Geometry for fun

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Figure 1: My avatar :D

Contents

1 Exercise 001

1.1 Problem

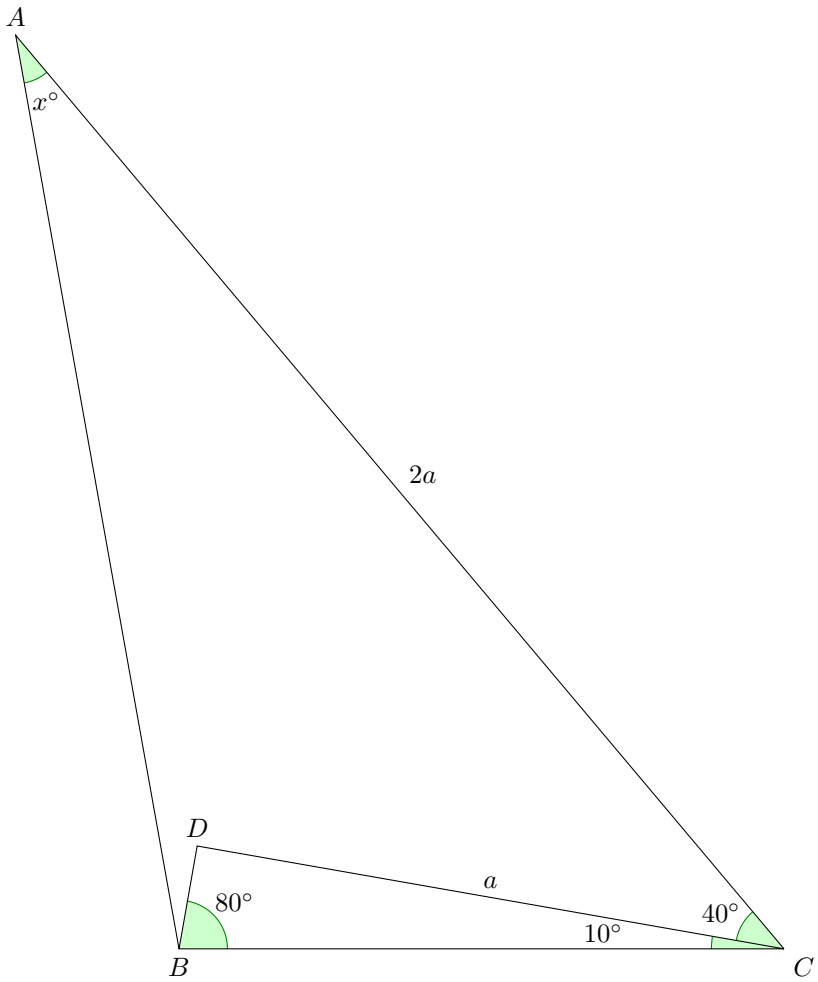

Figure 2: $\overline{DC} = a; \overline{AC} = 2a; \angle C\hat{B}D = 80°; \angle A\hat{C}D = 40°; \angle B\hat{C}D = 10°; \angle B\hat{A}C =?$

1.2 Solution 1

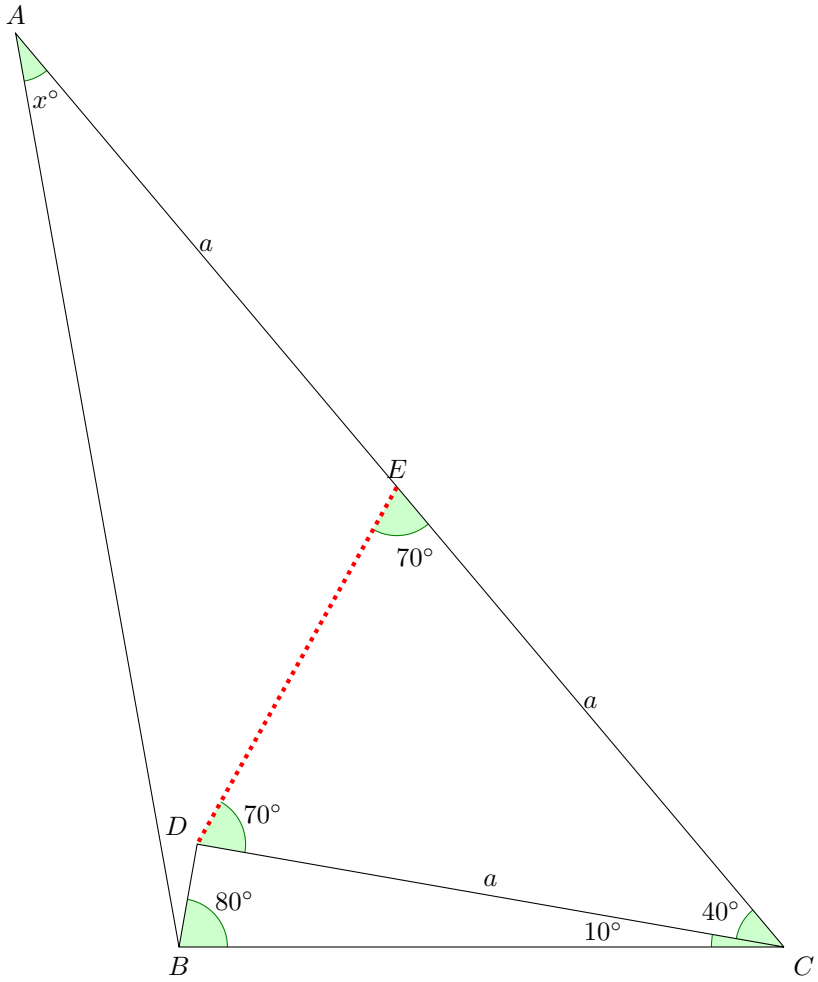

Figure 3: $\overline{CE} = \overline{CD} = \overline{AE} = a; \angle C\hat{E}D = \angle C\hat{D}E = 70°$

