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Tutorial 9

Theory of Computation

Questions marked (S) are self-test questions with solutions provided at

`http://infolab.stanford.edu/~ullman/ialcsols/sols.html`

Questions marked (A) are assignment questions.

Exercise 1 **$\mathcal{NP}$-Complete Problems** (S)

(Exercise 10.1.3) Suppose that there is an $\mathcal{NP}$-complete problem that has a deterministic solution that takes time $O(n^{\log_2 n})$. Note that this function lies between the polynomials and the exponentials, and is in neither class of functions. What could we say about the running time of any problem in $\mathcal{NP}$?

Exercise 2 **Satisfying Assignments** (S)

(Exercise 10.2.1) How many satisfying truth assignments does the following Boolean expression have?

$$x \wedge (y \vee \neg x) \wedge (z \vee \neg y)$$

Exercise 3 **Boolean Encodings of Graph Properties** (A)

(Exercise 10.2.2b–d) Suppose G is an undirected graph of four nodes: 1, 2, 3, and 4. Let x_{ij}, for $1 \leq i < j \leq 4$, be a Boolean variable that we interpret as saying "there is an edge between nodes i and j." The expression $x_{12}x_{23}x_{34}x_{14} + x_{13}x_{23}x_{24}x_{14} + x_{13}x_{34}x_{24}x_{12}$, for example, says that the graph G has a Hamilton circuit. In general, a Boolean expression over the x_{ij} variables describes a property of the graph in the sense that a truth assignment to the variables satisfies the expression if and only if it describes a graph having that property. Write expressions for the following properties:

1. G contains a clique of size 3 (i.e., a triangle).
2. G contains at least one node with no edges.
3. G is connected.

Exercise 4 **Conversion to 3CNF** (S)

(Exercise 10.3.1a) Put the following Boolean expression into 3CNF:

$$xy + \overline{x}z$$

Exercise 5 **Proving $\mathcal{NP}$-Completeness** (A)

(Exercise 10.4.4d) We know that the Node Cover problem is $\mathcal{NP}$-complete. Show that the following Dominating Set problem is $\mathcal{NP}$-complete: Given a graph G and an integer k, does there exist a subset S of at most k nodes of G such that each node is either in S or adjacent to a node of S?

Exercise 6 **Proving $\mathcal{NP}$-Completeness** (S)

(Exercise 10.4.4f) We know that it's $\mathcal{NP}$-complete to determine whether a graph G contains a clique of size k. Show that it's $\mathcal{NP}$-complete to determine whether a graph G contains a clique of size at least $m/2$, where m is the number of nodes of G.