Entropy (엔트로피)

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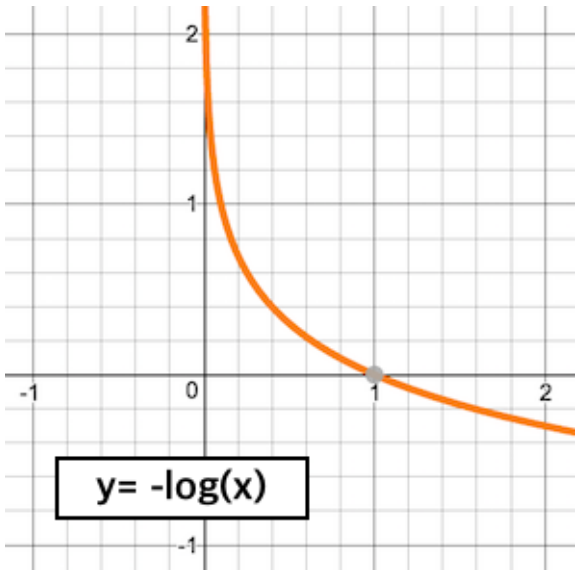
Information Gain (정보 이득량)

- 잘 일어나지 않는 사건은 자주 발생하는 사건보다 정보량이 많다고 정의
- 발생할 확률이 적을 수록 더 많은 정보량을 갖는다
  - 정보량이 많다 == 더 중요한 정보
- $I(x) = -\log P(x)$



Entropy

- 확률 변수의 평균 정보량
  - 결과 값을 예상하기 어려운 (불확실성) 정도를 의미
  - High Entropy → High Uncertainty
  - Low Entropy → Low Uncertainty

$$Entropy = E(-\log P(x)) = -\sum_{i=1}^{n} p_i \log_2 p_i$$

Entropy Calculation

동전 던지기	앞면: 50%, 뒷면: 50%	- (0.5*log0.5 + 0.5*log0.5)	Entropy : 1
해가 뜨는 방향	동: 100%, 서,남,북: 0%	- (1.0*log 1.0 + 3*(0.0*log 0.0))	Entropy : 0
사건 발생 확률	A: 90%, B: 10%, C: 0%	- (0.9*log 0.9 + 0.1*log 0.1 + 0)	Entropy : 0.47

Max Entropy

- Binary Classification

$$0 <= Entropy <= 1 \qquad (2^1 = 2)$$

- For 8 different classes

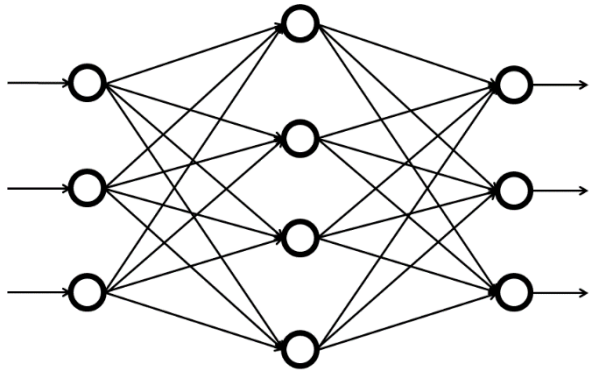
$$0 <= Entropy <= 3 \qquad (2^3 = 8)$$

- For 16 different classes

$$0 <= Entropy <= 4 \qquad (2^4 = 16)$$

Cross Entropy

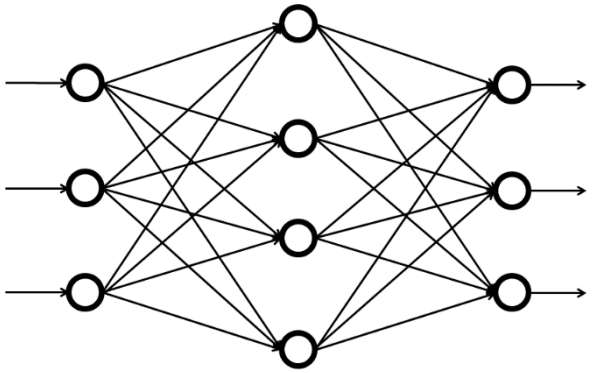
- 두 확률분포가 하나의 사건 X 에 대해 갖는 평균 정보량
- 분류 문제에서 비용함수/손실함수로 활용 가능



softmax	label	
0.8	1	고양이
0.1	0	강아지
0.1	0	토끼

Cross Entropy as Loss Function

- Q : Estimated Probability
- P : True Probability



Q		P	
0.8		1	고양이
0.1	cross entropy	0	강아지
0.1		0	토끼

$$Entropy = H(P) = -\sum_{i=1}^{n} p_i \log_2 p_i$$

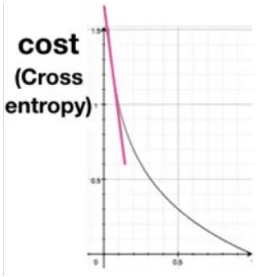
$$Cross\ Entropy = H(P,Q) = -\sum_{i=1}^{n} p_i \log_2 q_i$$