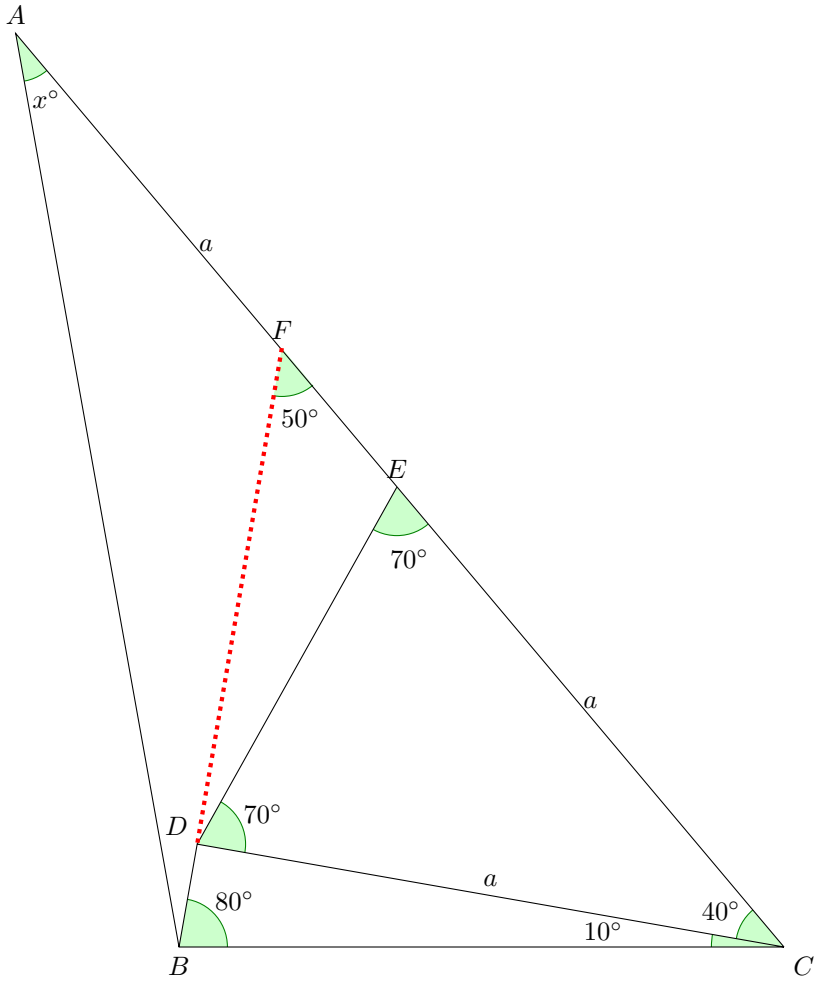

Figure 4: $\angle D\hat{E}C = 50°; \overline{BF} = \overline{BC}$

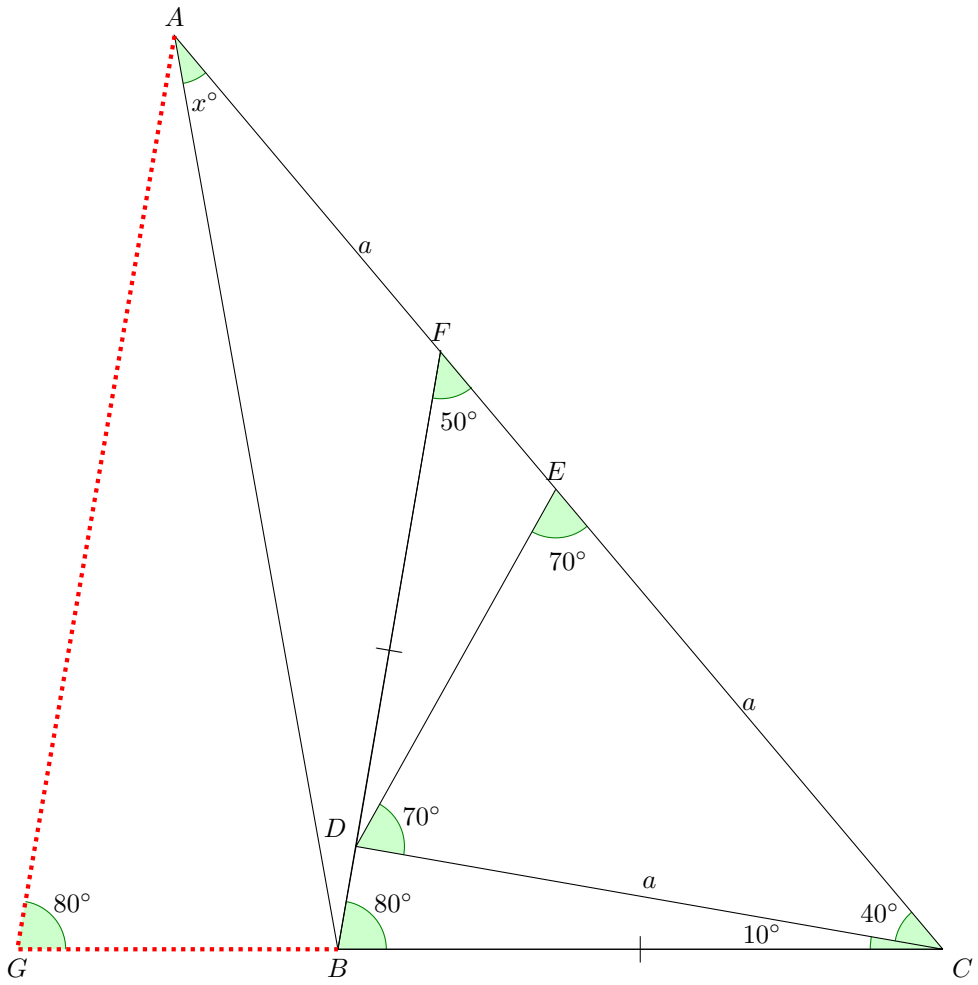

Figure 5: $\overline{GA}//\overline{BF}; \angle B\hat{G}A = \angle F\hat{B}C = 80°; \overline{AG} = \overline{CG}; \triangle FBC \approx \triangle AGC$

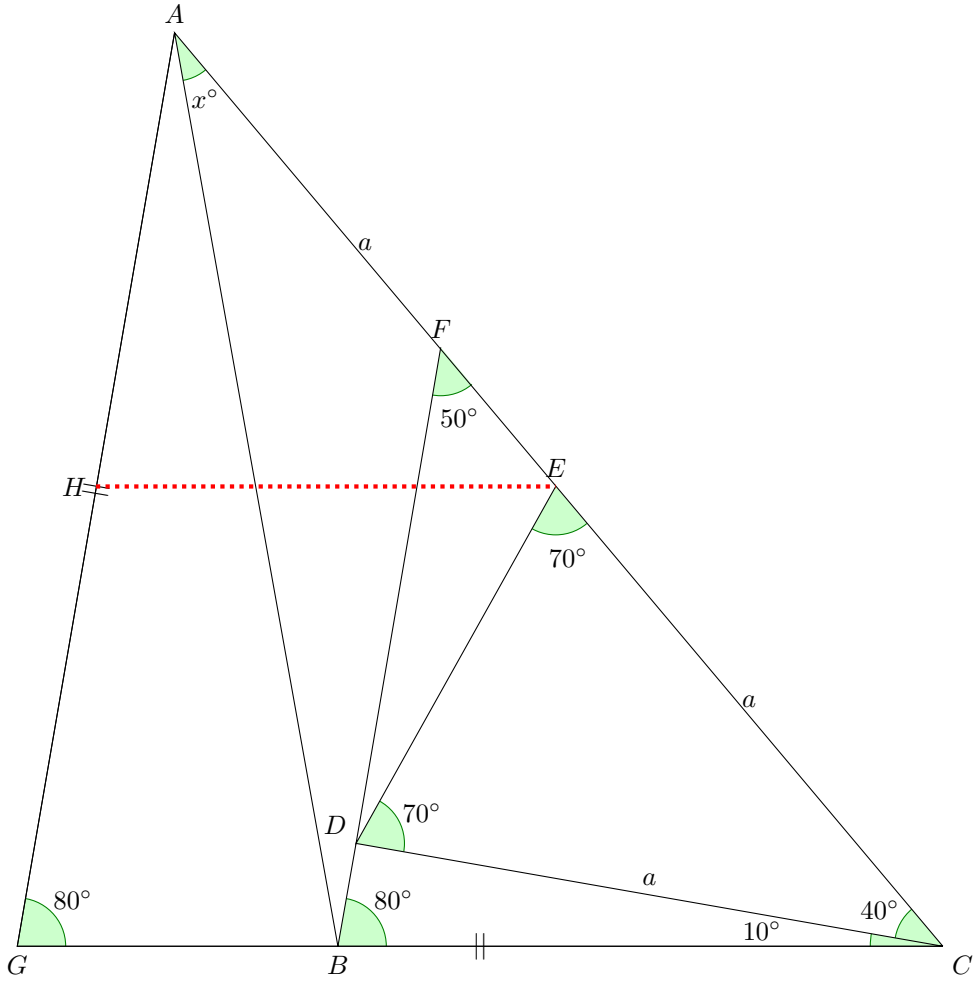

Figure 6: $\overline{HE}//\overline{GC}; \angle A\hat{H}E = 80°; H\hat{A}E = H\hat{E}A = 50°; \overline{GH} = \overline{HE} = \overline{HA}$

