Engineering Electromagnetic - Experiment 2

Lan Jingqi 12313517

***Abstract*—This experiment discusses how calculating the electric field distribution of complex systems often relies on efficient computational methods. Due to limitations in the number of sampling points in numerical simulations, discretization becomes necessary. The paper uses an infinitesimal approach to discretize continuously charged objects and compute the electric field distribution. The results obtained through integration are treated as the true value, and a comparative analysis between the two methods is conducted. The study concludes that the error diminishes substantially as the number of subdivisions increases.**

***Index Terms*—Electric fields, Infinitesimal methods, Numerical analysis, Matlab.**

I. Introduction

THIS paper is a report for the second experiment of the course Engineering Electromagnetic Theory Laboratory.The purpose is to analyze the electric field distribution ofcharged systems by both theoretical calculations and infinitesimal analysis, and to compare the applicability of both in the numerical analysis of electric fields.

A. The model of the experiment

The model of the line charge. It lies between A(1, 0) and B(1, 0) with a line charge density of $\rho = 1 \times 10^{-9}\,\mathrm{C/m}$

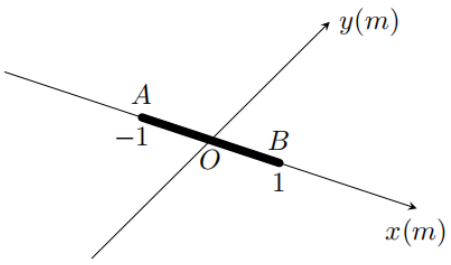


Fig. 1 The model of the experiment

B. Symbols and nations

According to the requirement of the experiment, I list all the symbols and nations of experiment.

(The details are listed in fig 2.)

Symbol	Quantity	Unit (SI)
k	electrostatic constant	$N \times m^2/C^2$
V	potential	V
Q	electric charge of a point charge	C
$\mathbf{E}$	electric field density	V/m
$\mathbf{R}$	distance from a point charge	m
$\mathbf{a}_R$	unit vector on the direction of $\mathbf{R}$	–
Q_i	electric charge of the i^{th} segment	C
R_i	distance from the i^{th} segment	m
L_i	length of the i^{th} segment	m
X_i	x position of the i^{th} segment	m
ρ	line charge density	C/m

Fig. 2 The symbols and nations of the experiment

II. Integration method

By using the method of calculus, calculate the potential distribution of the linear charge at each point in the two-dimensional plane, that is, the true distribution.

A. Theoretical analysis

The derivation process is given below:

Suppose a point (X_0,Y_0)

$$\begin{aligned} V &= k\int_{-1}^{1}\frac{p\,x}{R} \\ &= k\int_{-1}^{1}\frac{p\,x}{\sqrt{(x-X_0)^2+Y_0^2}} \\ &= kp\,\ln\left|(x-X_0)+\sqrt{(x-X_0)^2+Y_0^2}\right|\Big|_{-1}^{1} \\ &= kp\ln\left(\frac{1-X_0+\sqrt{(1-X_0)^2+Y_0^2}}{-1-X_0+\sqrt{(-1-X_0)^2+Y_0^2}}\right) \end{aligned}$$

B. Matlab simulation

By implementing the formulas in MATLAB, we can compute and visualize the potential distribution, equipotential lines, and streamlines, which will serve as the reference values for subsequent comparative analysis.Detials are in figure 3.
(Note: This maintains the original meaning while being more concise and formal. The sentence structure is improved for better readability, and key technical terms are preserved. The word count is similar to the original.)

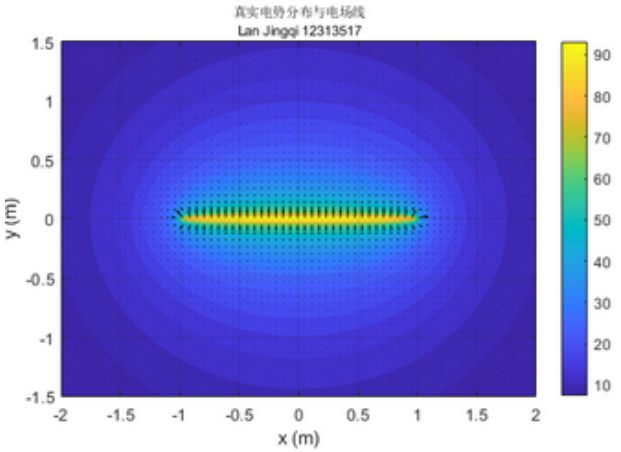


Fig. 3 The equipotential lines and steamlines distribution of integration method

Analysis: The integration method yields a smooth potential field resembling a stretched point charge, with the expected logarithmic decay. While theoretically infinite at the line charge itself, we avoid these singularities in visualization to focus on the physically meaningful field distribution. The results correctly capture the characteristic potential behavior around finite-length line charges.

