Statistical Learning and Data Mining
CS 363D/ SSC 358

Lecture: Association Rules I

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Adapted From: Pang-Ning Tan, Steinbach, Kumar

Association Rule Mining

- Given a set of transactions, find rules that will predict the occurrence of an item based on the occurrences of other items in the transaction

Market-Basket transactions

TID	*Items*
1	**Bread, Milk**
2	**Bread, Diaper, Beer, Eggs**
3	**Milk, Diaper, Beer, Coke**
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Example of Association Rules

$\{Diaper\} \rightarrow \{Beer\}$,
$\{Milk, Bread\} \rightarrow \{Eggs,Coke\}$,
$\{Beer, Bread\} \rightarrow \{Milk\}$,

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Example of Association Rules

{Diaper} → {Beer},
{Milk, Bread} → {Eggs,Coke},
{Beer, Bread} → {Milk},

Implication means co-occurrence, not causality!

Frequent Itemset

- **Itemset**
  - A collection of one or more items
    - Example: {Milk, Bread, Diaper}
  - k-itemset
    - An itemset that contains k items

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  - E.g. s({Milk, Bread, Diaper}) = 2/5

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- **Support**
  - Fraction of transactions that contain an itemset
  - E.g. s({Milk, Bread, Diaper}) = 2/5
- **Frequent Itemset**
  - An itemset whose support is greater than or equal to a *minsup* threshold

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Association Rule

- **Association Rule**
  - An implication expression of the form $X \rightarrow Y$, where X and Y are itemsets
  - Example:
    $\{\text{Milk, Diaper}\} \rightarrow \{\text{Beer}\}$

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- **Rule Evaluation Metrics**
  - Support (s)
    - Fraction of transactions that contain both X and Y
  - Confidence (c)
    - Measures how often items in Y appear in transactions that contain X

Example:

$$\{\text{Milk}, \text{Diaper}\} \Rightarrow \text{Beer}$$

$$s = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{|\text{T}|} =$$

$$c = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{\sigma(\text{Milk}, \text{Diaper})} =$$

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$$s = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{|\text{T}|} = \frac{2}{5} = 0.4$$

$$c = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{\sigma(\text{Milk}, \text{Diaper})} =$$

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$$c = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{\sigma(\text{Milk}, \text{Diaper})} = \frac{2}{3} = 0.67$$

Association Rule Mining

- Given a set of transactions T, the goal of association rule mining is to find all rules having
  - support ≥ *minsup* threshold
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- Given a set of transactions T, the goal of association rule mining is to find all rules having
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- Brute-force approach:
  - List all possible association rules
  - Compute the support and confidence for each rule
  - Prune rules that fail the *minsup* and *minconf* thresholds

  ⇒ Computationally prohibitive!

Mining Association Rules

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Example of Rules:

{Milk,Diaper} $\rightarrow$ {Beer} (s=0.4, c=0.67)
{Milk,Beer} $\rightarrow$ {Diaper} (s=0.4, c=1.0)
{Diaper,Beer} $\rightarrow$ {Milk} (s=0.4, c=0.67)
{Beer} $\rightarrow$ {Milk,Diaper} (s=0.4, c=0.67)
{Diaper} $\rightarrow$ {Milk,Beer} (s=0.4, c=0.5)
{Milk} $\rightarrow$ {Diaper,Beer} (s=0.4, c=0.5)

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{Diaper,Beer} → {Milk} (s=0.4, c=0.67)
{Beer} → {Milk,Diaper} (s=0.4, c=0.67)
{Diaper} → {Milk,Beer} (s=0.4, c=0.5)
{Milk} → {Diaper,Beer} (s=0.4, c=0.5)

Observations:

- All the above rules are binary partitions of the same itemset:
  {Milk, Diaper, Beer}

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Observations:

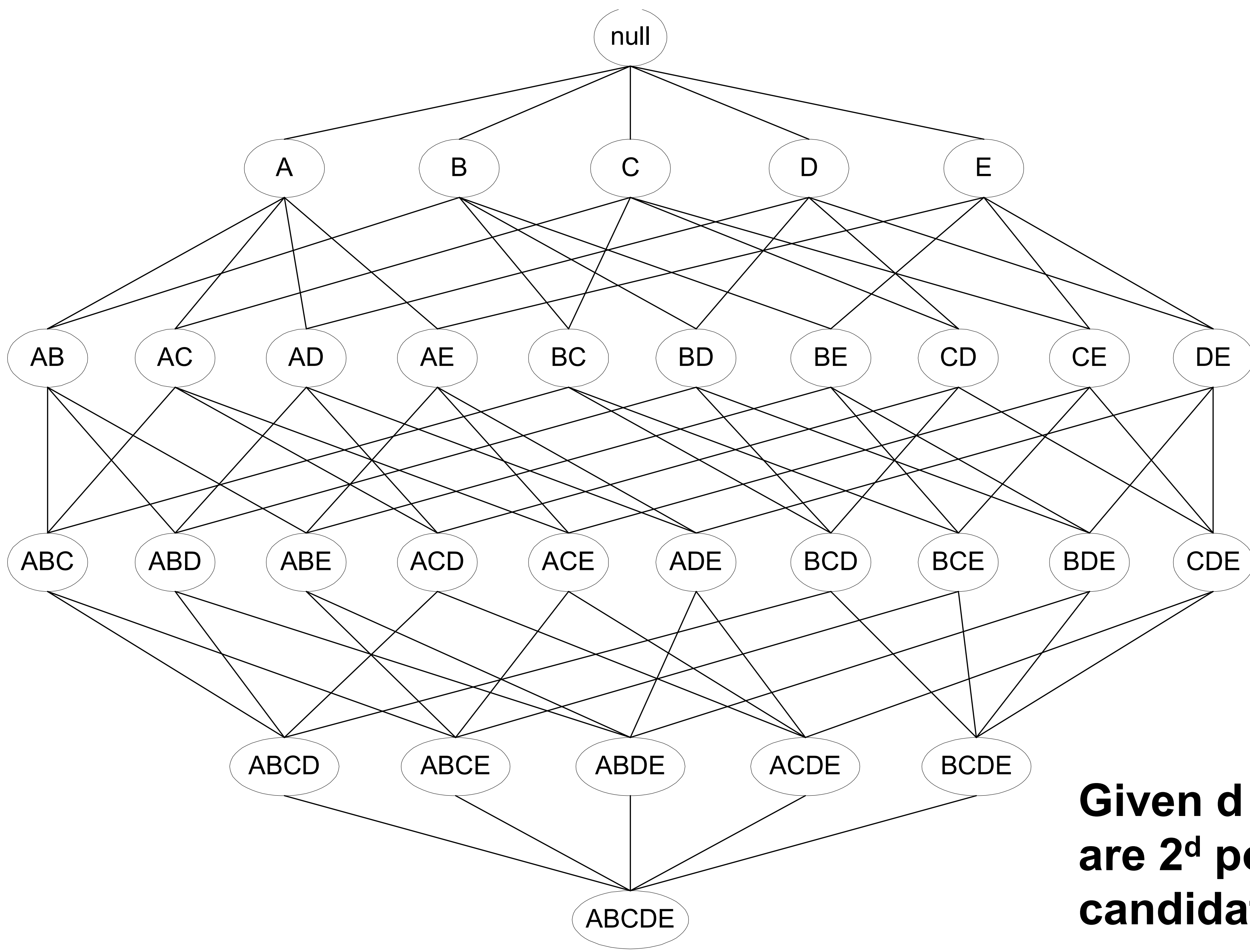
- All the above rules are binary partitions of the same itemset:
  {Milk, Diaper, Beer}
- Rules originating from the same itemset have identical support but can have different confidence
- Thus, we may decouple the support and confidence requirements

Mining Association Rules

- Two-step approach:
  1. Frequent Itemset Generation
     – Generate all itemsets whose support ≥ minsup
  2. Rule Generation
     – Generate high confidence rules from each frequent itemset, where each rule is a binary partitioning of a frequent itemset

- Frequent itemset generation is still computationally expensive

Frequent Itemset Generation



Given d items, there are 2^d possible candidate itemsets

Frequent Itemset Generation

- Brute-force approach:
  - Each itemset in the lattice is a candidate frequent itemset
  - Count the support of each candidate by scanning the database

Transactions

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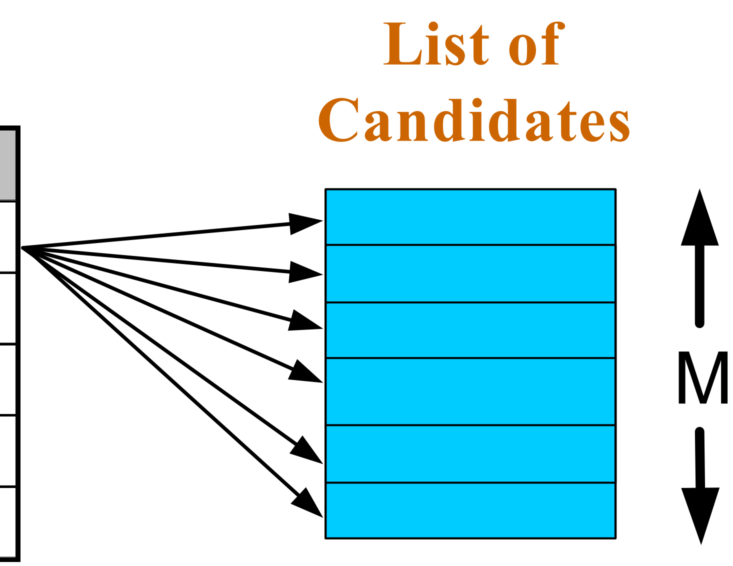
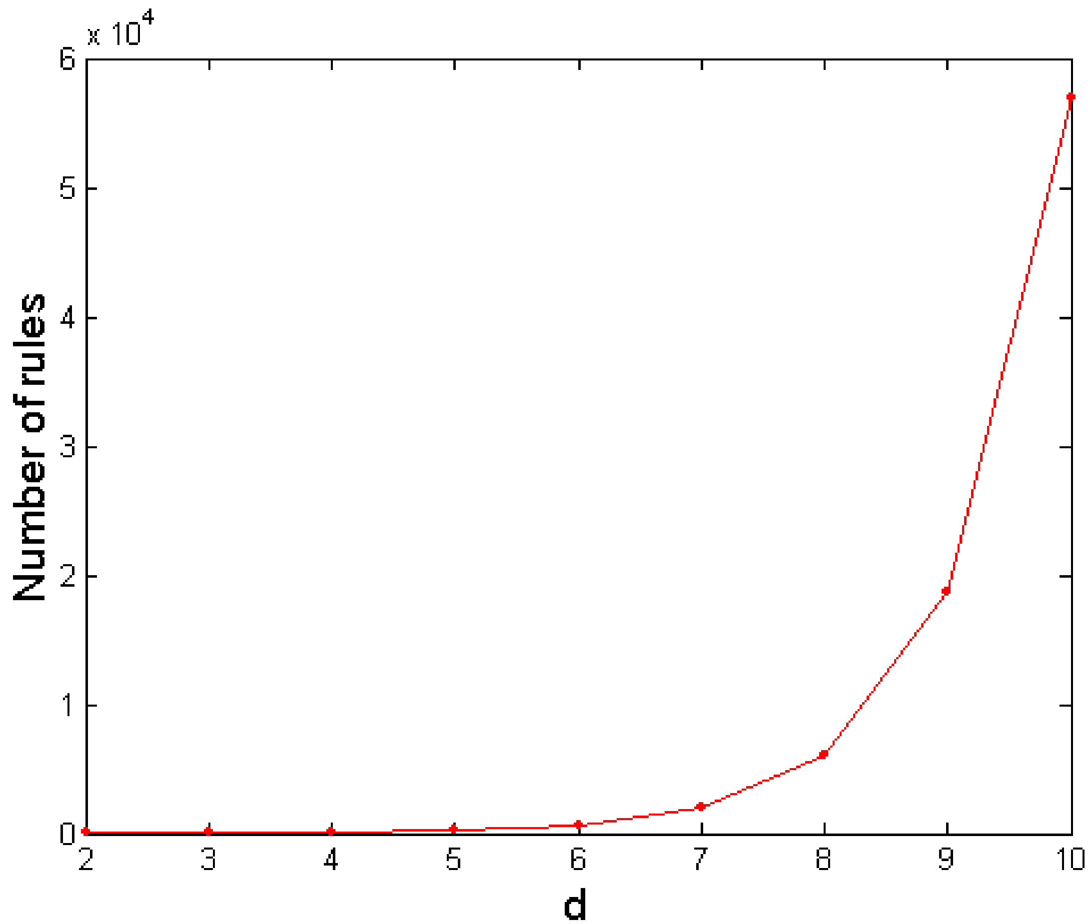