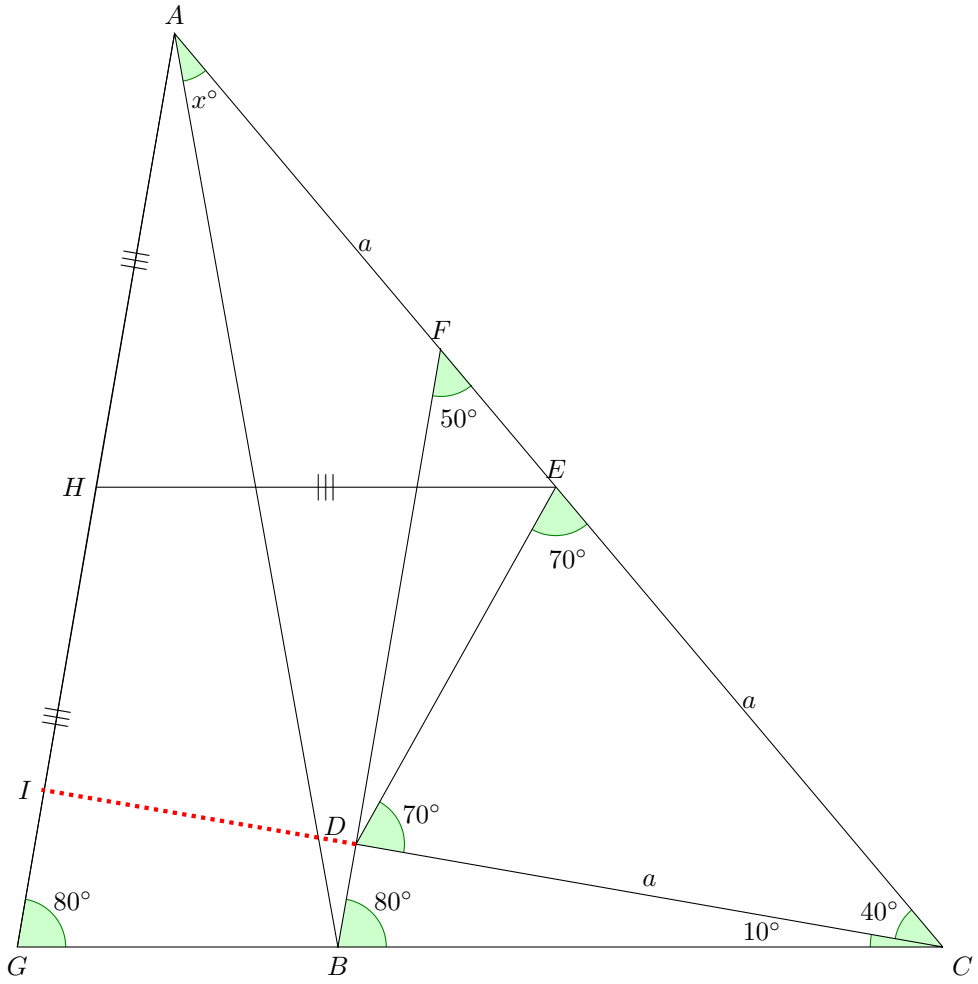

Figure 7: $\angle G\hat{I}C = \angle B\hat{D}C = 90°$

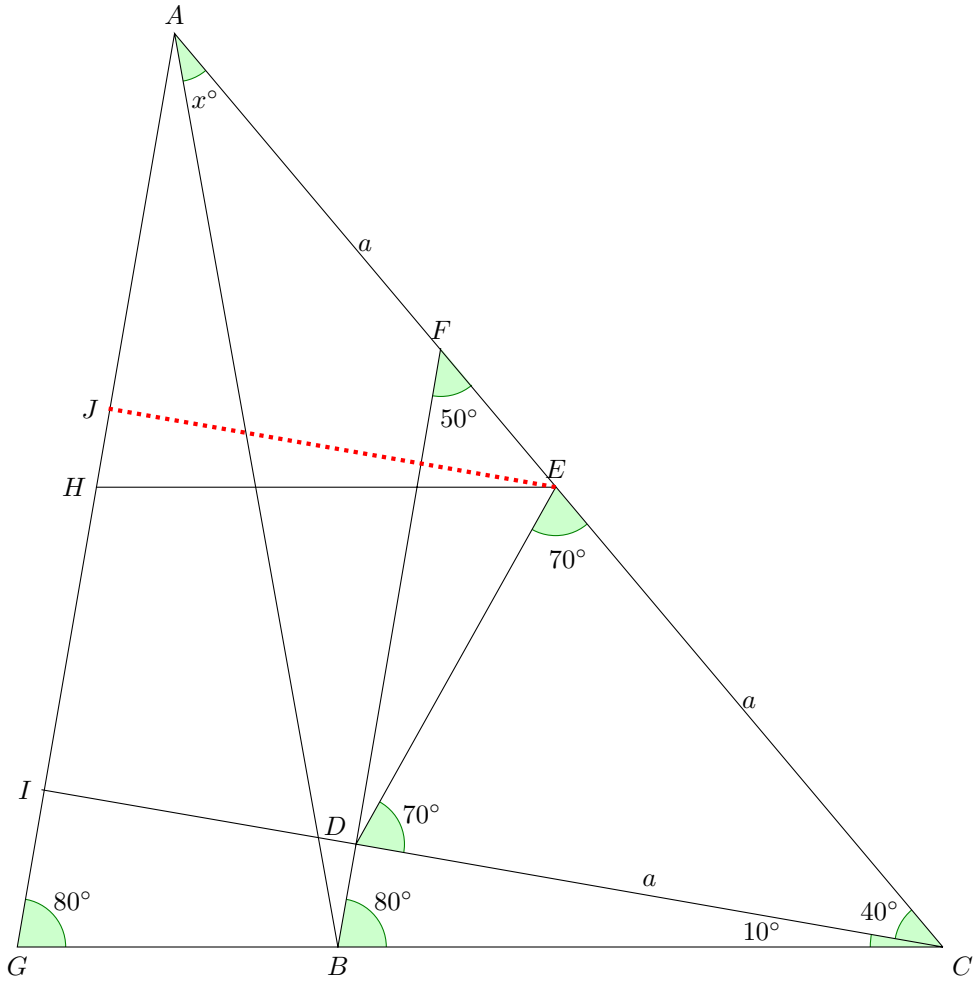

Figure 8: $\triangle HJE \approx \triangle BDC$

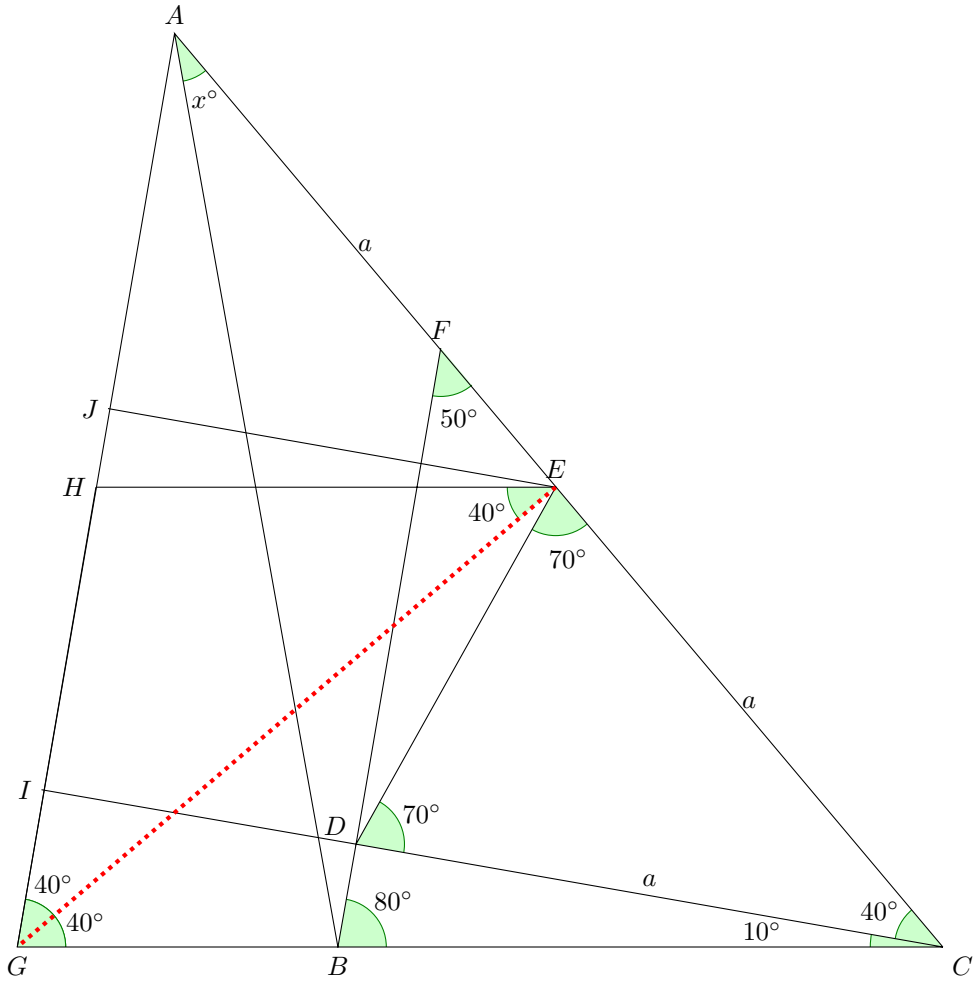

Figure 9: $\overline{GH} = \overline{HE} \therefore \angle H\hat{G}E = \angle H\hat{E}G = 40°$

