

# Statistical Learning and Data Mining

## CS 363D/ SSC 358

### Lecture: Nearest Neighbor Classifiers

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Adapted From: Pang-Ning Tan, Steinbach, Kumar

# Instance-Based Classifiers

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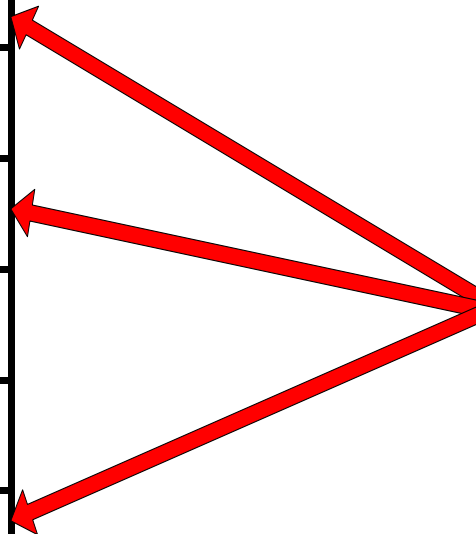
Set of Stored Cases

Atr1	.....	AtrN	Class
			A
			B
			B
			C
			A
			C
			B

- Store the training records
- Use training records to predict the class label of unseen cases

Unseen Case

Atr1	.....	AtrN



# Instance-Based Classifiers

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- Examples:
  - Rote-learner
    - ◆ Memorizes entire training data and performs classification only if attributes of record match one of the training examples exactly

# Instance-Based Classifiers

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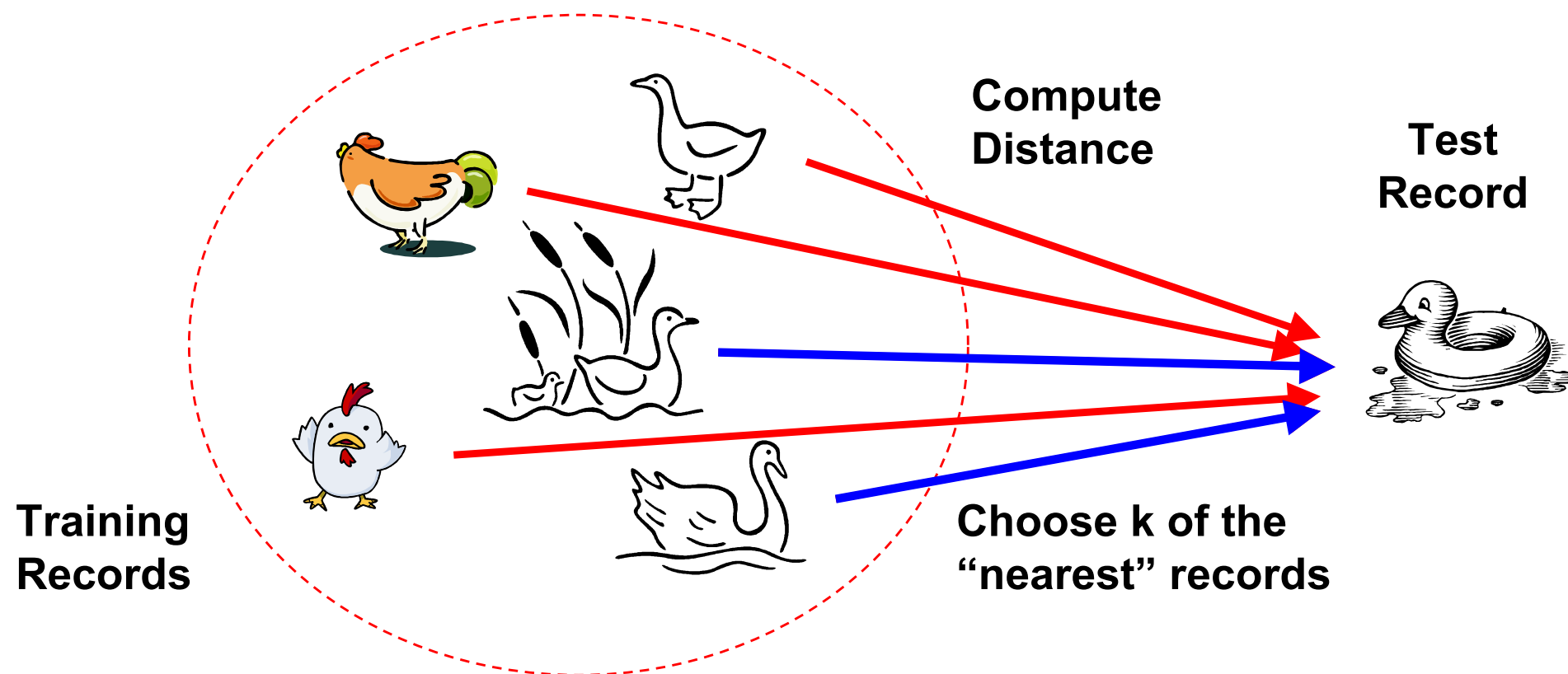
- Examples:
  - Rote-learner
    - ◆ Memorizes entire training data and performs classification only if attributes of record match one of the training examples exactly
  - Nearest neighbor
    - ◆ Uses  $k$  “closest” points (nearest neighbors) for performing classification

# Nearest-Neighbor Classifiers

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- Basic idea:

