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A network analysis of the volatility of high dimensional financial series

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Summary. Interconnectedness between stocks and firms plays a crucial role in the volatility contagion phenomena that characterize financial crises, and graphs are a natural tool in their analysis. We propose graphical methods for an analysis of volatility interconnections in the Standard & Poor's 100 data set during the period 2000–2013, which contains the 2007–2008 Great Financial Crisis. The challenges are twofold: first, volatilities are not directly observed and must be extracted from time series of stock returns; second, the observed series, with about 100 stocks, is high dimensional, and curse-of-dimensionality problems are to be faced. To overcome this double challenge, we propose a dynamic factor model methodology, decomposing the panel into a factor-driven and an idiosyncratic component modelled as a sparse vector auto-regressive model. The inversion of this auto-regression, along with suitable identification constraints, produces networks in which, for a given horizon h, the weight associated with edge (i, j) represents the h-step-ahead forecast error variance of variable i accounted for by variable j's innovations. Then, we show how those graphs yield an assessment of how *systemic* each firm is. They also demonstrate the prominent role of financial firms as sources of contagion during the 2007–2008 crisis.

Keywords: Dynamic factor models; Sparse vector auto-regression models; Standard & Poor's 100 index; Systemic risk; Volatility

1. Introduction

The study of networks as complex systems has been the subject of intensive research in recent years, in both the physics and the statistics communities (see, for example, Kolaczyk (2009) for a review of the main models, methods and results). Typically, the data sets that have been considered in that literature exhibit a 'natural' or prespecified network structure, such as world trade fluxes (Serrano and Boguñá, 2003; Barigozzi *et al*., 2010), co-authorship relations (Newman, 2001), power grids (Watts and Strogatz, 1998), social individual relationships (Zachary, 1977), fluxes of migrants (Fagiolo and Santoni, 2015) or political weblog data (Adamic and Glance, 2005). In all those studies, the network structure (as a collection of vertices and edges) is known and pre-exists the observations. More recently, in the aftermath of the Great Financial Crisis of 2007–2008, networks also have become a popular tool in financial econometrics and, more particularly, in the study of the interconnectedness of financial markets (Diebold and Yilmaz, 2014). In this case, however, the data under study—usually time series of stock returns

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and volatilities—do not have any particular prespecified network structure, and the graphical structure (i.e. the collection of edges) of interest must be recovered or estimated from the data.

In this paper, we focus on one particular network structure: the long-run variance decomposition network (LVDN). Following Diebold and Yilmaz (2014), the LVDN, jointly with appropriate identification assumptions, defines, for a given horizon h, a weighted and directed graph where the weight that is associated with edge (i, j) represents the proportion of h-step-ahead forecast error variance of variable i which is accounted for by the innovations in variable j. Therefore, by definition, LVDNs are completely characterized by the infinite vector moving average (VMA) representation given by Wold's classical representation theorem.

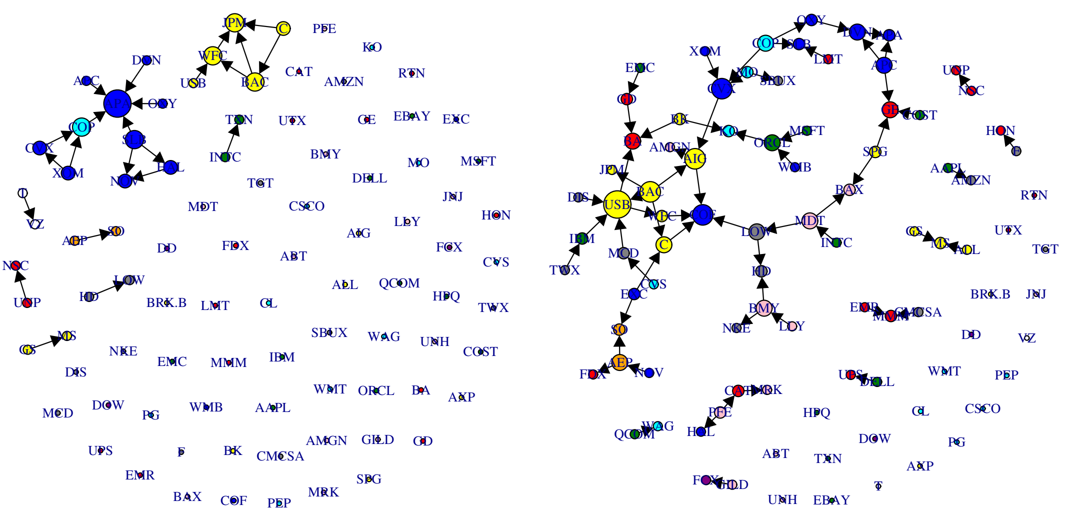
Classical network-related quantities such as *in*- and *out-node strength* or *centrality*, computed for a given LVDN, then admit an immediate economic interpretation in indicating, for instance, which are the stocks or the firms that are most affected by a global extreme event, and which are the stocks or the firms that, when hit by some extreme shock, are likely to transmit it and to spread it over to the whole market. The weights that are attached to each edge provide a quantitative assessment of the risks that are attached to such events. That type of network-based analysis is of particular relevance in financial econometrics—see, among others, Billio *et al*. (2012), Acemoglu *et al*. (2015), Hautsch *et al*. (2014, 2015) or Diebold and Yilmaz (2014).

Throughout, we are concentrating on the analysis of the LVDN that is associated with a panel of daily volatilities of the stocks constituting the Standard & Poor's 100 index S&P100. The period observed is from January 3rd, 2000, to September 30th, 2013. The stocks that are considered belong to 10 different sectors: consumer discretionary, consumer staples, energy, financial, healthcare, industrials, information technology, materials, telecommunication services and utilities.

Our main findings are illustrated in Fig. 1, which shows, for our data set, the estimated LVDNs associated

(a) with the period 2000–2013 and
(b) with the years 2007–2008, which witnessed the so-called Great Financial Crisis.

Inspection of these graphs reveals a main role of the energy (blue nodes) and financial (yellow nodes) sectors. The interconnections within and between those two sectors had a prominent role



(a) (b)

Fig. 1. Graphs of estimated LVDN for the S&P100 idiosyncratic volatilities: (a) 2000–2013; (b) 2007–2008

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in the period considered, due to the high energy prices in the years 2005–2007 and the Great Financial Crisis of the years 2007–2008. In particular, when focusing only on the 2007–2008 period, the financial stocks appear to be the most central (in the sense of the eigenvector centrality concept of Bonacich and Lloyd (2001)), and the connectedness of all network structures that are considered increases quite sizably, making the whole system considerably more prone to contagion. This increased connectedness is an unsurprising phenomenon, since volatility measures *fear* or *lack of confidence* of investors, which tends to spread during periods of high uncertainty.

The LVDNs in Fig. 1 are estimated in two steps. First, we obtain what we call the *idiosyncratic components* of volatilities by removing from the data the pervasive influence of global volatility shocks, which we refer to as *common shocks* or, given the present financial context, as *market shocks*. This is done by applying the general dynamic factor methodology that has recently been proposed by Forni *et al.* (2015a, b) and adapted by Barigozzi and Hallin (2016) to a study of volatilities. More precisely, the factor model structure that we are considering here is the generalized dynamic factor model (GDFM) which was originally proposed by Forni *et al.* (2000) and Forni and Lippi (2001). In a second step, the LVDN is obtained by estimating and inverting a sparse vector auto-regression (VAR) for the resulting idiosyncratic components, together with suitable identifying constraints. In particular, we consider VAR estimation based on three methods: the elastic net (Zou and Hastie, 2005), group lasso (Yuan and Lin, 2006) and adaptive lasso (Zou, 2006). We call this estimation approach the 'factor plus sparse VAR' approach.

This paper gives two main contributions to the existing financial literature on networks. First, we show that a combination of dynamic factor analysis and penalized regressions provides an ideal tool in the analysis of volatility and interconnectedness in large panels of financial time series. Second, we generalize to the high dimensional setting the LVDN estimation that was originally proposed by Diebold and Yilmaz (2014) for a small number of series.

There are strong reasons in favour of our approach controlling for market volatility shocks—as opposed to a direct sparse VAR analysis that does not control for those shocks. The main motivation is of an economic nature; but forecasting and empirical motivations are important as well.

1.1. Economic motivation

In the financial context, the pertinence of factor models is a direct consequence of arbitrage pricing theory and the related capital asset pricing model (Ross, 1976; Fama and French, 1993). These models allow us to disentangle and identify the main sources of variation driving large panels of financial time series:

(a) a strongly pervasive component that is typically driven by a few common shocks, or factors, affecting the whole market and
(b) an idiosyncratic weakly connected component that is driven by local, or sectoral, shocks.

In agreement with arbitrage pricing theory, the factor-driven, or common, component represents the *non-diversifiable* or *systematic* part of risk, whereas the idiosyncratic component becomes perfectly *diversifiable* as the dimension of the system grows (Chamberlain and Rothschild, 1983). Many studies, however, provide evidence that connectivity in the idiosyncratic component, although milder than in the common component, still may be quite non-negligible, even in large dimensional systems (see for example the empirical and theoretical results of Jovanovic (1987), Gabaix (2011) and Acemoglu *et al.* (2012)). This is bound to happen in highly interconnected systems like financial markets. Therefore, when an exceptionally large shock, such as bankruptcy,

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affects the idiosyncratic component of a particular stock, that shock, although idiosyncratic, subsequently is likely to spread, and hit, eventually, all idiosyncratic components across the system. Such events are called *systemic*, and diversification strategies against them might be ineffective. Studying the LVDN that is related to the idiosyncratic volatilities, and hence after controlling for market shocks, helps to identify the systemic elements of a panel of times series and provides the basis for an analysis of contagion mechanisms.

1.2. Forecasting motivation

It has been shown (see for example De Mol *et al.* (2008)) that forecasts that are obtained via penalized regression are highly unstable in the presence of collinearity. Thus, even though forecasting is not the main goal of this paper, removing the effect of common shocks before turning to sparse VAR estimation methods seems highly advisable.

1.3. Empirical motivation

When considering partial dependences, measured by partial spectral coherence, in the idiosyncratic component, i.e. after common components have been removed, hidden dependences between and within the financial and energy sectors are uncovered. This finding, which is documented in Section 5, is the empirical justification for preferring a factor plus sparse VAR approach rather than a direct application of sparse VAR methods.