  - Match each transaction against every candidate
  - Complexity ~ O(NMw) => Expensive since $M = 2^d$!!!

Computational Complexity

- Given d unique items:
  - Total number of itemsets = 2^d
  - Total number of possible association rules:



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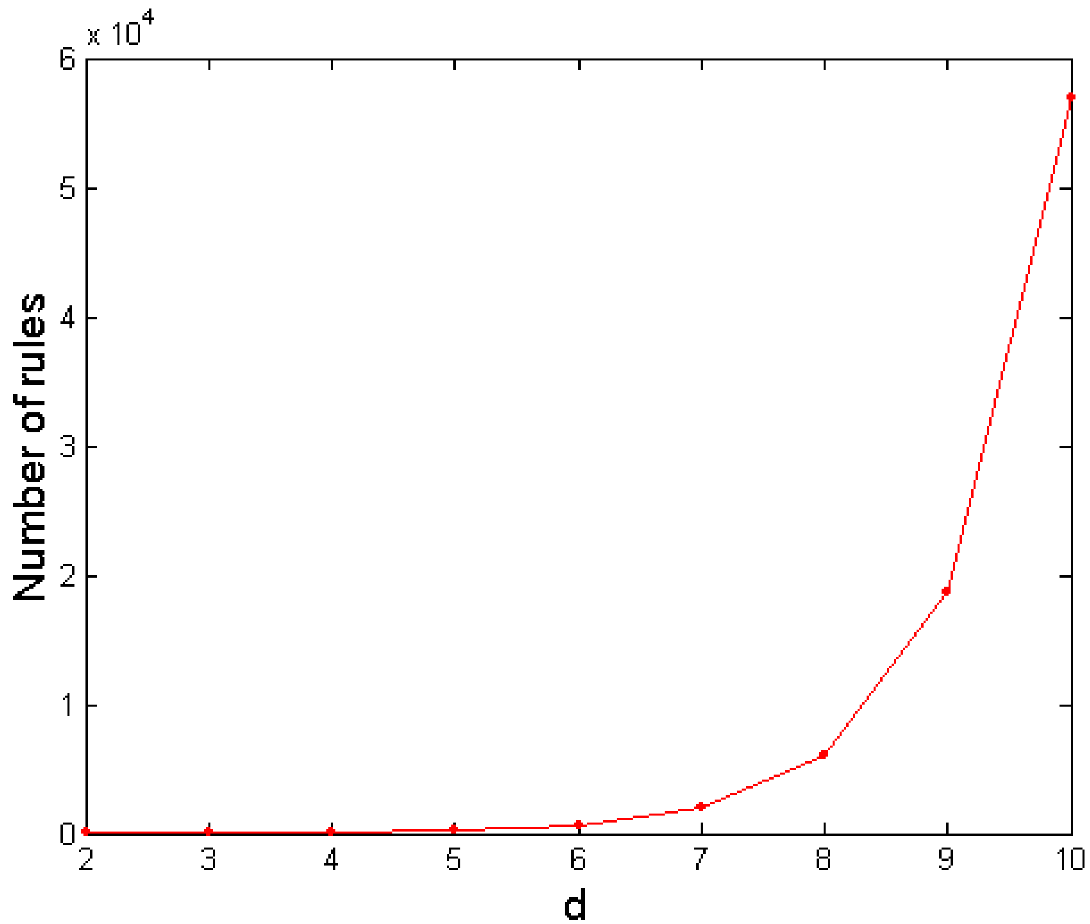

$$R = \sum_{k=1}^{d-1}\left[\binom{d}{k} \times \sum_{j=1}^{d-k}\binom{d-k}{j}\right] = 3^d - 2^{d+1} + 1$$

If d=6, R = 602 rules

Frequent Itemset Generation Strategies

- Reduce the number of candidates (M)
  - Complete search: $M=2^d$
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- Reduce the number of candidates (M)
  - Complete search: $M=2^d$
  - Use pruning techniques to reduce M
- Reduce the number of transactions (N)
  - Reduce size of N as the size of itemset increases
- Reduce the number of comparisons (NM)
  - Use efficient data structures to store the candidates or transactions
  - No need to match every candidate against every transaction