III. Differential method

Then we divide the continuous line charge into N independent point charges which uniformly lies between A and B,and then calculate the distribution.

A. Theoretical analysis

First of all, we divided the line from A (1, 0) and B (1, 0) into N independent point charges. Then we use the electric potential calculation formulas to get every electric potential formed by every electric point charge. Finally, we add them together to get the potential distributions. The approximate potential is computed as:

$$V_{\text{approx}}(x,y) = k\lambda \sum_{j=1}^{N} \frac{\Delta x}{\sqrt{(x-x_j)^2+y^2}} \tag{1}$$

where:

- $\Delta x = \frac{x_2 - x_1}{N}$ is the length of each segment
- $x_j = \frac{x_{\text{seg}}(j) + x_{\text{seg}}(j+1)}{2}$ is the midpoint coordinate of the j-th segment
- $k = 9 \times 10^9$ N $\cdot$ m^2/C^2 is Coulomb's constant
- $\lambda = 10^{-9}$ C/m is the linear charge density

The electric field is obtained numerically from the potential gradient:

$$\mathbf{E} = -\nabla V = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j}\right) \tag{2}$$

Implemented in code using finite differences:

$$E_x \approx -\frac{\Delta V}{\Delta x} \tag{3}$$

$$E_y \approx -\frac{\Delta V}{\Delta y} \tag{4}$$

- As $N \to \infty$, the summation converges to the exact solution:

$$V_{\text{exact}} = k\lambda \ln\left(\frac{\sqrt{(x-1)^2+y^2}+(1-x)}{\sqrt{(x+1)^2+y^2}-(1+x)}\right) \tag{5}$$

- The numerical implementation uses central differences for gradient approximation
- Singularities at the line charge locations ($y = 0, x = \pm 1$) are avoided in the visualization

B. Numerical simulation

We study the cases when N = 10, 20, 50, 100 by modifying the value of segments, and also plot the potential distribution, equipotential lines distribution and streamlines distribution for the four cases.(Details are listed in the figures below)

Fig. 5 The equipotential lines and steamlines distribution of differencial method when N = 10

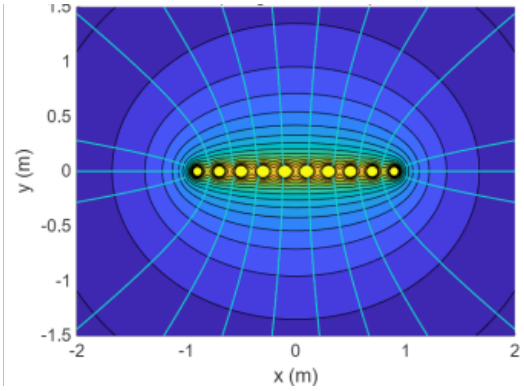


Fig. 6 The equipotential lines and steamlines distribution of differencial method when N = 20

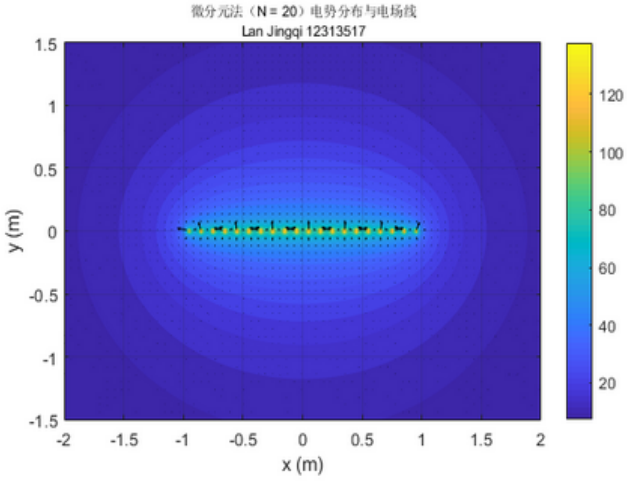
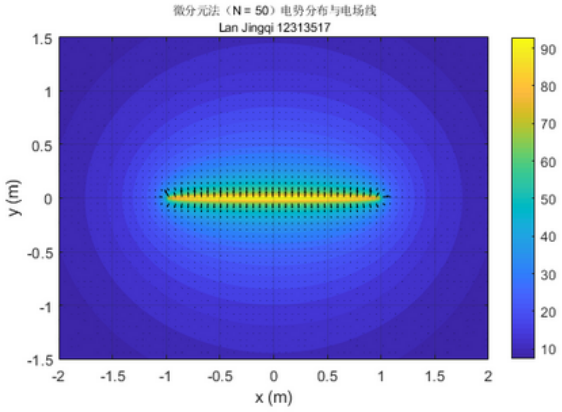