In Section 2, we introduce the GDFM for large panels of time series and the definition of an LVDN for the idiosyncratic components, allowing for a study of different sources of interdependences. In Section 3 we discuss estimation. In Section 4, following Barigozzi and Hallin (2016), we show how to extract from financial returns those volatility proxies which will be the object of our analysis. Section 5 presents the results for the S&P100 data set. A detailed list of the series considered and complementary results are provided in the Web-based supporting materials.

The data that are analysed in the paper and the programs that were used to analyse them can be obtained from

http://wileyonlinelibrary.com/journal/rss-datasets

2. Factors and networks in large panels of financial time series

We consider large panels of time series data, namely observed finite realizations, of the form $\{Y_{it}|i=1,\ldots,n,\ t=1,\ldots,T\}$, of some stochastic process $\mathbf{Y}:=\{Y_{it}|i\in\mathbb{N},\ t\in\mathbb{Z}\}$; i is a cross-sectional index and t stands for time. Both the cross-sectional dimension n and the sample size or series length T are large and, in asymptotics, we consider sequences of n and T values tending to ∞. The notation $\mathbf{Y}_n:=\{\mathbf{Y}_{nt}=(Y_{1t},Y_{2t},\ldots,Y_{nt})'|t\in\mathbb{Z}\}$ in what follows is used for the n-dimensional subprocess of $\mathbf{Y}$; the same notation is used for all n-dimensional vectors. In this section, $\mathbf{Y}_n$ stands for a generic panel of time series. Since our interest is to study connections and interdependences that are responsible for the contagion phenomena that might lead to financial crises, then, in Sections 4 and 5, we shall apply the definitions and results that are presented here to the case of financial volatilities.

In principle, as shown by Diebold and Yilmaz (2014), an LVDN can be estimated via classical VAR estimation. However, when dealing with high dimensional systems, VAR estimation is badly affected by curse-of-dimensionality problems, and adequate estimation techniques must be considered. The most frequent strategy, in the presence of large n data sets, is based on sparsity assumptions allowing for the application of penalized regression techniques (Hsu *et al.*, 2008; Abegaz and Wit, 2013; Nicholson *et al.*, 2014; Basu and Michailidis, 2015; Davis *et al.*, 2015;

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Kock and Callot, 2015; Barigozzi and Brownless, 2016; Gelper *et al.*, 2016). The presence of pervasive shocks affecting the large panels of time series considered in macroeconometrics and finance has motivated the development of another dimension reduction technique: the so-called dynamic factor methods. Various versions of those methods have become daily practice in many areas of econometrics and finance; among these the GDFM by Forni *et al.* (2000) and Forni and Lippi (2001) is the most general, whereas most other factor models that have been considered in the time series literature (Stock and Watson (2002), Bai and Ng (2002), Lam and Yao (2012) and Fan *et al.* (2013), to quote only a few) are particular cases.

2.1. The generalized dynamic factor model

To introduce the *dynamic factor representation* for $\mathbf{Y}$, we make the following assumptions.

Assumption 1. For all $n \in \mathbb{N}$, the vector process $\mathbf{Y}_n$ is second-order stationary, with mean 0 and finite variances.

Assumption 2. For all $n \in \mathbb{N}$, the spectral measure of $\mathbf{Y}_n$ is absolutely continuous with respect to the Lebesgue measure on $[-\pi, \pi]$, i.e. $\mathbf{Y}_n$ admits a full rank (for any n and θ) spectral density matrix $\Sigma_{\mathbf{Y};n}(\theta)$, $\theta \in [-\pi, \pi]$ with uniformly (in i, j, θ and n) bounded entries $\sigma_{\mathbf{Y};ij}(\theta)$.

Denote by $\lambda_{\mathbf{Y};n,1}(\theta), \ldots, \lambda_{\mathbf{Y};n,n}(\theta)$, $\theta \in [-\pi, \pi]$, the eigenvalues (in decreasing order of magnitude) of $\Sigma_{\mathbf{Y};n}(\theta)$; the mapping $\theta \mapsto \lambda_{\mathbf{Y};n,i}(\theta)$ is also called $\mathbf{Y}_n$'s ith *dynamic eigenvalue*. We say that $\mathbf{Y}$ admits a general dynamic factor representation with q factors if for all i the process $\{Y_{it}\}$ decomposes into a *common* component $\{X_{it}\}$ and an *idiosyncratic* component $\{Z_{it}\}$,

$$Y_{it} = X_{it} + Z_{it} =: \sum_{k=1}^{q} b_{ik}(L) u_{kt} + Z_{it}, \qquad i \in \mathbb{N}, \quad t \in \mathbb{Z}, \tag{1}$$

such that

(a) the q-dimensional vector process of factors $\mathbf{u} := \{\mathbf{u}_t = (u_{1t}\, u_{2t} \ldots u_{qt})' | t \in \mathbb{Z}\}$ is orthonormal zero-mean white noise,
(b) the filters $b_{ik}(L)$ are one sided and square summable for all $i \in \mathbb{N}$ and $k = 1, \ldots, q$,
(c) the qth dynamic eigenvalue $\lambda_{\mathbf{X};n,q}(\theta)$ of $\mathbf{X}_n$ diverges θ almost everywhere in the interval $[-\pi, \pi]$ as $n \to \infty$,
(d) the first dynamic eigenvalue $\lambda_{\mathbf{Z};n,1}(\theta)$ of $\mathbf{Z}_n$ is bounded (θ almost everywhere in $[-\pi, \pi]$) as $n \to \infty$,
(e) Z_{k,t_1} and u_{h,t_2} are mutually orthogonal for any k, h, t_1 and t_2, and
(f) q is minimal with respect to (a)–(e).

For any n, we can write model (1) in vector notation as

$$\mathbf{Y}_{nt} = \mathbf{X}_{nt} + \mathbf{Z}_{nt} =: B_n(L)\mathbf{u}_t + \mathbf{Z}_{nt}, \qquad n \in \mathbb{N}, \quad t \in \mathbb{Z}. \tag{2}$$

This actually defines the GDFM. In this model, the common and idiosyncratic components are identified by means of the following assumption on $\mathbf{Y}_n$'s dynamic eigenvalues.

Assumption 3. The qth eigenvalue $\lambda_{\mathbf{Y};n,q}(\theta)$ of $\Sigma_{\mathbf{Y};n}(\theta)$ diverges, θ almost everywhere in $[-\pi, \pi]$, whereas the $(q+1)$th eigenvalue, $\lambda_{\mathbf{Y};n,q+1}(\theta)$, is θ almost everywhere bounded, as $n \to \infty$.

More precisely, we know from Forni *et al.* (2000) and Forni and Lippi (2001) that, given assumptions 1 and 2, assumption 3 is necessary and sufficient for the process $\mathbf{Y}$ to admit the dynamic factor representation (1). Hallin and Lippi (2014) moreover provided very weak time domain primitive conditions under which model (1), and hence assumption 3, holds for some $q < \infty$.

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Finally, the idiosyncratic component $\mathbf{Z}_n$ always admits a Wold decomposition which, after adequate transformation, yields the VMA representation

$$\mathbf{Z}_{nt} := D_n(L)\mathbf{e}_{nt}, \qquad t \in \mathbb{Z}, \quad \mathbf{e}_{nt} \sim \text{wn}(\mathbf{0}, I_n), \tag{3}$$

where $D_n(L) = \Sigma_{k=0}^{\infty} D_{nk} L^k$ is a square summable power series in the lag operator L. Note that, although the magnitude of those coefficients is bounded by condition (d) above, no sparsity assumption is made.

2.2. The long-run variance decomposition network

To study the interdependences between series, and following traditional econometric analysis, we focus on the reactions of observed variables to unobserved shocks, i.e. impulse response functions. Large panels of financial time series are affected by market shocks that are, essentially, common to all stocks and represent the non-diversifiable components of risk, and by idiosyncratic shocks that are specific to one or a few stocks in the panel. The GDFM is the ideal tool for disentangling those two sources of variation:

(a) the q market shocks $\mathbf{u}$ and their impulse responses $B_n(L)$, defined in model (2), and
(b) the n idiosyncratic shocks $\mathbf{e}_n$ and their impulse responses $D_n(L)$, defined in model (3).

Our focus here is mainly on idiosyncratic shocks. Indeed, once we control for market effects, the study of interdependences between different stocks is strictly related to systemic risk measures, i.e. individual measures of how one given stock is likely to be affected by, and/or is likely to affect, all others.

For this, representation (3) is what we need, as it characterizes all (linear) interdependences between the components of $\mathbf{Z}_n$. In particular, D_{n0} characterizes contemporaneous dependences, whereas D_{nk} for $k>0$ characterizes the dynamic dependences with lag k. Denote by $d_{k,ij}$ the (i, j)th entry of D_{nk}. Then, following Diebold and Yilmaz (2014), we can summarize all dependences up to lag h by means of the forecast error variance decomposition and, more particularly, by the ratios

$$w_{ij}^h := 100 \frac{\sum_{k=0}^{h-1} d_{k,ij}^2}{\sum_{l=1}^{n} \sum_{k=0}^{h-1} d_{k,il}^2}, \qquad i, j = 1, \ldots, n. \tag{4}$$

The ratio w_{ij}^h is the percentage of the h-step-ahead forecast error variance of $\{Y_{it}\}$ accounted for by the innovations in $\{Y_{jt}\}$. Note that, by definition,

$$\frac{1}{100} \sum_{j=1}^{n} w_{ij}^h = 1 \qquad \text{for any } i;$$

hence

$$\frac{1}{100} \sum_{i,j=1}^{n} w_{ij}^h = n.$$

The LVDN is then defined by the set of edges

$$\mathcal{E}_{\text{LVDN}} := \{(i, j) \in \{1 \ldots n\}^2 |\ w_{ij}^{\text{LVDN}} := \lim_{h \to \infty} w_{ij}^h \neq 0\}. \tag{5}$$

In practice a horizon h must be chosen to compute those weights, and the operational definition of the LVDN therefore also depends on that h.

