The Austalian National University
Research School of Computer Science
Dirk Pattinson
Jinbo Huang

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Assignment 2

Theory of Computation

Release Date: Monday 15 April 2013

Due Date: Monday 29 April 2013

Submission: Hand in to Jinbo Huang in class.

Note: Hand written answers are acceptable if written neatly. Correct answers may be given less than full credit if unnecessarily complicated.

Exercise 1 **Proof by Induction** (A)

(Exercise 5.1.7) Consider the CFG G defined by productions:

$$S \to aS \,|\, Sb \,|\, a \,|\, b$$

1. Prove by induction on the string length that no string in $L(G)$ has ba as a substring.
2. Describe $L(G)$ informally. Justify your answer using part (1).

Exercise 2 **Unambiguous Grammars** (A)

(Exercise 5.4.7) The following grammar generates *prefix* expressions with operands x and y and binary operators $+, -$, and $*$:

$$E \to +EE \,|\, *EE \,|\, -EE \,|\, x \,|\, y$$

1. Give leftmost and rightmost derivations, and a parse tree for the string +*-xyxy.
2. Prove that this grammar is unambiguous.

Exercise 3 **Acceptance by Final State vs. Empty Stack** (A)

(Exercise 6.2.6) Consider the PDA P from Exercise 1.

a) Convert P to another PDA P_1 that accepts by empty stack the same language that P accepts by final state; i.e., $N(P_1) = L(P)$.

b) Find a PDA P_2 such that $L(P_2) = N(P)$; i.e., P_2 accepts by final state what P accepts by empty stack.

Exercise 4 **From Grammar to PDA** (A)

(Exercise 6.3.2 extended) Convert the grammar

$$S \to aAA$$
$$A \to aS \,|\, bS \,|\, a$$

to a PDA that accepts the same language by empty stack, and show an accepting sequence of IDs for input string $aabaaa$.

Exercise 5 **Use of CFL Pumping Lemma** (A)

Use the CFL pumping lemma to show the following language not to be context-free: $\{a^i b^j c^k \,|\, i \times j = k\}$.

Exercise 6 **Closures Properties of CFLs** (A)

(Exercise 7.3.3b) Show that the CFLs are *not* closed under the following operation:

$max(L) = \{w \,|\, w$ is in L and for no x other than ϵ is wx in $L\}$.