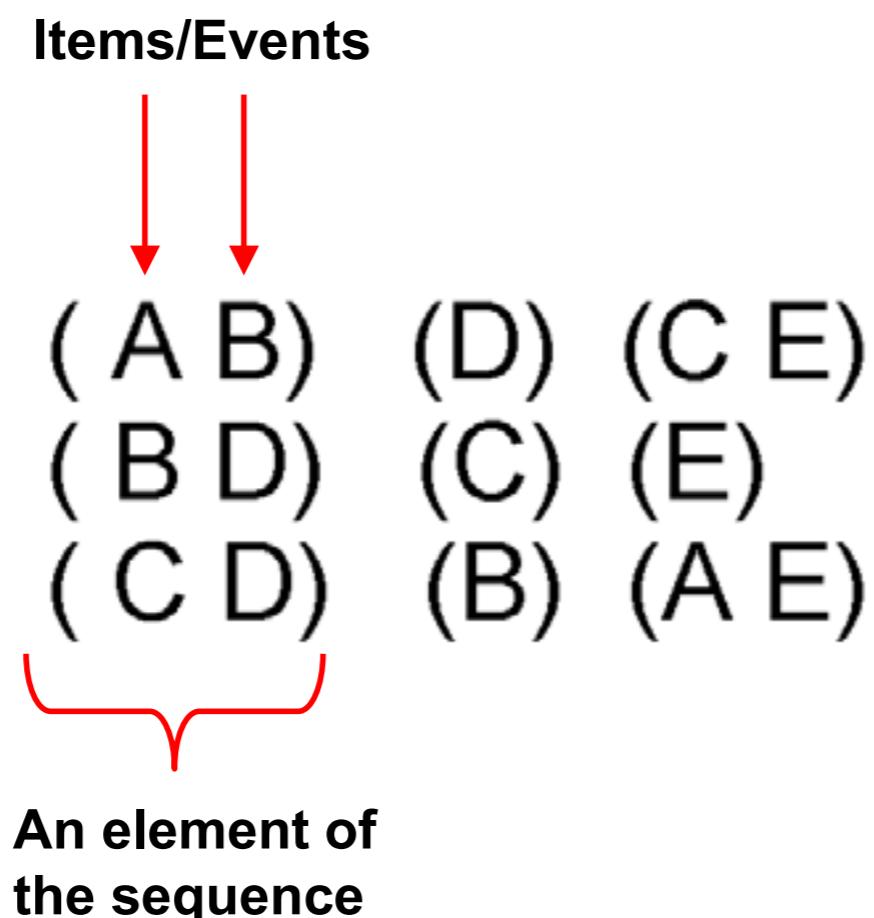
Graph Data II

- Benzene Molecule: C_6H_6



Ordered Data: Sequential Data

- Sequences of transactions



Ordered Data: Sequence Data

- Genomic sequence data

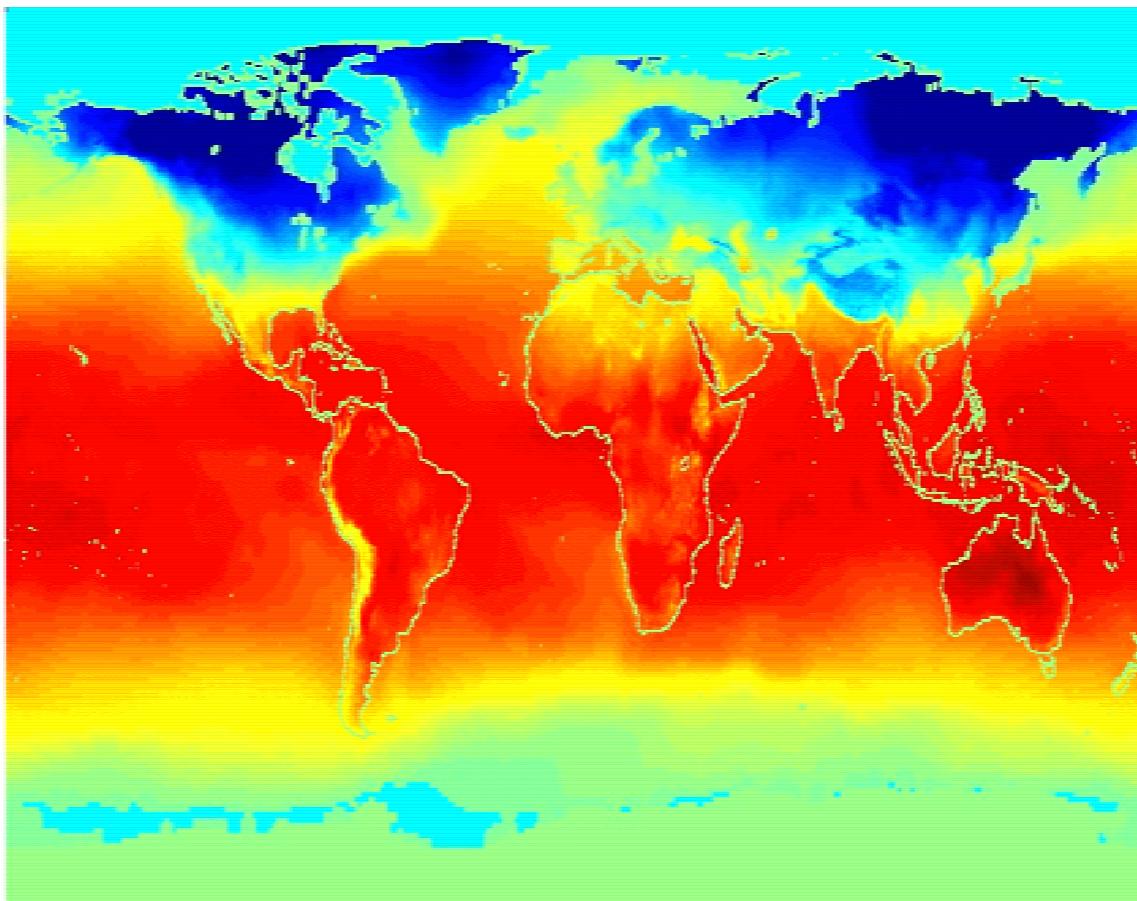
**GGTTCCGCCTTCAGCCCCGCGGCC
CGCAGGGCCCGCCCCGCGCCGTC
GAGAAGGGCCCGCCTGGCGGGCG
GGGGGAGGCAGGGCCGCCGAGC
CCAACCGAGTCCGACCAGGTGCC
CCCTCTGCTCGGCCTAGACCTGA
GCTCATTAGGCGGCAGCGGACAG
GCCAAGTAGAACACGCGAAGCGC
TGGGCTGCCTGCTGCGACCAGGG**

