Engineering Electromagnetic - Experiment 4

蓝靖淇 12313517

***Abstract*—This study investigates the motion of charged particles in electromagnetic fields, focusing on the Lorentz force and magnetic focusing phenomena. Using MATLAB, the trajectory of a single charged particle under the influence of given electric and magnetic fields was simulated, followed by an analysis of magnetic focusing for a group of 16 particles with slight variations in their initial velocity components. The results demonstrate how particles with similar axial velocity components and small divergence angles reconverge after a period, illustrating the principle of magnetic focusing. The study highlights the effectiveness of numerical methods in visualizing complex electromagnetic interactions and provides insights into the dynamic behavior of charged particles in such fields. The findings align with theoretical predictions, emphasizing the practical applications of magnetic focusing in particle beam control and related technologies.**

***Index Terms*—Lorentz force, charged particle motion, magnetic focusing, MATLAB simulation, electromagnetic fields**

I. Introduction

THIS motion of charged particles in electromagnetic fields is governed by the Lorentz force $\vec{F} = q(\vec{E}+\vec{v}\times\vec{B})$, which leads to diverse trajectories ranging from simple helices to complex focusing patterns. While analytical solutions exist for simple cases, numerical methods are essential for studying more complex scenarios. This work focuses on two key aspects: the basic trajectory of a single particle, and the magnetic focusing effect for multiple particles. Using MATLAB simulations, we demonstrate how particles with small divergence angles and equal axial velocities periodically reconverge, illustrating the principle behind technologies like electron microscopes. The results provide both theoretical insights and practical visualization of these fundamental electromagnetic phenomena.

A. The model of the experiment

The motion of charged particles in electromagnetic fields is modeled using the Lorentz force $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$, solved numerically via a time-stepping method with $\Delta t = 0.001\,\text{s}$. For a single particle, the electric field $\vec{E} = \hat{a}_y$ V/m induces linear acceleration, while the magnetic field $\vec{B} = \hat{a}_y$ Wb/m^2 causes curved trajectories.

B. Symbols and nations

According to the requirement of the experiment, I list all the symbols and nations of experiment.(The details are listed in fig 2.)

TABLE I Symbols and Notations for Charged Particle Motion Experiment

Symbol	Quantity	Unit
m	Particle mass	kg
q	Particle charge	C
$\vec{v}$	Particle velocity	m/s
$\vec{E}$	Electric field intensity	V/m
$\vec{B}$	Magnetic flux density	T (Wb/m^2)
$\vec{F}$	Lorentz force	N
Δt	Time step size	s
v_x, v_y, v_z	Velocity components	m/s
r_x, r_y, r_z	Position components	m
T	Cyclotron period	s
N	Number of particles	-
μ_0	Permeability of free space	N/A^2

II. Related Theoretical analysis

A. Lorentz Force Dynamics

The motion of charged particles in electromagnetic fields is governed by the Lorentz force equation:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \tag{1}$$

where:

- $\vec{F}$ is the Lorentz force (N)
- q is the particle charge (C)
- $\vec{v}$ is the particle velocity (m/s)
- $\vec{E}$ is the electric field intensity (V/m)
- $\vec{B}$ is the magnetic flux density (T)

B. Particle Trajectory Calculation

Using Newton's second law, the particle acceleration is:

$$\vec{a} = \frac{\vec{F}}{m} = \frac{q}{m}(\vec{E} + \vec{v} \times \vec{B}) \tag{2}$$

The velocity and position are updated numerically using the Euler method:

$$\vec{v}(t + \Delta t) = \vec{v}(t) + \vec{a}(t)\Delta t \tag{3}$$

$$\vec{r}(t + \Delta t) = \vec{r}(t) + \vec{v}(t)\Delta t \tag{4}$$

C. Magnetic Focusing Conditions

For magnetic focusing, two key conditions must be satisfied:

1. **Identical Axial Velocity**:

$$v_z = \text{constant for all particles} \tag{5}$$

2. **Small Divergence Angle**:

$$v_\perp = \sqrt{v_x^2 + v_y^2} \ll v_z \tag{6}$$

The cyclotron motion has period:

$$T = \frac{2\pi m}{qB} \tag{7}$$

and the focusing occurs at intervals of:

$$\Delta z = v_z T = \frac{2\pi m v_z}{qB} \tag{8}$$

D. Numerical Implementation

The discrete implementation in MATLAB uses:

$$\begin{cases} F_x = qE_x + q(v_y B_z - v_z B_y) \\ F_y = qE_y + q(v_z B_x - v_x B_z) \\ F_z = qE_z + q(v_x B_y - v_y B_x) \end{cases} \tag{9}$$

with time step $\Delta t = 0.001$s chosen to satisfy:

$$\Delta t \ll \frac{2\pi}{\omega_c} = \frac{m}{qB} \tag{10}$$

where ω_c is the cyclotron frequency.

