

Critical Thinking (Math 110) Fall 2022 Schedule

MWF 11:30 am to 12:20pm in VEP 5107

September 2022						
◀ August						October ▶
Sun	Mon	Tue	Wed	Thu	Fri	Sat
	Aug. 29 First Day of Classes Introductions and expectations What is critical thinking?	Aug. 30	Aug. 31 What is critical thinking? Inductive and deductive argument	1	2 Mathematical logic/thinking Read pages 1-11 before class	3
4	5 Labor Day, no class	6	7 Mathematical logic/thinking Read chapter 1 and 2.1 (pages 12-28) before class	8	9 Mathematical logic/thinking Read chapter 2.2 through exercises 2.2.2 (pages 28-32) before class	10
11 Homework 1 Due at 11:59 pm on Gradescope	12 Mathematical logic/thinking Finish chapter 2.2 (pages 32-36) before class	13	14 Mathematical logic/thinking Read chapter 2.3 through Exercises 2.3.1 (pages 36-38) before class	15	16 Mathematical logic/thinking Read this webpage to the integral sign. Skim 2.3 and 2.4 of Devlin pay attention to <i>equivalence, necessary, sufficient, there Exists, and for All</i>	17
18	19 Mathematical logic/thinking Read 3.1 and 3.2 before class	20	21 Mathematical logic/thinking Read 3.3 and 3.4 before class	22	23 Essay Outline Due Monday Mathematical logic/thinking Read 3.5 and try to understand induction before class	24
25 Homework 2 Due at 11:59 pm on Gradescope	26 Essay Outline Due in class: "Compare and contrast mathematical argument and 'popular argument.'" Workshop Essays	27	28 Essay writing in class	29	30 Essay Draft Due in class: "Compare and contrast mathematical argument and 'popular argument.'" Workshop Essays	

October 2022

◀ September

November ▶

Sun	Mon	Tue	Wed	Thu	Fri	Sat
	Sept. 26 Essay Outline Due in class: “Compare and contrast mathematical argument and ‘conventional argument.’” Workshopping Essays		Sept. 28 Essay writing in class		Sept. 30 Essay Draft Due in class: “Compare and contrast mathematical argument and ‘conventional argument.’” Workshopping Essays	1
2	3 Essay Due: “Compare and contrast mathematical argument and ‘conventional argument.’” <u>Weapons of Math Destruction/Algorithms of Oppression</u>	4	5 <u>Weapons of Math Destruction (scan)</u> Read Introduction and Chapter 1 before class Go to <u>Library Reserves</u> to access full text online	6	7 <u>Weapons of Math Destruction</u> Read Chapter 2 before class Go to <u>Library Reserves</u> to access full text online	8
9	10 <u>Algorithms of Oppression</u> Read Introduction and Chapter 1 before class Go to <u>Library Reserves</u> to access full text online	11	12 <u>Algorithms of Oppression</u> Reread Introduction and Chapter 1 before class Go to <u>Library Reserves</u> to access full text online	13	14 Perspectives on algorithms: <u>Read the Atlantic article After Babel</u> by Jonathan Haidt before class.	15
16	17 <u>Intro to Number Theory</u> Read Chapter 1 of before class of <u>A Friendly Introduction to Number Theory</u> before class.	18	19 <u>Intro to Number Theory</u> Read Chapter 2 of before class of <u>A Friendly Introduction to Number Theory</u> before class.	20	21 <u>Intro to Number Theory</u>	22
23 Mini-Homework 2.5 Due at 11:59 pm on Gradescope	24 Essay Due: “How has mathematics been used on you and how can you respond to it?” You must cite at least two of our readings for evidence. Intro to Number Theory	25	26 Intro to Number Theory	27	28 Intro to Number Theory	29
30 Homework 3 Due at 11:59 pm on Gradescope	31 Intro to Number Theory 					