Ordered Data

- Spatio-Temporal Data

Average Monthly Temperature of land and ocean

Jan



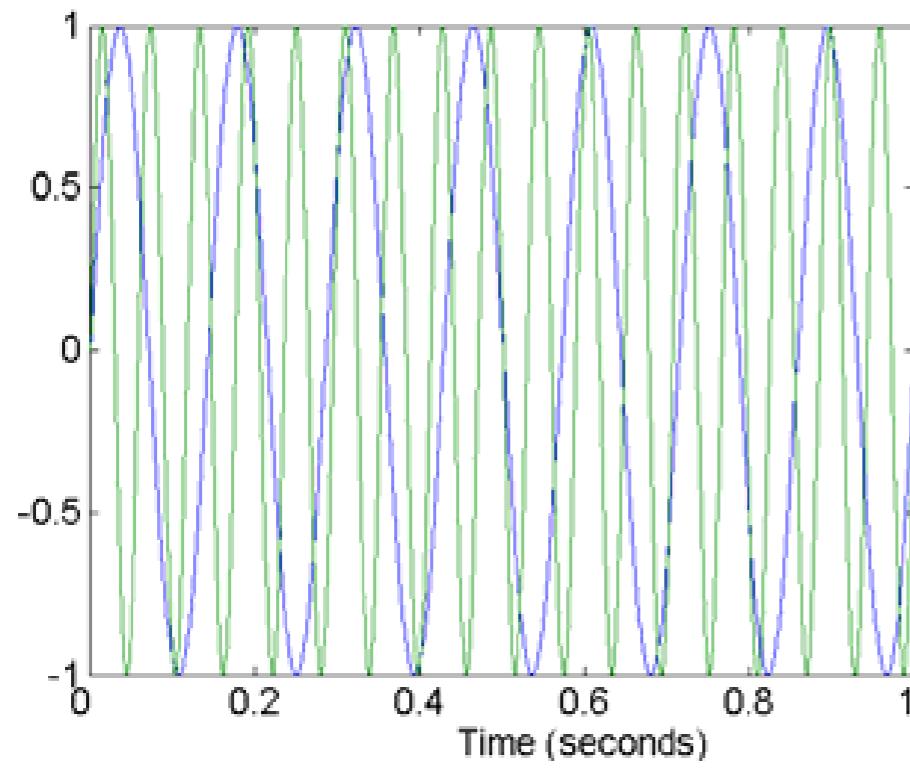
Data Quality

Data Quality

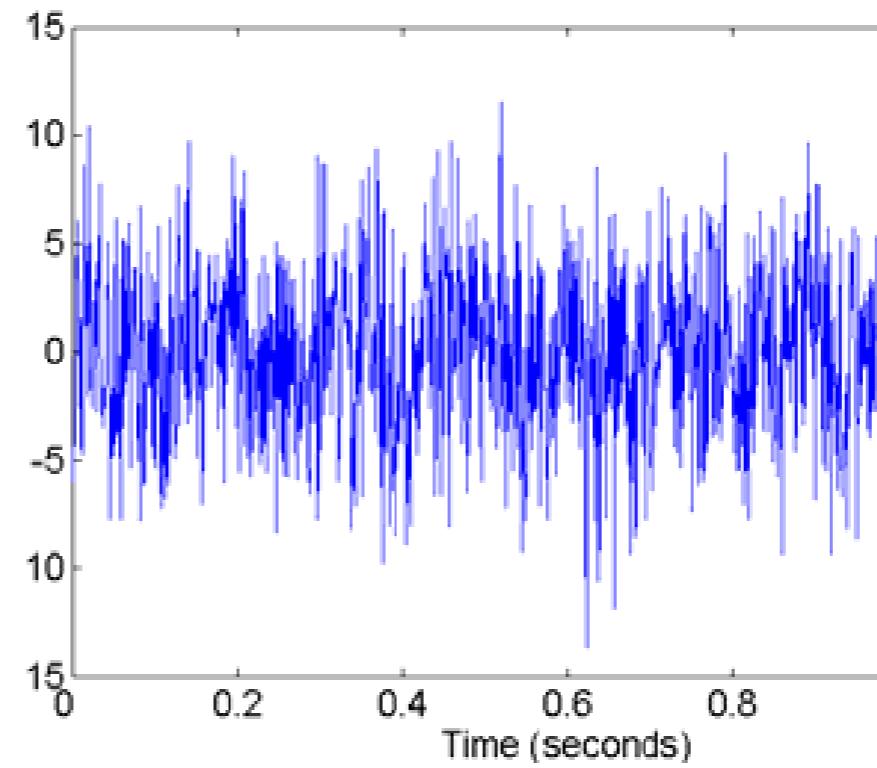
- What kinds of data quality problems?
 - How can we detect problems with the data?
 - What can we do about these problems?
-
- Examples of data quality problems:
 - Noise and outliers
 - missing values
 - duplicate data

Noise

- Noise refers to modification of original values
 - Examples: distortion of a person's voice when talking on a poor phone and “snow” on television screen



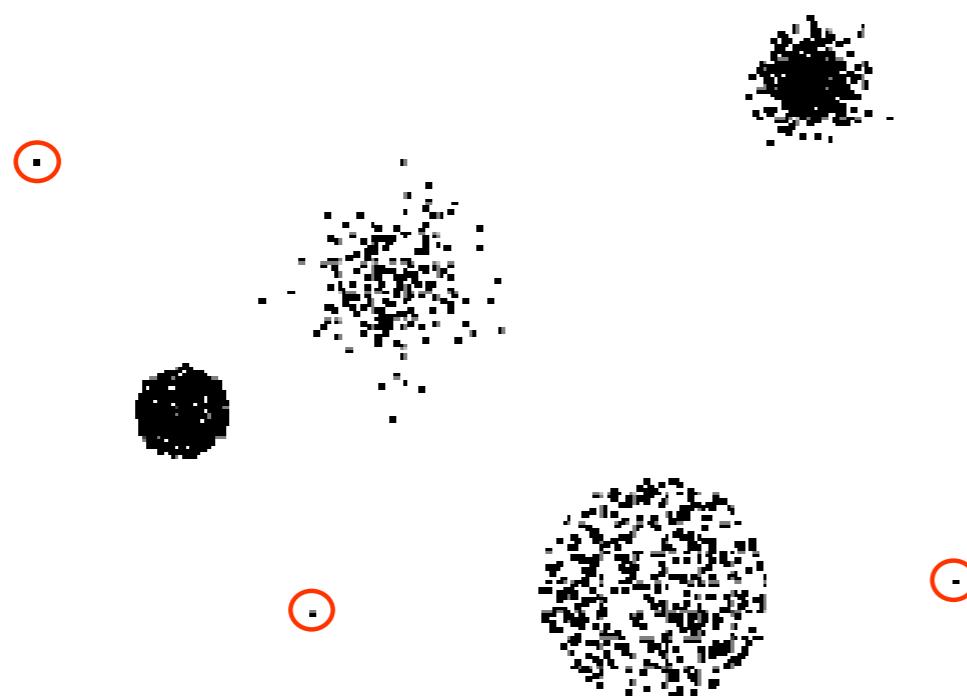
Two Sine Waves



Two Sine Waves + Noise

Outliers

- Outliers are data objects with characteristics that are considerably different than most of the other data objects in the data set



Missing Values

● Reasons for missing values

- Information is not collected
(e.g., people decline to give their age and weight)
- Attributes may not be applicable to all cases
(e.g., annual income is not applicable to children)

Missing Values

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- Information is not collected
(e.g., people decline to give their age and weight)
- Attributes may not be applicable to all cases
(e.g., annual income is not applicable to children)

● Handling missing values

- Eliminate Data Objects
- Estimate Missing Values
- Ignore the Missing Value During Analysis
- Replace with all possible values (weighted by their probabilities)

Duplicate Data

- Data set may include data objects that are duplicates, or almost duplicates of one another
 - Major issue when merging data from heterogeneous sources
- Examples:
 - Same person with multiple email addresses
- Data cleaning
 - Process of dealing with duplicate data issues

Data Preprocessing

Data Preprocessing

- Aggregation
- Sampling
- Dimensionality Reduction
- Feature subset selection
- Feature creation
- Discretization and Binarization
- Attribute Transformation

Aggregation

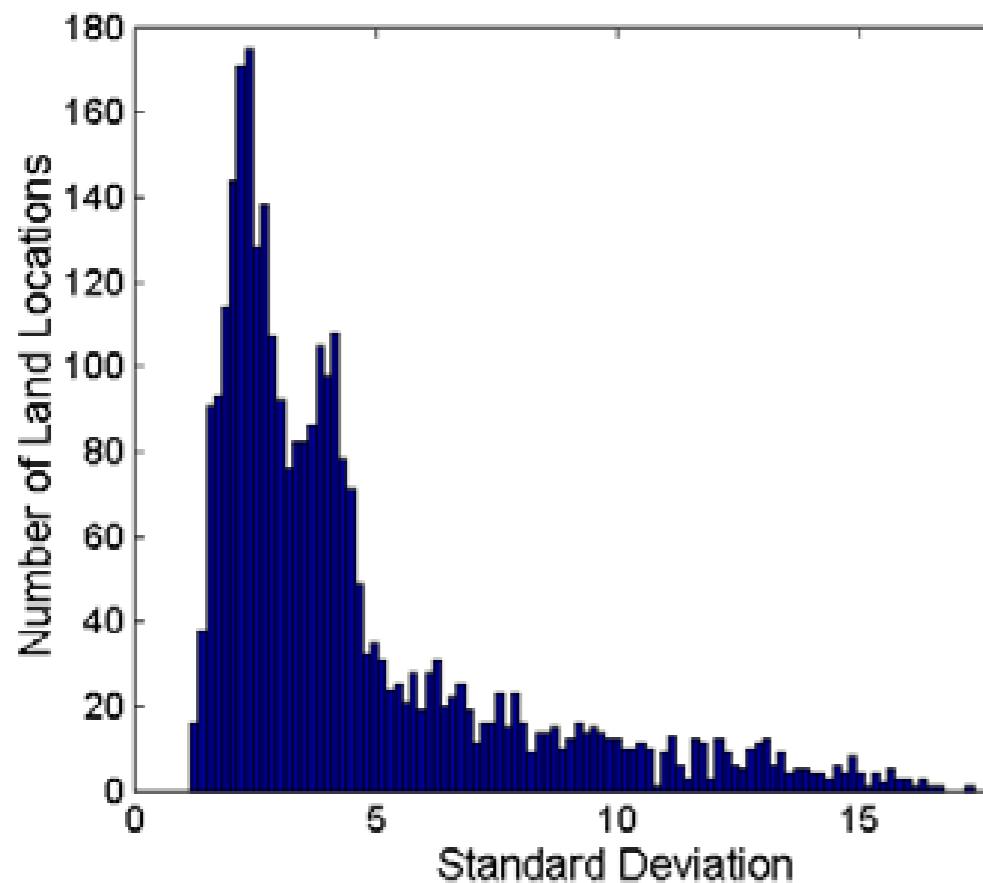
- Combining two or more attributes (or objects) into a single attribute (or object)