- If it walks like a duck, quacks like a duck, then it's probably a duck



# Nearest-Neighbor Classifiers

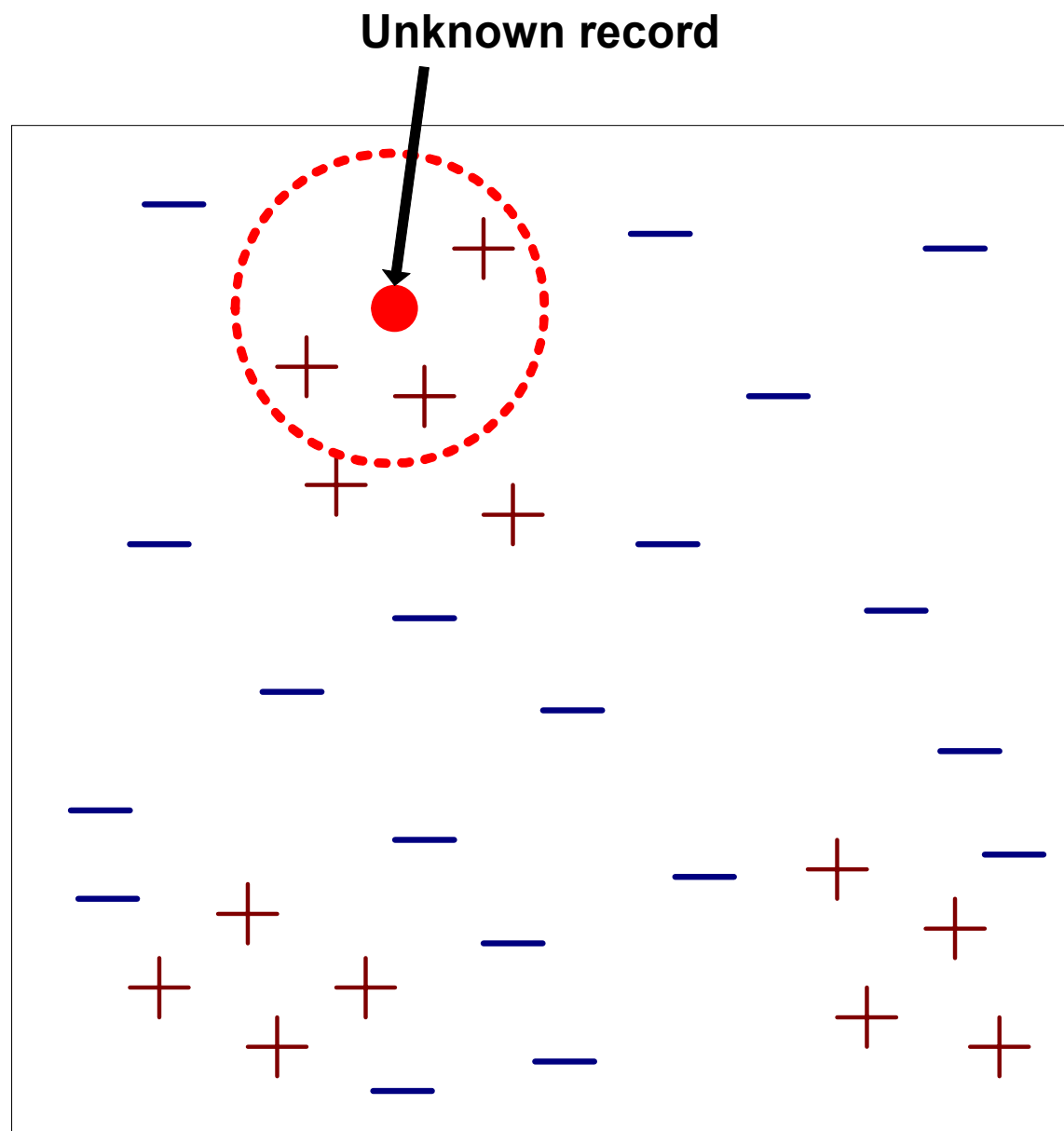
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- Basic idea:



# Nearest-Neighbor Classifiers

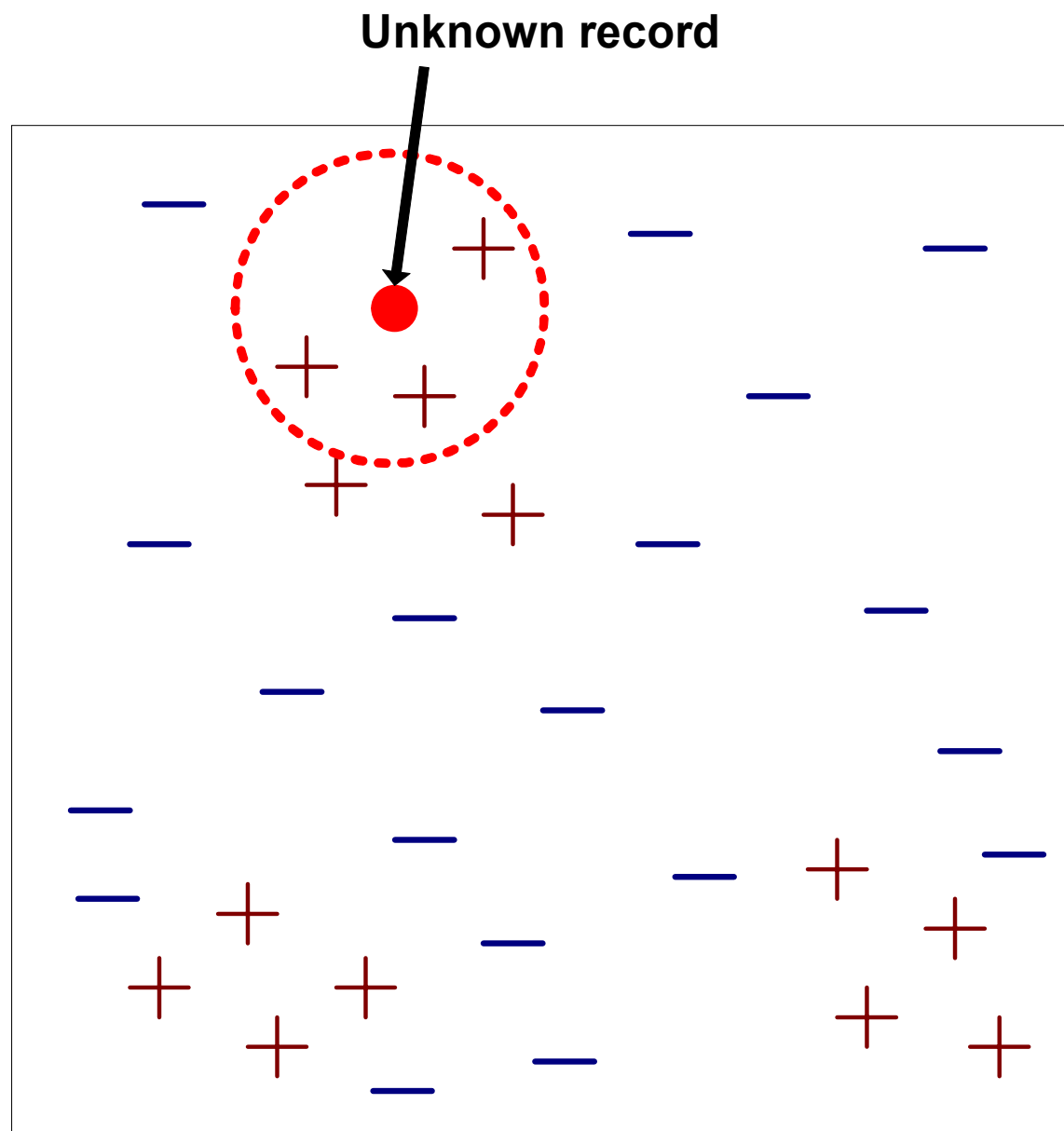
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- Requires three things
  - The set of stored records
  - Distance Metric to compute distance between records
  - The value of  $k$ , the number of nearest neighbors to retrieve