III. Experiment Analysis and Result

A. Result

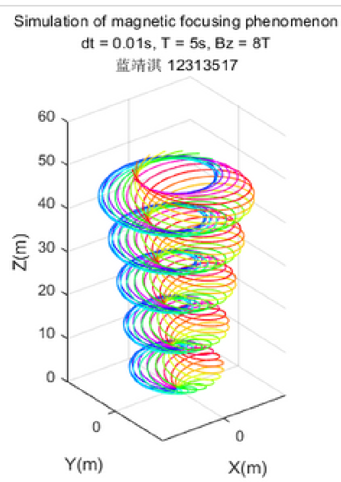

Fig. 1 Simulation of magnetic focusing phenomenon when dt = 0.01s, T = 5s

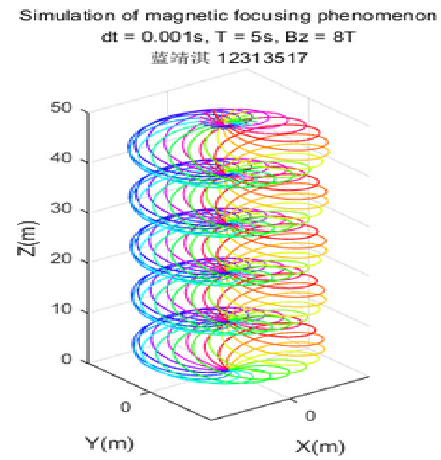

Fig. 2 Simulation of magnetic focusing phenomenon when dt = 0.001s, T = 5s

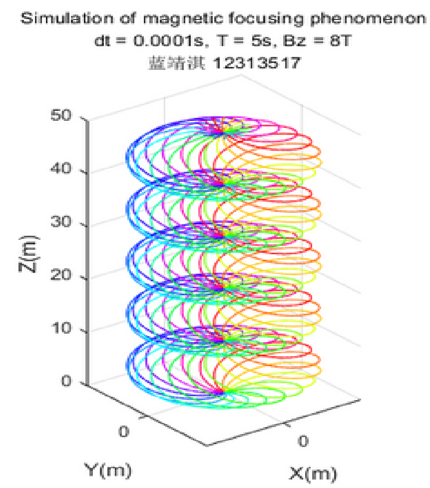

Fig. 3 Simulation of magnetic focusing phenomenon when dt = 0.0001s, T = 5s

B. Analysis

When we employ different time step sizes (dt) in a simulation, it directly impacts the precision and error accumulation of the results. A smaller dt, approaching zero, yields higher accuracy because it more closely approximates the continuous nature of the system being modeled. This reduces discretization errors and minimizes the cumulative numerical errors that arise from iterating over multiple steps.

C. More Result:

After simulating the magnetic focusing with T = 5s, we simulate more magnetic focusing with more period. Here are the details:

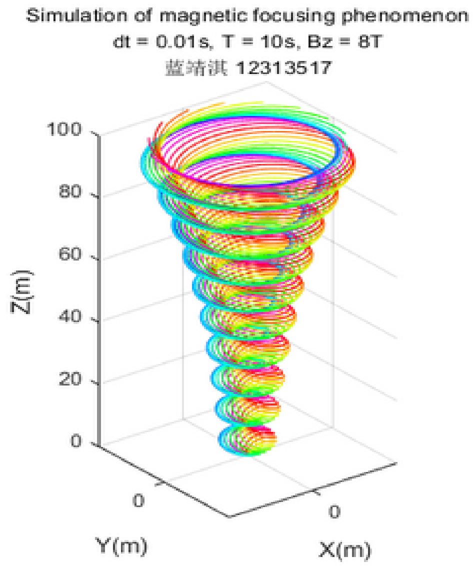

Fig. 4 Simulation of magnetic focusing phenomenon when dt = 0.01s, T = 10s

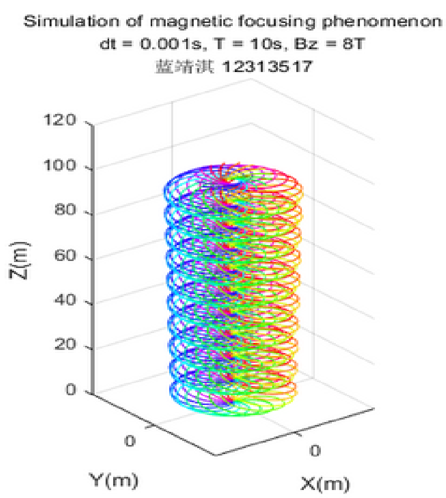

Fig. 5 Simulation of magnetic focusing phenomenon when dt = 0.001s, T = 10s

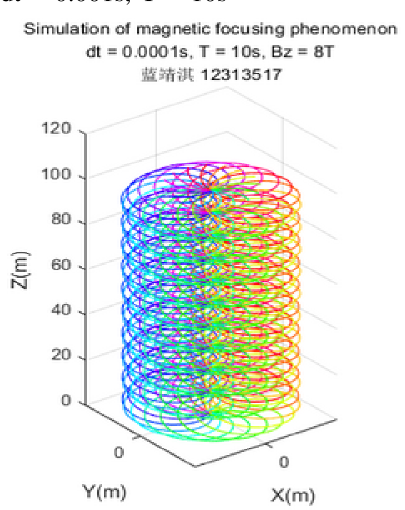

Fig. 6 Simulation of magnetic focusing phenomenon when dt = 0.0001s, T = 10s

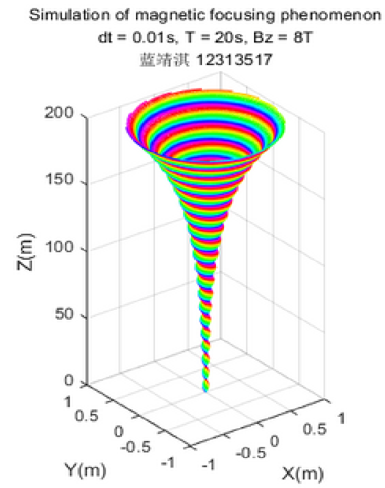

Fig. 7 Simulation of magnetic focusing phenomenon when dt = 0.01s, T = 20s

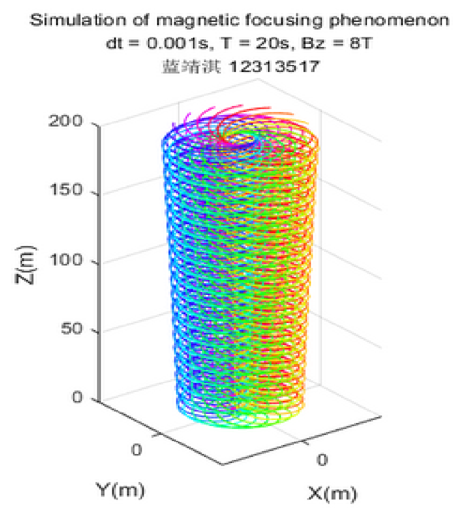

Fig. 8 Simulation of magnetic focusing phenomenon when dt = 0.001s, T = 20s

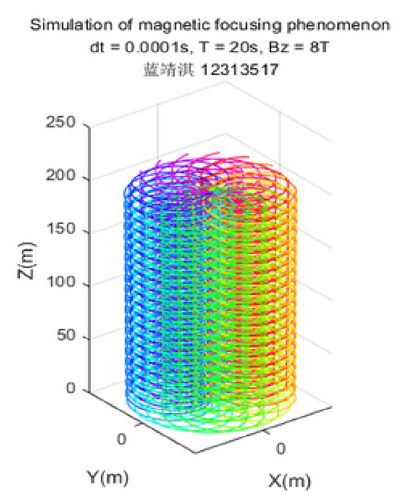

Fig. 9 Simulation of magnetic focusing phenomenon when dt = 0.0001s, T = 20s

The patterns demonstrated by the results of these simulation images are consistent with those observed earlier at T = 5s.