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Three measures of connectedness can be based on the quantities that are defined in expression (4). First, we define the *from-degree* of component i and *to-degree* of component j (also called the *in-strength* of node i and *out-strength* of node j) as

$$\begin{aligned} \delta_i^{\text{From}} &:= \sum_{j=1, j\neq i}^{n} w_{ij}^h, \qquad i=1,\ldots,n, \\ \delta_j^{\text{To}} &:= \sum_{i=1, i\neq j}^{n} w_{ij}^h, \qquad j=1,\ldots,n, \end{aligned} \tag{6}$$

respectively. As pointed out by Diebold and Yilmaz (2014), these two measures are closely related to two classical measures of systemic risk that have been considered in the financial literature. The from-degree is directly related to the so-called *marginal expected shortfall* and *expected capital shortfall* (of series i), which measure the exposure of component i to extreme events affecting all other components (see Acharya *et al.* (2012) for a definition of these measures). As for the to-degree, it is related to *co-value at risk*, which measures the effect on the whole panel of an extreme event affecting component j (see Adrian and Brunnermeier (2016) for a definition). Finally, we can define an overall measure of connectedness by summing all from-degrees (and, equivalently, all to-degrees):

$$\delta^{\text{Tot}} := \frac{1}{n}\sum_{i=1}^{n} \delta_i^{\text{From}} = \frac{1}{n}\sum_{j=1}^{n} \delta_j^{\text{To}}. \tag{7}$$

Given the economic interpretation of these quantities, the LVDN of the idiosyncratic component $\mathbf{Z}_n$ seems to be an ideal tool for studying systemic risk and, for this reason, in the empirical study of Section 5, we mainly focus on the LVDN of volatilities (see also Section 4 for a motivation).

An LVDN also can be constructed for the common component $\mathbf{X}_n$ by using a definition that is analogous to expression (4), but based on the entries of the matrix polynomial $B_n(L)$, defined in model (2). However, because of the singularity of $B_n(L)$, definition (4) in this case does not measure the proportion of the h-step-ahead forecast error variance of variable i accounted for by the innovations in variable j, but rather the proportion of the same forecast error variance explained by the jth market shock $\{u_{jt}\}$.

2.3. Vector auto-regression representations

The LVDN of the idiosyncratic component $\mathbf{Z}_n$ is defined from the coefficients of the VMA representation (3). That representation can be estimated as an inverted sparse VAR. We accordingly make the following assumption.

Assumption 4. The idiosyncratic component $\mathbf{Z}_n$ admits, for some p that does not depend on n, the VAR(p) representation

$$F_n(L)\mathbf{Z}_{nt} = \mathbf{v}_{nt}, \qquad t \in \mathbb{Z}, \quad \mathbf{v}_{nt} \sim \text{wn}(\mathbf{0}, C_n^{-1}), \tag{8}$$

where $F_n(L) = \Sigma_{k=0}^{p} F_{nk} L^k$ with $F_{n0} = I_n$ and $\det\{F_n(z)\} \neq 0$ for any $z \in \mathbb{C}$ such that $|z| \leqslant 1$, and C_n has full rank. Moreover, denoting by $f_{k,ij}$ and c_{ij} the (i, j)th entries of F_{nk} and C_n,

$$\max_{j=1,\ldots,n} \sum_{i=1}^{n} \mathbb{I}_{(f_{k,ij}\neq 0)} = o(n), \qquad k=1,\ldots,p, \quad n \in \mathbb{N}, \tag{9}$$

$$\max_{i=1,\ldots,n} \sum_{j=1}^{n} \mathbb{I}_{(f_{k,ij}\neq 0)} = o(n), \qquad k=1,\ldots,p, \quad n \in \mathbb{N}, \tag{10}$$

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$$\max_{j=1,\ldots,n} \sum_{i=1}^{n} \mathbb{I}_{(c_{ij}\neq 0)} = o(n), \qquad n \in \mathbb{N}. \tag{11}$$

The first part (8) of this assumption is quite mild, provided that p can be chosen sufficiently large. The second part requires some further clarification. In expression (9)–(10), we require the VAR coefficient matrices in expression (8) to have only a small number of non-zero entries. In this sense we say that the VAR representation (8) is sparse (see, for example, the definitions of sparsity in Bickel and Levina (2008) and Cai and Liu (2011)). That assumption is needed for the consistent estimation of expression (8) in the large n setting, and it is justified by the idea that in a GDFM, once we control for common shocks, most interdependences between the elements of $\mathbf{Z}_n$ are quite weak (since the corresponding dynamic eigenvalues are bounded as $n \to \infty$). However, note that, whereas a sparse VAR is related to conditional second moments, the GDFM assumptions on the idiosyncratic component are based on unconditional second moments. For this reason, the GDFM assumptions do not imply a sparse VAR representation for $\mathbf{Z}_n$, and expressions (9) and (10) are needed. Finally, for convenience, we parameterize the covariance matrix of the VAR innovations by means of its inverse C_n and in expression (11) we require this matrix to be sparse also, in accordance with the idea of a sparse global conditional dependence structure of idiosyncratic components.

As a by-product, a long-run Granger causality network (LGCN) can be defined by the set of edges

$$\mathcal{E}_{\mathrm{LGCN}} := \left\{ (i,j) \in \{1\ldots n\}^2 \mid w_{ij}^{\mathrm{LGCN}} := \sum_{k=0}^{p} f_{k,ij} \neq 0 \right\}. \tag{12}$$

This network captures the leading or lagging conditional dependences of a given panel of time series. Such graphical representations of VAR dependences were initially proposed by Dahlhaus and Eichler (2003) and Eichler (2007), and extend to a time series context the graphical models for independent data that were considered by Dempster (1972), Meinshausen and Bühlmann (2006), Friedman *et al.* (2008) and Peng *et al.* (2009), to quote only a few.

Two comments are in order here. A network is said to be sparse if its weight matrix has many 0-entries. First, note that, under assumption 4, the LGCN is likely to be sparse. In contrast, the GDFM assumptions do not guarantee sparsity of the LVDN but only some weaker restrictions on the magnitude of its entries, as dictated by the boundedness of the eigenvalues of $\mathbf{Z}_n$'s spectral density matrix. Second, the economic interpretation of the LGCN is not as straightforward as that of the LVDN, and the LGCN therefore is of lesser interest for the analysis of financial systems: mainly, it will be a convenient tool in the derivation of the LVDN. This is in line with traditional macroeconomic analysis where impulse response functions, i.e. VMA coefficients, rather than VAR coefficients, are the object of interest for policy makers.

As for the common component $\mathbf{X}_n$, Forni *et al.* (2015a) showed that it admits the singular VAR representation

$$A_n(L)\mathbf{X}_{nt} = H_n \mathbf{u}_t, \qquad t \in \mathbb{Z}, \quad \mathbf{u}_t \sim \mathrm{wn}(\mathbf{0}, I_q). \tag{13}$$

Assuming, without loss of generality, that $n = m(q+1)$ for some integer m, the VAR operator $A_n(L)$ in model (13) is block diagonal, with $(q+1)\times(q+1)$-dimensional diagonal blocks of the form $A^{(i)}(L) = \Sigma_{k=0}^{p_i} A_k^{(i)} L^k$ such that, for any $i = 1, \ldots, m$, $A_0^{(i)} = I_n$ and $\det\{A^{(i)}(z)\} \neq 0$ for any $z \in \mathbb{C}$ such that $|z| \leqslant 1$. Moreover, H_n is a full rank $n \times q$ matrix, and $\mathbf{u}$ is the q-dimensional process of common shocks defined in model (2).

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2.4. Identification

Starting from $\mathbf{Z}_n$'s VAR representation (8) and comparing it with representation (3), we have

$$D_n(L) = F_n(L)^{-1} R_n, \tag{14}$$

where the full rank matrix R_n is making the shocks $R_n^{-1}\mathbf{v}_n =: \mathbf{e}_n$ orthonormal (such matrices under assumption 4 exist). In other words, the LVDN is obtained from the inversion of the VAR in expression (8) by selecting a suitable transformation R_n meeting the required identification constraints. The simplest choice for R_n follows from a Choleski decomposition of the covariance C_n^{-1} of the shocks (see Sims (1980))—namely, selecting R_n as the lower triangular matrix such that $C_n^{-1} = R_n R_n'$. Such a choice is appealing, as it is purely data driven, but it depends on the ordering of the variables or, equivalently, on the ordering of the components of the shocks vector $\mathbf{v}_n$.

Many orderings are possible, and the ordering that we propose is based on $\mathbf{v}_n$'s partial correlation structure. More precisely, assuming that C_n has full rank, the partial correlation between $\{v_{it}\}$ and $\{v_{jt}\}$ is

$$\rho^{ij} := \frac{-c_{ij}}{\surd(c_{ii}c_{jj})}, \qquad i, j = 1, \ldots, n, \tag{15}$$

where c_{ij} is the (i, j)th entry of C_n. Associated with this concept of partial correlation is the partial correlation network (PCN), with edges

$$\mathcal{E}_{\mathrm{PCN}} := \{(i, j) \in \{1 \ldots n\}^2 \mid w_{ij}^{\mathrm{PCN}} := \rho^{ij} \neq 0\}, \tag{16}$$

which, by assumption 4, is a sparse network. The PCN can be studied by means of the n linear regressions

$$v_{it} = \sum_{j=1, j\neq i}^{n} \beta_{ij} v_{jt} + \nu_{it}, \qquad t \in \mathbb{Z}, \quad \nu_{it} \sim \mathrm{wn}(0, \sigma_i^2), \quad i = 1, \ldots, n. \tag{17}$$

Indeed, it can be shown that $\beta_{ij} = \rho^{ij}\surd(\sigma_i^2/\sigma_j^2)$ (see, for example, lemma 1 in Peng *et al.* (2009)), so that $(i, j) \in \mathcal{E}_{\mathrm{PCN}}$ if and only if $\beta_{ij} \neq 0$. Thus, the inverse covariance matrix C_n of the VAR shocks is directly related to the partial correlation matrix of the VAR innovations, and this matrix in turn can be seen as the PCN weight matrix. We then order the shocks by decreasing order of eigenvector centrality (as in Bonacich (1987)) in that PCN, the most central component receiving label 1, etc.

The centrality measure that is considered defines each node's centrality as the sum of the centrality values of its neighbouring nodes. It is easily seen that the mathematical translation of this idea leads to an eigenvector-related concept of centrality. That concept of eigenvector centrality differs from that of degree centrality (the number of neighbours of a given node); indeed, a node receiving many links does not necessarily have high eigenvector centrality, whereas a node with high eigenvector centrality is not necessarily highly linked (it might have few but important linkers).

It should be noted that usually eigenvector centrality is defined for networks in which the sign of the weight that is associated with a given edge is not taken into account, so negative and positive partial correlations have the same importance. However, on the basis of the idea that contagion is more likely between nodes which are linked through positive weights rather than negative weights, we can also consider eigenvector centrality for a signed network, thus preserving the information about weights' signs. Both approaches are considered in the empirical analysis that follows.

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This identification strategy seems well suited to the study of financial contagion. Indeed, in an impulse response exercise, we study the propagation of shocks through the system starting from lag 0 up to a given lag $h > 0$. Thus, a given order of shocks defines which component we choose to hit first. By ordering nodes according to their centrality in the PCN of VAR residuals, we are considering the case in which the most contemporaneously interconnected node is firstly affected by an unexpected shock, and then, by means of the subsequent impulse response analysis, we study the propagation of such a shock through the whole system. The corresponding row of the LVDN adjacency matrix gives a summary, in terms of explained variance, of the effect of this propagation mechanism after h lags.