# Nearest-Neighbor Classifiers

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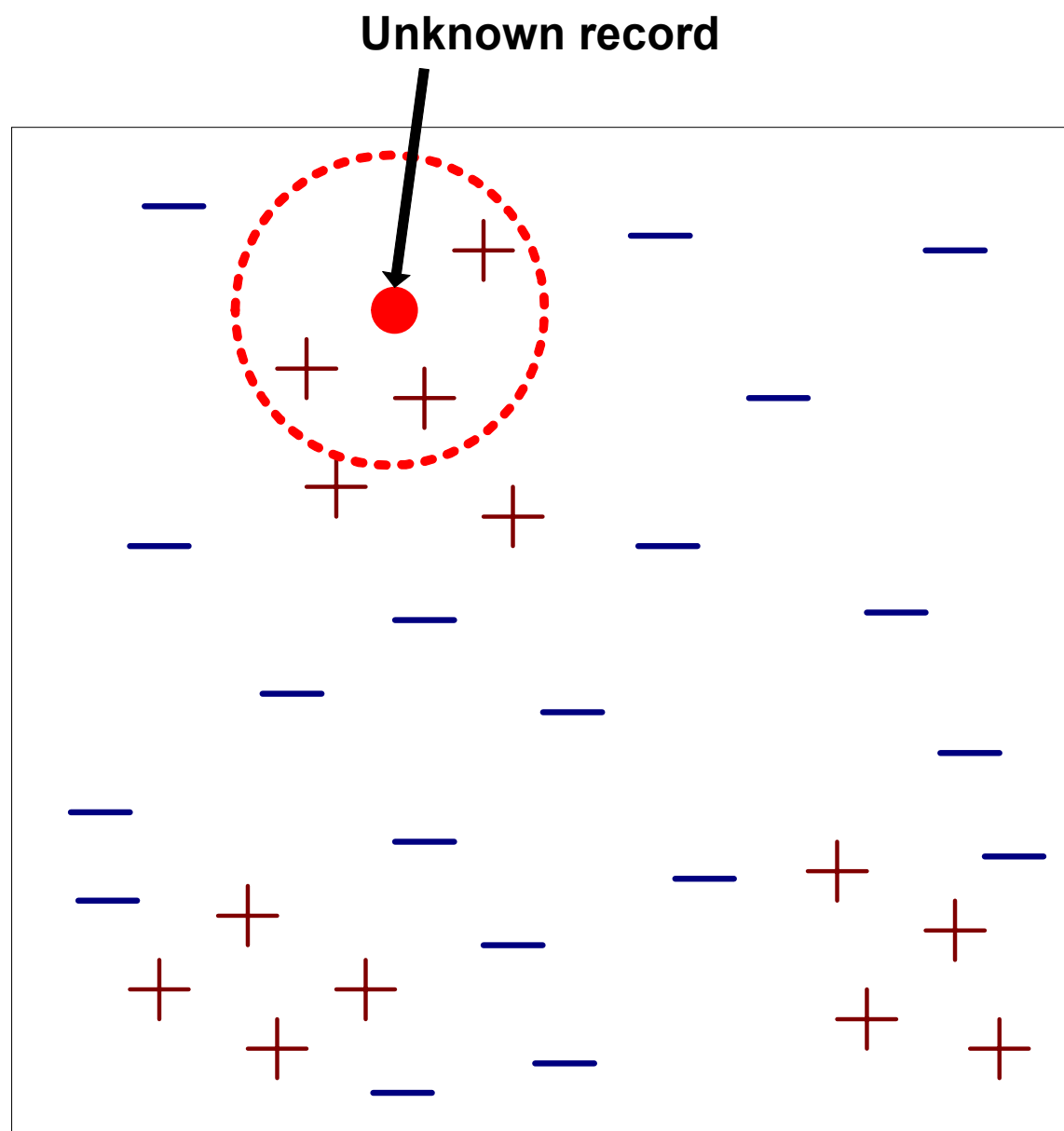


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- To classify an unknown record:



# Nearest-Neighbor Classifiers

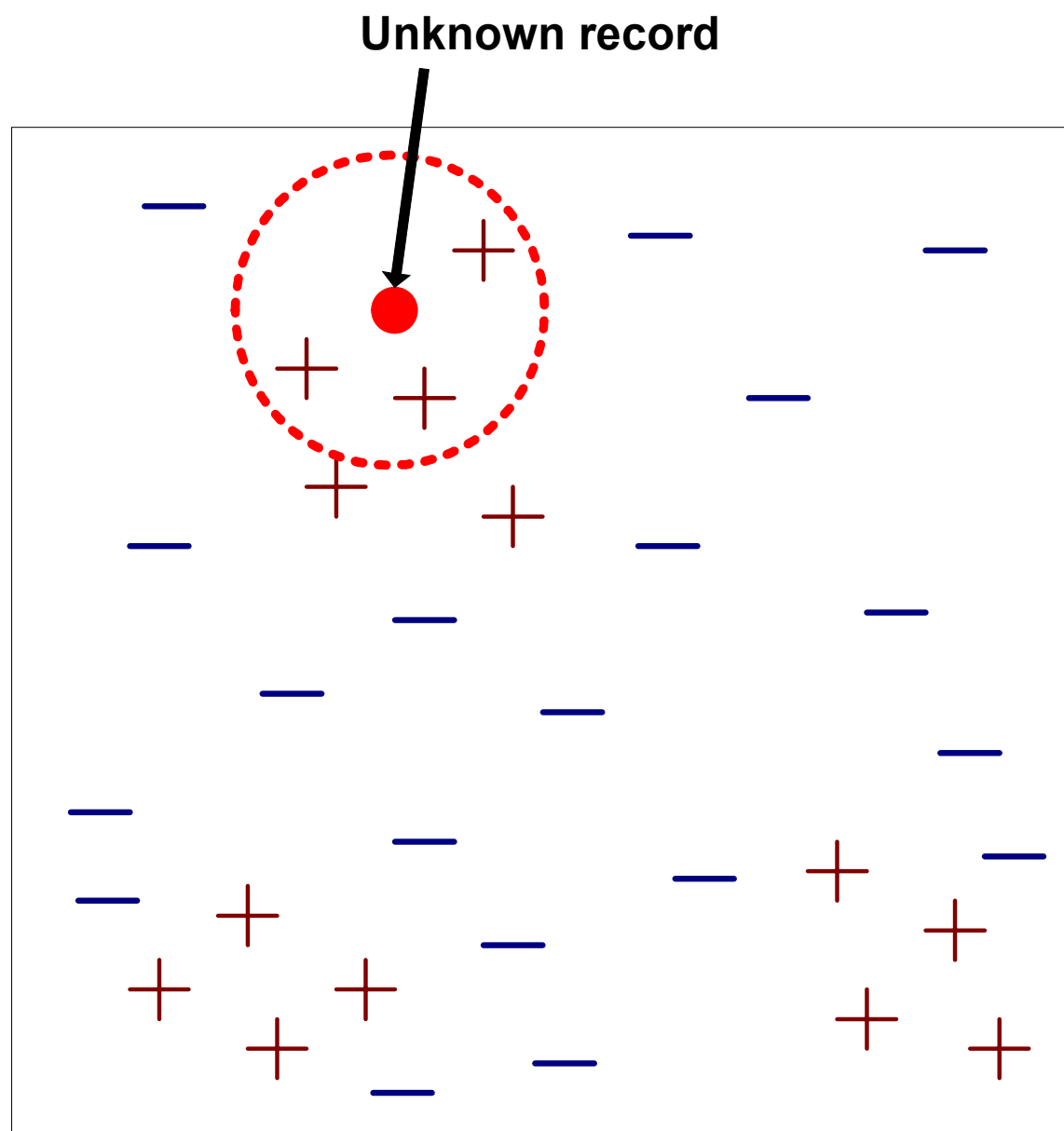
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- Requires three things
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- To classify an unknown record:
  - Compute distance to other training records

# Nearest-Neighbor Classifiers

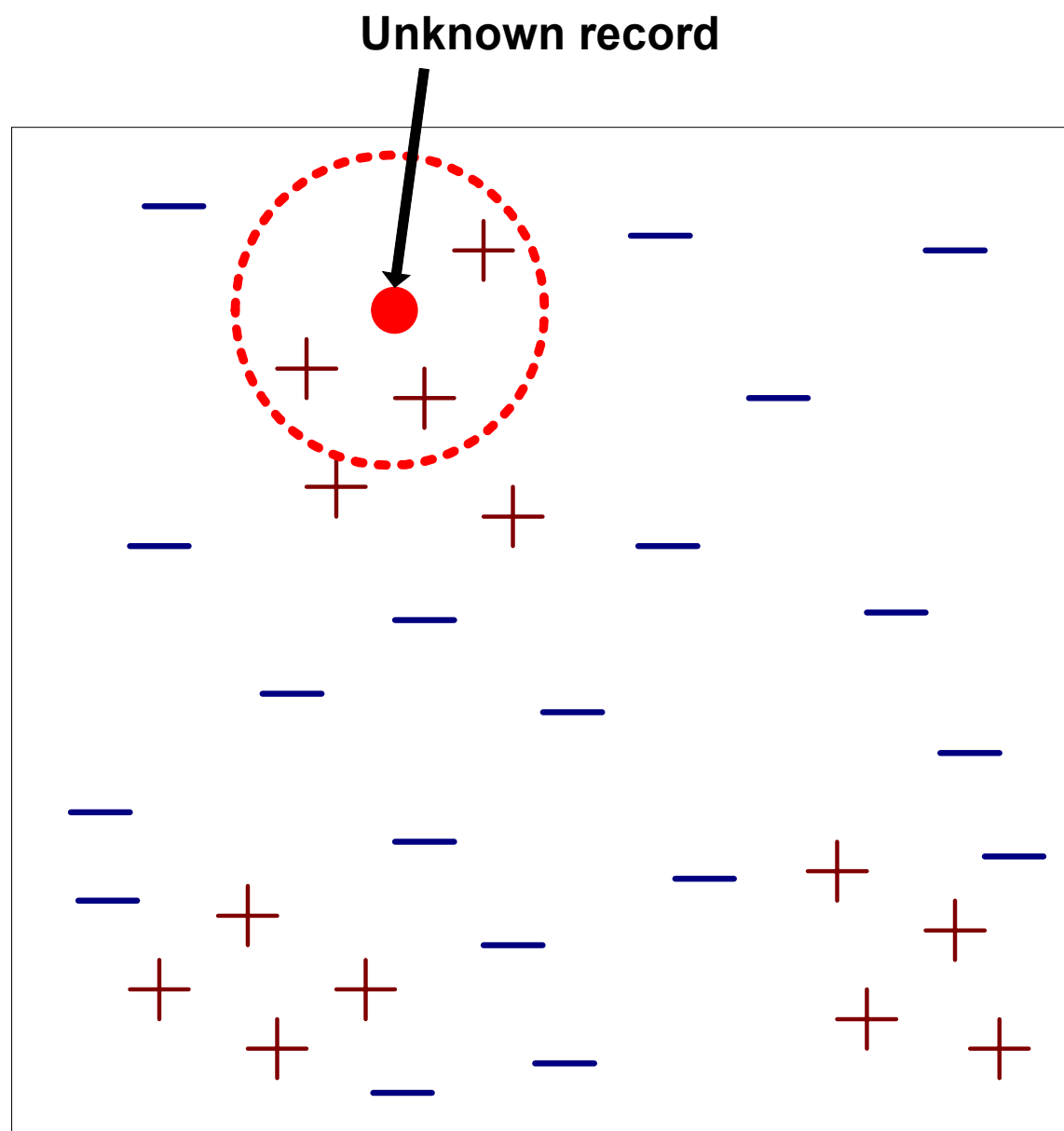
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- To classify an unknown record:
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  - Identify  $k$  nearest neighbors