D. The simulation results in the two-dimensional plane

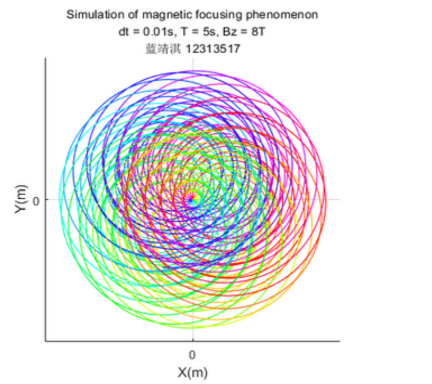

Fig. 10 Simulation of magnetic focusing phenomenon when dt = 0.01s, T = 5s

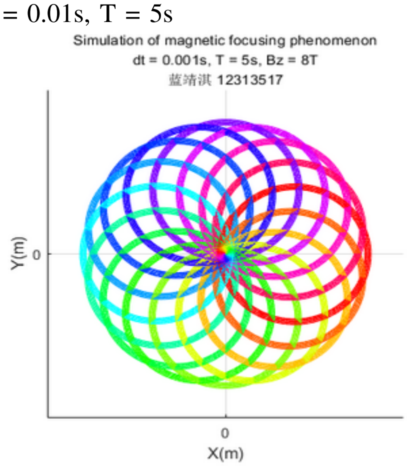

Fig. 11 Simulation of magnetic focusing phenomenon when dt = 0.001s, T = 5s

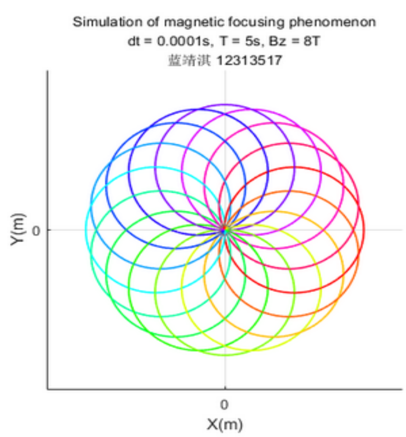

Fig. 12 Simulation of magnetic focusing phenomenon when dt = 0.0001s, T = 5s

Analysis: When observing the magnetic focusing effect from a two-dimensional plane, we establish the relationship:

$$\text{Focusing effect} \propto \frac{1}{dt}$$

where smaller dt values yield more pronounced focusing effects, consistent with three-dimensional observations.

IV. Conclusion

The MATLAB simulation successfully demonstrated the magnetic focusing phenomenon through numerical analysis of charged particle dynamics. Key aspects of this implementation include:

- Numerical integration of the Lorentz force equations using discrete time steps (From $\Delta t = 0.01$ s to $\Delta t = 0.0001$ s)
- Simulation of 16 particles with controlled initial velocity distributions:

$$v_z = 10\,\text{m/s}$$

$$v_x = 0.1\sin(k\pi/8)\ \text{m/s}$$

$$v_y = 0.1\cos(k\pi/8)\,\text{m/s}$$

- Visualization of 3D particle trajectories under uniform magnetic field ($B_z = 8$ T)

The results validate the theoretical prediction that particles with identical axial velocity components will reconverge periodically, confirming the magnetic focusing effect. We find that in this experiment, the smaller the dt we use, the more pronounced the magnetic focusing effect observed in the experiment becomes.This computational approach provides an effective method for studying charged-particle behavior in electromagnetic fields, with applications in particle beam optics and accelerator physics.

V. MATLAB codes

The magnetic focusing simulation was implemented in MATLAB as follows:

```
m = 0.02;
q = 1.6e-2;
dt = ;
T = ; % Changed by yourself
t = 0:dt:T;

Ex = 0; Ey = 0; Ez = 0;
Bx = 0; By = 0; Bz = 8;

num_particles = 16;
vx = zeros(num_particles, length(t));
vy = zeros(num_particles, length(t));
vz = zeros(num_particles, length(t));
rx = zeros(num_particles, length(t));
ry = zeros(num_particles, length(t));
rz = zeros(num_particles, length(t));

for k = 0:num_particles-1
```

```
    vz(k+1,1) = 10;
    vx(k+1,1) = 0.1 * sin(k*pi/8);
    vy(k+1,1) = 0.1 * cos(k*pi/8);
    rx(k+1,1) = 0;
    ry(k+1,1) = 0;
    rz(k+1,1) = 0;
end

for i = 1:length(t)-1
    for k = 1:num_particles
        Fx = q*Ex + q*(vy(k,i)*Bz - vz(k,i)*By);
        Fy = q*Ey + q*(vz(k,i)*Bx - vx(k,i)*Bz);
        Fz = q*Ez + q*(vx(k,i)*By - vy(k,i)*Bx);

        ax = Fx/m;
        ay = Fy/m;
        az = Fz/m;

        vx(k,i+1) = vx(k,i) + ax*dt;
        vy(k,i+1) = vy(k,i) + ay*dt;
        vz(k,i+1) = vz(k,i) + az*dt;

        rx(k,i+1) = rx(k,i) + vx(k,i)*dt;
        ry(k,i+1) = ry(k,i) + vy(k,i)*dt;
        rz(k,i+1) = rz(k,i) + vz(k,i)*dt;
    end
end

figure;
hold on;
grid on;
pbaspect([1 1 2]);
title("Simulation of magnetic focusing phenomenon(dt = " + dt + "s, T = " + T + "s)");
xlabel('X(m)');
ylabel('Y(m)');
zlabel('Z(m)');
xticks(-1:0.5:1);
yticks(-1:0.5:1);
view(2);

colors = hsv(num_particles);
for k = 1:num_particles
    plot3(rx(k,:), ry(k,:), rz(k,:), 'Color', colors(k,:), 'LineWidth', 0.8);
end
```

Listing 1 Magnetic focusing simulation code