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Tutorial 3

Theory of Computation

Questions marked (S) are self-test questions with solutions provided at

`http://infolab.stanford.edu/~ullman/ialcsols/sols.html`

Questions marked (A) are assignment questions.

Exercise 1 **Use of Pumping Lemma** (S)

(Exercise 4.1.2a) Prove that the following is not a regular language: $\{0^n \mid n \text{ is a perfect square}\}$.

Exercise 2 **Use of Pumping Lemma** (A)

(Exercise 4.1.2e–h) Prove that the following are not regular languages.

1. The set of strings of 0's and 1's that are of the form ww, that is, some string repeated.
2. The set of strings of 0's and 1's that are of the form ww^R, that is, some string followed by its reverse.
3. The set of strings of 0's and 1's of the form $w\overline{w}$, where $\overline{w}$ is formed from w by replacing all 0's by 1's, and vice versa; e.g., $\overline{011} = 100$, and 011100 is an example of a string in the language.
4. The set of strings of the form $w1^n$, where w is a string of 0's and 1's of length n.

Exercise 3 **Closure Properties** (S)

(Exercise 4.2.2) If L is a language, and a is a symbol, then L/a, the *quotient* of L and a, is the set of strings w such that wa is in L. For example, if $L = \{a, aab, baa\}$, then $L/a = \{\epsilon, ba\}$. Prove that if L is regular, so is L/a. ***Hint***: Start with a DFA for L and consider the set of accepting states.

Exercise 4 **Closure Properties** (S)

(Exercise 4.2.8) Let L be a language. Define $half(L)$ to be the set of first halves of strings in L, that is, $\{w \mid \text{ for some } x$ such that $|x| = |w|$, we have wx in $L\}$. For example, if $L = \{\epsilon, 0010, 011, 010110\}$ then $half(L) = \{\epsilon, 00, 010\}$. Notice that odd-length strings do not contribute to $half(L)$. Prove that if L is a regular language, so is $half(L)$.

Exercise 5 **Closure Properties** (A)

(Exercise 4.2.6) Show that the regular languages are closed under the following operations. ***Hint***: Start with a DFA for L and perform a construction to get the desired language.

1. $min(L) = \{w \mid w \in L, \text{ but no proper prefix of } w \in L\}$
2. $max(L) = \{w \mid w \in L, wx \notin L \text{ for any nonempty } x\}$
3. $init(L) = \{w \mid wx \in L \text{ for some } x\}$

Exercise 6 **Decision Properties** (S)

(Exercise 4.3.1) Give an algorithm to tell whether a regular language L is infinite. ***Hint***: Use the pumping lemma to show that if the language contains any string whose length is above a certain lower limit, then the language must be infinite.

Exercise 7 **DFA Minimization** (A)

	0	1
$\rightarrow A$	B	E
B	C	F
$*C$	D	H
D	E	H
E	F	I
$*F$	G	B
G	H	B
H	I	C
$*I$	A	E

(Exercise 4.4.2) The above is the transition table of a DFA (with "*" marking accepting states).

1. Draw the table of distinguishabilities for this automaton.
2. Construct the minimum-state equivalent DFA.