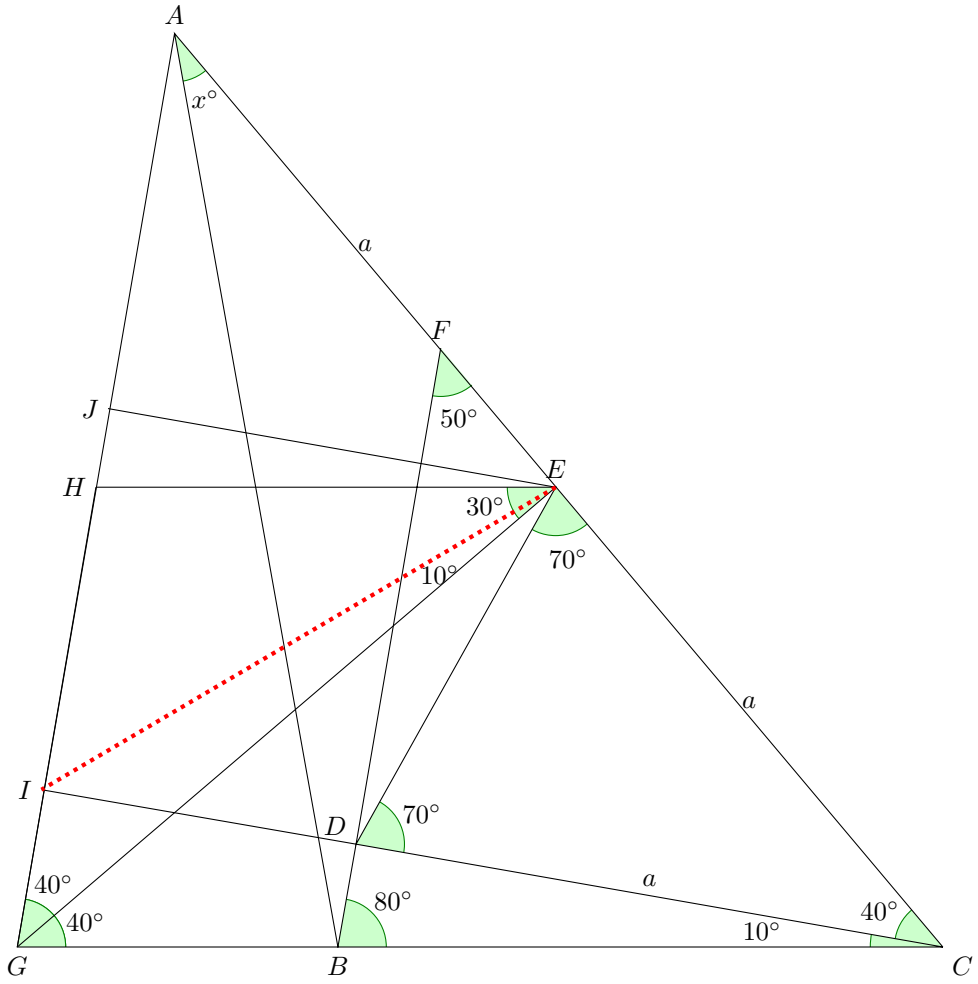

Figure 10: $\overline{AE} = \overline{IE} = \overline{EC}$

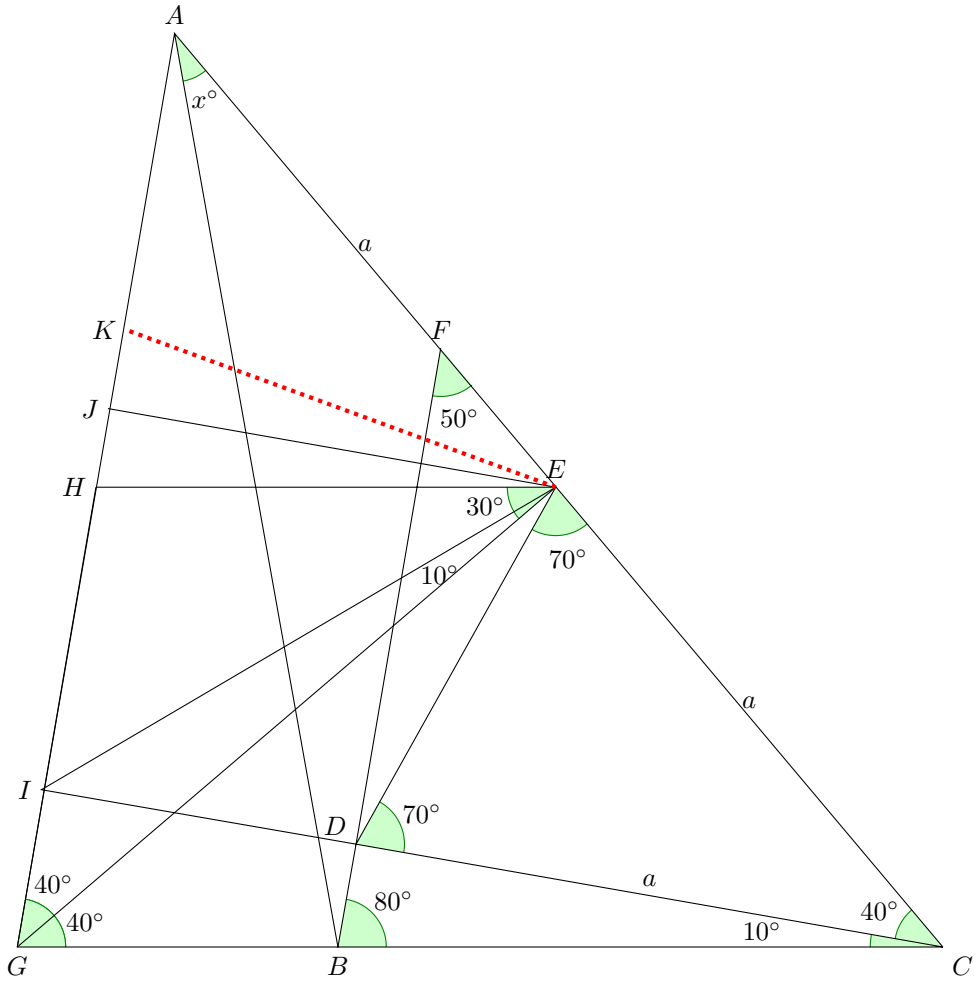

Figure 11: $\triangle HJE \equiv \triangle KJE; \triangle KIE \approx \triangle BCF$

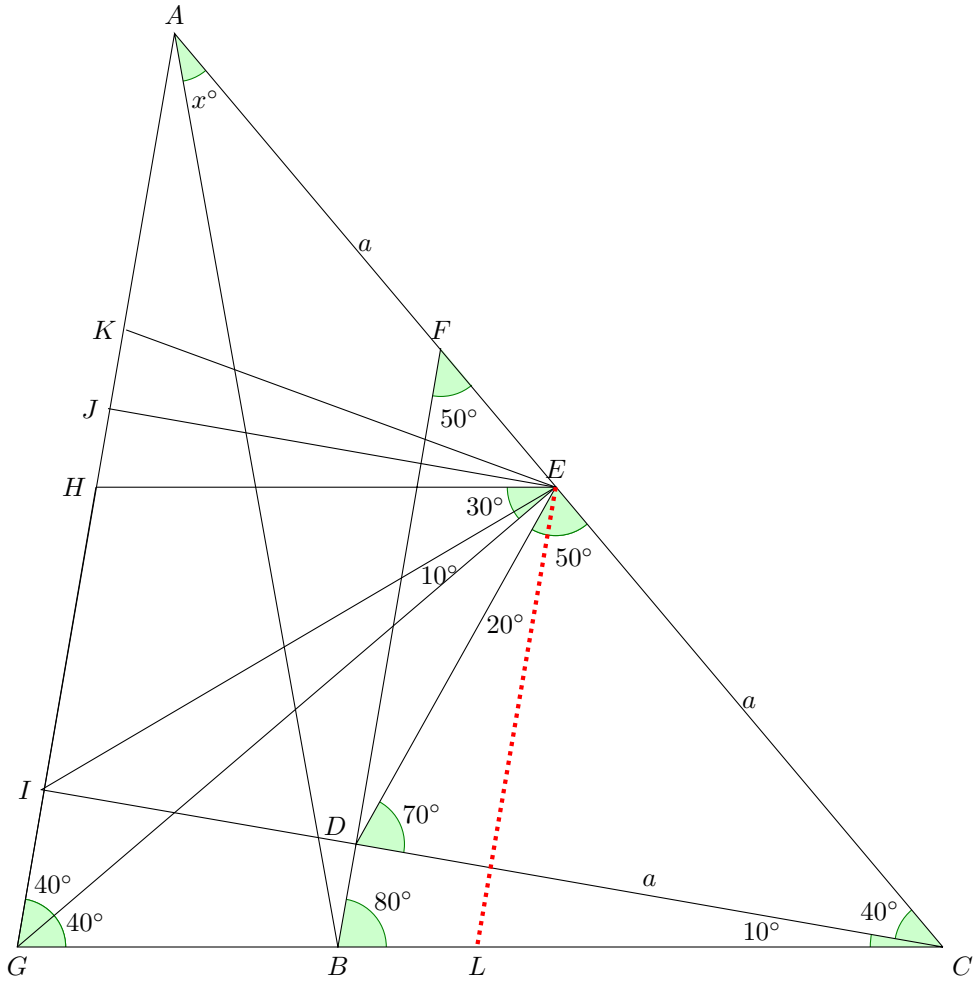

Figure 12: $\triangle HJE \equiv \triangle KJE; \triangle KIE \approx \triangle BCF$