Finally, it is worth mentioning a few other possible methods for identifying the shocks in the LVDN. One could rank the series based on endogenous characteristics of the firms that are considered, such as market capitalization. Another data-driven approach was taken in Swanson and Granger (1997), who proposed to test the overidentifying restrictions that are implied by orthogonal shocks. That approach is closely related to ours as those restrictions involve partial correlations of the shocks; however, as the dimension n of the problem increases, the related computational problems rapidly become non-tractable. Diebold and Yilmaz (2014) adopted generalized variance decompositions, which are a very popular method that was originally proposed by Koop *et al.* (1996). This approach, however, is only valid if the shocks have a Gaussian distribution, which is an assumption which is unlikely to hold true for the S&P100 data set.

Turning to the common component $\mathbf{X}_n$, its LVDN is obtained by inverting the VAR representation (13). Namely, there is a $q \times q$ invertible matrix K such that

$$B_n(L) = A_n(L)^{-1} H_n K. \tag{18}$$

Here K is required to identify the orthonormal market shocks $\mathbf{u}$. However, if $q = 1$ as in the empirical analysis of Section 5, the choice of K reduces to that of a sign and a scale.

In the next sections, we first discuss the estimation of GDFMs and LVDNs under the general definition of this section, and then adapt to the particular case of unobserved volatilities.

3. Estimation

In this section we review estimation of equations (14) and (18). The numbers of factors throughout are determined via the information criterion that was proposed by Hallin and Liška (2007) and based on the behaviour of dynamic eigenvalues as the panel size grows from 1 to n. In this section we consider a generic observed panel $\{Y_{it} \mid i = 1, \ldots, n,\ t = 1, \ldots, T\}$ of time series, with sample size T (in the next section these would be either returns or volatilities). Hereafter, we use the superscript T to denote estimated quantities.

3.1. Generalized dynamic factor model estimation

First we recover the common component by using its auto-regressive representation (13). For a given number of factors q, the method, which was described in detail by Forni *et al.* (2015a, b), is based on the following steps.

(a) Estimate the spectral density matrix of $\mathbf{Y}_n$, which is denoted as $\Sigma^T_{\mathbf{Y};n}(\theta)$, e.g. by using the smoothed periodogram estimator.

(b) Use the q largest dynamic principal components of $\Sigma^T_{\mathbf{Y};n}(\theta)$ to extract the spectral density matrix of $\mathbf{X}_n$, which is denoted as $\Sigma^T_{\mathbf{X};n}(\theta)$ (see Brillinger (1981)).

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(c) Compute the autocovariances of $\mathbf{X}_n$ by inverse Fourier transform of $\Sigma^T_{\mathbf{X};n}(\theta)$ and use these to compute the Yule–Walker estimator of the VAR filters $A^T_n(L)$; denote by $\epsilon^T_n := \{\epsilon^T_{it} \mid i = 1, \ldots, n,\ t = 1, \ldots, T\}$ the corresponding residuals.

(d) From the sample covariance of ϵ^T_n, compute the eigenvectors corresponding to its q largest eigenvalues (these are the columns of the estimator H^T_n); then, by projecting ϵ^T_n onto the space that is spanned by the columns of H^T_n, obtain the q-dimensional vector $\{\mathbf{u}^T_t \mid t = 1, \ldots, T\}$.

(e) The estimated LVDN of the common component is given by $A^T_n(L)^{-1} H^T_n K$, where K is a generic $q \times q$ invertible matrix such that $H^T_n K$ consistently estimates H_n.

(f) The estimated common and idiosyncratic components are respectively given by

$$\mathbf{X}^T_{nt} = A^T_n(L)^{-1} H^T_n \mathbf{u}^T_t \quad \text{and} \quad \mathbf{Z}^T_{nt} = \mathbf{Y}_{nt} - \mathbf{X}^T_{nt}, \qquad t = 1, \ldots, T.$$

Details on each step and asymptotic properties of the estimators are given in Forni *et al.* (2015b). In particular, the parameters of the model are estimated consistently as n and T tend to ∞, at $O_P\{\max(n^{-1/2}, T^{-1/2})\}$ rate.

3.2. Sparse vector auto-regressive estimation

Once we have an estimator of the n idiosyncratic components, we estimate the VAR(p) representation (8) by minimizing the penalized quadratic loss

$$\mathcal{L}_T = \sum_{t=1}^{T} \left(Z^T_{it} - \sum_{k=1}^{p} \mathbf{f}'_{k,i} \mathbf{Z}^T_{nt-k} \right)^2 + \mathcal{P}(\mathbf{f}_{1,i} \ldots, \mathbf{f}_{p,i}), \qquad i = 1, \ldots, n, \tag{19}$$

where $\mathbf{f}'_{k,i}$ is the ith row of F_{kn} and $\mathcal{P}(\cdot)$ is some given penalty which depends on the estimation method chosen. In particular, we consider three alternative strategies:

(a) the elastic net, as defined by Zou and Hastie (2005), where the penalty is a weighted average of ridge and lasso penalties,

$$\mathcal{P}^{\text{EN}}(\mathbf{f}_{1,i}, \ldots, \mathbf{f}_{p,i}) = \lambda \|(\mathbf{f}'_{1,i} \ldots \mathbf{f}'_{p,i})'\|_1 + (1 - \lambda) \|(\mathbf{f}'_{1,i} \ldots \mathbf{f}'_{p,i})'\|_2^2;$$

(b) the adaptive lasso, as defined by Zou (2006), where the penalty is a lasso penalty but conditioned on pre-estimators $\widetilde{\mathbf{f}}_{k,i}$ of the parameters (typically given by ridge or least squares estimators),

$$\mathcal{P}^{\text{AL}}(\mathbf{f}_{1,i}, \ldots, \mathbf{f}_{p,i}) = \lambda \frac{\|(\mathbf{f}'_{1,i} \ldots \mathbf{f}'_{p,i})'\|_1}{\|(\widetilde{\mathbf{f}}'_{1,i} \ldots \widetilde{\mathbf{f}}'_{p,i})'\|_1};$$

(c) the group lasso, as defined by Yuan and Lin (2006), where the explanatory variables are grouped before penalizing; thus in a VAR context the groups are given by the lags of each variable, and thus there are n groups, each of p elements, and we have

$$\mathcal{P}^{\text{GL}}(\mathbf{f}_{1,i}, \ldots, \mathbf{f}_{p,i}) = \lambda \sqrt{p} \sum_{j=1}^{n} \|(f_{1,ij} \ldots f_{p,ij})'\|_2.$$

The penalization constant λ, in all three methods, and the maximum VAR lag p are determined by minimizing, over a grid of possible values, a Bayesian information criterion type of criterion.

The elastic net and adaptive lasso are particularly useful in a time series context since they are known to stabilize a simple lasso estimator which might suffer instability due to serial dependence in the data. To the best of our knowledge, the elastic net so far has not been considered in the estimation of high dimensional VARs, whereas the adaptive lasso has been

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used by Kock and Callot (2015) and Barigozzi and Brownless (2016), among others. In contrast, the group lasso in principle is likely to make the LGCN more sparse than will the other two methods and has been used, for VAR estimation, by Nicholson *et al.* (2014) and Gelper *et al.* (2016). Other possible penalized VAR estimators, which we do not consider here, include the simple lasso and smoothly clipped absolute deviation (see, for example, Hsu *et al.* (2008) and Abegaz and Wit (2013)). The consistency of all those methods is proved in those references, when both n and T tend to ∞, under a variety of technical assumptions which we do not report here.

3.3. Identification of the long-run variance decomposition network

Once an estimator of the VAR coefficients has been obtained, we can estimate, in a last step, the PCN of the VAR residuals, as defined in expression (16), which in turn can be used for LVDN identification. This again can be performed by means of several regularization methods. Here we chose the approach that was proposed by Peng *et al.* (2009), which we refer to for technical details. More precisely, the weights of the PCN are obtained by estimating the regressions in expression (17) via the traditional lasso (Tibshirani, 1996) to ensure sparsity as required by assumption 4. Alternatively, the PCN of the residuals can be estimated jointly with the LGCN as originally proposed by Rothman *et al.* (2010) for cross-sectional regressions and, in a time series context, by Abegaz and Wit (2013), Gelper *et al.* (2016) or (with adaptive lasso penalty) Barigozzi and Brownless (2016). Once we have estimated the PCN of VAR residuals, we can order them according to their centrality in the network. We then estimate the matrix R_n^T, which is needed for identification, as the Choleski factor of the sample covariance matrix of the ordered residuals. Finally, from R_n^T and the estimated VAR operators $F_n^T(L)$, we compute the VMA operator $D_n^T(L) = F_n^T(L)^{-1} R_n^T$. The estimated LVDNs weights, which are denoted as w_{ij}^{hT}, readily follow from expression (4).

It should be noted that, whereas by estimating a sparse VAR the LGCN by construction is sparse, this is not so for the LVDN which, being derived from the *inverse* of a sparse VAR, does not necessarily have to be sparse. In other words, the assumption of sparsity of VAR coefficients is made for dealing with the curse of dimensionality and does not entail sparsity of the corresponding LVDN. The matrix W^{hT} with entries w_{ij}^{hT} (the LVDN adjacency matrix) is therefore not necessarily sparse. Nevertheless, since most of its entries are quite small, considering a sparse thresholded version W_τ^{hT} with threshold $\tau > 0$ is very natural. Here, we chose the threshold τ minimizing the sum $\|(W^{hT} W^{hT'})^{-1/2}(W_\tau^{hT} W_\tau^{hT'})(W^{hT} W^{hT'})^{-1/2} - I_n\|_2$ of squared errors (see for example Fan *et al.* (2013) for a similar approach, although in a different context).

4. Network analysis of financial volatilities

The available data sets, in the study of interdependences of financial institutions, in general, are (large) panels of stock returns. If our interest is in the systematic, i.e. market-driven, and systemic components of risk, then volatilities are what we need, not returns. Financial crises typically are characterized by unusually high levels of volatility generated by some major systemic events such as the bankruptcy of some major institutions. Analysing interdependences in volatility panels is the first step to a study of financial contagion (see Diebold and Yilmaz (2014) for a thorough discussion).