Reducing Number of Candidates

- Apriori principle:
  - If an itemset is frequent, then all of its subsets must also be frequent

Reducing Number of Candidates

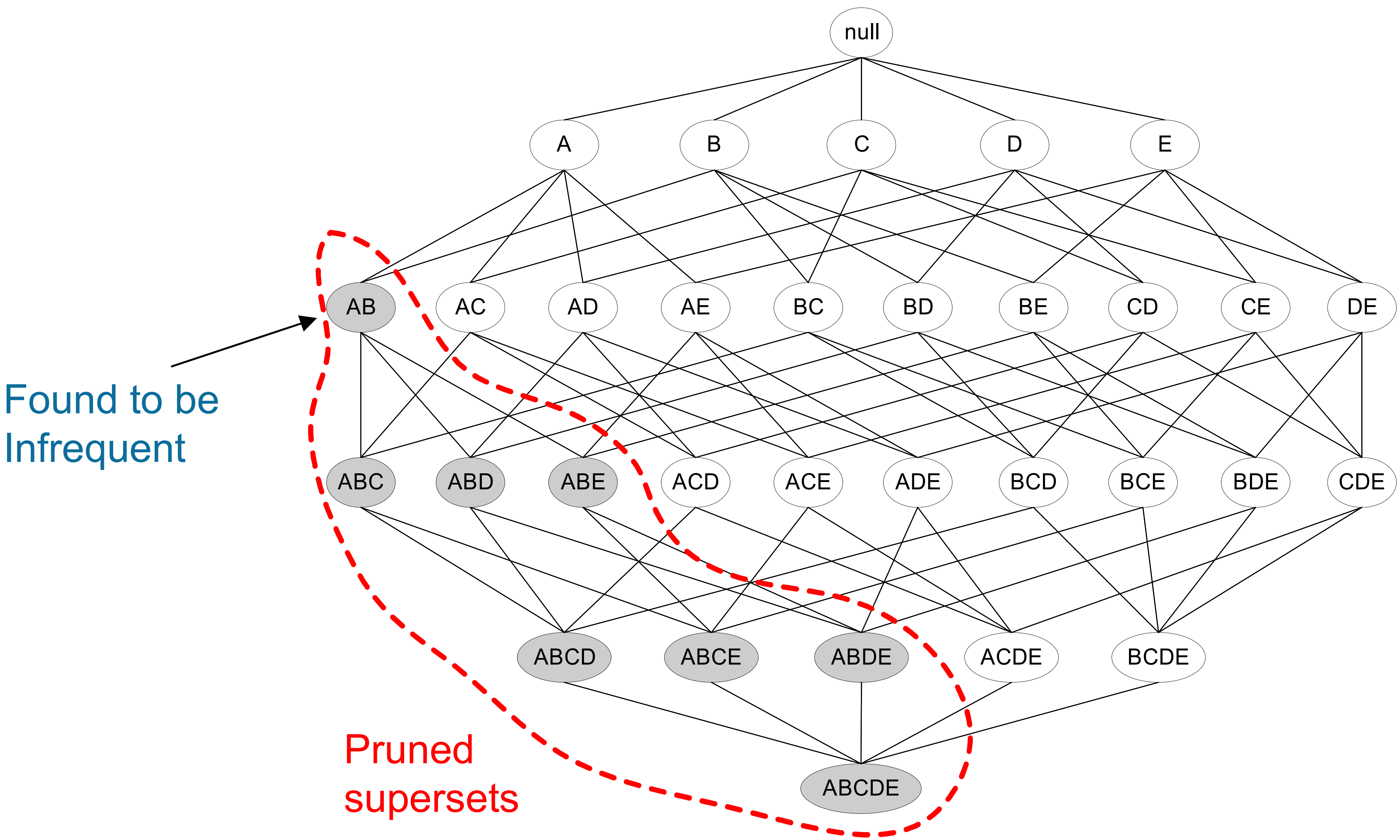
- Apriori principle:
  - If an itemset is frequent, then all of its subsets must also be frequent

- Apriori principle holds due to the following property of the support measure:

$$\forall X, Y : (X \subseteq Y) \Rightarrow s(X) \geq s(Y)$$

  - Support of an itemset never exceeds the support of its subsets
  - This is known as the anti-monotone property of support

Illustrating Apriori Principle



Illustrating Apriori Principle

Item	Count
Bread	**4**
Coke	**2**
Milk	**4**
Beer	**3**
Diaper	**4**
Eggs	**1**

Items (1-itemsets)

Minimum Support = 3

Illustrating Apriori Principle

Item	Count
Bread	**4**
Coke	**2**
Milk	**4**
Beer	**3**
Diaper	**4**
Eggs	**1**

Items (1-itemsets)

Itemset	Count
{Bread,Milk}	**3**
{Bread,Beer}	**2**
{Bread,Diaper}	**3**
{Milk,Beer}	**2**
{Milk,Diaper}	**3**
{Beer,Diaper}	**3**

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Minimum Support = 3

Illustrating Apriori Principle

Items (1-itemsets)

Item	Count
Bread	**4**
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Milk	**4**
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(No need to generate candidates involving Coke or Eggs)

Itemset	Count
{Bread,Milk}	**3**
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Triplets (3-itemsets)

Itemset	Count
{Bread,Milk,Diaper}	**3**

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Minimum Support = 3

Triplets (3-itemsets)

Itemset	Count
{Bread,Milk,Diaper}	**3**

If every subset is considered,
$^6C_1 + {}^6C_2 + {}^6C_3 = 41$
With support-based pruning,
$6 + 6 + 1 = 13$