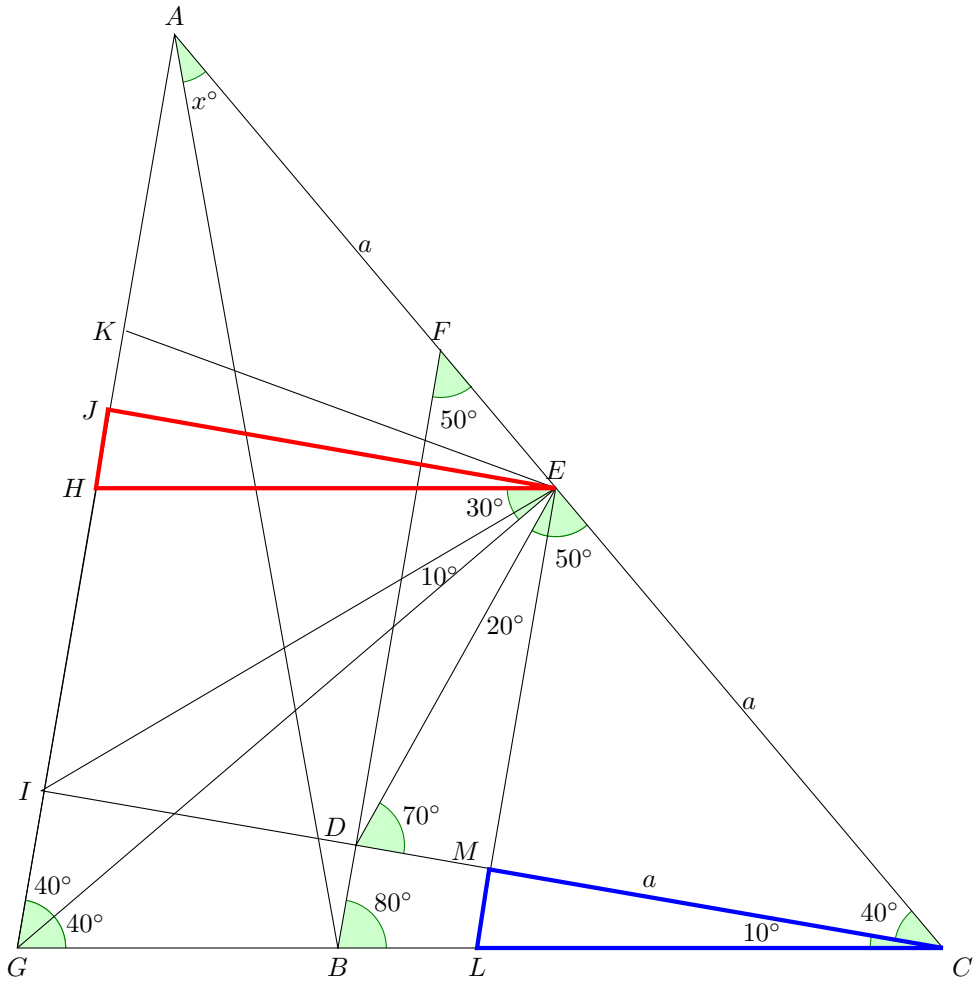

Figure 13: $\overline{JH} = \overline{ML} = \overline{KJ} = \frac{\overline{IG}}{2} \therefore \overline{KH} = \overline{ML}$

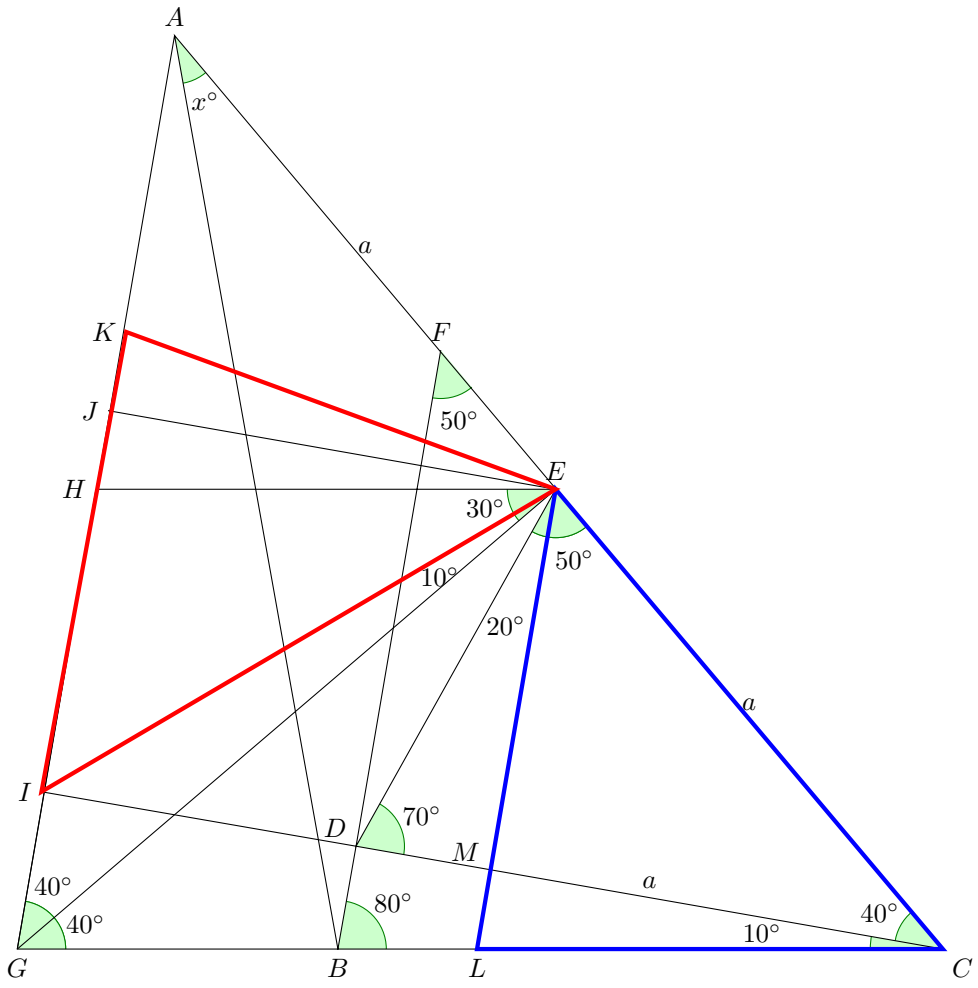

Figure 14: $\overline{AG}//\overline{EL} \therefore \overline{AG} = 2\overline{EL}; \overline{KI} = \overline{LC}, \overline{KE} = \overline{LE}, \angle K\hat{I}E = \angle L\hat{C}E = 50° \therefore \triangle KIE \equiv \triangle LCE$

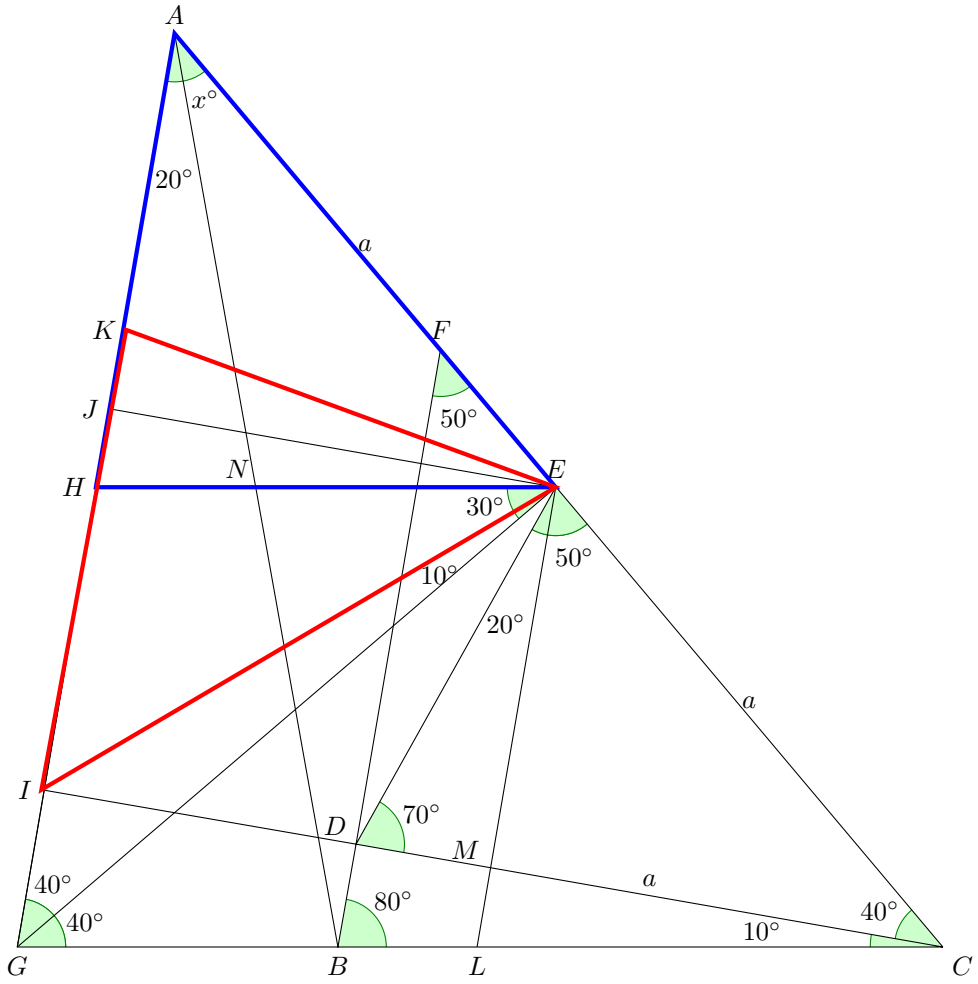

Figure 15: $\triangle KIE \equiv \triangle HEA \therefore \overline{AH} = \overline{AM} \therefore \angle G\hat{A}B = 20° \therefore \angle B\hat{A}C = 30°$

2 Exercise 002

2.1 Problem

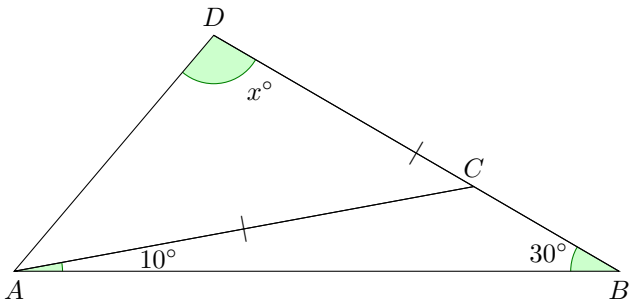

Figure 16: $\overline{DB} = \overline{AC}; \angle C\hat{A}B = 10°; \angle A\hat{B}D = 30°; \angle A\hat{D}B = x°$