# Nearest-Neighbor Classifiers

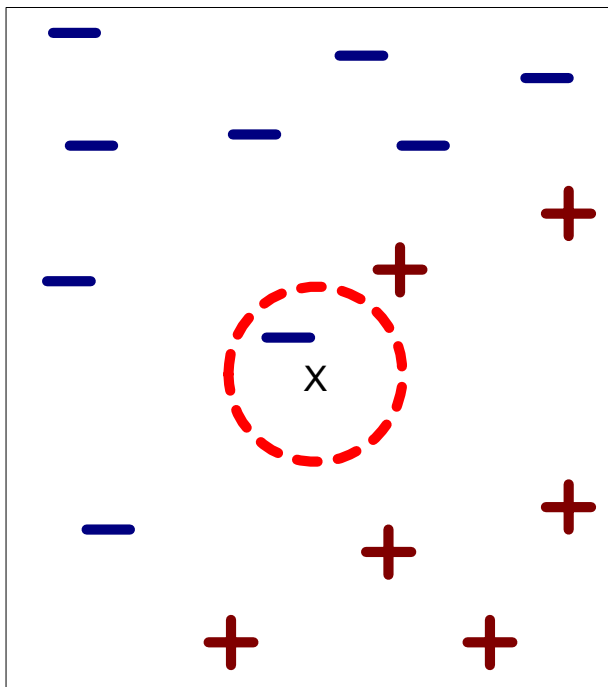
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- Requires three things
  - The set of stored records
  - Distance Metric to compute distance between records
  - The value of  $k$ , the number of nearest neighbors to retrieve
- To classify an unknown record:
  - Compute distance to other training records
  - Identify  $k$  nearest neighbors
  - Use class labels of nearest neighbors to determine the class label of unknown record (e.g., by taking majority vote)

# Definition of Nearest Neighbor

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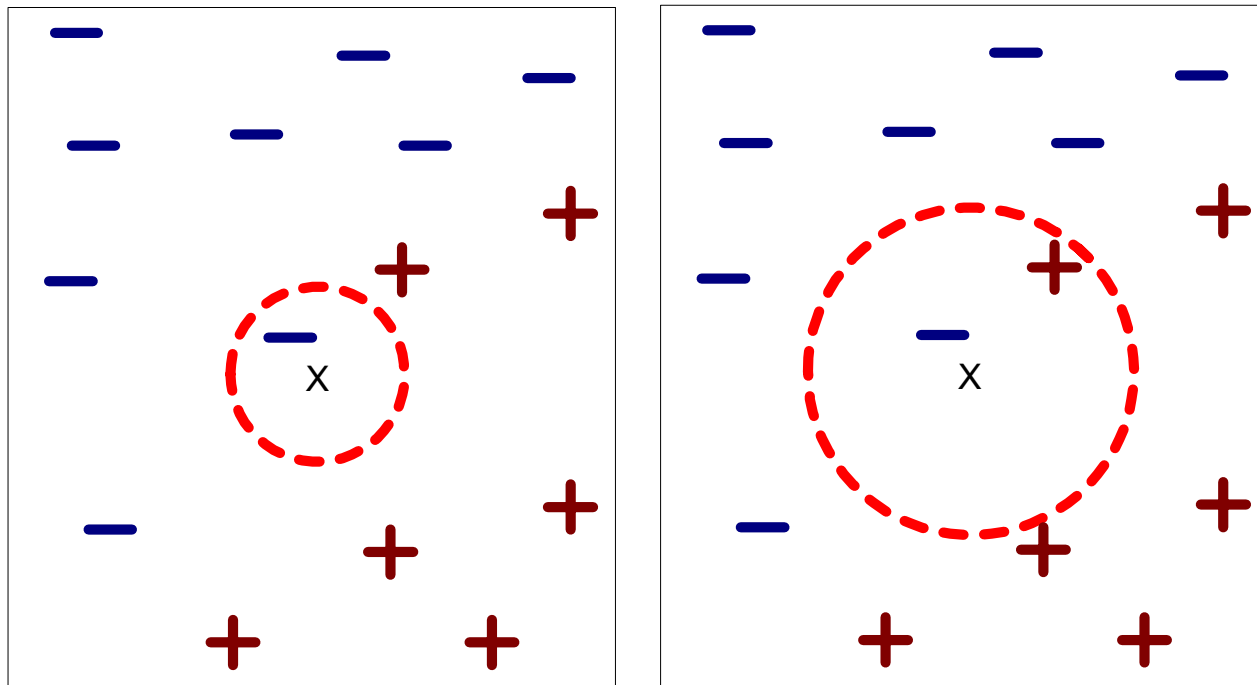


(a) 1-nearest neighbor

K-nearest neighbors of a record  $x$  are data points that have the  $k$  smallest distance to  $x$