Apriori Algorithm

- Method:
  - Let k=1
  - Generate frequent itemsets of length 1
  - Repeat until no new frequent itemsets are identified
    - Generate length (k+1) candidate itemsets from length k frequent itemsets
    - Prune candidate itemsets containing subsets of length k that are infrequent
    - Count the support of each candidate by scanning the DB
    - Eliminate candidates that are infrequent, leaving only those that are frequent

Reducing Number of Comparisons

- Candidate counting:
  - Scan the database of transactions to determine the support of each candidate itemset
  - To reduce the number of comparisons, store the candidates in a hash structure
    - Instead of matching each transaction against every candidate, match it against candidates contained in the hashed buckets

Transactions

N

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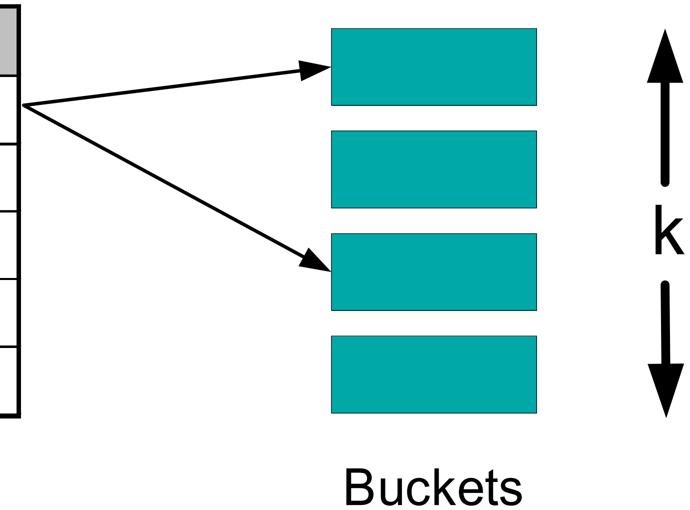
Hash Structure



Factors Affecting Computational Complexity

- Choice of minimum support threshold
  - lowering support threshold results in more frequent itemsets
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- Size of database
  - since Apriori makes multiple passes, run time of algorithm may increase with number of transactions
- Average transaction width
  - transaction width increases with denser data sets

Compact Representation of Frequent Itemsets

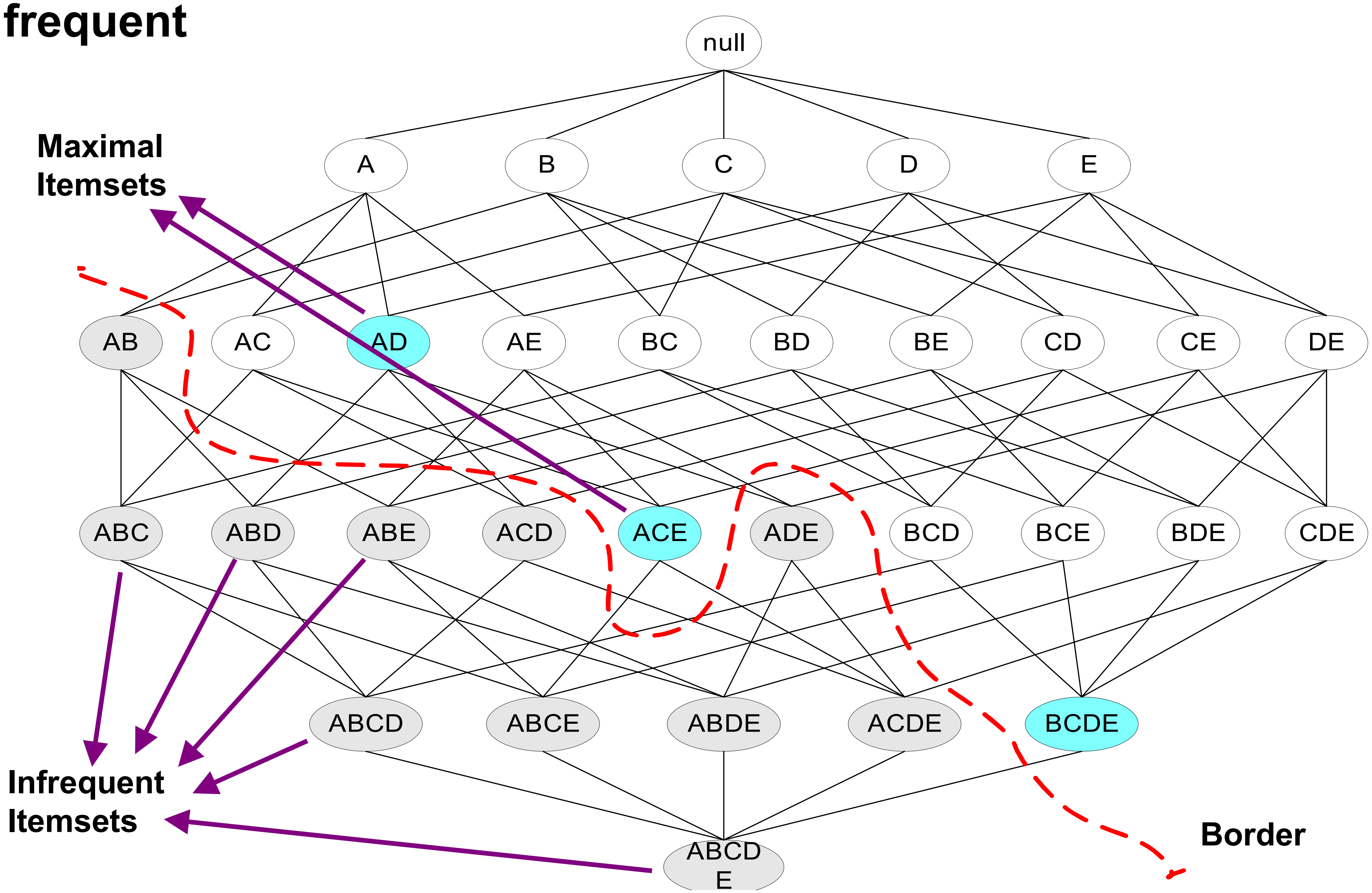
- Some itemsets are redundant because they have identical support as their supersets