November 2022						
◀ October						December ▶
Sun	Mon	Tue	Wed	Thu	Fri	Sat
	Oct. 31 Intro to Number Theory	1	2 Intro to Number Theory	3	4 Intro to Number Theory: Division theorem and Euclid's algorithm.	5
6	7 Midterm Intro to Number Theory	8	9 Intro to Number Theory: Proving fundamental theorem of arithmetic	10	11 Veterans Day, no class	12
13	14 Euclid's <i>Elements</i> and geometry: Intro and Proposition I.	15	16 Euclid's <i>Elements</i> and geometry: Euclid's Lemma. Midterm Correction Due (11:59 pm Gradescope)	17	18 Euclid's <i>Elements</i> and geometry: Infinitude of the primes.	19
20 Homework 4 Due (11:59 pm Gradescope)	21 Euclid's <i>Elements</i> and geometry: The Pythagorean theorem.	22	23 Asynchronous Class. Watch <i>The Proof</i> , a documentary about the proof of Fermat's Last Theorem	24 Thanksgiving holiday, no class	25 Thanksgiving holiday, no class	26
27	28 Midterm 2 Euclid's <i>Elements</i> and geometry	29	30 Plato's <i>The Allegory of the Cave</i>			

December 2022						
◀ November						January ▶
Sun	Mon	Tue	Wed	Thu	Fri	Sat
	Nov. 28 Midterm 2 Euclid's <i>Elements</i> and geometry		Nov. 30 Plato's <i>Meno</i>	1	2 Plato's <i>The Allegory of the Cave</i>	3
4	5 Descartes's <i>Discourse on Method</i>	6	7 <u>The Declaration of Independence</u>	8	9 Last day of classes What does mathematics look like?	10
11	12 Exam Week	13 Exam Week	14 Exam Week Essay Due: "Compare and contrast the axiomatic approaches of Euclid's <i>Elements</i> and of the Declaration of Independence. How do the axiomatic approaches of these documents affect our world today?"	15 Exam Week	16 Exam Week	17 Exam Week
18	19	20	21	22 Grades due from instructors	23	24
25	26	27	28	29	30	31

Critical Thinking Learning Outcomes

- **A3.1: Distinguish matters of fact from issues of judgment or opinion and derive factual or judgmental inferences from unambiguous statements of knowledge or belief.**
- **A3.2: Judge the reliability and credibility of sources.**
- **A3.3: Effectively argue a point of view by clarifying the issues, focusing on the pertinent issues, and staying relevant to the topic.**
- **A3.4: Understand the nature of inductive and deductive reasoning, identify formal and informal fallacies of reasoning, and employ various methods for testing the strength, soundness, and validity of different argument forms.**
- **A3.5: Understand the basic concepts of meaning (sense, reference, connotation, etc.) and identify different methods of word definition.**
- **A3.6: Understand logic and its relationship to language by identifying the basic components of reasoning, including the propositional content of statements, the functions of premises and conclusions in the makeup of arguments, the linkage between evidence and inference, and the rules of inference and logical equivalence.**

Math-Specific Learning Outcome

The critical thinking learning outcomes are involved and occupy a large portion of the course. For this reason, there is only one math-specific learning outcomes.

- Develop a general familiarity with elements of mathematical language and argument in the context of number theory and geometry. (Number theory and geometry are merely here to provide a context and meaning for this learning. The mathematical ideas that students learn will be fundamental and widely applicable in other mathematical fields and STEM disciplines.)

Further mathematical motivation: Students will develop a big-picture understanding of the dynamic landscape of mathematics. At the end of the course, students will know there are a myriad of mathematical research areas and questions that are being investigated by a wide variety of people from different cultures and backgrounds.

Note: There appears to be a large degree of ambiguity in the literature regarding the definitions of *learning outcomes*, *learning goals*, and *learning objectives*. In mathematics precise terminology is of the utmost importance, so my training leads me to have an aversion to poorly defined terms. We adopt the following terminology: **Learning outcomes** and **learning goals** are synonymous. They are the large-scale goals/outcomes that diligent students should expect to have made progress toward after completing the course. They are big-picture goals and as such can be hard to assess or achieve in an absolute sense. For example, distinguishing matters of fact from issues of judgment or opinion is not a skill that can be mastered in a semester. This is something that learned people spend a lifetime working toward and improving upon. However, marked improvement in this skill after a semester means that a student has met this learning outcome. By contrast, **learning objectives** are smaller-scale, concrete, and measurable aspects of a course that build towards meeting learning outcomes. For example, writing an essay comparing and contrasting mathematical argument and “conventional argument” is an immediate and readily attainable objective for a student. In turn, one can see how a student’s essay addressing this prompt helps them understand logic and its relationship to language. Thus, the small-scale objectives come together to serve the large-scale learning goals. Lastly, we should mention that objectives and assignments are quite similar. In the example given above, the assignment (the essay) and the objective (the student addressing the prompt of the essay) are nearly synonymous. Prosaically, the objectives are the verbiage required to justify how each assignment is helping meet all the learning outcomes it addresses.