# Definition of Nearest Neighbor

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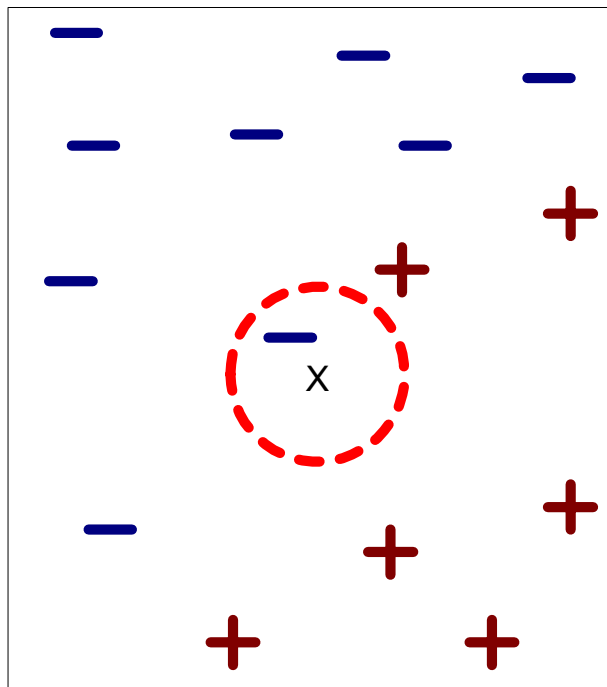
(a) 1-nearest neighbor

(b) 2-nearest neighbor

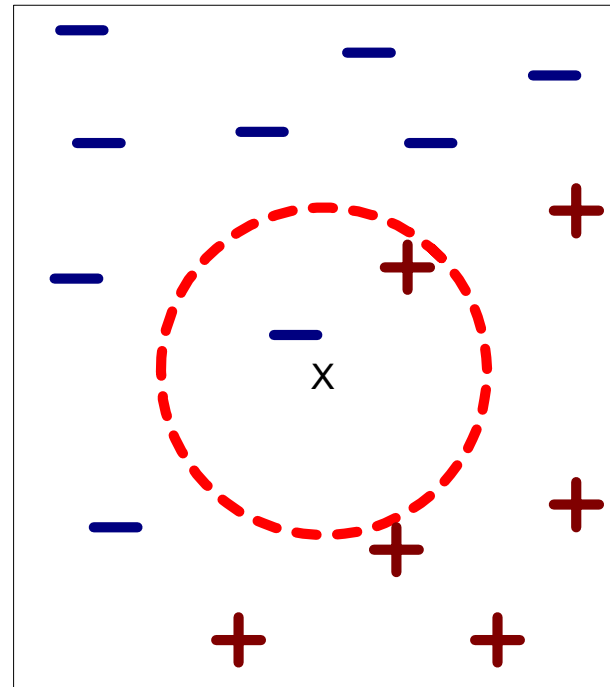
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# Definition of Nearest Neighbor

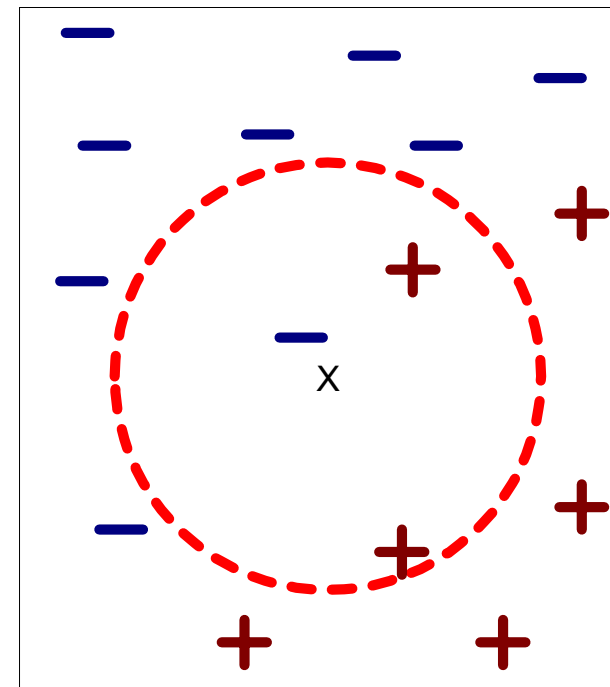
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(a) 1-nearest neighbor