Volatilities unfortunately are unobserved and therefore must be estimated from the panels of returns. Many volatility proxies can be constructed from the series of returns such as the adjusted log-range (Parkinson, 1980) from daily returns, or log-realized volatilities (Andersen *et al.*, 2003) based on intradaily returns. Those proxies are generally treated as observed quantities,

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and nothing is said about the associated estimation error. However, volatilities can also be estimated from conditionally heteroscedastic models for financial returns such as multivariate generalized auto-regressive conditional heteroscedastic models (see, for instance, the survey by Bauwens *et al*. (2006)). Those are, however, parametric models which, because of the curse-of-dimensionality problem, cannot be handled in the present large n setting. We therefore follow the approach in Barigozzi and Hallin (2016) in adopting a global point of view, with a joint analysis of returns and volatilities in a high dimensional setting. That analysis is based on a two-step dynamic factor procedure: first a GDFM procedure, applied to the panel of returns, is extracting a (double) panel of volatility proxies, which, in a second step, is analysed via a second GDFM. The LVDNs that we are interested in are those of the common and idiosyncratic volatility components resulting from this second step.

More precisely, we consider a panel $\mathbf{r}_n := \{\mathbf{r}_{nt} = (r_{1t} r_{2t} \ldots r_{nt})' | t \in \mathbb{Z}\}$ of n stock returns (such as the constituents of the S&P100-index). We assume that $\mathbf{r}_n$ satisfies assumptions 1–3, i.e. admits the GDFM decomposition

$$\mathbf{r}_{nt} = \boldsymbol{\chi}_{nt} + \boldsymbol{\xi}_{nt}, \qquad t \in \mathbb{Z}, \tag{20}$$

where $\boldsymbol{\chi}_n$ is driven by q common shocks and $\boldsymbol{\xi}_n$ is idiosyncratic—call them *level common* and *level idiosyncratic* components respectively.

When applied to the real data in Section 5, the Hallin and Liška (2007) criterion very clearly yields $q = 1$, i.e. a unique level common shock. Thus the level common component $\boldsymbol{\chi}_n$ admits (see expression (13)) the auto-regressive representation

$$\mathcal{A}_n(L)\boldsymbol{\chi}_{nt} =: \boldsymbol{\eta}_{nt} =: (\eta_{1t}, \ldots, \eta_{nt})', \qquad t \in \mathbb{Z}, \tag{21}$$

where $\mathcal{A}_n(L)$ is a one-sided square summable stable block diagonal auto-regressive filter with blocks of size 2×2, and $\boldsymbol{\eta}_n$ is an n-dimensional white noise process with a singular covariance matrix of rank 1.

We then assume that assumption 4 holds for the level idiosyncratic component $\boldsymbol{\xi}_n$, which thus admits the sparse VAR representation

$$\mathcal{F}_n(L)\boldsymbol{\xi}_{nt} = \mathbf{v}_{nt} =: (v_{1t}, \ldots, v_{nt})', \qquad t \in \mathbb{Z}, \tag{22}$$

where $\mathbf{v}_n$ is an n-dimensional white noise process and $\mathcal{F}_n(L)$ is a one-sided stable VAR filter with sparse coefficients, the rows of which have a finite number of non-zero terms. Estimators of $\boldsymbol{\eta}_n$ and $\mathbf{v}_n$ are obtained by applying the methodology that was described in the previous section.

Turning to volatilities, define two panels of volatility proxies,

$$\boldsymbol{\sigma}_{nt} := \log(\boldsymbol{\eta}_{nt}^2) \quad \text{and} \quad \boldsymbol{\omega}_{nt} := \log(\mathbf{v}_{nt}^2), \qquad t \in \mathbb{Z}, \tag{23}$$

with $\log(\boldsymbol{\eta}_{nt}^2) := (\log(\eta_{1t}^2), \ldots, \log(\eta_{nt}^2))'$ and $\log(\mathbf{v}_{nt}^2) := (\log(v_{1t}^2), \ldots, \log(v_{nt}^2))'$. After due centring, we assume that those two panels of volatility proxies in turn satisfy assumptions 1–3, and hence admit the GDFM decompositions

$$\mathring{\boldsymbol{\sigma}}_{nt} := \boldsymbol{\sigma}_{nt} - E[\boldsymbol{\sigma}_n] = \boldsymbol{\chi}_{\sigma,nt} + \boldsymbol{\xi}_{\sigma,nt}, \qquad t \in \mathbb{Z}, \tag{24}$$

$$\mathring{\boldsymbol{\omega}}_{nt} := \boldsymbol{\omega}_{nt} - E[\boldsymbol{\omega}_n] = \boldsymbol{\chi}_{\omega,nt} + \boldsymbol{\xi}_{\omega,nt}, \qquad t \in \mathbb{Z}, \tag{25}$$

where $\boldsymbol{\chi}_{\sigma,n}$ and $\boldsymbol{\chi}_{\omega,n}$ are driven by q_σ and q_ω common shocks respectively, and $\boldsymbol{\xi}_{\sigma,n}$ and $\boldsymbol{\xi}_{\omega,n}$ are idiosyncratic—call them the volatility common and idiosyncratic components respectively.

Traditional factor models for volatilities, derived from factor models of returns, assume that $\mathring{\boldsymbol{\sigma}}_n$ has no idiosyncratic component and, more importantly, that $\mathring{\boldsymbol{\omega}}_n$ has no common component. Such an assumption is quite unlikely to hold, as there is no reason for market shocks only to

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affect the volatility $\mathring{\boldsymbol{\sigma}}_n$ of level common returns. The empirical results in Barigozzi and Hallin (2016) indeed amply confirm that a non-negligible proportion of the variance of $\mathring{\boldsymbol{\omega}}_n$ is explained by the same market shocks also driving $\mathring{\boldsymbol{\sigma}}_n$.

The two volatility common components $\boldsymbol{\chi}_{\sigma,n}$ and $\boldsymbol{\chi}_{\omega,n}$ jointly define a $2n$-dimensional panel made of two blocks. These blocks might be driven by $q_{\sigma\omega}$ shocks, some of which are common in both blocks, and some others common only in one of them (see Hallin and Liška (2011) for a general theory of factor models with block structure). However, when analysing the real data in Section 5, we find that $q_{\sigma\omega}=q_\sigma=q_\omega=1$, and therefore there is evidence of a unique common shock, which is denoted as $\{\varepsilon_t\}$, driving both blocks, and thus unambiguously qualifying as the market volatility shock. The auto-regressive representation of these two common components then reads (see also expression (13))

$$\begin{pmatrix} A_{\sigma,n}(L) & \mathbf{0}_n \\ \mathbf{0}_n & A_{\omega,n}(L) \end{pmatrix} \begin{pmatrix} \boldsymbol{\chi}_{\sigma,n} \\ \boldsymbol{\chi}_{\omega,n} \end{pmatrix} = \begin{pmatrix} H_{\sigma,n} \\ H_{\omega,n} \end{pmatrix} \varepsilon_t, \qquad t \in \mathbb{Z}, \quad \varepsilon_t \sim \mathrm{wn}(0,1), \tag{26}$$

where $A_{\sigma,n}(L)$ and $A_{\omega,n}(L)$ are one-sided square summable block diagonal stable filters with blocks of size 2×2, and $H_{\sigma,n}$ and $H_{\omega,n}$ are n-dimensional column vectors. All parameters in expression (26) can be estimated as described in Section 3.1.

The singular LVDNs for the common components of volatilities thus can be built from the VMA filters (see expression (18))

$$\begin{aligned} B_{\sigma,n}(L) &:= A_{\sigma,n}(L)^{-1} H_{\sigma,n} K_\sigma, \\ B_{\omega,n}(L) &:= A_{\omega,n}(L)^{-1} H_{\omega,n} K_\omega, \end{aligned} \tag{27}$$

where K_σ and K_ω in this case are just scalars that are needed to identify the scale and sign of the market shocks. For a given horizon h, the LVDN weights that are defined in expression (4) provide the percentages of h-step-ahead forecast error variance of series i accounted for by the common market shock.

For the two volatility idiosyncratic components $\boldsymbol{\xi}_{\sigma,n}$ and $\boldsymbol{\xi}_{\omega,n}$, we assume that assumption 4 holds, yielding the sparse VAR representations

$$F_{\sigma,n}(L)\boldsymbol{\xi}_{\sigma,n} = \boldsymbol{\nu}_{\sigma,nt}, \qquad t\in\mathbb{Z}, \quad \boldsymbol{\nu}_{\sigma,nt} \sim \mathrm{wn}(\mathbf{0}, C_{\sigma,n}^{-1}), \tag{28}$$

$$F_{\omega,n}(L)\boldsymbol{\xi}_{\omega,n} = \boldsymbol{\nu}_{\omega,nt}, \qquad t\in\mathbb{Z}, \quad \boldsymbol{\nu}_{\omega,nt} \sim \mathrm{wn}(\mathbf{0}, C_{\omega,n}^{-1}), \tag{29}$$

where $F_{\sigma,n}(L)$ and $F_{\omega,n}(L)$ are one-sided stable filters with sparse coefficients, the rows of which have a finite number of non-zero terms. All parameters in expressions (28) and (29) can be estimated as described in Section 3.2.

Inverting those auto-regressive representations, we obtain the VMA filters (see also expression (14))

$$\begin{aligned} D_{\sigma,n}(L) &:= F_{\sigma,n}(L)^{-1} R_{\sigma,n}, \\ D_{\omega,n}(L) &:= F_{\omega,n}(L)^{-1} R_{\omega,n}. \end{aligned} \tag{30}$$

Here, $R_{\sigma,n}$ and $R_{\omega,n}$ are $n\times n$ invertible matrices such that $R_{\sigma,n}^{-1}\boldsymbol{\nu}_{\sigma,n}$ and $R_{\omega,n}^{-1}\boldsymbol{\nu}_{\omega,n}$ are orthonormal. As explained in Section 2, we choose those matrices to be the Choleski factors of the covariances of the VAR shocks $\boldsymbol{\nu}_{\sigma,n}$ and $\boldsymbol{\nu}_{\omega,n}$, ordered according to their centrality in the PCN that is induced by $C_{\sigma,n}$ and $C_{\omega,n}$ (see also expressions (15)–(17)). From expression (30), and for a given horizon h, the LVDN weights computed from expression (4) then provide the shares of h-step-ahead forecast error variance for the idiosyncratic volatility of series i accounted for by innovations in the idiosyncratic volatility of series j. As already explained in Section 3.3, most of those weights are quite small, and naturally can be thresholded, yielding a sparse network.