2.2 Solution 1

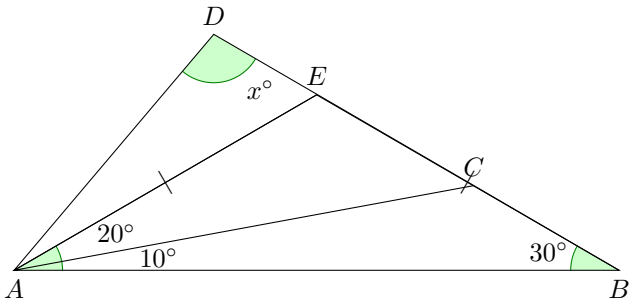

Figure 17: $\angle E\hat{A}C = 20°; \angle hatEAB = \angle E\hat{B}A \therefore \overline{AE} = \overline{EB}$

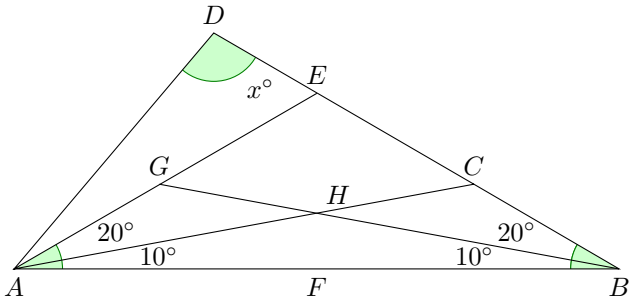

Figure 18: $\angle G\hat{B}A = 10°; \overline{GB} = \overline{AC}$

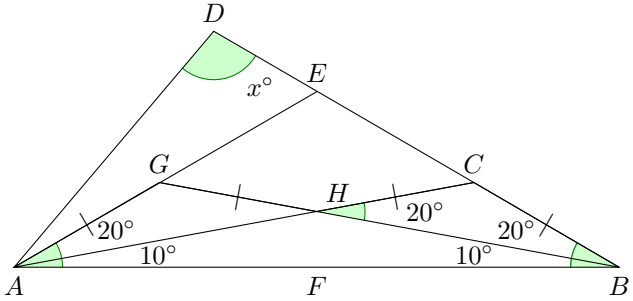

Figure 19: $\angle H\hat{A}B + \angle H\hat{B}A = \angle C\hat{H}B = \angle G\hat{H}A = 20°$

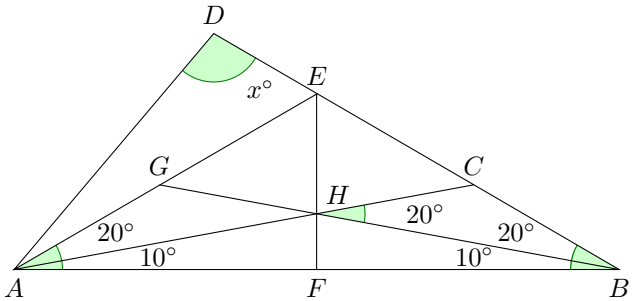

Figure 20: $\overline{EF} \perp \overline{AB}; \angle E\hat{F}B = \angle E\hat{F}A = 90°; \triangle AEF \equiv \triangle BEF$

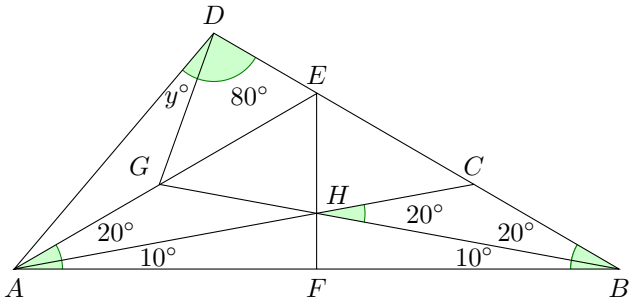

Figure 21: $\overline{BG} = \overline{BD} \therefore \angle D\hat{G}B = 80°; \angle G\hat{A}H = \angle G\hat{H}A = 20° \therefore \angle E\hat{G}H = 40° \therefore \angle D\hat{G}E = 40°$

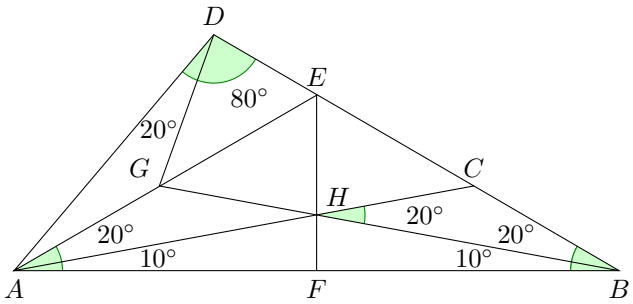

Figure 22: $\angle D\hat{G}E = \angle H\hat{G}E = 40° and \angle E\hat{D}G = \angle E\hat{H}G = 80° \therefore \triangle DEG = \triangle HEG \therefore \overline{DG} = \overline{HG} = \overline{AG} \therefore \angle D\hat{A}G = \angle A\hat{D}G = 20°$

3 Triangulo Russo

3.1 Problem

3.2 Solution 1

3.3 Solution 2

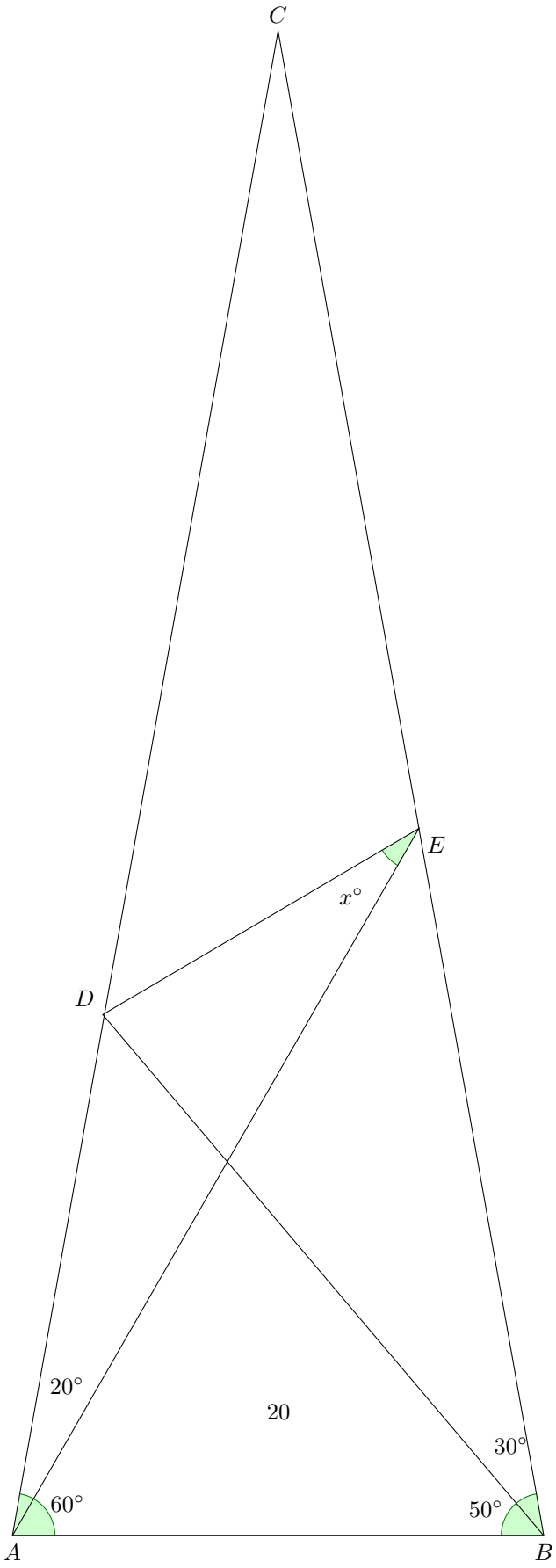

Figure 23: $\angle D\hat{A}E = 20°; \angle E\hat{A}B = 60°; \angle D\hat{B}A = 50°; \angle C\hat{B}D = 30°$