(b) 2-nearest neighbor



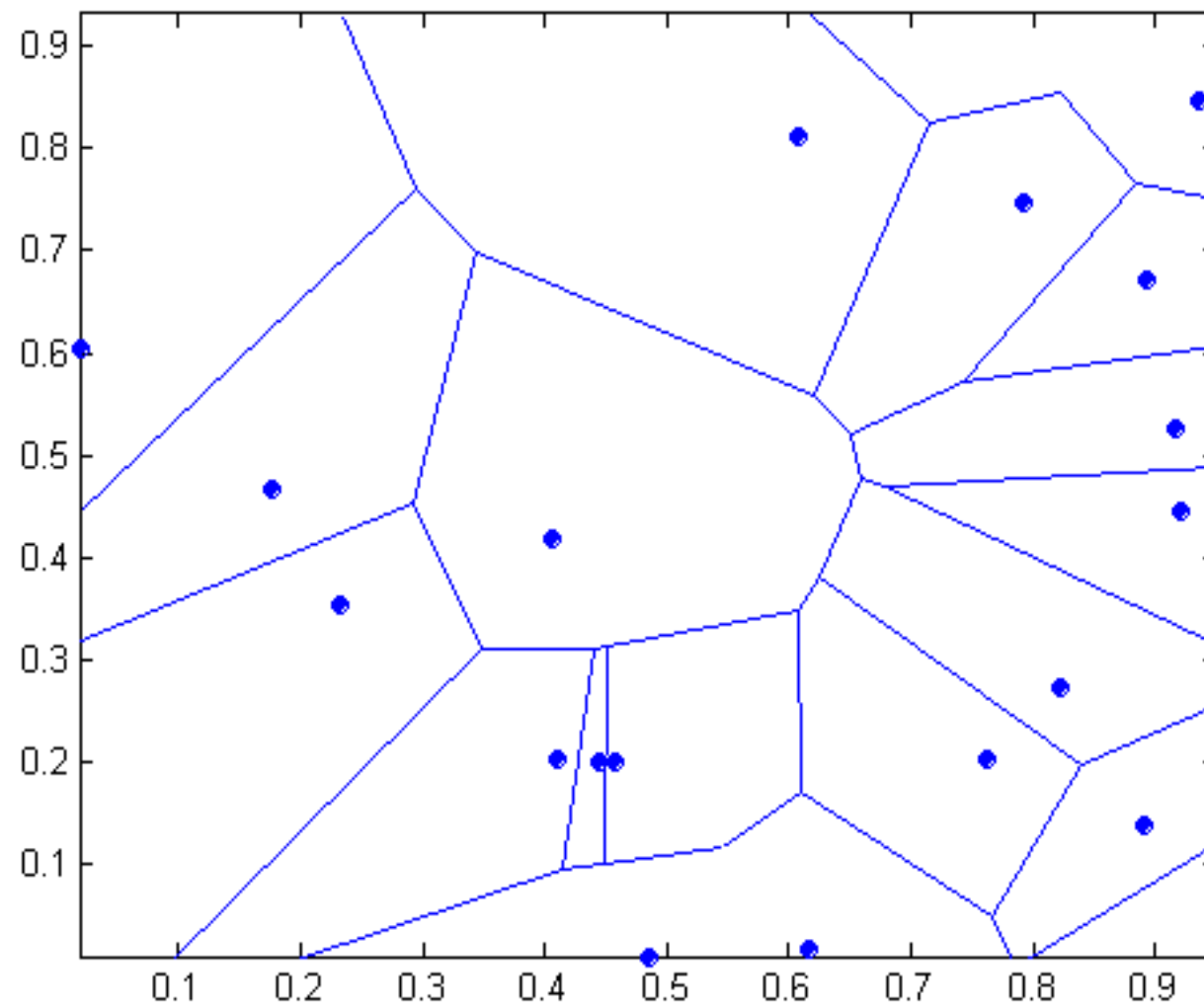
(c) 3-nearest neighbor

K-nearest neighbors of a record  $x$  are data points that have the  $k$  smallest distance to  $x$

# 1 Nearest-Neighbor

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## Voronoi Diagram



# Nearest Neighbor Classification

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- Compute distance between two points:
  - Euclidean distance

$$d(p, q) = \sqrt{\sum_i (p_i - q_i)^2}$$

- Determine the class from nearest neighbor list



# Nearest Neighbor Classification

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  - take the majority vote of class labels among the k-nearest neighbors

# Nearest Neighbor Classification

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- Compute distance between two points:

- Euclidean distance

$$d(p, q) = \sqrt{\sum_i (p_i - q_i)^2}$$

- Determine the class from nearest neighbor list
  - take the majority vote of class labels among the k-nearest neighbors
  - Weigh the vote according to distance
    - ◆ weight factor,  $w = 1/d^2$

# Nearest Neighbor Classification

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- Choosing the value of  $k$ :
  - If  $k$  is too small,

# Nearest Neighbor Classification

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- Choosing the value of  $k$ :
  - If  $k$  is too small, sensitive to noise points

# Nearest Neighbor Classification

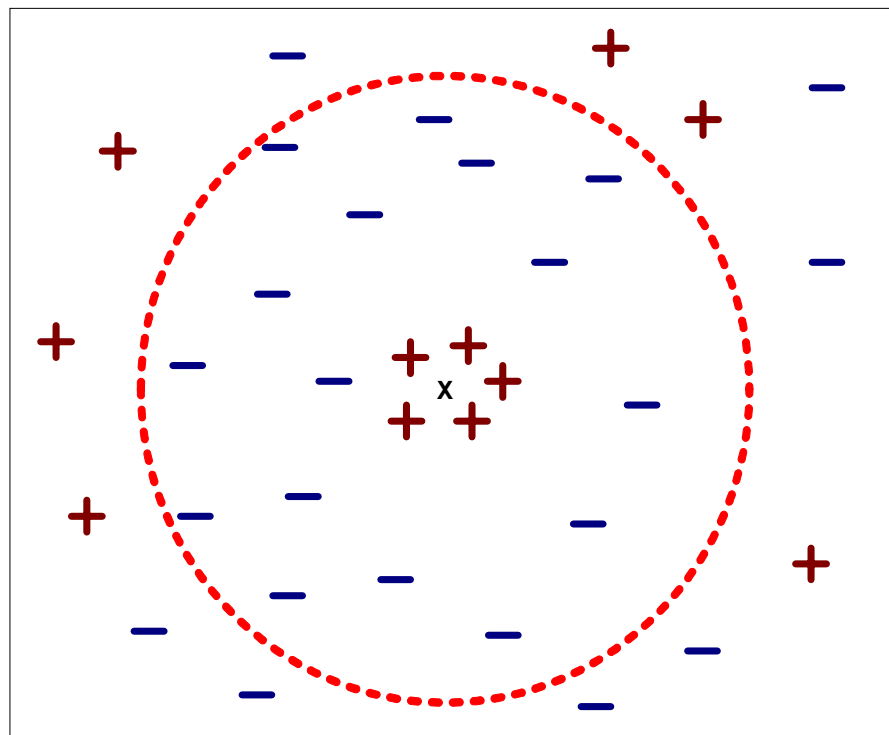
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- Choosing the value of  $k$ :
  - If  $k$  is too small, sensitive to noise points
  - If  $k$  is too large,

# Nearest Neighbor Classification

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- Choosing the value of  $k$ :
  - If  $k$  is too small, sensitive to noise points
  - If  $k$  is too large, neighborhood may include points from other classes



# Nearest Neighbor Classification

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- Scaling issues

- Attributes may have to be scaled to prevent distance measures from being dominated by one of the attributes
- Example:
  - ◆ height of a person may vary from 1.5m to 1.8m
  - ◆ weight of a person may vary from 90lb to 300lb
  - ◆ income of a person may vary from \$10K to \$1M