- Purpose

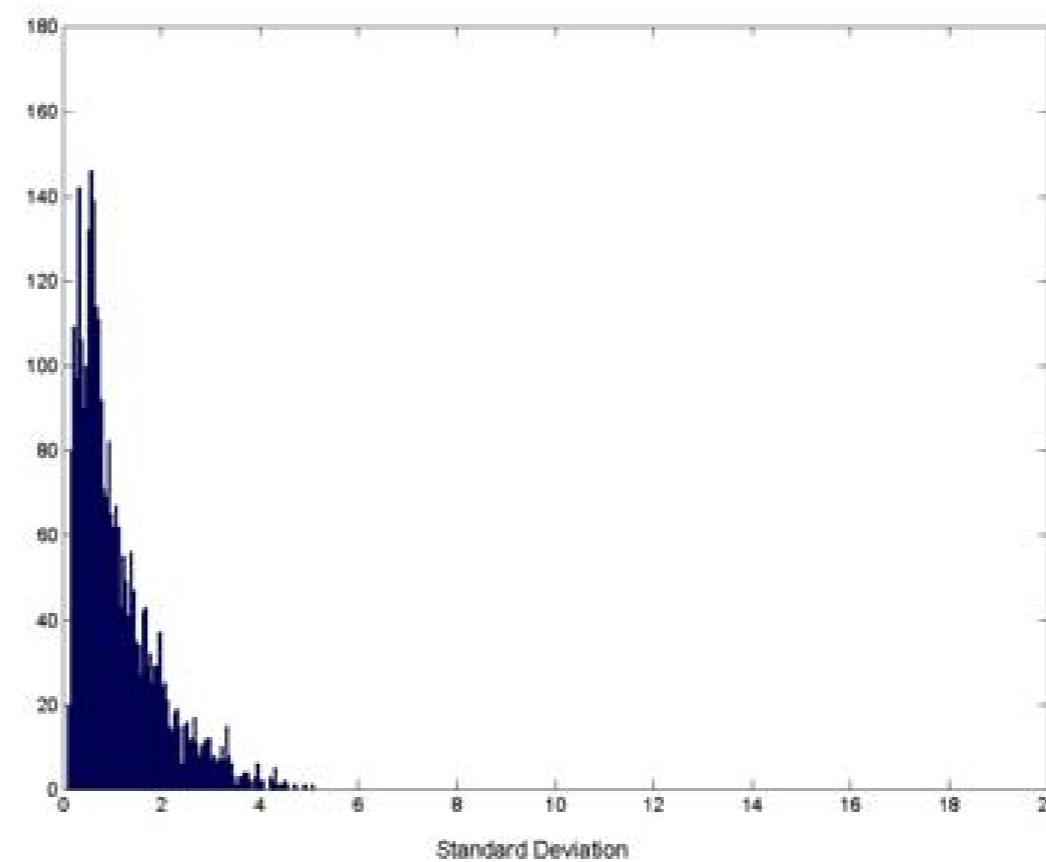
- Data reduction
 - ◆ Reduce the number of attributes or objects
 - Change of scale
 - ◆ Cities aggregated into regions, states, countries, etc
 - More “stable” data
 - ◆ Aggregated data tends to have less variability

Aggregation: More Stable Data

Variation of Precipitation in Australia



**Standard Deviation of Average
Monthly Precipitation**



**Standard Deviation of Average
Yearly Precipitation**

Sampling

- Sampling is the main technique employed for data selection.
 - It is often used for both the preliminary investigation of the data and the final data analysis.
- Statisticians sample because obtaining the entire set of data of interest is too expensive or time consuming.
- Sampling is used in data mining because processing the entire set of data of interest is too expensive or time consuming.

Sampling

- The key principle for effective sampling is the following:
 - using a sample will work almost as well as using the entire data sets, if the sample is representative

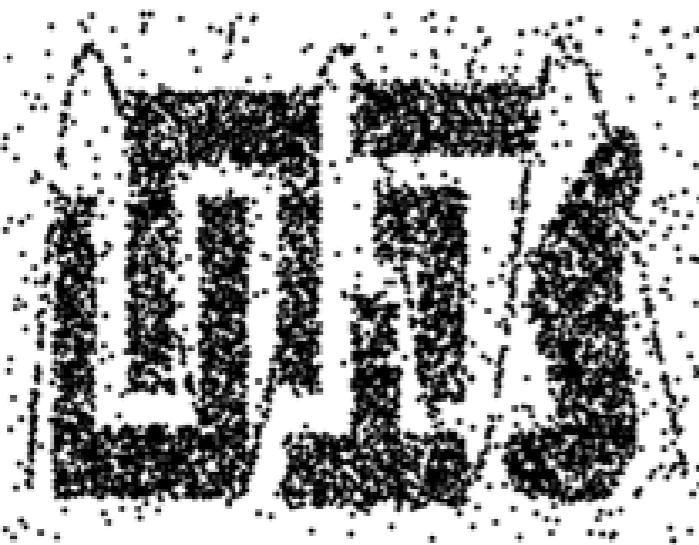
Sampling

- The key principle for effective sampling is the following:
 - using a sample will work almost as well as using the entire data sets, if the sample is representative
 - A sample is representative if it has approximately the same property (of interest) as the original set of data

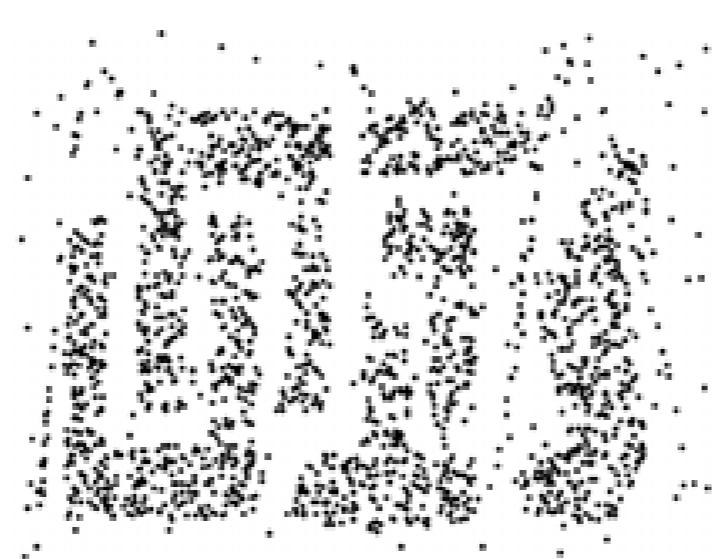
Types of Sampling

- Simple Random Sampling
 - There is an equal probability of selecting any particular item
- Sampling without replacement
 - As each item is selected, it is removed from the population
- Sampling with replacement
 - Objects are not removed from the population as they are selected for the sample.
 - ◆ In sampling with replacement, the same object can be picked up more than once
- Stratified sampling
 - Split the data into several partitions; then draw random samples from each partition

Sample Size: How many samples?