Outcomes and course design: Outcomes A3.4, A3.5, and A3.6 will be confronted extensively in the first four weeks of the course from the standpoint of mathematics. However, the text used for this portion of the course engages with mathematical thinking by contrast with “conventional” (often inductive and anecdotal) reasoning.

Thus, outcomes A3.1-A3.3 will be addressed in a secondary manner in the first four weeks of the course. Week five will be focused on writing an essay addressing the following prompt: “Compare and contrast mathematical argument and ‘conventional argument.’” This will sharpen student’s understanding of logic, reasoning, and meaning (A3.4-6) while simultaneously asking them to define critical thinking in contrast to more passive, everyday “thinking.”

With weeks six and seven, the course will pivot to looking at various critiques, condemnations, and celebrations of current corporate manifestations of mathematical thinking. With their understanding of mathematical thinking well-developed from the first five weeks of the course, students will have to engage with these sources to separate matters of fact from opinion (A3.1), evaluate the reliability of the evidence presented (A3.2), and argue their own thesis (A2.2) answering, “How has mathematics been used on you and how can you respond to it?” Students will be prepared for this task not only by reading the material, but by in-class discussions with their peers.

Weeks eight through ten will be devoted to an introduction to number theory. This more traditional mathematical material will build towards outcome A3.4-A3.6 while also giving students skills that will help them in future STEM courses. Students will demonstrate these skills with an in-class midterm.

After the midterm, we will shift to Euclid’s *Elements*. This text is one of the pillars on which modern mathematics is built; however, it is also over 2,000 years old and does not always meet the standards of modern mathematical reasoning. We will read and discuss this text critically in class so as to simultaneously understand the mathematical ideas and tools Euclid is building while also being skeptical of the work. This critical discussion of a fundamental historical math text is an objective that naturally builds toward all six learning outcomes (A3.1-A3.6) with a special emphasis toward the first three outcomes (A3.1-A3.3). It is a daunting, but immensely rewarding task to use one’s budding mathematical skills to critique an ancient text that is one of the pillars of Western thought.

After three weeks of Euclid, we will spend weeks 14 and 15 analyzing how the *Elements* has impacted Western thought by reading Plato, Descartes, and the Declaration of Independence. These two weeks will help students develop the ideas to write the final essay of the course where they will answer how the axiomatic approach of Euclid has affected our world today. As objectives, the discussions of these final two weeks blend skills that are requisite for all six learning outcomes (A3.1-A3.6) for the course. Moreover, students will produce a final essay that shows a high level of mastery of and significant progress toward all six learning outcomes. This essay will simultaneously allow

students to create an authentic definition of ‘critical thinking’ in comparison with Western thought. Thus, there is an important aspect of meta-learning whereby students must think critically in their own context in order to deconstruct the westernized notion of what constitutes “critical thinking.”

In this course, our taxonomy of learning will most often take the following shape: Students first confront a given idea or concept by reading it on their own. When they come to class, they will have identified this concept. Through in-class discussions and presentations, they will not only understand, but analyze this concept. These discussions will ideally culminate with students comparing the concept with others and integrating it into their own mental framework of the topic we are studying. Finally, in an essay, students will create their own arguments critiquing and judging the concept and produce their own synthesis of the relevant topic.

The taxonomy of the pure mathematical topics of the course will follow a slightly different framework.

Grading

Homework	15%
Geometry Quiz	10%
Midterm	25%
Essay 1	15%
Essay 2	15%
Essay 3	20%

Course Description from Catalog:

Critical thinking in decision-making. Formal and informal fallacies of language and thought; the often unreliable guide of common-sense reasoning; analysis and criticism of ideas; distinction between fact and judgment, belief and knowledge; inductive and deductive arguments; and effective techniques of decision-making. Students will learn critical thinking skills to apply to common issues of everyday life.