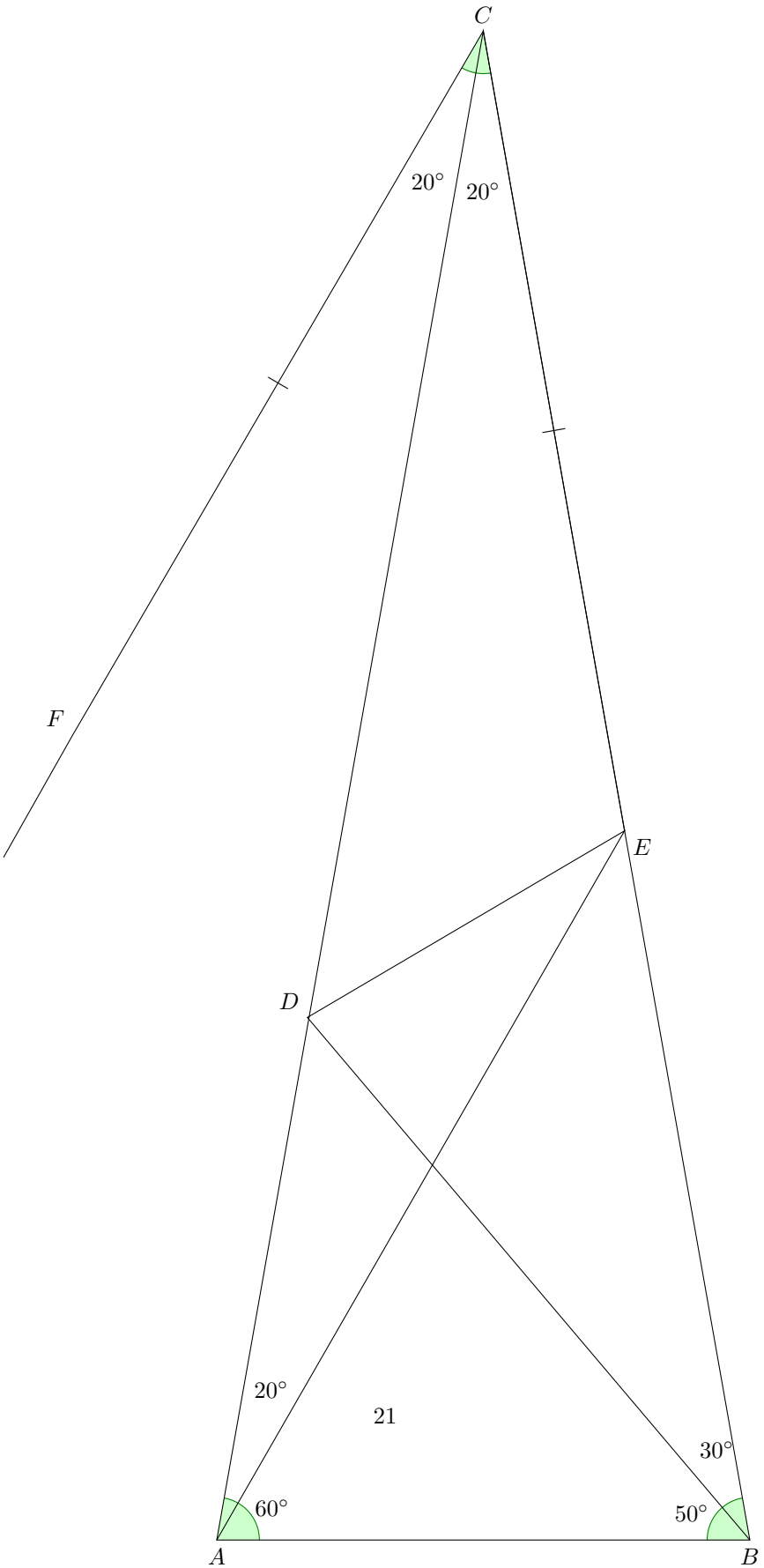

Figure 24: $\angle F\hat{C}D = 20°; \overline{CF} = \overline{CE}; \overline{FC} \parallel \overline{AE}$