# Nearest Neighbor Classification

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- Problem with Euclidean measure:
  - High dimensional data
    - ◆ curse of dimensionality



# Nearest Neighbor Classification

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- Problem with Euclidean measure:
  - High dimensional data
    - ◆ **curse of dimensionality**
  - Can produce counter-intuitive results

1 1 1 1 1 1 1 1 1 1 1 0	vs	1 0 0 0 0 0 0 0 0 0 0 0
0 1 1 1 1 1 1 1 1 1 1 1		0 0 0 0 0 0 0 0 0 0 0 1
$d = 1.4142$		$d = 1.4142$

# Nearest Neighbor Classification

---

- Problem with Euclidean measure:
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0 1 1 1 1 1 1 1 1 1 1 1		0 0 0 0 0 0 0 0 0 0 0 1
$d = 1.4142$		$d = 1.4142$

- ◆ Solution: Normalize the vectors to unit length

# Nearest Neighbor Classification

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- k-NN classifiers are lazy learners
  - It does not build models explicitly
  - Unlike eager learners such as decision tree induction and rule-based systems
  - Classifying unknown records are relatively expensive

# Ensemble Methods

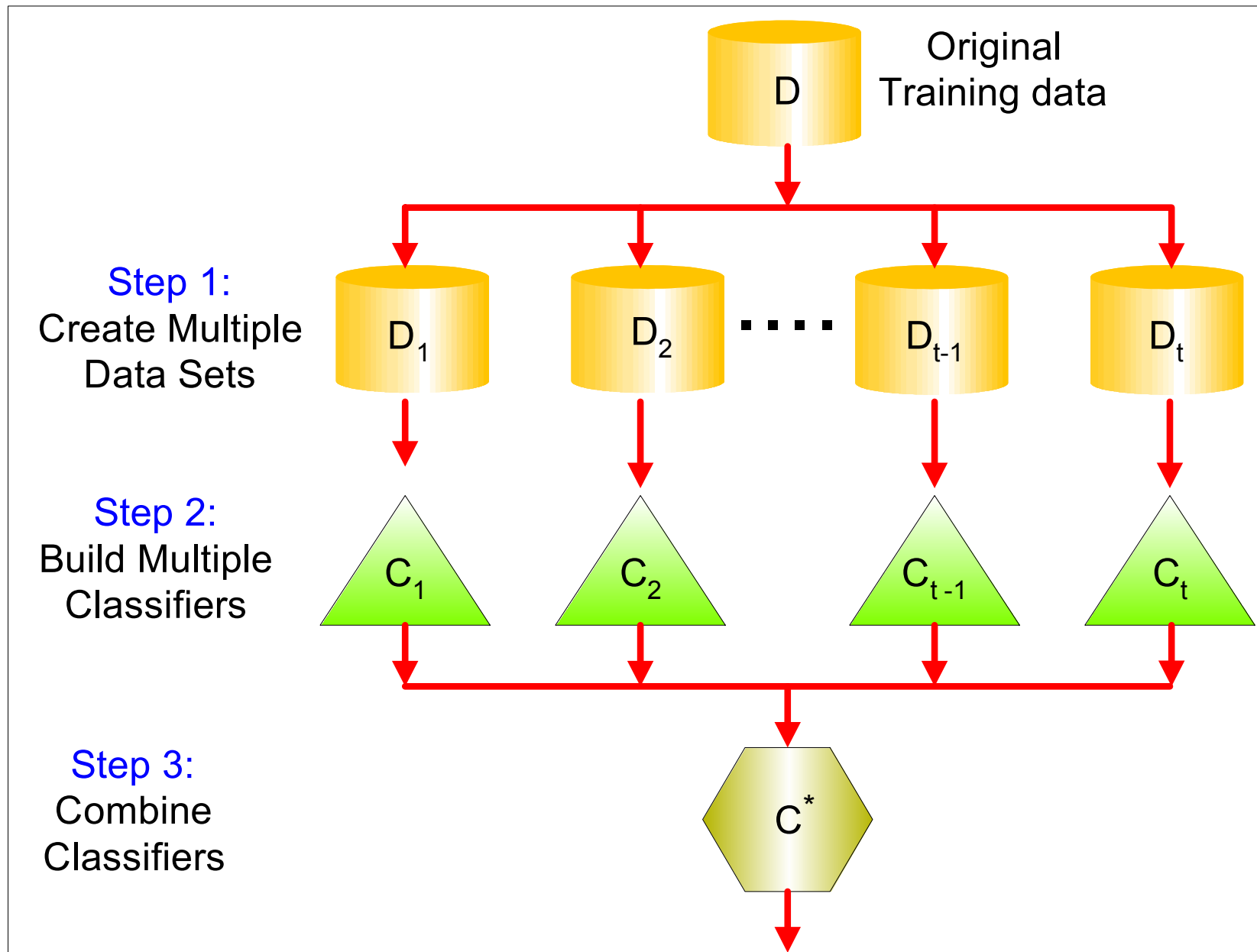
# Ensemble Methods

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- Construct a set of classifiers from the training data
- Predict class label of previously unseen records by aggregating predictions made by multiple classifiers

# General Idea

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# Why does it work?

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- Suppose there are 25 base classifiers
  - Each classifier has error rate,  $\varepsilon = 0.35$
  - Assume classifiers are independent
  - Probability that the ensemble classifier makes a wrong prediction:

$$\sum_{i=13}^{25} \binom{25}{i} \varepsilon^i (1 - \varepsilon)^{25-i} = 0.06$$

# Many Approaches to Step 1 (Creating Multiple Datasets)

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- Copy the dataset multiple times
- Partitioning the dataset
- Bagging
- Boosting



# Bagging

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- Sampling with replacement

Original Data	1	2	3	4	5	6	7	8	9	10
Bagging (Round 1)	7	8	10	8	2	5	10	10	5	9
Bagging (Round 2)	1	4	9	1	2	3	2	7	3	2
Bagging (Round 3)	1	8	5	10	5	5	9	6	3	7

- Build classifier on each bootstrap sample