THE UNIVERSITY OF TEXAS
AT AUSTIN

EE381V LARGE SCALE OPTIMIZATION

Problem Set 7

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1.1 Petersen Graph

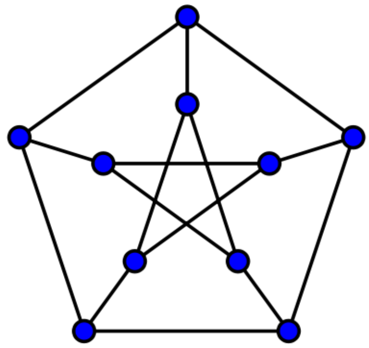

Figure 1: Petersen Graph

1.2 Planar Graph I

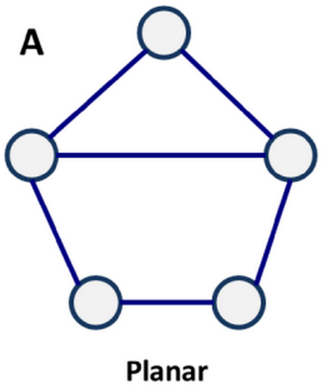

Figure 2: Planar Graph I

1.3 Planar Graph II

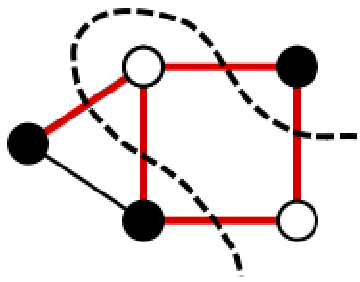

Figure 3: Planar Graph II

Part II
Written Assignment

1 Network Congestion Control

1.1 Problem Formulation

The overall system problem – to maximize utility minus cost – can be formulated as a convex optimization problem:

$$\begin{array}{ll} \text{maximize} & \sum_{s\in S} U_s(x_s) - \sum_{j\in J} C_j(f_j) \\ \text{subject to} & Hy = x, Ay \le f \\ \text{over} & x, y \ge 0 \end{array} \tag{1}$$

1.2 Problem Decoupling

Lagrangian:

$$\begin{aligned} L(x,y;\lambda,\mu) &= \sum_{s\in S} U_s(x_s) - \sum_{j\in J} C_j(f_j) - \lambda^T(x - Hy) + \mu^T(f - Ay - z) && (2)\\ &= \sum_{s\in S}\big(U_s(x_s) - \lambda_s x_s\big) - \sum_{r\in R} y_r\big(\lambda_{s(r)} - \sum_{j\in r}\mu_j\big) + \sum_{j\in J}\mu_j(f_j - z_j) - \sum_{j\in J} C_j(f_j) && (3) \end{aligned}$$

where λ and μ are lagrange multipliers.
According to optimality conditions

$$\frac{\partial L}{\partial x_s} = U_s'(x_s) - \lambda_s \tag{4}$$

$$\frac{\partial L}{\partial y_r} = \lambda_{s(r)} - \sum_{j\in r}\mu_j \tag{5}$$

$$\frac{\partial L}{\partial z_j} = -\mu_j \tag{6}$$

$$\lambda \ge U_s'(x_s), Hy = x, (\lambda - U'(x))^x = 0 \tag{7}$$

$$\mu \ge 0, Ax \le C, \mu^T(C - Ax) = 0 \tag{8}$$

$$\lambda^T H \le \mu^T A, y \ge 0, (\mu^T A - \lambda^T H)y = 0 \tag{9}$$

$USER_s(U_s;\lambda_s)$

$$\begin{array}{ll} \text{maximize} & \sum_{s\in S} U_s(x_s) - \lambda_s x_s \\ \text{subject to} & x_s \ge 0 \end{array} \tag{10}$$

$NETWORK(H,F;\lambda)$

$$\begin{array}{ll} \text{maximize} & \sum_{s\in S} \lambda_s x_s - \sum_{j\in J} C_j(f_j) \\ \text{subject to} & Hy = x, Ay \le f \\ \text{over} & x, y \ge 0 \end{array} \tag{11}$$

Theorem 1. *There exists a price vector $\lambda = (\lambda_s, s \in S)$ such that the vector $x = (x_s, s \in S)$, formed from the unique solution x_s to $USER_s(U_s;\lambda_s)$ for each $s \in S$, solves $NETWORK(H,A,C;\lambda)$. The vector x then also solves $SYSTEM(U,H,A,f)$.*

Proof. First note that $USER_s(U_s;\lambda_s)$ has unique solution for each s. Then we observe that the lagrangian form for $NETWORK(H,F;\lambda)$ is

$$L(x,y;\lambda,\mu) = \sum_{s\in S} U_s(x_s) - \sum_{j\in J} C_j(f_j) - p^T(x - Hy) + q^T(f - Ay - z) \tag{12}$$

$$= \sum_{s\in S} \big(U_s(x_s) - p_s x_s\big) - \sum_{r\in R} y_r\big(p_{s(r)} - \sum_{j\in r} q_j\big) + \sum_{j\in J} q_j(f_j - z_j) - \sum_{j\in J} C_j(f_j) \tag{13}$$

Hence, any quadruple (λ,μ,x,y), which satisfies optimality of **??**-**??** (solution of $SYSTEM$) identifies $p=\lambda$ and $q=\mu$, which establish that (x,y) solves $NETWORK(H,F;\lambda)$.
Conversely, for any solution x to $NETWORK(H,F;\lambda)$, then exists a p and q, where $x_s \geq 0$ then $p_s = \lambda_s$ and if $x_s = 0$, then $p_s \geq \lambda_s$. Thus if x_s solves $USER_s(U_s;\lambda_s)$, then it also solves $USER_s(U_s;p_s)$. Based on p and q, we can then construct a quadruple that satisfies optimality of **??**-**??**. This quadruple gives x that solves $SYSTEM(U,H,A,f)$. □

2 Problem 7.12

3 Problem 7.13

A Codes Printout

A.1 SDP-relaxation for MaxCut