8000 points



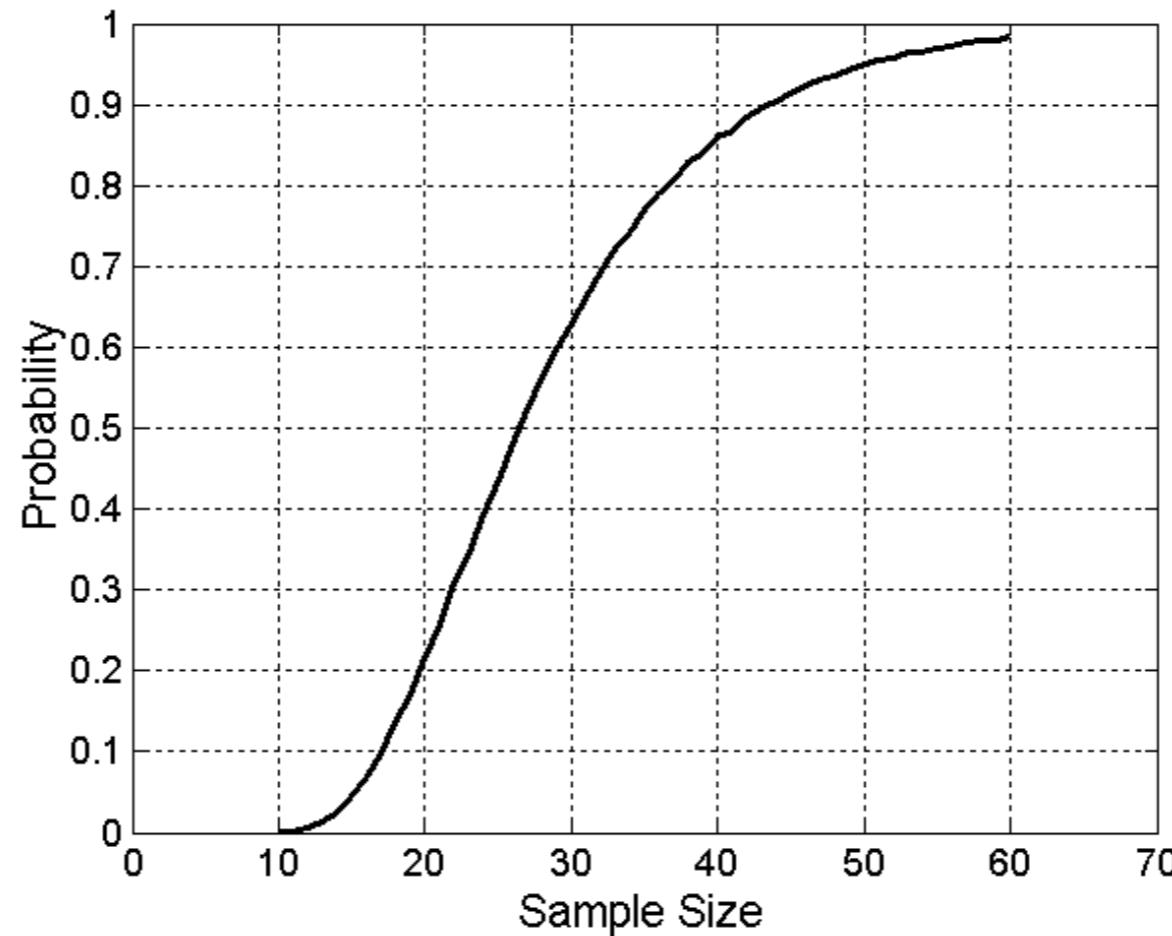
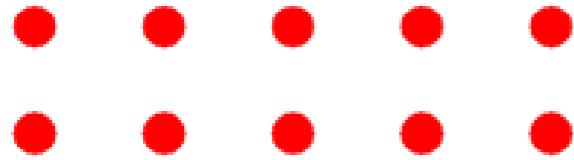
2000 Points



500 Points

Sample Size

- What sample size is necessary to get at least one object from each of 10 groups.



Dimensionality Reduction

Curse of Dimensionality

- When dimensionality increases, data becomes increasingly sparse in the space that it occupies
- Definitions of density and distance between points, which is critical for clustering and outlier detection, become less meaningful

Dimensionality Reduction

- Purpose:

- Avoid curse of dimensionality
- Reduce amount of time and memory required by data mining algorithms
- Allow data to be more easily visualized
- May help to eliminate irrelevant features or reduce noise

- Techniques

- Principle Component Analysis
- Singular Value Decomposition
- Others: supervised and non-linear techniques

Dimensionality Reduction

- PCA, SVD, etc.: These “create” new features via linear combination of existing features
 - ▶ We will look at these soon after a review lecture on linear algebra
- Easier Method: Just throw away features!
 - ▶ Feature Subset Selection

Feature Subset Selection

- Another way to reduce dimensionality of data
- Redundant features
 - duplicate much or all of the information contained in one or more other attributes
 - Example: purchase price of a product and the amount of sales tax paid
- Irrelevant features
 - contain no information that is useful for the data mining task at hand
 - Example: students' ID is often irrelevant to the task of predicting students' GPA

Feature Subset Selection

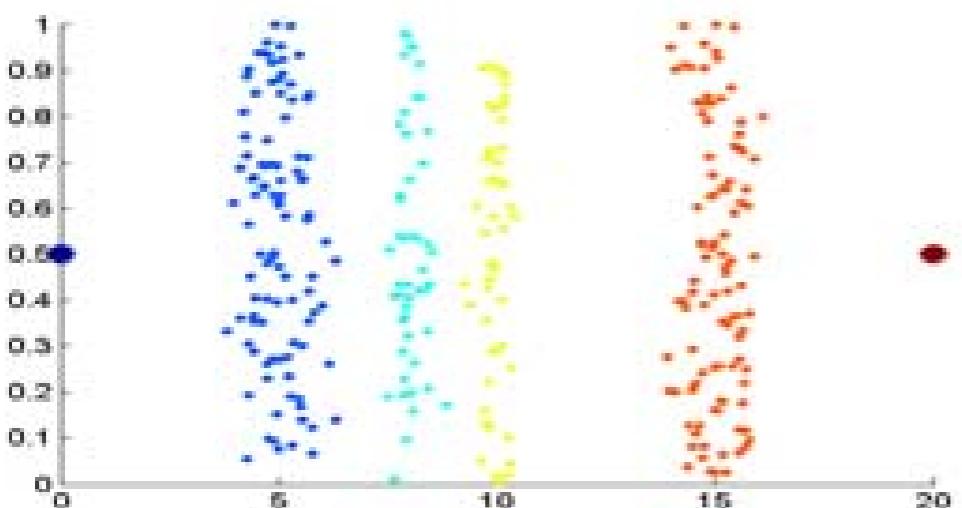
- Techniques:

- Brute-force approach:
 - ◆ Try all possible feature subsets as input to data mining algorithm
- Embedded approaches:
 - ◆ Feature selection occurs naturally as part of the data mining algorithm
- Filter approaches:
 - ◆ Features are selected before data mining algorithm is run
- Wrapper approaches:
 - ◆ Use the data mining algorithm as a black box to find best subset of attributes

Discretization

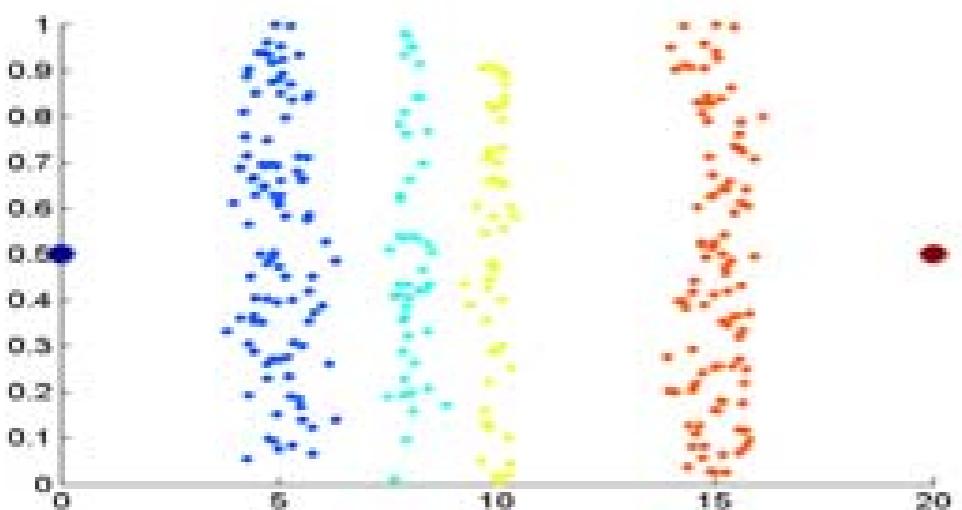
- Some data mining methods can only work with discrete data
- So have to “discretize” real-valued attribute values
 - ▶ Take the continuous attribute value and substitute in a discrete value