TID	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
1	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**

- Need a compact representation

Maximal Frequent Itemset

An itemset is maximal frequent if none of its immediate supersets is frequent



Closed Itemset

- An itemset is closed if none of its immediate supersets has the same support as the itemset

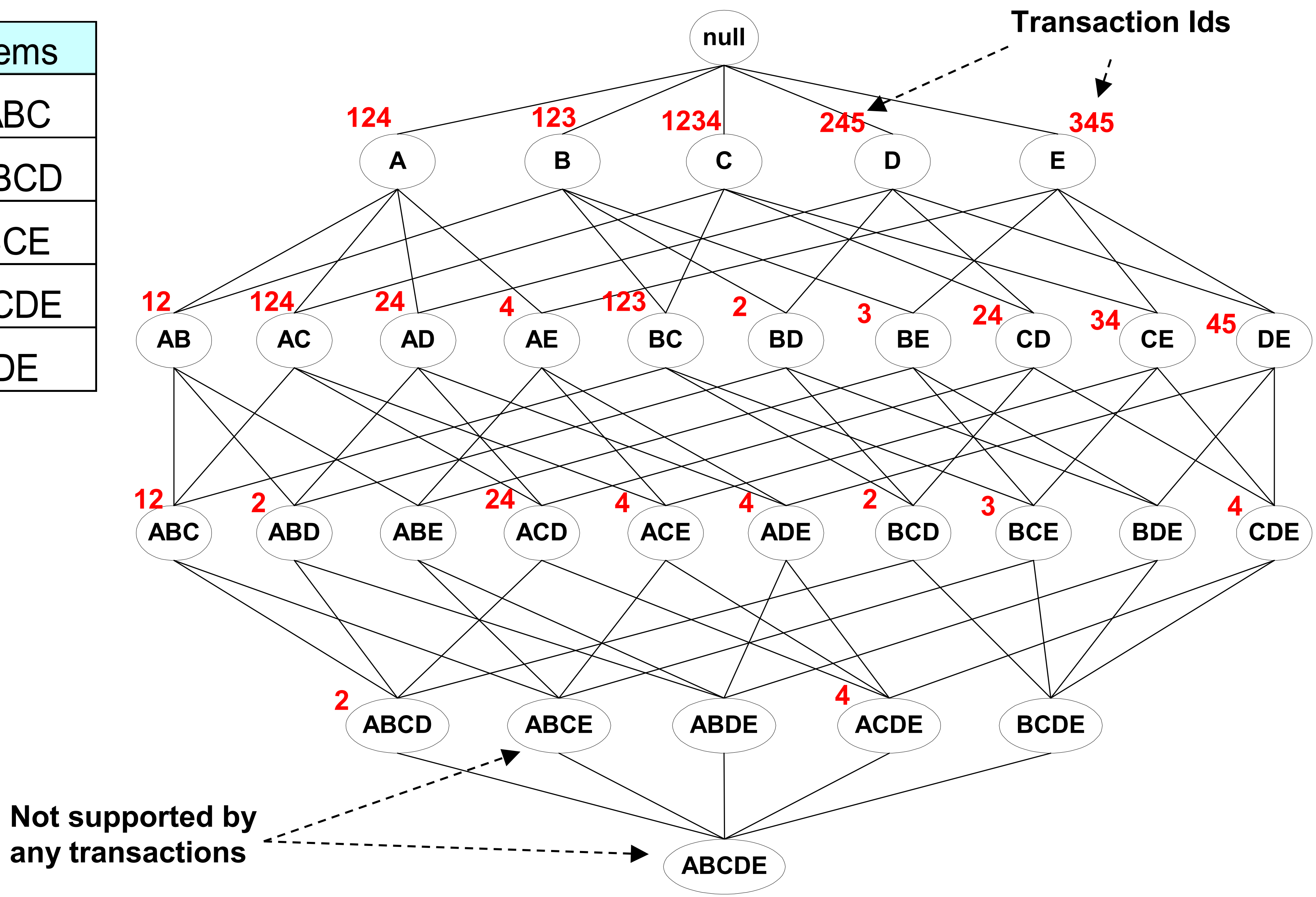
TID	Items
1	{A,B}
2	{B,C,D}
3	{A,B,C,D}
4	{A,B,D}
5	{A,B,C,D}

Itemset	Support
{A}	4
{B}	5
{C}	3
{D}	4
{A,B}	4
{A,C}	2
{A,D}	3
{B,C}	3
{B,D}	4
{C,D}	3