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5. The network of S&P100-volatilities

In this section, we consider the panel of stocks that is used in the construction of the S&P100-index and, on the basis of the daily adjusted closing prices $\{p_{it} | \ i=1,\dots,n, \ t=1,\dots,T\}$, we compute the panel of percentage daily log-returns

$$r_{it} := 100 \log(p_{it}/p_{it-1}), \qquad i=1,\dots,n, \quad t=1,\dots,T,$$

which are our observed 'levels' or 'returns'. The observation period is $T=3457$ days, from January 3rd, 2000, to September 30th, 2013, and, since not all 100 constituents of the index were traded during the observation period, we end up with a panel of $n=90$ time series. A detailed list of the series that were considered is provided in the Web-based supporting materials. All results in this section are reported for the whole period 2000–2013, and for the period 2007–2008 corresponding to the Great Financial Crisis. Networks are represented by heat maps showing the entries of the corresponding adjacency matrices. In all those plots we highlight the energy and financial sectors which correspond to the index values 22–33 and 34–46 respectively. Moreover, all LVDNs considered report shocks' effects over $h=20$ periods, corresponding to a 1-month horizon (20 days plus the contemporaneous effects). Results for $h=5, 10$ are reported in the Web-based supporting materials.

As already explained, the Hallin and Liška (2007) criterion yields $q^T=1$, i.e. the level common component is driven by a one-dimensional common shock. By estimating expressions (20) and (21), we recover the estimated level common component $\boldsymbol{\chi}_n^T$, the rank 1 shocks $\boldsymbol{\eta}_n^T$ and the level idiosyncratic component $\boldsymbol{\xi}_n^T$. The contribution of the level common component to the total variation of returns, computed as the ratio between the sum of the (empirical) variances of the estimated level common components $\boldsymbol{\chi}_n^T$ to the sum of the (empirical) variances of the observed returns, is 0.36. Turning to the level idiosyncratic component $\boldsymbol{\xi}_n^T$, we recover an n-dimensional innovation vector $\mathbf{v}_n^T$ by fitting a sparse VAR.

The panels $\boldsymbol{\sigma}_n^T$ and $\boldsymbol{\omega}_n^T$ of level common and level idiosyncratic volatilities are constructed, as explained in expression (23), from the $\boldsymbol{\eta}_n^T$s and $\mathbf{v}_n^T$s; the centred panels $\mathring{\boldsymbol{\sigma}}_n^T$ and $\mathring{\boldsymbol{\sigma}}_n^T$ are obtained by subtracting the sample means. Applying the Hallin and Liška (2007) criterion again, we obtain $q_\sigma^T=q_\omega^T=1$ and, for the global panel, $q_{\sigma\omega}^T=1$. This implies a unique market shock, which is common to the two subpanels. We then compute the estimators $\boldsymbol{\chi}_{\sigma,n}^T$ and $\boldsymbol{\xi}_{\sigma,n}^T$ of the two level common volatility components of the GDFM decomposition (24), and the estimators $\boldsymbol{\chi}_{\omega,n}^T$ and $\boldsymbol{\xi}_{\omega,n}^T$ of the two level idiosyncratic volatility components of the GDFM decomposition (25). The overall contribution of the common components (driven by the unique market shock, and hence non-diversifiable) to the total variances of $\boldsymbol{\sigma}_n^T$ and $\boldsymbol{\omega}_n^T$ are 0.60 and 0.17 respectively.

5.1. The long-run variance decomposition network of volatility common components

We now turn to the LVDNs of the two common components $\boldsymbol{\chi}_{\sigma,n}^T$ and $\boldsymbol{\chi}_{\omega,n}^T$, given by expression (27). Since both panels are driven by the same unique common shock, the networks are identified once we impose a sign and a scale on the shocks. The sign is set in such a way that the sample correlation between the estimated market shock $\{\varepsilon_t^T\}$ and the cross-sectional average of all common components is positive. The scale is set in such a way that the following results represent the effect of 1-standard-deviation market volatility shock, i.e. the sequence of moving average loading coefficients is divided by the standard error of the market shock. We then obtain the estimated filters $B_{\sigma,n}^T(L)$ and $B_{\omega,n}^T(L)$.

Since LVDNs in this case are singular, we do not show a graph but rather report, in Table 1, for each panel $\boldsymbol{\chi}_{\sigma,n}^T$ and $\boldsymbol{\chi}_{\omega,n}^T$, the percentages of sectoral 20-step-ahead forecast error

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Table 1. Percentages of 20-step-ahead forecast error variances due to the market shock

Sector	*Results for 2000–2013*		*Results for 2007–2008*	
	$w^T_{\chi\sigma,j}$	$w^T_{\chi\omega,j}$	$w^T_{\chi\sigma,j}$	$w^T_{\chi\omega,j}$
Consumer discretionary	10.10	7.63	9.91	7.73
Consumer staples	10.38	10.70	9.83	10.45
Energy	9.86	13.35	9.83	27.04
Financial	10.12	13.66	10.61	18.19
Healthcare	9.92	8.84	9.77	6.24
Industrials	9.41	7.59	9.59	6.34
Information technology	10.01	10.00	10.11	3.76
Materials	9.92	6.77	10.05	9.53
Telecommunication services	10.33	9.80	10.29	5.79
Utilities	9.96	11.67	10.01	4.93
Total	100	100	100	100

variance accounted for by the unique market shock. More precisely, for the 10 sectors that were considered, the figures in Table 1 are the ratios

$$w^T_{\chi\sigma,j} := 100\frac{\sum_{i=1}^{n_j}\sum_{k=0}^{20}(b^T_{\sigma,k,i})^2}{\sum_{i=1}^{n}\sum_{k=0}^{20}(b^T_{\sigma,k,i})^2} \quad\text{and}\quad w^T_{\chi\omega,j} := 100\frac{\sum_{i=1}^{n_j}\sum_{k=0}^{20}(b^T_{\omega,k,i})^2}{\sum_{i=1}^{n}\sum_{k=0}^{20}(b^T_{\omega,k,i})^2}, \qquad j=1,\ldots,10,$$

where $b^T_{\sigma,k,i}$ and $b^T_{\omega,k,i}$ are the ith entries of $B^T_{\sigma,nk}$ and $B^T_{\omega,nk}$ respectively, and n_j is the number of stocks in sector j. As the single shock obviously explains all the variance of the common component, we normalize the figures in each column so that their sum equals 100.

In both periods that were considered, the common component of level common volatility is affected uniformly across all sectors by a market shock (the $w^T_{\chi\sigma,j}$-columns in Table 1), whereas the common components of level idiosyncratic volatilities exhibit some interesting intersectoral differences (the $w^T_{\chi\omega,j}$-columns in Table 1). In particular, when looking at the $w^T_{\chi\omega,j}$-column, the energy and financial sectors are the most affected. The effect of a market shock during the crisis is even heavier on those two sectors. We refer to Barigozzi and Hallin (2016) for further results on the volatility of common components.

5.2. The long-run variance decomposition network of the volatility idiosyncratic components

Turning to the idiosyncratic volatilities $\boldsymbol{\xi}^T_{\omega,n}$ and $\boldsymbol{\xi}^T_{\sigma,n}$, it appears that $\boldsymbol{\xi}^T_{\sigma,n}$ is essentially uncorrelated, both serially and cross-sectionally. Therefore, we focus only on the idiosyncratic volatility $\boldsymbol{\xi}^T_{\omega,n}$ of level idiosyncratic components. On the basis of a Bayesian information criterion, we estimate a sparse VAR(5) model for $\boldsymbol{\xi}_{\omega,n}$. The weight that is associated with the (i,j) edge of the LGCN of $\boldsymbol{\xi}^T_{\omega,n}$ then is the (i,j) entry of $F^T_{\omega,n}(1)=\Sigma_{k=0}^5 F^T_{\omega,nk}$.

We define the *edge density* of a network with set of edges $\mathcal{E}$ as the proportion $\#(\mathcal{E})/(n^2-n)$ of couples (i,j) in $\mathcal{E}$. When considering the elastic net estimation approach, we obtain, for the period 2000–2013, an LGCN with edge density 53% whereas, for the period 2007–2008,

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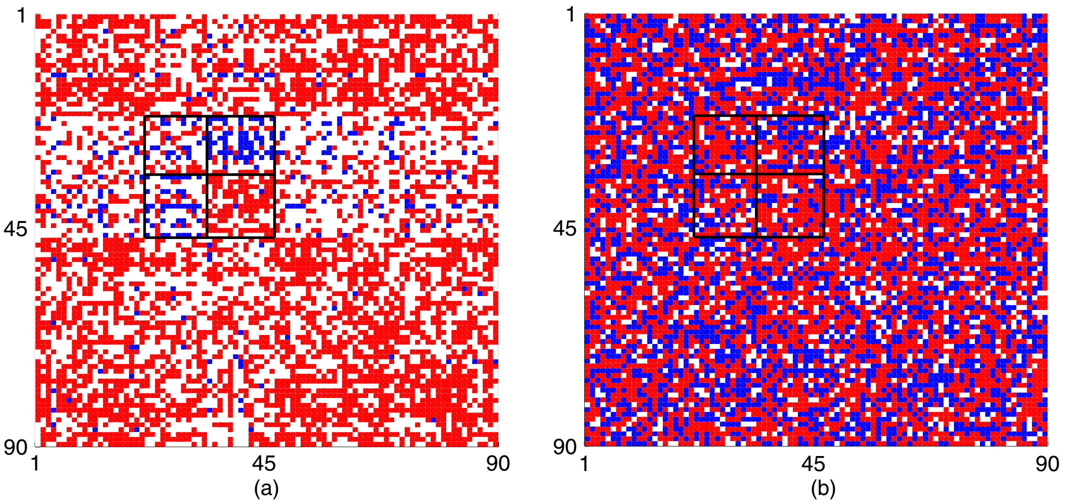

Fig. 2. LGCN for $\xi^T_{\omega,n}$ (■, negative weights; ■, positive weights): (a) 2000–2013; (b) 2007–2008

that density is close to 86%. The corresponding LGCNs are shown in Fig. 2. As expected, the LGCNs are much sparser when resulting from group lasso estimation, with densities 14% for the period 2000–2013, and 37% for the period 2007–2008. Nevertheless, since the subsequent results on LVDNs turn out to be qualitatively similar regardless of the way that we estimate the VAR, we show here only the elastic net results, deferring group and adaptive lasso results to the Web-based supporting materials.