Discretization: Methods

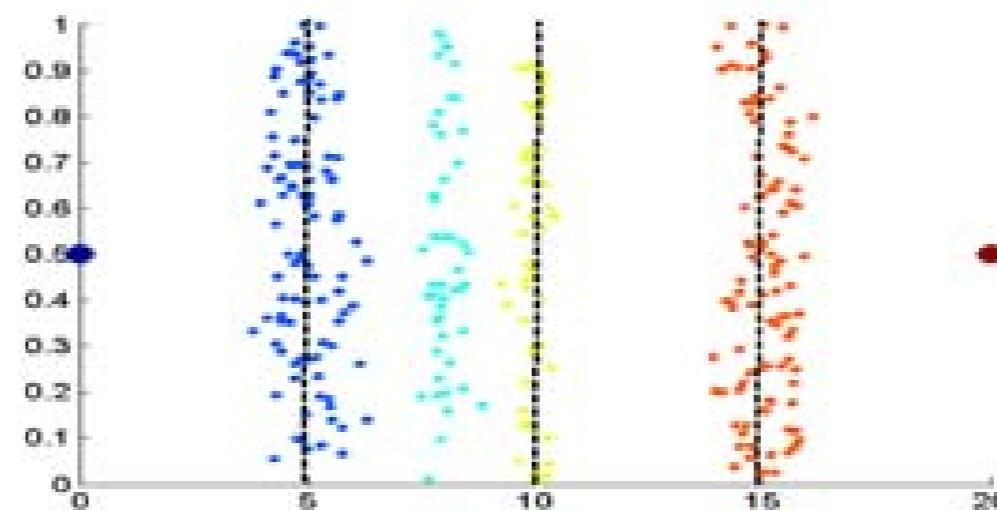


Data

Discretization: Methods

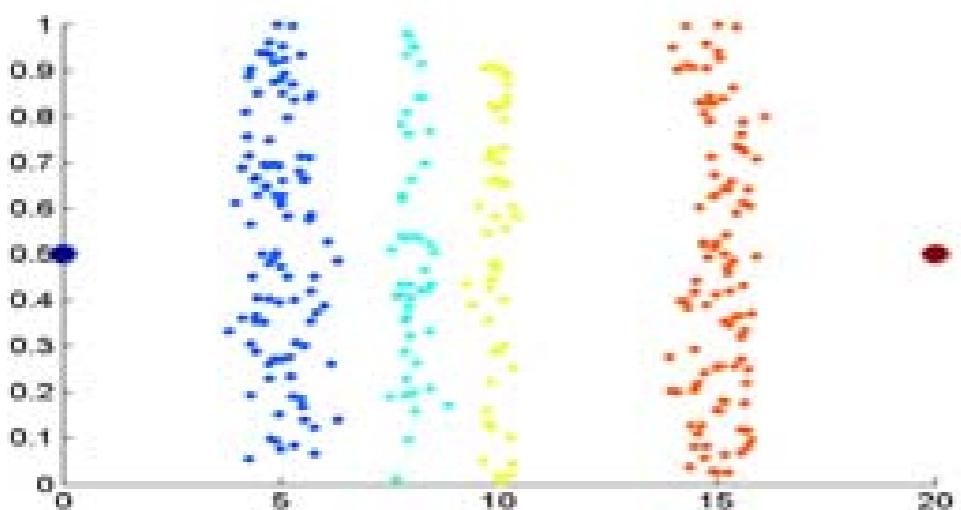


Data

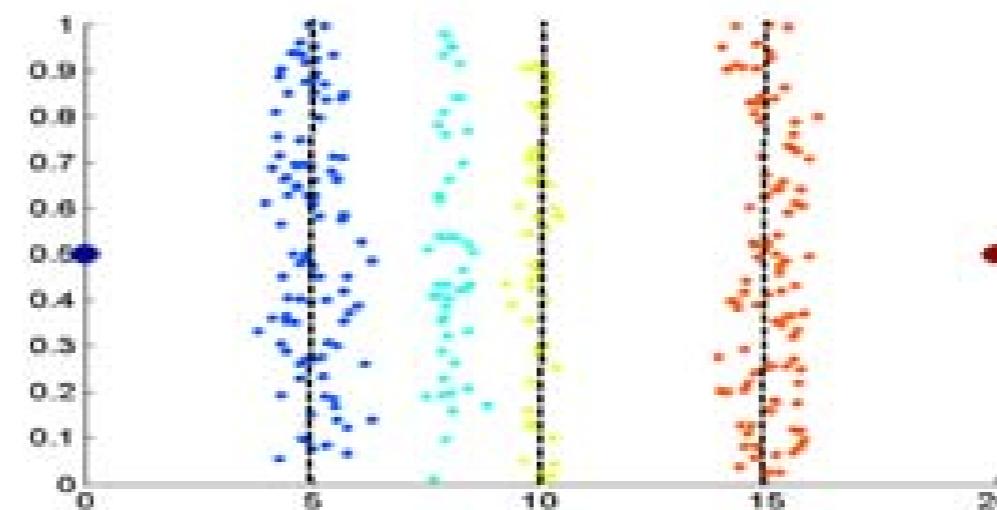


Equal interval width

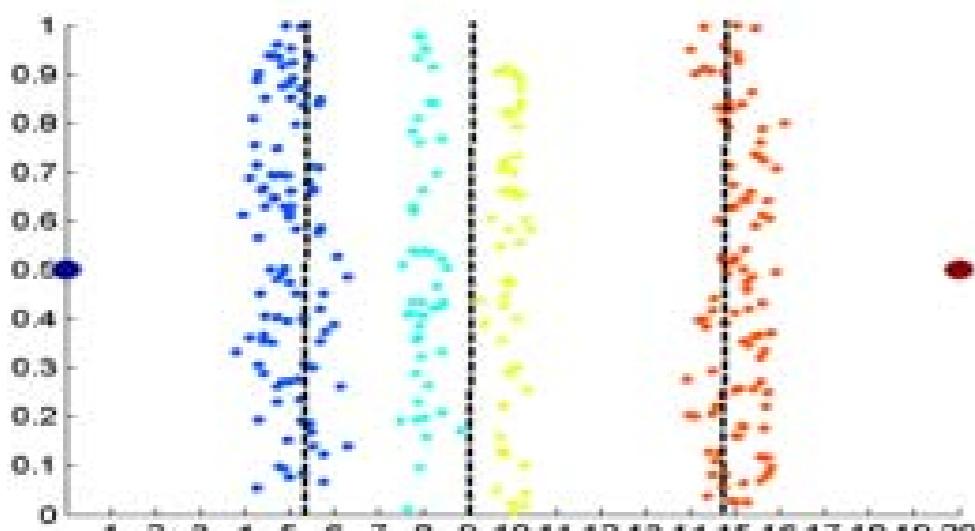
Discretization: Methods



Data

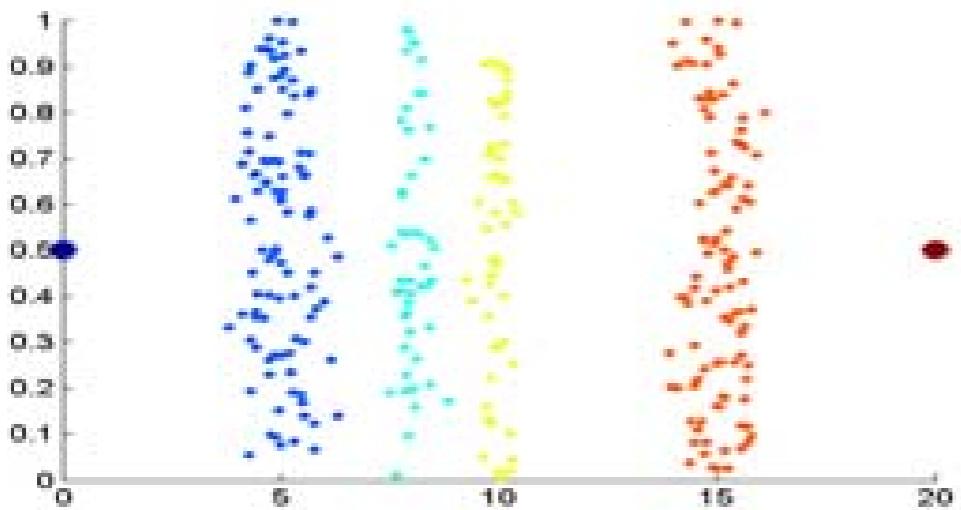


Equal interval width

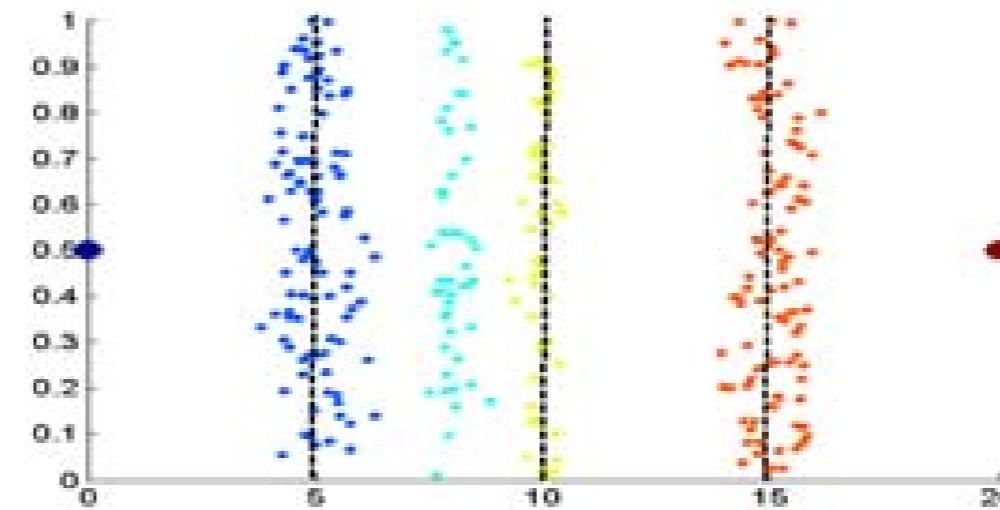


Equal frequency

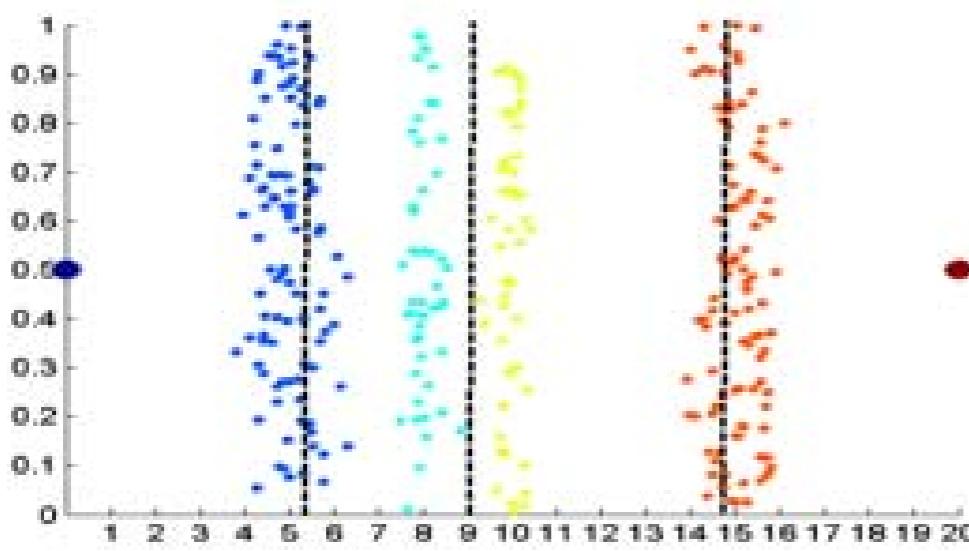
Discretization: Methods



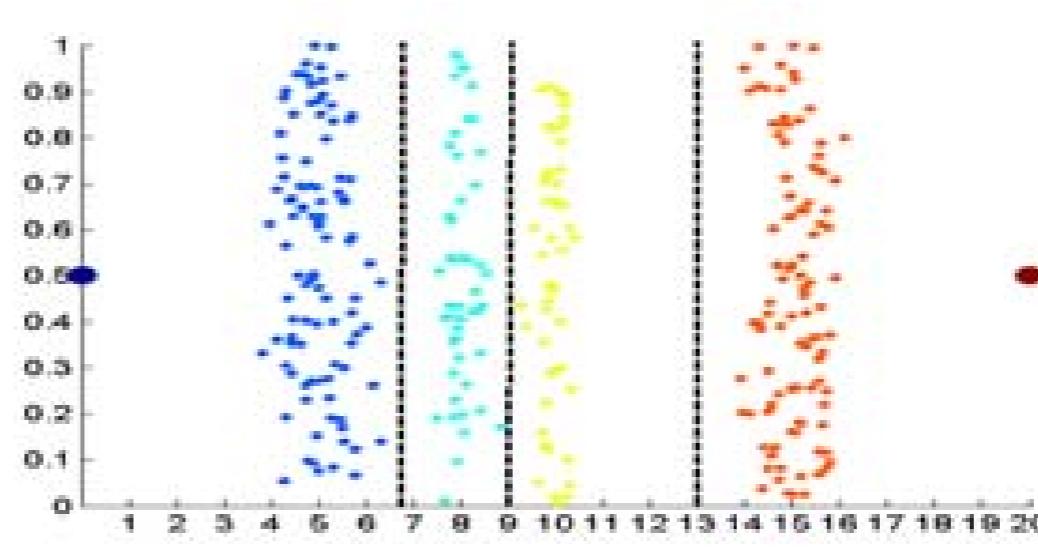
Data



Equal interval width



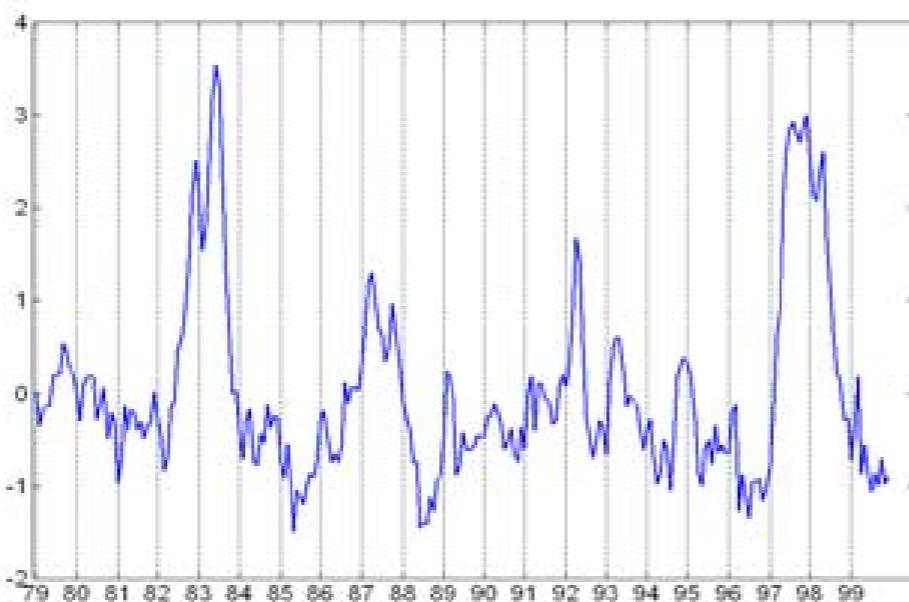
Equal frequency



K-means

Attribute Transformation

- A function that maps the entire set of values of a given attribute to a new set of replacement values such that each old value can be identified with one of the new values
 - Simple functions: x^k , $\log(x)$, e^x , $|x|$
 - Standardization and Normalization



Attribute Transformation: Standardization

Let \bar{x} be the mean (average),
and s_x the standard deviation of the attribute values.

Then “standardizing” a variable x entails
using the transformed variable $x' := (x - \bar{x})/s_x$.

Allows us to talk about variables/attributes at different scales
e.g. income and age are at different scales.

Proximity: Similarity, Dissimilarity

Similarity and Dissimilarity

- Measures of how similar or dissimilar two objects are
 - ▶ very useful in most data mining applications
 - ▶ in fact, is typically all that you need: can even throw away the data after computing the similarity values between all pairs of records/objects

Similarity and Dissimilarity

● Similarity

- Numerical measure of how alike two data objects are.
- Is higher when objects are more alike.
- Often falls in the range [0,1]

● Dissimilarity

- Numerical measure of how different are two data objects
- Lower when objects are more alike
- Minimum dissimilarity is often 0
- Upper limit varies

● Proximity refers to a similarity or dissimilarity

Similarity/Dissimilarity for Simple Attributes

p and q are the attribute values for two data objects.