Cross Entropy Examples

$$Cross\ Entropy = H(P,Q) = -\sum_{i=1}^{n} p_i \log_2 q_i$$

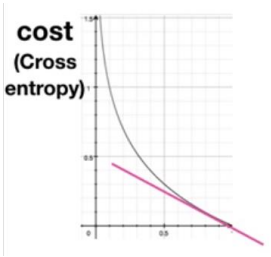
Q	P
0.0	1
1.0	0
0.0	0

$$-(\ 1 * \log_2 0.0 + 0 * \log_2 1.0 + 0 * \log_2 0.0\ ) = infinity$$



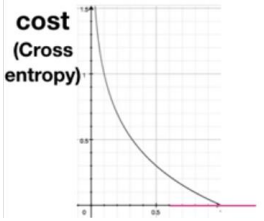
Q	P
0.8	1
0.1	0
0.1	0

$$-(\ 1 * \log_2 0.8 + 0 * \log_2 0.1 + 0 * \log_2 0.1\ ) = 0.32$$



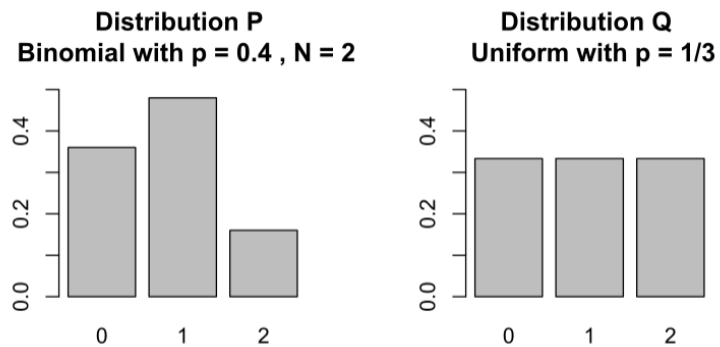
Q	P
1.0	1
0.0	0
0.0	0

$$-(\ 1 * \log_2 1.0 + 0 * \log_2 0.0 + 0 * \log_2 0.0\ ) = 0.0$$



KL-Divergence (쿨백-라이블러 발산)

- Kullback Leibler (KL) divergence
- 두 확률 분포의 다른 정도를 측정하는 방법



x	0	1	2
Distribution $P(x)$	0.36	0.48	0.16
Distribution $Q(x)$	0.333	0.333	0.333

KL Divergence : Relative Entropy

- $D(P||Q)$ is relative entropy of P with respect to Q
- To measure how one probability distribution is different from another
  - KL divergence is zero if two distributions are identical

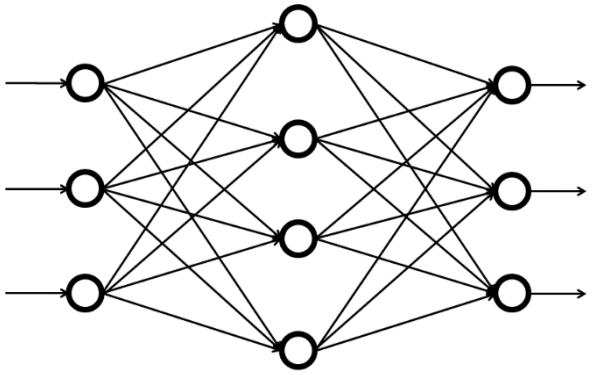
$$KL\ Divergence\ =\ Cross\ Entropy\ -\ Entropy$$

$$D(P||Q) = H(P,Q) - H(P)$$

$$D(P||Q) = -\sum_{i=1}^{n} p_i \log_2 q_i + \sum_{i=1}^{n} p_i \log_2 p_i$$

$$D(P||Q) \; ! = D(Q||P)$$

KL Divergence for deep learning model?



Q		P	
0.8		1	고양이
0.1	$D(P\|\|Q)$	0	강아지
0.1		0	토끼

$$D(P||Q) = Cross\ Entropy - Entropy$$