As explained in the previous sections, identification of the LVDN requires some choices. The Choleski ordering is appealing, since it is data driven, but requires an ordering of the cross-sectional items (the stocks). Here, we use an identification method that is based on the partial correlation of the VAR residuals $\nu^T_{\omega,n}$. As explained in Section 2, we ordered the stocks according to the concept of eigenvector centrality for undirected networks (see Bonacich (1987)) in the

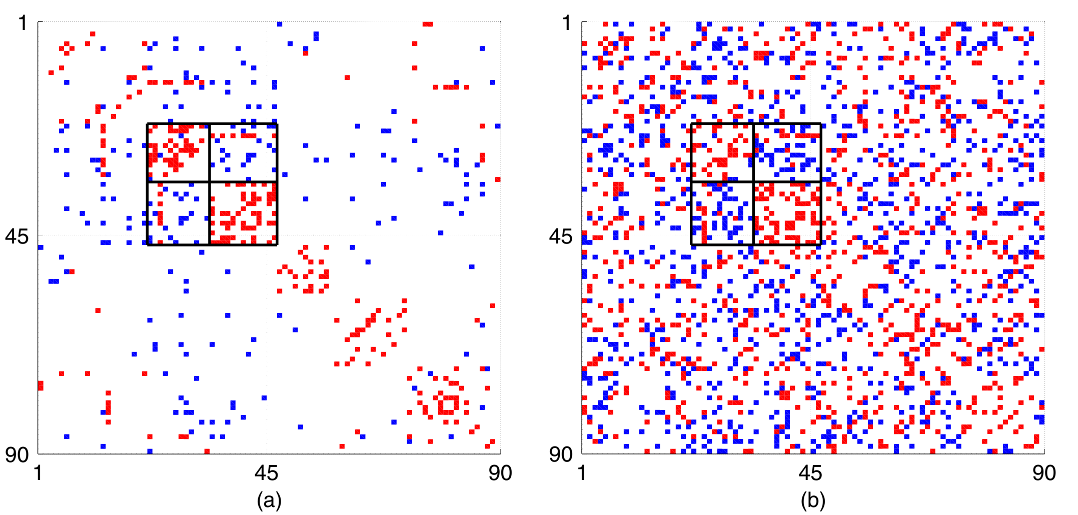

Fig. 3. PCN for $\nu^T_{\omega,n}$ (■, negative weights; ■, positive weights): (a) 2000–2013; (b) 2007–2008

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estimated PCN. The resulting network is shown in Fig. 3 for the two periods under study. The densities of those networks are 6% for the period 2000–2013 and 24% for the period 2007–2008. The 10 most central stocks are reported in Table 2. For results by using other identification methods we refer to the Web-based supporting material.

Summing up, from the estimation of the VAR model and the analysis of its residuals we see that

(a) the Great Financial Crisis has considerably blown up the dynamic interdependences between stocks and
(b) the energy and financial stocks appear as the most interconnected stocks, with more intrasectoral dependences rather than intersectoral.

The LVDN for $\boldsymbol{\xi}^T_{\omega,n}$ now can be computed on the basis of the ordering that is (partially) shown in Table 2. That identification defines (see expression (30)) the matrix $R^T_{\omega,n}$ as the Choleski factor of the sample covariance matrix of the ordered shocks. The weight of edge (i, j) of the $\boldsymbol{\xi}^T_{\omega,n}$ LVDN is

$$w^T_{\xi_\omega,ij} = 100 \frac{\sum_{k=0}^{20} (d^T_{\omega,k,ij})^2}{\sum_{j=1}^{n} \sum_{k=0}^{20} (d^T_{\omega,k,ij})^2}, \qquad i, j = 1, \ldots, n,$$

where $d^T_{\omega,k,ij}$ is the (i, j) entry of $D^T_{\omega,nk}$ such that $D^T_{\omega,n}(L) = \Sigma_{k=0}^{20} D^T_{\omega,nk} L^k$ as defined in expression (30). The resulting network is directed and weighted, but it is not sparse, meaning that in principle all its weights can be different from 0. Still, only a small proportion of edges has weights larger than 1, corresponding to a proportion of variance larger than 1%. In particular, out of the total number $(n(n-1) = 8010)$ of possible edges, only 0.4% have weights that are larger than 1% over the period 2000–2013, whereas this number increases considerably, up to 5%, during the period 2007–2008. Selected percentiles are given in Table 3.

Fig. 4 shows the $\boldsymbol{\xi}^T_{\omega,n}$ LVDN weights. Inspection reveals that LVDNs, although not sparse, have many entries that are close to 0. A thresholded version, as described in Section 3, is reported in Fig. 5. The resulting plots are highly sparse and indeed the optimal value of the thresholding parameter τ is found to be 1.17 for the period 2000–2013 and 1.87 for the period 2007–2008. Plots for other values of τ are provided in the Web-based supporting materials. Given the few remaining links, this is conveniently visualized, and the corresponding networks are

Table 2. Eigenvector centrality in the PCN for $\boldsymbol{\nu}^T_{\omega,n}$

2000–2013	*2007–2008*
JPM, JP Morgan Chase & Co.	BAC, Bank of America Corp.
C, Citigroup Inc.	USB, US Bancorp
BAC, Bank of America Corp.	JPM, JP Morgan Chase & Co.
APA, Apache Corp.	MS, Morgan Stanley
WFC, Wells Fargo	WFC, Wells Fargo
COP, Conoco Phillips	DVN, Devon Energy
OXY, Occidental Petroleum Corp.	GS, Goldman Sachs
DVN, Devon Energy	AXP, American Express Inc.
SLB, Schlumberger	COF, Capital One Financial Corp.
MS, Morgan Stanley	UNH, United Health Group Inc.

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Table 3. Selected percentiles of $\xi^T_{\omega,n}$ LVDN weights

Percentiles	*Results for 2000–2013*	*Results for 2007–2008*
50th	0.02	0.17
90th	0.13	0.71
95th	0.20	1.00
97.5th	0.29	1.28
99th	0.48	1.76
Maximum	4.29	4.53

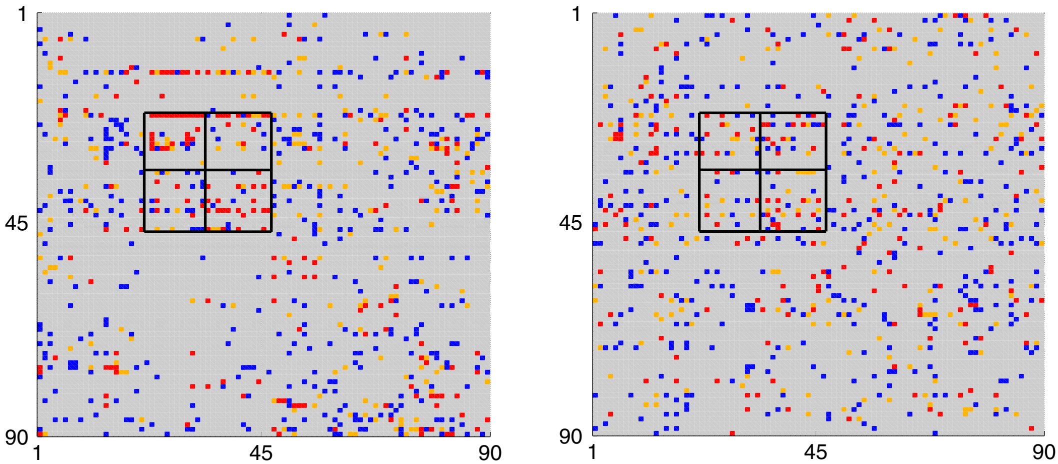

Fig. 4. LVDN for $\xi^T_{\omega,n}$, weights below the 95th percentile (■), between the 95th and 97.5th percentiles (■) between the 97.5th and 99th percentiles (■), and above the 99th percentile (■): (a) 2000–2013; (b) 2007–2008

shown in Fig. 1. Note that financial (yellow nodes) and energy (blue nodes) stocks are the most interconnected. When considering the whole sample, there are almost no intersectoral links; however, during the Great Financial Crisis, the degree of interconnectedness of the financial sector with other sectors quite dramatically increases.

Those findings can be quantified by computing from- and to-degrees, as defined in expression (6). As explained in Section 2, the from-degree measures the exposure of a given firm to shocks coming from all other firms, whereas the to-degree measures the effect of a shock to a given firm on all others. In Table 4, we show sectoral averages of from- and to-degrees when computed both for the non-sparse and the thresholded LVDNs. We also report sectoral total connectedness and overall total connectedness as defined in expression (7). Finally, an overall measure of how systemic is an institution is given by its centrality in the network. Table 5 shows, for the two periods that were considered, the rankings of firms according to a measure of eigenvector centrality adapted to weighted directed networks (see Bonacich and Lloyd (2001)). For the period 2000–2013, only very few stocks are connected and qualify as central.

All results amply demonstrate that the Great Financial Crisis quite spectacularly tightened up the links between firms. In particular, when considering the thresholded LVDN version, the

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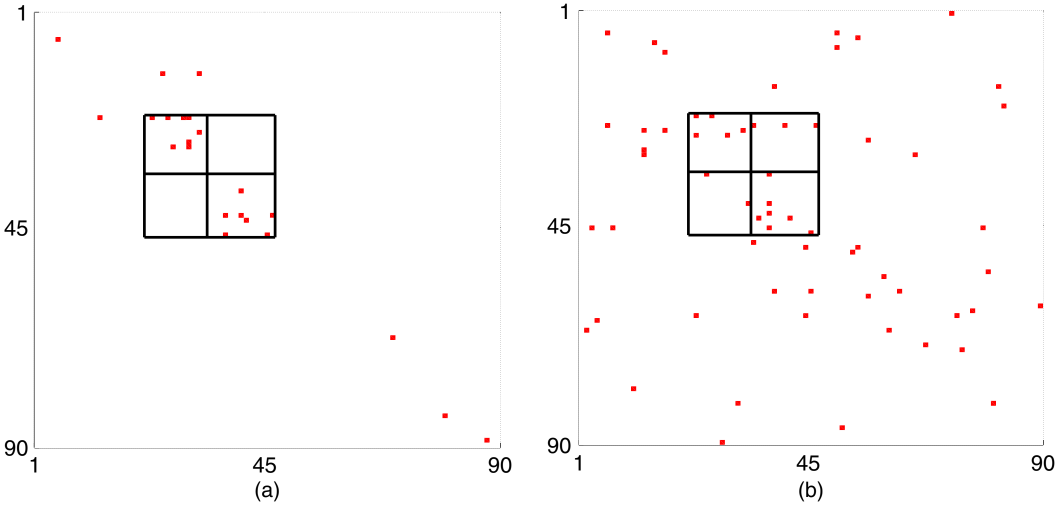

Fig. 5. Thresholded LVDN for $\xi^T_{\omega,n}$ (■, non-zero weights): (a) 2000–2013; (b) 2007–2008