Attribute Type	Dissimilarity	Similarity
Nominal	$d = \begin{cases} 0 & \text{if } p = q \\ 1 & \text{if } p \neq q \end{cases}$	$s = \begin{cases} 1 & \text{if } p = q \\ 0 & \text{if } p \neq q \end{cases}$
Ordinal	$d = \frac{ p-q }{n-1}$ (values mapped to integers 0 to $n-1$, where n is the number of values)	$s = 1 - \frac{ p-q }{n-1}$
Interval or Ratio	$d = p - q $	$s = -d, s = \frac{1}{1+d}$ or $s = 1 - \frac{d - \min_d}{\max_d - \min_d}$

Euclidean Distance

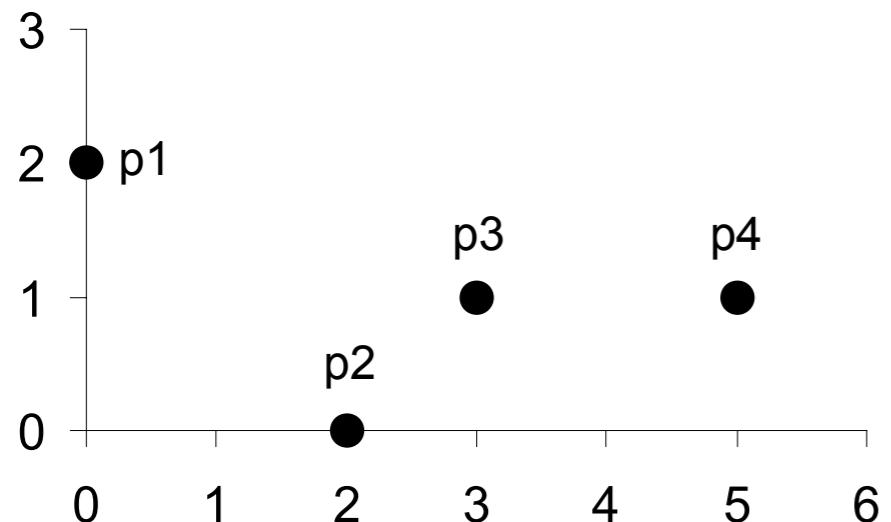
- Euclidean Distance

$$dist = \sqrt{\sum_{k=1}^n (p_k - q_k)^2}$$

Where n is the number of dimensions (attributes) and p_k and q_k are, respectively, the k^{th} attributes (components) or data objects p and q .

- Standardization is necessary, if scales differ.

Euclidean Distance



point	x	y
p1	0	2
p2	2	0
p3	3	1
p4	5	1

	p1	p2	p3	p4
p1	0	2.828	3.162	5.099
p2	2.828	0	1.414	3.162
p3	3.162	1.414	0	2
p4	5.099	3.162	2	0

Distance Matrix

Minkowski Distance

- Minkowski Distance is a generalization of Euclidean Distance

$$dist = \left(\sum_{k=1}^n |p_k - q_k|^r \right)^{\frac{1}{r}}$$

Where r is a parameter, n is the number of dimensions (attributes) and p_k and q_k are, respectively, the k th attributes (components) or data objects p and q .

Minkowski Distance: Examples

- $r = 1$. City block (Manhattan, taxicab, L_1 norm) distance.
 - A common example of this is the Hamming distance, which is just the number of bits that are different between two binary vectors
- $r = 2$. Euclidean distance
- $r \rightarrow \infty$. “supremum” (L_{\max} norm, L_∞ norm) distance.
 - This is the maximum difference between any component of the vectors
- Do not confuse r with n , i.e., all these distances are defined for all numbers of dimensions.

Minkowski Distance: Examples

point	x	y
p1	0	2
p2	2	0
p3	3	1
p4	5	1

L1	p1	p2	p3	p4
p1	0	4	4	6
p2	4	0	2	4
p3	4	2	0	2
p4	6	4	2	0

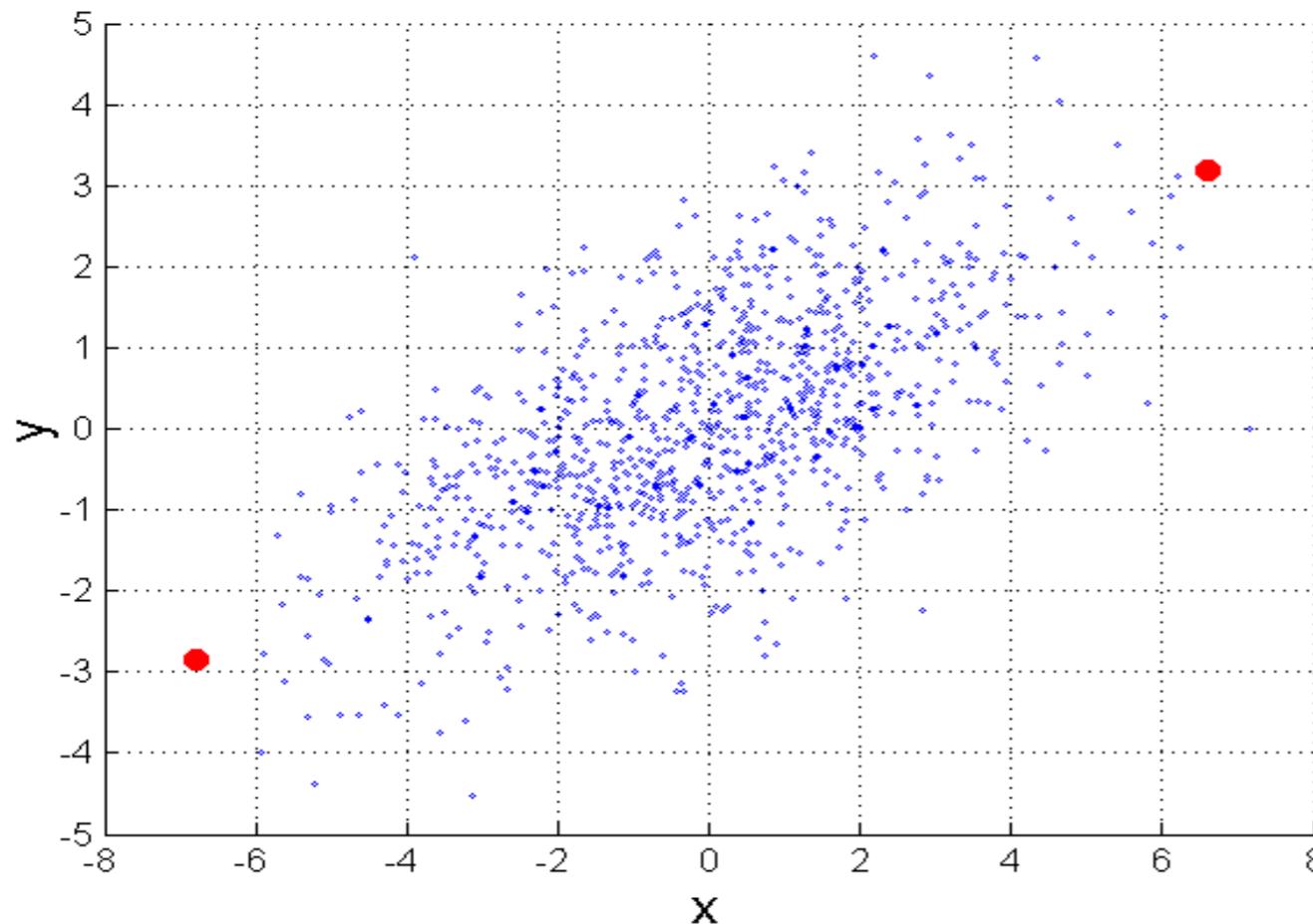
L2	p1	p2	p3	p4
p1	0	2.828	3.162	5.099
p2	2.828	0	1.414	3.162
p3	3.162	1.414	0	2
p4	5.099	3.162	2	0

