THE UNIVERSITY OF TEXAS
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CS331 ALGORITHM

Assignment 06

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Contents

1 Exercise 2

Let i, j, and k be distinct integers in $[n]$ such that j lies between i and k, i.e., either $i < j < k$ and $k < j < i$. Assume that bids $\alpha(i)$ and $\alpha(j)$ are linear. Prove that if $gap(p, i, j) = 0$ and $g(j, k) = 0$, then $g(i, j) = 0$.

Based on the known condition as follows,

$$gap(p, i, j) = 0 \tag{1}$$
$$gap(p, j, k) = 0 \tag{2}$$

And the assumed condition that

$$i, j, k \in [n] \tag{3}$$
$$\alpha(j) \text{ is linear} \tag{4}$$
$$\alpha(i) \text{ is linear} \tag{5}$$

We have

$$max(0, s(i) \cdot g(i, j) - f(p, i, j)) = 0 \tag{6}$$
$$max(0, s(j) \cdot g(j, k) - f(p, j, k)) = 0 \tag{7}$$

That is,

$$s(i) \cdot g(i, j) - f(p, i, j) \leq 0 \tag{8}$$
$$s(j) \cdot g(j, k) - f(p, j, k) \leq 0 \tag{9}$$

We sum up (8) and (9), then get

$$s(i) \cdot g(i, j) - f(p, i, j) + s(j) \cdot g(j, k) - f(p, j, k) \leq 0 \tag{10}$$

This can be simplified to be

$$s(i) \cdot g(i, k) - f(p, i, k) \leq g(j, k) \cdot (s(i) - s(j)) \tag{11}$$

According to the known condition, we have

$$i < j < k \text{ or } k < j < i \tag{12}$$

In the case of $i < j < k$, we have $g(j, k) = q_{\beta(k)} - q_{\beta(j)} \geq 0$ and $s(i) - s(j) \leq 0$ since $\alpha(i)$ and $\alpha(j)$ are matched to βi and βj in MWMCM M, respectively. Similarly, in the case of $k < j < i$, we have $g(i, j) = q_{\beta(j)} - q_{\beta(i)} \leq 0$ and $s(i) - s(j) \geq 0$. Therefore, it can be concluded that in either case,

$$g(j, k) \cdot (s(i) - s(j)) \leq 0 \tag{13}$$

Hence, we have

$$s(i) \cdot g(i, k) - f(p, i, k) \leq g(j, k) \leq 0 \tag{14}$$

Since $\alpha(i)$ is linear bid and $i \in [n]$

$$gap(p, i, k) = 0 \tag{15}$$

2 Exercise 5