Itemset	Support
{A,B,C}	2
{A,B,D}	3
{A,C,D}	2
{B,C,D}	3
{A,B,C,D}	2

Example: Closed Itemsets

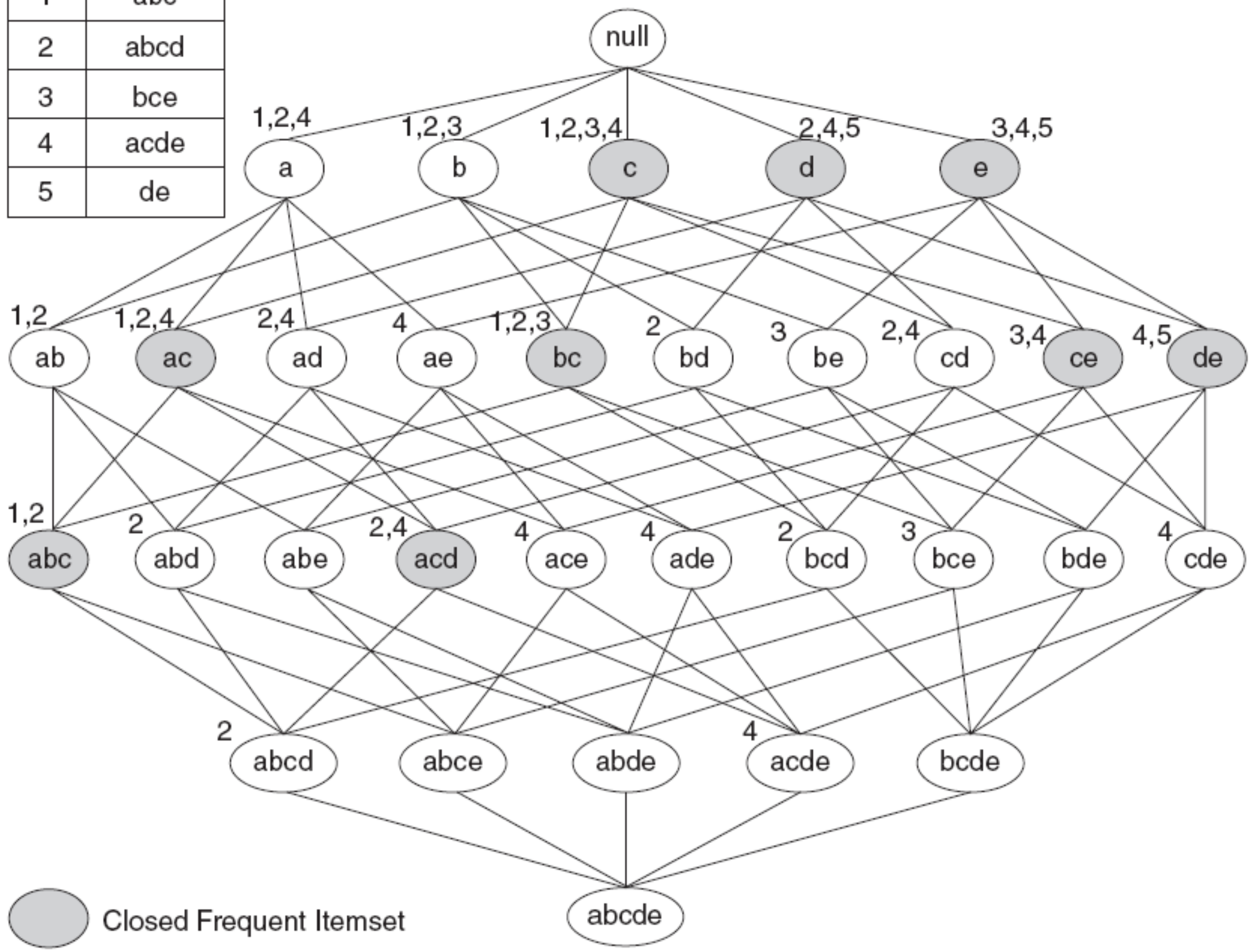
TID	Items
1	ABC
2	ABCD
3	BCE
4	ACDE
5	DE



Example: Closed Frequent Itemsets

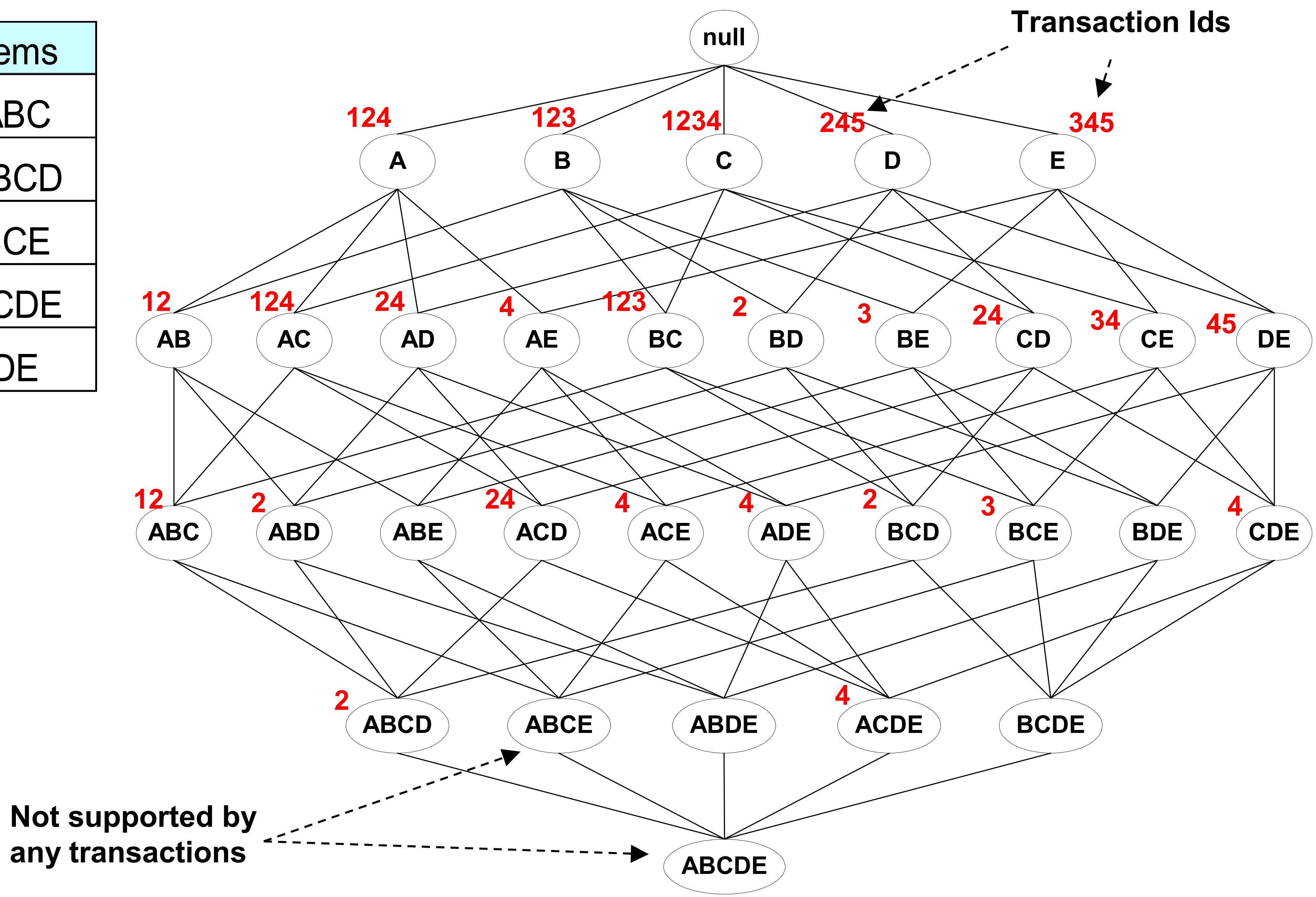
TID	Items
1	abc
2	abcd
3	bce
4	acde
5	de