L ∞	p1	p2	p3	p4
p1	0	2	3	5
p2	2	0	1	3
p3	3	1	0	2
p4	5	3	2	0

Distance Matrix

Mahalanobis Distance

$$\text{mahalanobi } s(p, q) = (p - q) \Sigma^{-1} (p - q)^T$$

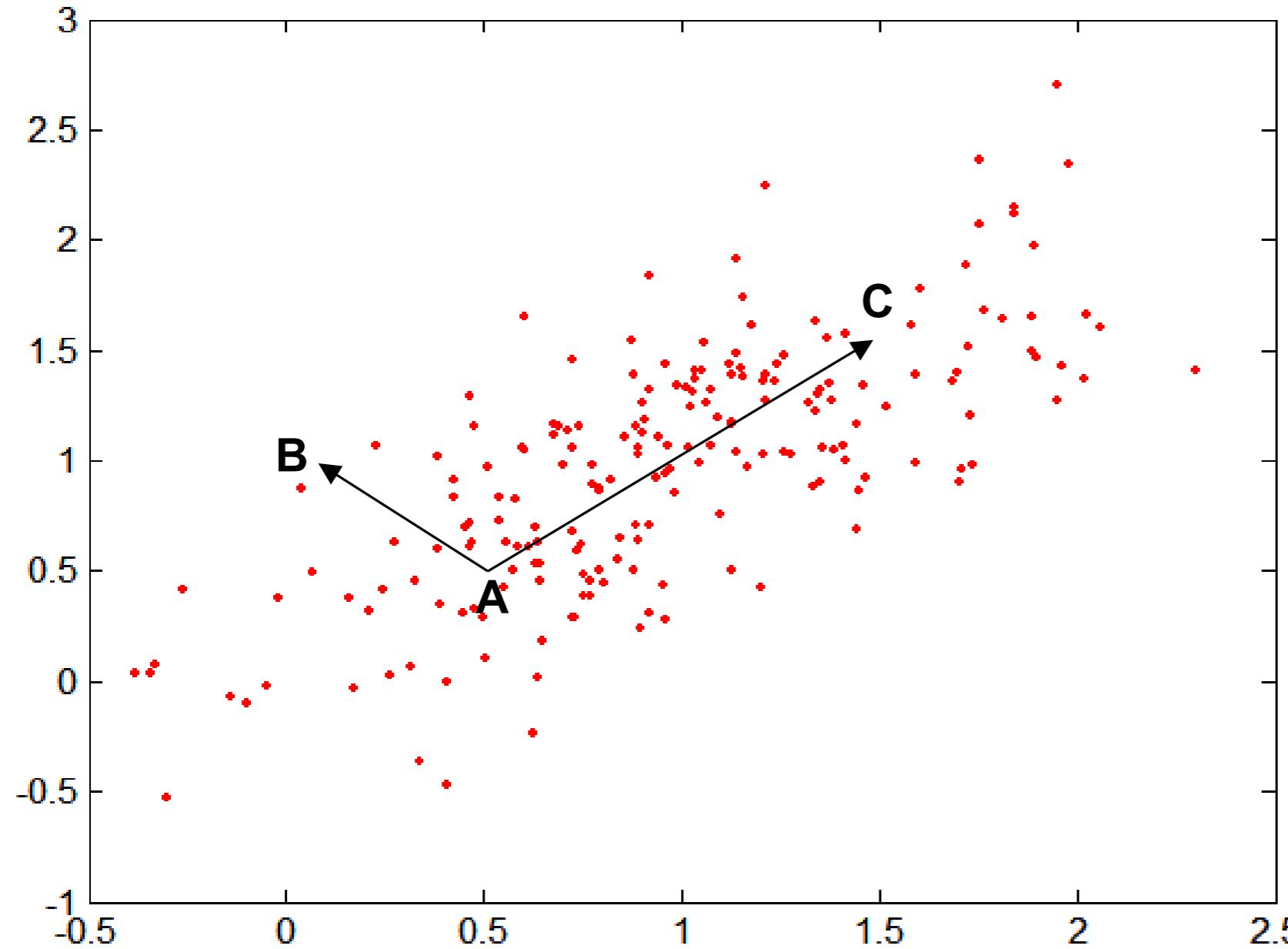


Σ is the covariance matrix of the input data X

$$\Sigma_{j,k} = \frac{1}{n-1} \sum_{i=1}^n (X_{ij} - \bar{X}_j)(X_{ik} - \bar{X}_k)$$

For red points, the Euclidean distance is 14.7, Mahalanobis distance is 6.

Mahalanobis Distance



Covariance Matrix:

$$\Sigma = \begin{bmatrix} 0.3 & 0.2 \\ 0.2 & 0.3 \end{bmatrix}$$

A: (0.5, 0.5)

B: (0, 1)

C: (1.5, 1.5)

Mahal(A,B) = 5

Mahal(A,C) = 4

Similarity Between Binary Vectors

- Common situation is that objects, p and q , have only binary attributes

Similarity Between Binary Vectors

- Common situation is that objects, p and q , have only binary attributes
- Compute similarities using the following quantities

M_{01} = the number of attributes where p was 0 and q was 1

M_{10} = the number of attributes where p was 1 and q was 0

M_{00} = the number of attributes where p was 0 and q was 0

M_{11} = the number of attributes where p was 1 and q was 1

- Simple Matching and Jaccard Coefficients

SMC = number of matches / number of attributes

$$= (M_{11} + M_{00}) / (M_{01} + M_{10} + M_{11} + M_{00})$$

J = number of 11 matches / number of not-both-zero attributes values

$$= (M_{11}) / (M_{01} + M_{10} + M_{11})$$

SMC vs Jaccard: Example

$p = 1000000000$

$q = 000001001$

$M_{01} = 2$ (the number of attributes where p was 0 and q was 1)

$M_{10} = 1$ (the number of attributes where p was 1 and q was 0)

$M_{00} = 7$ (the number of attributes where p was 0 and q was 0)

$M_{11} = 0$ (the number of attributes where p was 1 and q was 1)

$$\text{SMC} = (M_{11} + M_{00}) / (M_{01} + M_{10} + M_{11} + M_{00}) = (0+7) / (2+1+0+7) = 0.7$$

$$J = (M_{11}) / (M_{01} + M_{10} + M_{11}) = 0 / (2 + 1 + 0) = 0$$

Cosine Similarity

- If d_1 and d_2 are two document vectors, then

$$\cos(d_1, d_2) = (d_1 \bullet d_2) / \|d_1\| \|d_2\|,$$

where \bullet indicates vector dot product and $\|d\|$ is the length of vector d .

- Example:

$$d_1 = 3 \ 2 \ 0 \ 5 \ 0 \ 0 \ 0 \ 2 \ 0 \ 0$$

$$d_2 = 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 2$$

$$d_1 \bullet d_2 = 3*1 + 2*0 + 0*0 + 5*0 + 0*0 + 0*0 + 0*0 + 2*1 + 0*0 + 0*2 = 5$$

$$\|d_1\| = (3^2 + 2^2 + 0^2 + 5^2 + 0^2 + 0^2 + 0^2 + 2^2 + 0^2 + 0^2)^{0.5} = (42)^{0.5} = 6.481$$

$$\|d_2\| = (1^2 + 0^2 + 0^2 + 0^2 + 0^2 + 0^2 + 0^2 + 1^2 + 0^2 + 2^2)^{0.5} = (6)^{0.5} = 2.245$$

$$\cos(d_1, d_2) = .3150$$

Cosine Similarity

- If d_1 and d_2 are two document vectors, then

$$\cos(d_1, d_2) = (d_1 \bullet d_2) / \|d_1\| \|d_2\|,$$

where \bullet indicates

- Example:

$$d_1 = [3, 1, 2]$$

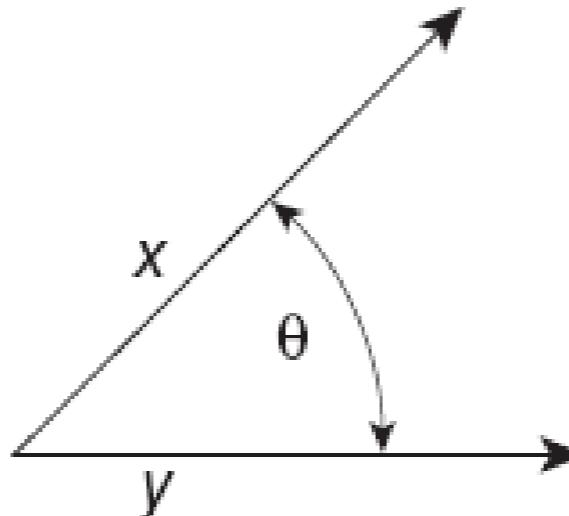
$$d_2 = [1, 0, 2]$$

$$d_1 \bullet d_2 = 3*1 + 1*0 + 2*2 = 7$$

$$\|d_1\| = \sqrt{3^2 + 1^2 + 2^2} = \sqrt{14}$$

$$\|d_2\| = \sqrt{1^2 + 0^2 + 2^2} = \sqrt{5}$$

$$\cos(d_1, d_2) = .3150$$



Geometric illustration of the cosine measure.