

Statistical Learning and Data Mining

CS 363D/ SSC 358

Lecture: Bayes Classification

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Adapted From: Pang-Ning Tan, Steinbach, Kumar

Recap: Bayes Theorem

- Conditional Probability:

$$P(C | A) = \frac{P(A, C)}{P(A)}$$

$$P(A | C) = \frac{P(A, C)}{P(C)}$$

- Bayes theorem:

$$P(C | A) = \frac{P(A | C)P(C)}{P(A)}$$

Recap: Bayes Theorem

- Given:

- A doctor knows that meningitis causes stiff neck 50% of the time
- Prior probability of any patient having meningitis is $1/50,000$
- Prior probability of any patient having stiff neck is $1/20$

- If a patient has stiff neck, what's the probability he/she has meningitis?

Recap: Bayes Theorem

- Given:

- A doctor knows that meningitis causes stiff neck 50% of the time
- Prior probability of any patient having meningitis is $1/50,000$
- Prior probability of any patient having stiff neck is $1/20$

- If a patient has stiff neck, what's the probability he/she has meningitis?

$$P(M | S) = \frac{P(S | M)P(M)}{P(S)} = \frac{0.5 \times 1/50000}{1/20} = 0.0002$$

Bayesian Classifiers

- Consider each attribute and class label as random variables
- Given a record with attributes (A_1, A_2, \dots, A_n)
 - Goal is to predict class C
 - Specifically, we want to find the value of C that maximizes $P(C | A_1, A_2, \dots, A_n)$
- Can we estimate $P(C | A_1, A_2, \dots, A_n)$ directly from data?

Bayesian Classifiers

- Approach:

- compute the posterior probability $P(C \mid A_1, A_2, \dots, A_n)$ for all values of C using the Bayes theorem

$$P(C \mid A_1 A_2 \dots A_n) = \frac{P(A_1 A_2 \dots A_n \mid C) P(C)}{P(A_1 A_2 \dots A_n)}$$

- Choose value of C that maximizes $P(C \mid A_1, A_2, \dots, A_n)$
- Equivalent to choosing value of C that maximizes $P(A_1, A_2, \dots, A_n \mid C) P(C)$

- How to estimate $P(A_1, A_2, \dots, A_n \mid C)$?

Naive Bayes Classifier

- Assume independence among attributes A_i when class is given:
 - $P(A_1, A_2, \dots, A_n | C) = P(A_1 | C_j) P(A_2 | C_j) \dots P(A_n | C_j)$
 - Can estimate $P(A_i | C_j)$ for all A_i and C_j .
 - New point is classified to C_j if $P(C_j) \prod P(A_i | C_j)$ is maximal.

Estimating Probabilities from Data

<i>Tid</i>	Refund	Marital Status	Taxable Income	Evade
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

- Class: $P(C) = N_c/N$

- e.g., $P(\text{No}) = 7/10$,
 $P(\text{Yes}) = 3/10$

- For discrete attributes:

$$P(A_i | C_k) = |A_{ik}| / N_{C_k}$$

- where $|A_{ik}|$ is number of instances having attribute A_i and belongs to class C_k
 - Examples:

$$P(\text{Status}=\text{Married}|\text{No}) = 4/7$$
$$P(\text{Refund}=\text{Yes}|\text{Yes})=0$$

Estimating Probabilities from Data

- For continuous attributes:
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Estimating Probabilities from Data

- For continuous attributes:
 - Discretize the range into bins
 - Two-way split: $(A < v)$ or $(A > v)$
 - Probability density estimation:
 - ◆ Assume attribute follows a normal distribution
 - ◆ Use data to estimate parameters of distribution (e.g., mean and standard deviation)
 - ◆ Once probability distribution is known, can use it to estimate the conditional probability $P(A_i|c)$

Estimating Probabilities from Data

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- Normal distribution:

$$P(A_i | c_j) = \frac{1}{\sqrt{2\pi\sigma_{ij}^2}} e^{-\frac{(A_i - \mu_{ij})^2}{2\sigma_{ij}^2}}$$

- One for each (A_i, c_i) pair

- For (Income, Class=No):

- If Class=No

◆ sample mean = 110

◆ sample variance = 2975

$$P(\text{Income} = 120 | \text{No}) = \frac{1}{\sqrt{2\pi(54.54)}} e^{-\frac{(120-110)^2}{2(2975)}} = 0.0072$$

Example: Naive Bayes Classifier

Given a Test Record:

$$X = (\text{Refund} = \text{No}, \text{Married}, \text{Income} = 120\text{K})$$

naive Bayes Classifier:

$$P(\text{Refund}=\text{Yes}|\text{No}) = 3/7$$

$$P(\text{Refund}=\text{No}|\text{No}) = 4/7$$

$$P(\text{Refund}=\text{Yes}|\text{Yes}) = 0$$

$$P(\text{Refund}=\text{No}|\text{Yes}) = 1$$

$$P(\text{Marital Status}=\text{Single}|\text{No}) = 2/7$$

$$P(\text{Marital Status}=\text{Divorced}|\text{No}) = 1/7$$

$$P(\text{Marital Status}=\text{Married}|\text{No}) = 4/7$$

$$P(\text{Marital Status}=\text{Single}|\text{Yes}) = 2/7$$

$$P(\text{Marital Status}=\text{Divorced}|\text{Yes}) = 1/7$$

$$P(\text{Marital Status}=\text{Married}|\text{Yes}) = 0$$

For taxable income:

If class=No: sample mean=110
 sample variance=2975

If class=Yes: sample mean=90
 sample variance=25

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For taxable income:

If class=No: sample mean=110
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If class=Yes: sample mean=90
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- $$\begin{aligned} P(X|\text{Class}=\text{No}) &= P(\text{Refund}=\text{No}|\text{Class}=\text{No}) \\ &\quad \times P(\text{Married}|\text{Class}=\text{No}) \\ &\quad \times P(\text{Income}=120\text{K}|\text{Class}=\text{No}) \\ &= 4/7 \times 4/7 \times 0.0072 = 0.0024 \end{aligned}$$

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For taxable income:

If class=No: sample mean=110
 sample variance=2975
If class=Yes: sample mean=90
 sample variance=25

- $P(X|\text{Class}=\text{No}) = P(\text{Refund}=\text{No}|\text{Class}=\text{No})$
 $\times P(\text{Married}|\text{Class}=\text{No})$
 $\times P(\text{Income}=120\text{K}|\text{Class}=\text{No})$
 $= 4/7 \times 4/7 \times 0.0072 = 0.0024$
- $P(X|\text{Class}=\text{Yes}) = P(\text{Refund}=\text{No}|\text{Class}=\text{Yes})$
 $\times P(\text{Married}|\text{Class}=\text{Yes})$
 $\times P(\text{Income}=120\text{K}|\text{Class}=\text{Yes})$
 $= 1 \times 0 \times 1.2 \times 10^{-9} = 0$

Example: Naive Bayes Classifier

Given a Test Record:

$$X = (\text{Refund} = \text{No}, \text{Married}, \text{Income} = 120\text{K})$$

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- $P(X|\text{Class}=\text{No}) = P(\text{Refund}=\text{No}|\text{Class}=\text{No})$
 $\times P(\text{Married}|\text{Class}=\text{No})$
 $\times P(\text{Income}=120\text{K}|\text{Class}=\text{No})$
 $= 4/7 \times 4/7 \times 0.0072 = 0.0024$
- $P(X|\text{Class}=\text{Yes}) = P(\text{Refund}=\text{No}|\text{Class}=\text{Yes})$
 $\times P(\text{Married}|\text{Class}=\text{Yes})$
 $\times P(\text{Income}=120\text{K}|\text{Class}=\text{Yes})$
 $= 1 \times 0 \times 1.2 \times 10^{-9} = 0$

Since $P(X|\text{No})P(\text{No}) > P(X|\text{Yes})P(\text{Yes})$

Therefore $P(\text{No}|X) > P(\text{Yes}|X)$
 $\Rightarrow \text{Class} = \text{No}$

Naive Bayes Classifier

- If one of the conditional probability is zero, then the entire expression becomes zero
- Probability estimation:

$$\text{Original : } P(A_i | C) = \frac{N_{ic}}{N_c}$$

$$\text{Laplace : } P(A_i | C) = \frac{N_{ic} + 1}{N_c + v}$$

v : number of values taken by att. i

Naive Bayes Classifier

- If one of the conditional probability is zero, then the entire expression becomes zero
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$$\text{Original : } P(A_i | C) = \frac{N_{ic}}{N_c}$$

$$\text{Laplace : } P(A_i | C) = \frac{N_{ic} + 1}{N_c + v}$$

$$\text{m - estimate : } P(A_i | C) = \frac{N_{ic} + mp}{N_c + m}$$

v : number of values taken by att. i

p : prior probability

m : parameter

Example: Naive Bayes Classifier

Name	Give Birth	Can Fly	Live in Water	Have Legs	Class
human	yes	no	no	yes	mammals
python	no	no	no	no	non-mammals
salmon	no	no	yes	no	non-mammals
whale	yes	no	yes	no	mammals
frog	no	no	sometimes	yes	non-mammals
komodo	no	no	no	yes	non-mammals
bat	yes	yes	no	yes	mammals
pigeon	no	yes	no	yes	non-mammals
cat	yes	no	no	yes	mammals
leopard shark	yes	no	yes	no	non-mammals
turtle	no	no	sometimes	yes	non-mammals
penguin	no	no	sometimes	yes	non-mammals
porcupine	yes	no	no	yes	mammals
eel	no	no	yes	no	non-mammals
salamander	no	no	sometimes	yes	non-mammals
gila monster	no	no	no	yes	non-mammals
platypus	no	no	no	yes	mammals
owl	no	yes	no	yes	non-mammals
dolphin	yes	no	yes	no	mammals
eagle	no	yes	no	yes	non-mammals

Give Birth	Can Fly	Live in Water	Have Legs	Class
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A: attributes

M: mammals

N: non-mammals

$$P(A | M) = \frac{6}{7} \times \frac{6}{7} \times \frac{2}{7} \times \frac{2}{7} = 0.06$$

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$$P(A | M) = \frac{6}{7} \times \frac{6}{7} \times \frac{2}{7} \times \frac{2}{7} = 0.06$$

$$P(A | N) = \frac{1}{13} \times \frac{10}{13} \times \frac{3}{13} \times \frac{4}{13} = 0.0042$$

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$$P(A | M)P(M) = 0.06 \times \frac{7}{20} = 0.021$$

$$P(A | N)P(N) = 0.004 \times \frac{13}{20} = 0.0027$$

Give Birth	Can Fly	Live in Water	Have Legs	Class
yes	no	yes	no	?

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$$P(A|M)P(M) > P(A|N)P(N)$$

=> Mammals

Give Birth	Can Fly	Live in Water	Have Legs	Class
yes	no	yes	no	?

Summary: Naive Bayes

- Robust to isolated noise points
- Handle missing values by ignoring the instance during probability estimate calculations
- Robust to irrelevant attributes
- Independence assumption may not hold for some attributes