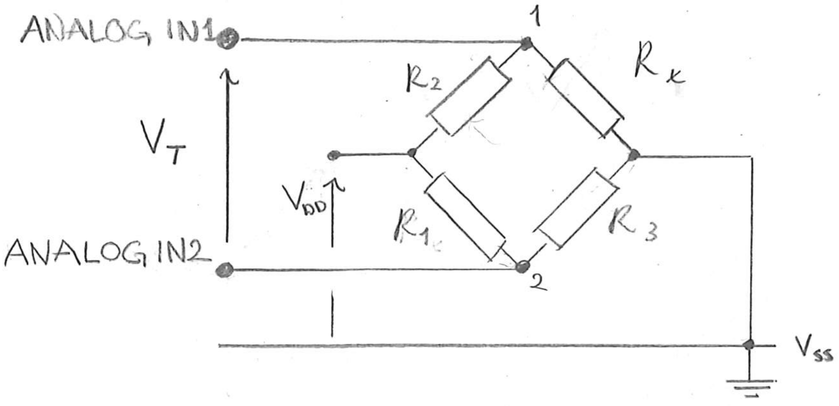
Analog Temp. Circuit



$$V_{DD} - V_{SS} = V \quad (V_{SS} = 0V)$$

$$V_T = V_1 - V_2 = \frac{R_x}{R_2 + R_x} V - \frac{R_3}{R_1 + R_3} V$$

$$\left[\frac{V_T}{V} + \frac{R_3}{R_1 + R_3}\right](R_2 + R_x) = R_x$$

$$\left[\frac{V_T}{V} + \frac{R_3}{R_1 + R_3}\right] R_2 = R_x \left[1 - \frac{V_T}{V} - \frac{R_3}{R_1 + R_3}\right]$$

$$\Rightarrow \underline{\underline{R_x}} = \frac{R_2\left(\frac{V_T}{V} + \frac{R_3}{R_1 + R_3}\right)}{1 - \frac{V_T}{V} - \frac{R_3}{R_1 + R_3}} = \boxed{\frac{R_2}{\frac{V(R_1 + R_3)}{V_T(R_1 + R_3) + V R_3} - 1}}$$

$$\left(\frac{V_T}{V} + \frac{R_3}{R_1 + R_3}\right)^{-1} = \frac{V(R_1 + R_3)}{V_T(R_1 + R_3) + V R_3}$$

$$\boxed{R_x = \frac{R_2}{\frac{R_1/R_3 + 1}{V_T/V\,(R_1/R_3 + 1) + 1} - 1}}$$

Pt100, Pt500 and Pt1000 resistors belong to a standardised series. The document defining this standard is the European "EN 60751".

Pt100 - 100 Ω @ 0°C
Pt500 - 500 Ω @ 0°C
Pt1000 - 1000 Ω @ 0°C

The relation of temperature-to-resistance is polynomial:

$$R(T) = \begin{cases} R_0(1 + AT + BT^2 + C[T - 100°C]T^3), & T \in (-200°C, 0°C) \\ R_0(1 + AT + BT^2), & T \in (0°C, 850°C) \end{cases}$$

where

$A = 3.90802 \times 10^{-3}$ °C^{-1},
$B = -5.775 \times 10^{-7}$ °C^{-2},
$C = -4.2735 \times 10^{-12}$ °C^{-4},
T is temperature in °C,
R_0 is resistance @ 0°C in Ω

Resistance-to-temperature:

Above 0°C:

$$T(R) = \frac{-R_0 A + \sqrt{(RA)^2 - 4B(R_0 - R)}}{2 R_0 B}$$

Iteration for below 0°C:

$$T_{i+1} = T_i - \frac{R(T_i) - R}{R'(T_i)} = T_i - \frac{R_0(1 + AT_i + BT_i^2 + C[T_i - 100°C]T_i^3 - R}{R_0(A + 2BT_i + C[3T_i^2(T_i - 100°C) + T_i^3])}$$

To set appropriate bridge resistor values, we need to set a range to be measured. Setting $\pm 50°C$, we get the range $(R(-50°C), R(50°C))$.

$R(-50°C) =$	$80.3\,\Omega$	Pt100
	$401.5\,\Omega$	Pt500
	$803.1\,\Omega$	Pt1000
$R(50°C) =$	$119.4\,\Omega$	Pt100
	$597.0\,\Omega$	Pt500
	$1194.0\,\Omega$	Pt1000

Looking back at the equation for V_T (measured voltage):

$$V_T = V\left(\frac{R_x}{R_1+R_x} - \frac{R_3}{R_2+R_3}\right)$$

Setting $R_1 = R_2$ and $R_3 = R_x(T)|_{T=T_{min}}$, we get $V_T = 0V$ at T_{min}, the smallest measurable temperature.

Now to set $R = R_1 = R_2$. We want maximum V_T available. Looking at the above equation and taking the derivative w.r.t. R:

$$\frac{dV_T/V}{dR} = -\frac{R_x}{(R+R_x)^2} + \frac{R_3}{(R+R_3)^2} = 0$$

$$\Rightarrow R_x(R^2 + 2RR_3 + R_3^2) = R_3(R^2 + 2RR_x + R_x^2)$$

$$\Rightarrow R^2(R_x - R_3) + R_xR_3^2 - R_3R_x^2 = 0$$

$$\Rightarrow R = \left[\frac{R_3R_x^2 - R_xR_3^2}{R_x - R_3}\right]^{1/2}$$

We now have three cases (Pt100, Pt500, Pt1000) for a given temperature range. The three cases are compared for total current consumption and output voltage range V_T.

The range, given that $V_T = 0V \big|_{T_{min}}$, is $V_T \big|_{T_{max}}$.

After extensive MATLAB analysis, the following is concluded:

1. The thermistors have current limits to be met;

from datasheet

	Limit (mA)	Recommended (mA)
Pt100	7	1
Pt500	3	0.1
Pt1000	1	0.1

2. In order to meet these ratings, current-limiting resistors placed in series with R_4 and $R_x(T)$ are needed.

3. Additionally, $R = R_1 = R_2$ need to be increased by about a factor 4 to reach the current ratings, relative $R\big|_{V_{T_{max}}}$.

4. $V_{T_{max}}$ resulting in the $10^2 \sim 10^0$ mV range necessitates amplification. The low supply voltage and output resistance can be compensated by a low-power, rail-to-rail instrumentation amplifier

So, the result is:

Pt 1000 JUMO 00409849

MAX 4460 Instrumentation Amplifier*

$R = R1 = R2 = (8.2k\Omega \| 270k\Omega) \approx 7955.75\Omega$

$R3 = 1.8k\Omega$

Desired gain ~ 29.4 ⇒ R_{FB2}, R_{FB1} setting the gain

→ $R_{FB2} = 330k\Omega$, $R_{FB1} = 12k\Omega$, $G = 28.5$

or

→ $R_{FB2} = 100k\Omega$, $R_{FB1} = 2.7k\Omega + 820\Omega$, $G = 29.41$

$R_{FB2} = 330k\Omega$, $R_{FB1} = (12k\Omega \| 390k\Omega)$, $G = 29.35$

0.1 µF in-amp decoupler

$R_{limiter} = 1k\Omega$