Table 4. From- and to-degree sectoral averages in the $\xi^T_{\omega,n}$ LVDN

Sector	*Non-thresholded averages*				*Thresholded averages*			
	2000–2013		*2007–2008*		*2000–2013*		*2007–2008*	
	From	*To*	*From*	*To*	*From*	*To*	*From*	*To*
Consumer discretionary	4.32	2.37	26.31	26.41	0.24	0.24	1.96	3.29
Consumer staples	3.98	4.65	27.47	22.65	0.00	0.44	2.80	1.36
Energy	5.52	7.92	21.91	33.72	1.90	1.54	4.01	6.10
Financial	4.74	6.22	24.42	35.56	0.77	0.77	5.23	6.69
Healthcare	5.00	2.51	28.06	22.36	0.00	0.00	4.27	1.83
Industrials	4.43	3.21	27.26	25.81	0.21	0.21	2.43	3.53
Information technology	5.03	4.89	29.90	19.98	0.00	0.00	3.35	1.58
Materials	3.24	4.62	26.86	27.01	0.00	0.00	2.99	0.81
Telecommunications services	6.50	7.26	27.44	16.52	1.91	1.91	0.00	0.00
Utilities	5.15	8.74	29.49	21.54	0.00	0.00	4.94	2.38
Total degree	4.73		26.54		0.47		3.40	

shocks to financial stocks still account for almost 7% of the total idiosyncratic variance during the crisis, whereas shocks from other sectors explain about 5% of the financial intrasectoral variability (see the last two columns of Table 4). The financial sector appears to be the most central in the network, the most vulnerable to shocks coming from all other sectors and the most systemic in the sense that a shock to the financial stocks is most likely to affect the whole panel strongly.

A few more comments are in order in relation to the results in the Web-based supporting materials. First, when comparing the results for $h=20$ with those obtained at shorter horizons ($h=5, 10$), we see that connectivity increases considerably with the forecast horizon, indicating that the effect of a shock keeps on propagating for a long time. This result is consistent with the fact that volatilities in financial data, although stationary, tend to have long memory (see

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Table 5. Eigenvector centrality in the $\xi^T_{\omega,n}$ thresholded LVDN

2000–2013	*2007–2008*
BAC, Bank of America Corp.	USB, US Bancorp
JPM, JP Morgan Chase & Co.	BAC, Bank of America Corp.
WFC, Wells Fargo	COF, Capital One Financial Corp.
C, Citigroup Inc.	AIG, American International Group Inc.
USB, US Bancorp	C, Citigroup Inc.
APA, Apache Corp.	WFC, Wells Fargo
SLB, Schlumberger	BA, Boeing Co.
COP, Conoco Phillips	CVX, Chevron
—	MCD, McDonalds Corp.
—	IBM, International Business Machines

Barigozzi and Hallin (2016) for a detailed analysis of this aspect). Second, group lasso and elastic net VAR estimation yield qualitatively similar results. However, despite sparser LGCNs, the group lasso yields estimated LVDNs that are more tightly connected than those resulting from the elastic net. This is particularly true during the crisis period when financial and energy stocks are the most central, but also the healthcare sector seems to have a decisive role.

To conclude, we provide some empirical justification of the factor plus sparse VAR approach that is adopted in this paper by comparing the conditional dependences in the volatility panel $\boldsymbol{\omega}^T_n$ with those of its idiosyncratic component $\boldsymbol{\xi}^T_{\omega;n}$. To do this, we consider *partial spectral coherence* (*PSC*), which is the analogue of partial correlation, but in the spectral domain, and is strictly related to the coefficients of a VAR representation (see Davis *et al.* (2015) for a definition).

In line with the long-run spirit of the LVDN definition, and since volatilities have strong persistence, we first consider the PSC values at frequency $\theta = 0$, thus looking at long-run conditional dependences. Selected percentiles of the distributions of the absolute value of the PSC entries for $\boldsymbol{\omega}^T_n$ and $\boldsymbol{\xi}^T_{\omega;n}$ and the distribution of the absolute value of their differences are shown in Table 6. Both PSC values have many small (in absolute value) entries, which is consistent with our sparsity assumptions.

The two PSCs are shown in Figs 6(a) and 6(b) whereas in Fig. 6(c) we show the absolute values of their differences. Inspection of these results clearly indicates that

(a) after removal of the market shock the idiosyncratic component still contains important dependences and

Table 6. Distribution of absolute value of PSC entries

Percentile	$\lvert PSC_{\omega^T_n}(\theta=0)\rvert$	$\lvert PSC_{\xi^T_{\omega;n}}(\theta=0)\rvert$	$\lvert PSC_{\omega^T_n}(\theta=0) - PSC_{\xi^T_{\omega;n}}(\theta=0)\rvert$
50th	0.06	0.06	0.02
90th	0.16	0.16	0.06
95th	0.18	0.19	0.08
97.5th	0.22	0.22	0.12
99th	0.25	0.25	0.15
Maximum	0.35	0.34	0.24

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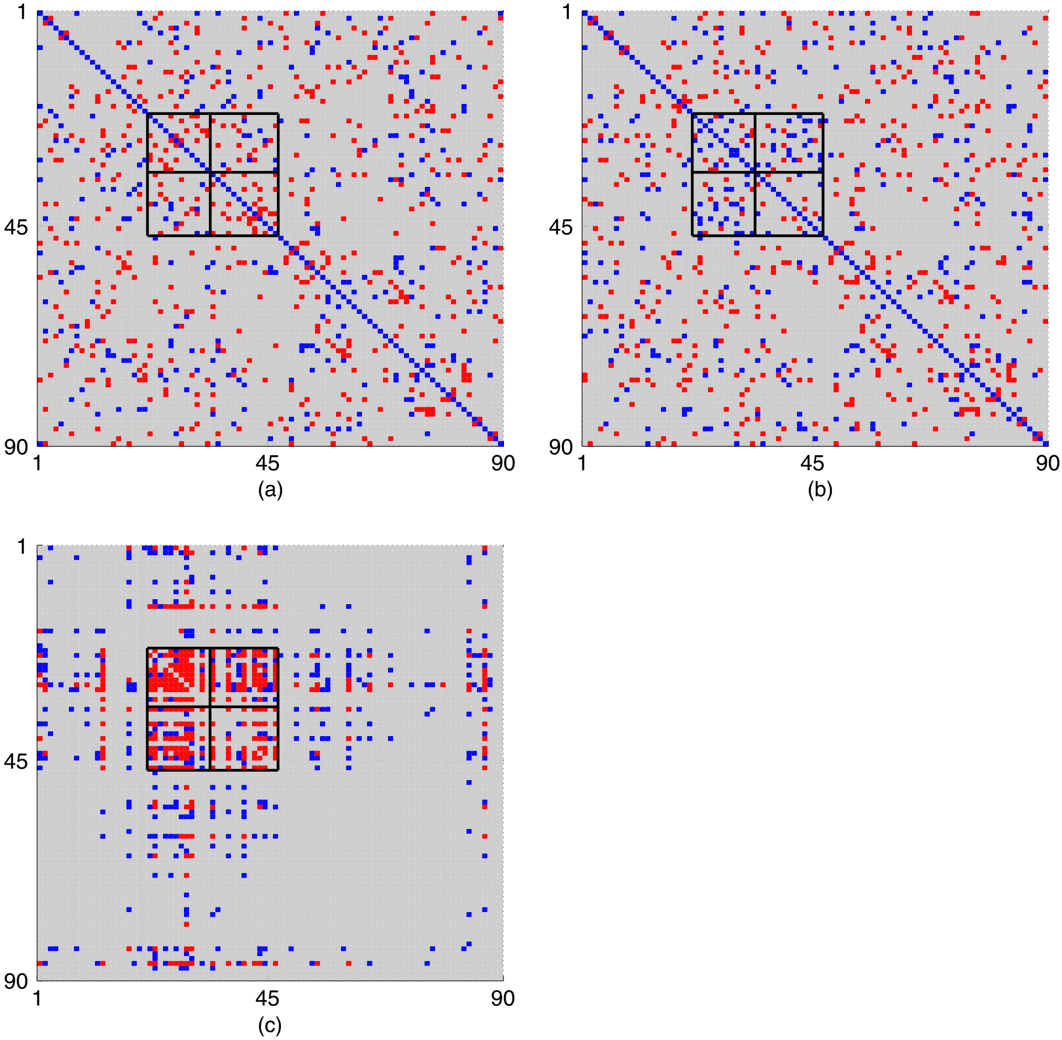

Fig. 6. PSCs at frequency $\theta=0$: (a) $\mathrm{PSC}_{\omega_n^T}(\theta=0)$ (■, weights in absolute values below the 90th percentile; ■, weights above the 90th percentile; ■, weights below the 10th weights percentile); (b) $\mathrm{PSC}_{\xi_{\omega;n}^T}(\theta=0)$ (■, weights in absolute values below the 90th percentile; ■, weights above the 90th percentile; ■, weights below the 10th percentile); (c) $|\mathrm{PSC}_{\omega_n^T}(\theta=0)-\mathrm{PSC}_{\xi_{\omega;n}^T}(\theta=0)|$ (■, weights below the 90th percentile; ■, weights between the 90th and 95th percentiles; ■, weights above the 95th percentile)

(b) an important benefit of our factor plus sparse VAR approach is to uncover the hidden dependences between and within the financial and energy sectors.

Moreover, when repeating the same analysis at other frequencies (e.g. $\theta=\pi/2,\ \pi$), no significant difference emerges between the two PSCs; the benefits of our approach thus are relevant mainly in the long run.

Similar conclusions can be derived by considering spectral densities (at different frequencies) and the squared partial spectral coherence (averaged over all frequencies), which is a measure which is non-zero if and only if two series are uncorrelated at all leads and lags after taking

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into account the (linear) effects of all other series in the panel. Results are in the Web-based supporting materials.

6. Conclusions

In this paper, we extend the study of interconnectedness of volatility panels that was initiated by Diebold and Yilmaz (2014) to the high dimensional setting where a factor structure can be assumed for the data. We determine and quantify the various sources of variation driving a panel of volatilities of S&P100-stocks over the period 2000–2013. Our analysis highlights the key role of the financial sector, which appears to be particularly important during the Great Financial Crisis. Other sectors such as energy, and in some cases also healthcare, seem to have an important role also. Moreover, we show that, contrary to a direct sparse VAR approach, our factor plus sparse VAR method can unveil crucial intersectoral dependences, which can be of tantamount importance for investors' decisions in the context of risk management.

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