minsup = 40%

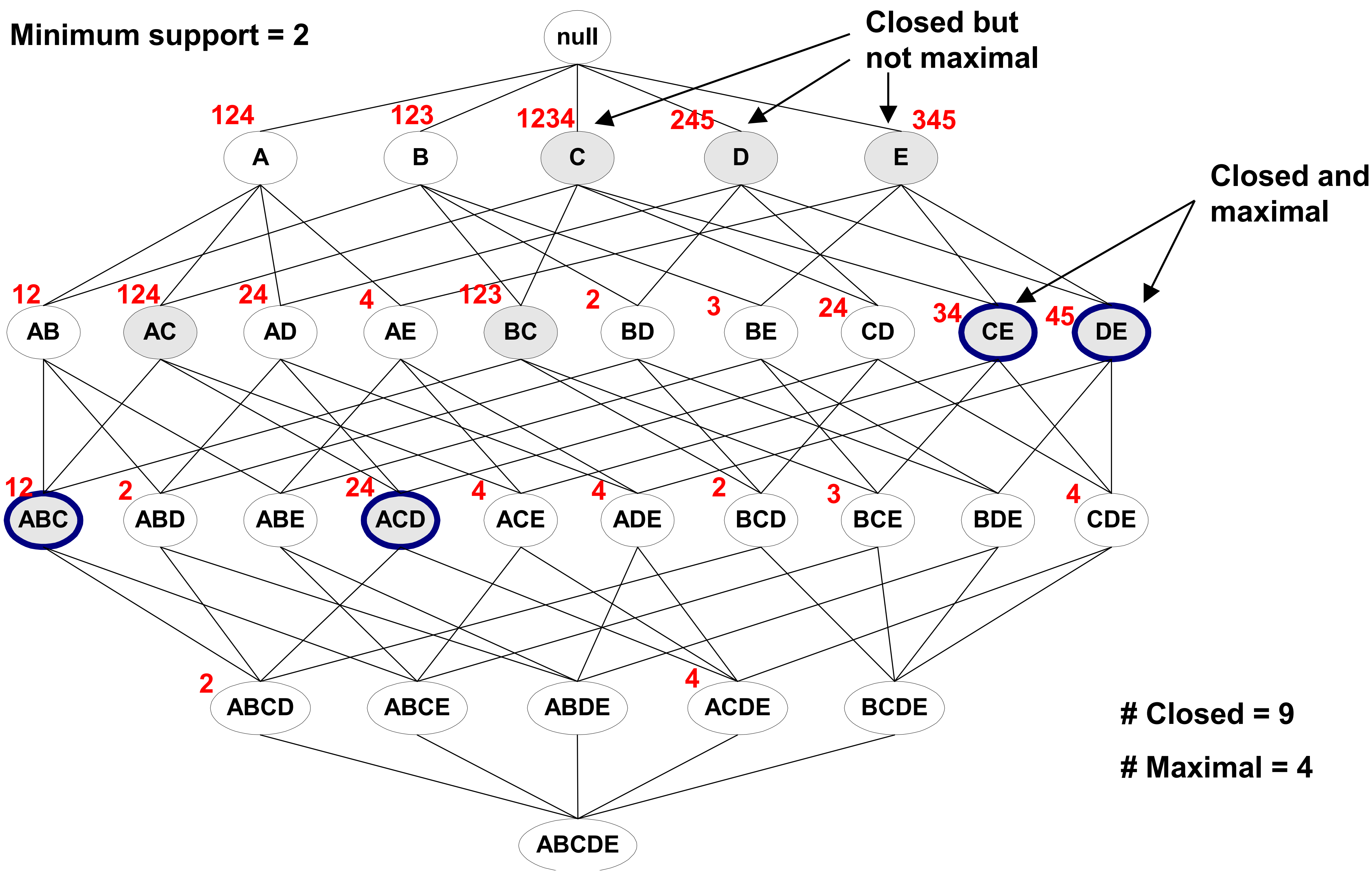


Maximal vs Closed Itemsets

TID	Items
1	ABC
2	ABCD
3	BCE
4	ACDE
5	DE



Maximal vs Closed Itemsets



Closed = 9

Maximal = 4

Maximal vs Closed Frequent Itemsets