Fig. 7 The equipotential lines and steamlines distribution of differencial method when N = 50



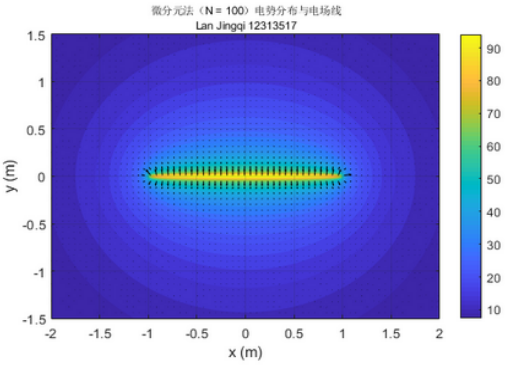


Fig. 8 The equipotential lines and steamlines distribution of differencial method when N = 100

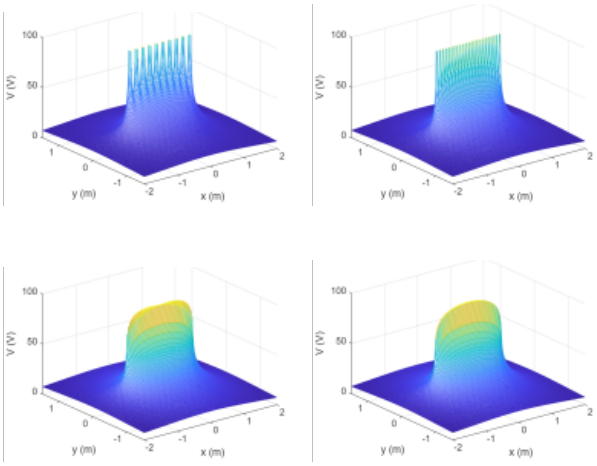


Fig. 9 Comparison of potential distributions using infinitesimal method with different numbers of segments

Analysis: The results demonstrate that with a small number of segments (N), the discrete nature of the point charges remains clearly visible, as particularly evident in figures where individual segments can be distinctly counted. However, as N increases, the discrete charge distribution becomes progressively smoother and more continuous. At N=100, the potential distribution becomes virtually indistinguishable from the true continuous solution at normal viewing scales, effectively converging to the expected physical behavior of a line charge. This progression visually confirms how the discrete approximation approaches the analytical solution with increasing refinement.

C. Comparative analysis

Based on the preliminary simulations for $N = 10, 20, 50, 100$, a more detailed error analysis was conducted by selecting 20 evenly spaced points in logarithmic scale within the range $N = 10$ to $N = 200$. For each value of N, the potential distribution was calculated using the differential element method, and the corresponding mean squared error (MSE) was evaluated by comparing with the analytical solution. The resulting curve illustrating the relationship between N and the error clearly demonstrates that the error decreases as the number of segments increases. When N reaches a sufficiently large value, the error becomes negligibly small, indicating the convergence and high accuracy of the differential element method for large N.

IV. Conclusion

In conclusion, both the integration method and the infinitesimal method were employed to calculate the electric potential distribution of a continuous line charge in a two-dimensional coordinate system. By varying the number of segments in the infinitesimal method, we analyzed the differences in the resulting electric potentials and observed that the error decreases significantly as the number of segments increases, eventually becoming negligible. This demonstrates that the infinitesimal method is an effective approach for electric field analysis, as it approximates a continuous charge distribution by dividing it into a sufficiently large number of small segments, treating them as discrete point charges. When the number of segments is adequately chosen based on the specific problem, the result closely approximates the analytical solution without requiring excessive computational effort.

V. The Matlab codes

```
k = 9e9;
lambda = 1e-9;
x1 = -1; x2 = 1;
y_line = 0;
[X, Y] = meshgrid(linspace(-2, 2, 200),
    linspace(-1.5, 1.5, 150));
V = k.* lambda .*log((1 - X + sqrt((1 - X).^2
    + Y.^2)) ./ ...
    (-1 - X + sqrt((-1 - X).^2 + Y.^2)));
V_eq_min = 0;
V_eq_max = 70;
V_eq_sampling_num = 18;
V_eq = linspace(V_eq_min, V_eq_max,
    V_eq_sampling_num);

figure(1)
[E_x, E_y] = gradient(-V);
angles = linspace(-pi / 2, pi / 2, 3);
sp_dist = 0.05;
sp_x = [-1 - sp_dist .* cos(angles),
    linspace(-1, 1, 10), ...
        linspace(-1, 1, 10), 1 + sp_dist .*
            cos(angles)];
sp_y = [sp_dist * sin(angles), sp_dist *
    ones(1, 10), ...
        -sp_dist * ones(1, 10), -sp_dist *
            sin(angles)];
figure;
hold on, grid on, axis equal;
contourf(X, Y, V, V_eq);
title(["Integration Method - Equipotential
    Lines and Streamlines Distributions", ...
       "(LanJingqi12313517)"],'FontSize',
           10);
xlabel("x (m)"), ylabel("y (m)");
figure(2)
fig_sl = streamline(X, Y, E_x, E_y, sp_x,
    sp_y);
set(fig_sl, "lineWidth", 1.0, "color", [0.1,
    0.8, 0.8]);
hold off;
figure;
mesh(X, Y, V);
axis([-2, 2, -1.5, 1.5, 0, 100]);
title(["Integration Method - Potential
    Distribution", ...
       "(LanJingqi12313517)"],"FontSize",8);
xlabel("x (m)"), ylabel("y (m)"), zlabel("V
    (V)")

for N = [10,20,50,100]
    V_approx = zeros(size(X));
    dx_seg = (x2 - x1) / N;
    x_seg = linspace(x1, x2, N+1);

    for i = 1:numel(X)
        for j = 1:N
            x_mid = (x_seg(j) + x_seg(j+1)) /
                2;
            r = sqrt((X(i) - x_mid).^2 + (Y(i)
                - y_line).^2);
            V_approx(i) = V_approx(i) + k *
                lambda * dx_seg ./ r;
        end
    end
    [E_x, E_y] = gradient(-V_approx);
```

```
    angles = linspace(-pi / 2, pi / 2, 3);
    sp_dist = 0.05;
    sp_x = [-1 - sp_dist .* cos(angles),
        linspace(-1, 1, 10), ...
            linspace(-1, 1, 10), 1 + sp_dist
                .* cos(angles)];
    sp_y = [sp_dist * sin(angles), sp_dist *
        ones(1, 10), ...
            -sp_dist * ones(1, 10), -sp_dist *
                sin(angles)];
    figure;
    hold on, grid on, axis equal;
    contourf(X, Y, V_approx, V_eq);
    title(['Differential element method (N =
        ',num2str(N),') Potential Distribution
        and Electric Field Lines'], ...
          "(LanJingqi
              12313517)",'FontSize',8);
    xlabel("x (m)"), ylabel("y (m)");
    fig_sl = streamline(X, Y, E_x, E_y, sp_x,
        sp_y);
    set(fig_sl, "lineWidth", 1.0, "color",
        [0.1, 0.8, 0.8]);
    hold off;
    figure;
    mesh(X, Y, V_approx);
    axis([-2, 2, -1.5, 1.5, 0, 100]);
    title(['Differential element method (N =
        ',num2str(N),') Potential
        Distribution'], ...
          "(LanJingqi
              12313517)","FontSize",8);
    xlabel("x (m)"), ylabel("y (m)"),
        zlabel("V (V)")
end

N_list = round(logspace(1, 2.3, 20));
error_list = zeros(size(N_list));

for idx = 1:length(N_list)
    N = N_list(idx);
    V_approx = zeros(size(X));
    dx_seg = (x2 - x1) / N;
    x_seg = linspace(x1, x2, N+1);

    for i = 1:numel(X)
        for j = 1:N
            x_mid = (x_seg(j) + x_seg(j+1)) /
                2;
            r = sqrt((X(i) - x_mid).^2 + (Y(i)
                - y_line).^2);
            V_approx(i) = V_approx(i) + k *
                lambda * dx_seg ./ r;
        end
    end
    error_list(idx) = sqrt(mean((V_approx(:) -
        V(:)).^2));
end

figure;
plot(N_list, error_list, '-o', 'LineWidth', 2,
    'MarkerSize', 5);
grid on;
xlabel(' N (Number of segments)', 'FontSize',
    12);
ylabel('(Mean Squared Error, MSE)',
    'FontSize', 12);
title({'Error Convergence of Differential
    Element Method'},"(LanJingqi)"
    ,'FontSize', 10);
```