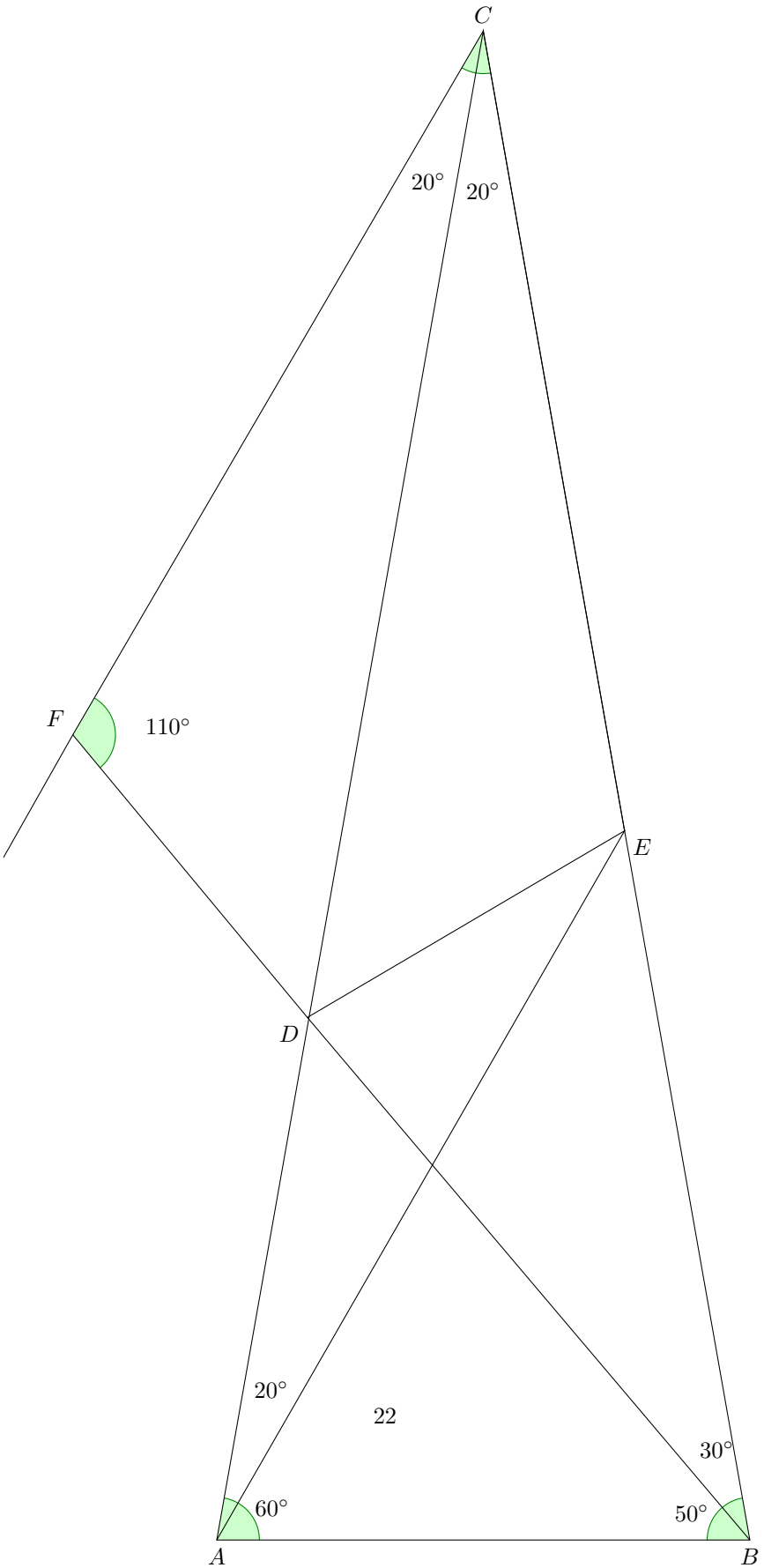

Figure 25: $F \in \overline{BD}$

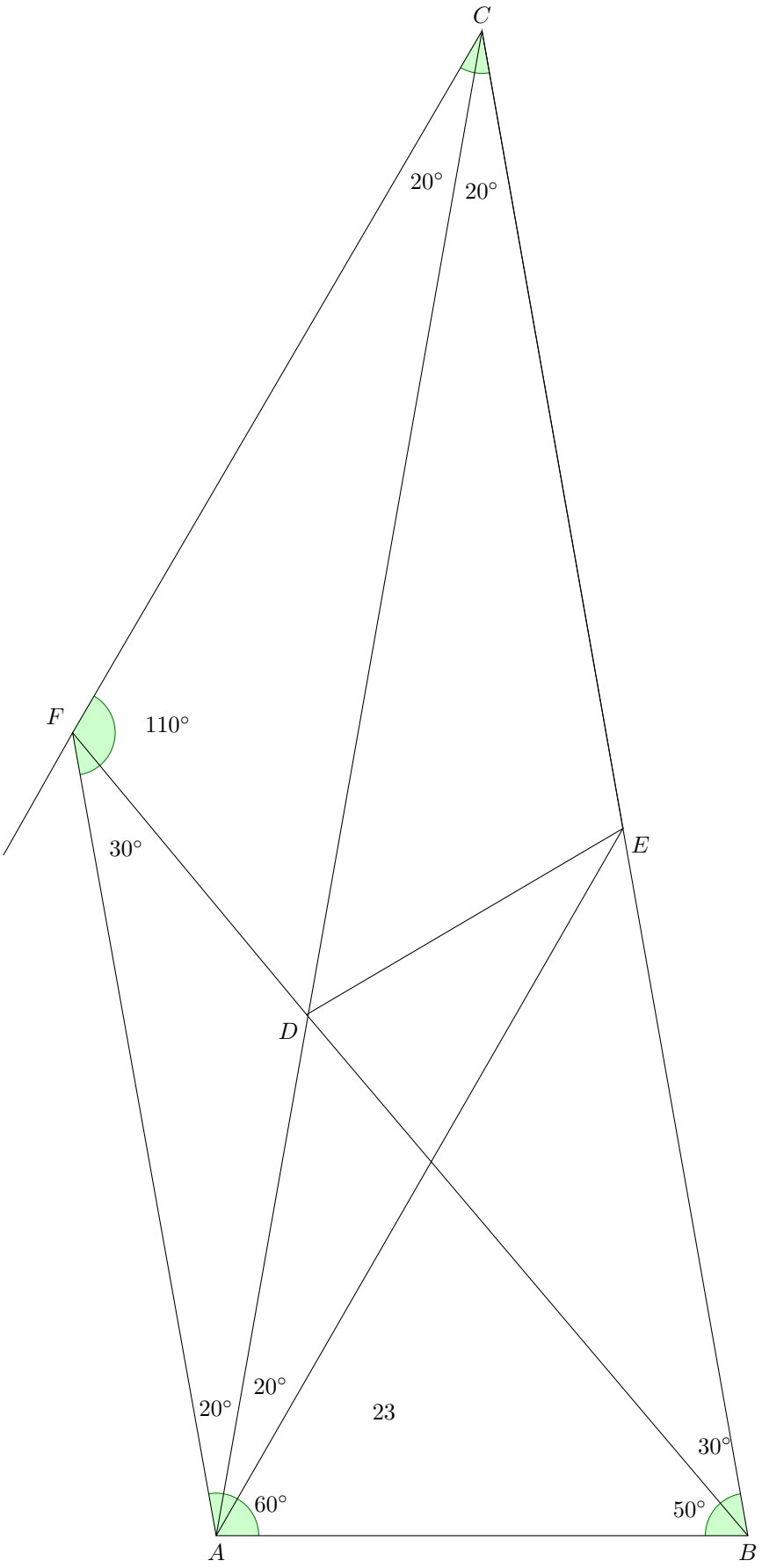

Figure 26: $\triangle CDB \approx \triangle ADF \therefore \angle C\hat{A}F = A\hat{C}E; \angle C\hat{B}F = \angle B\hat{F}A$

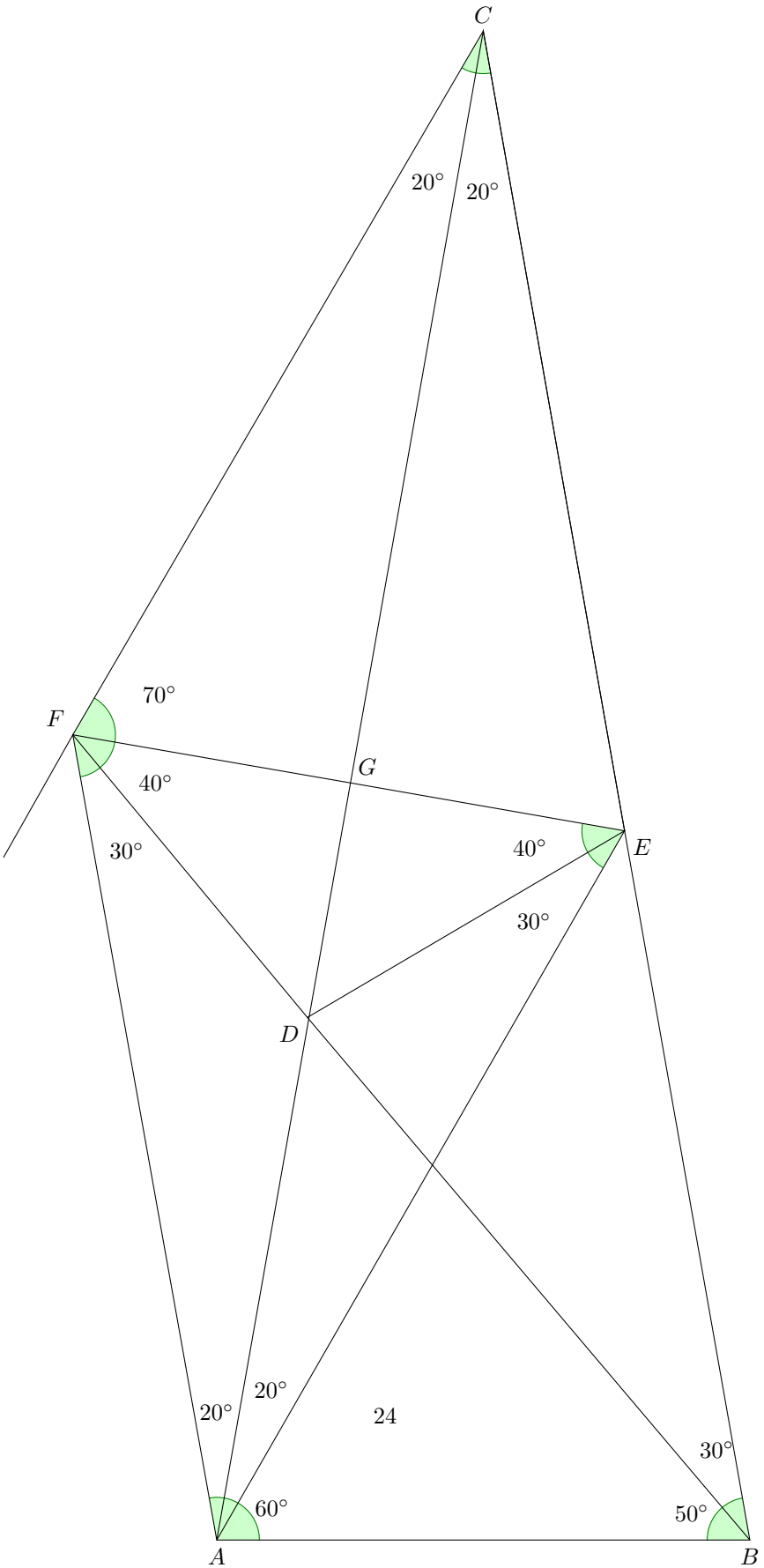

Figure 27: $\triangle FGD \equiv \triangle EGD \therefore \angle G\hat{E}D = 40° \therefore \angle D\hat{E}A = 30°$

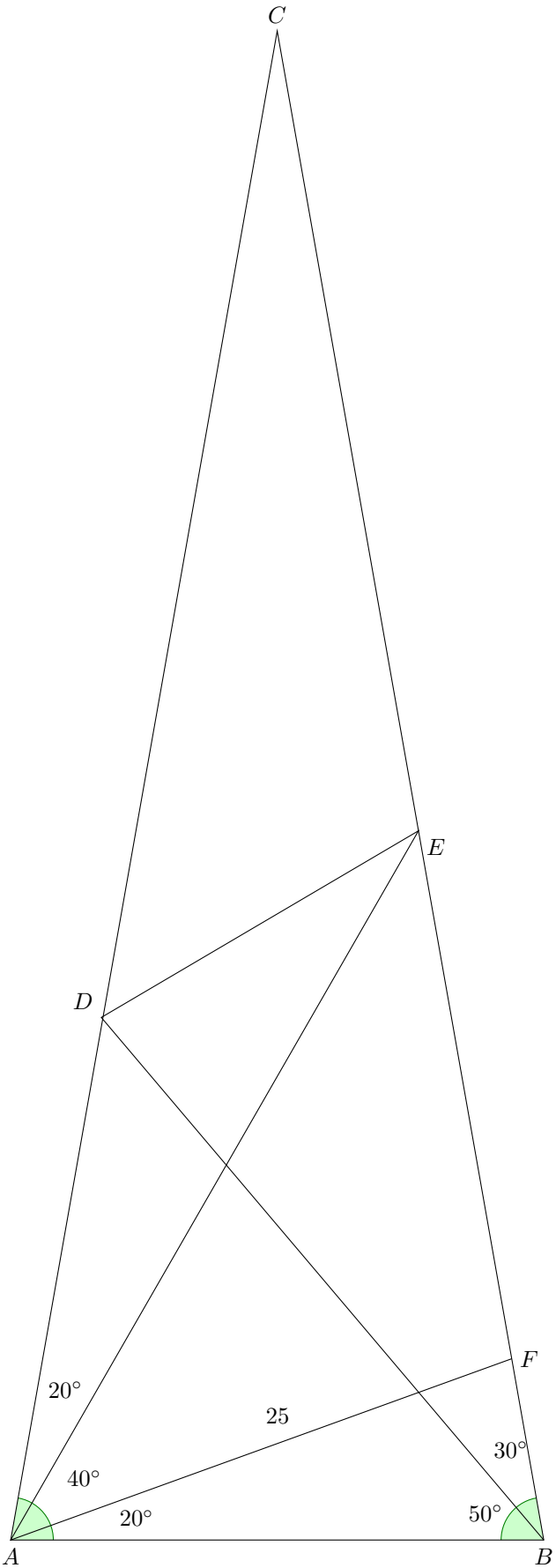

Figure 28: $\overline{AD} = \overline{AB} = \overline{AF}; \angle F\hat{A}B = 20°; \angle E\hat{A}F = 40°$

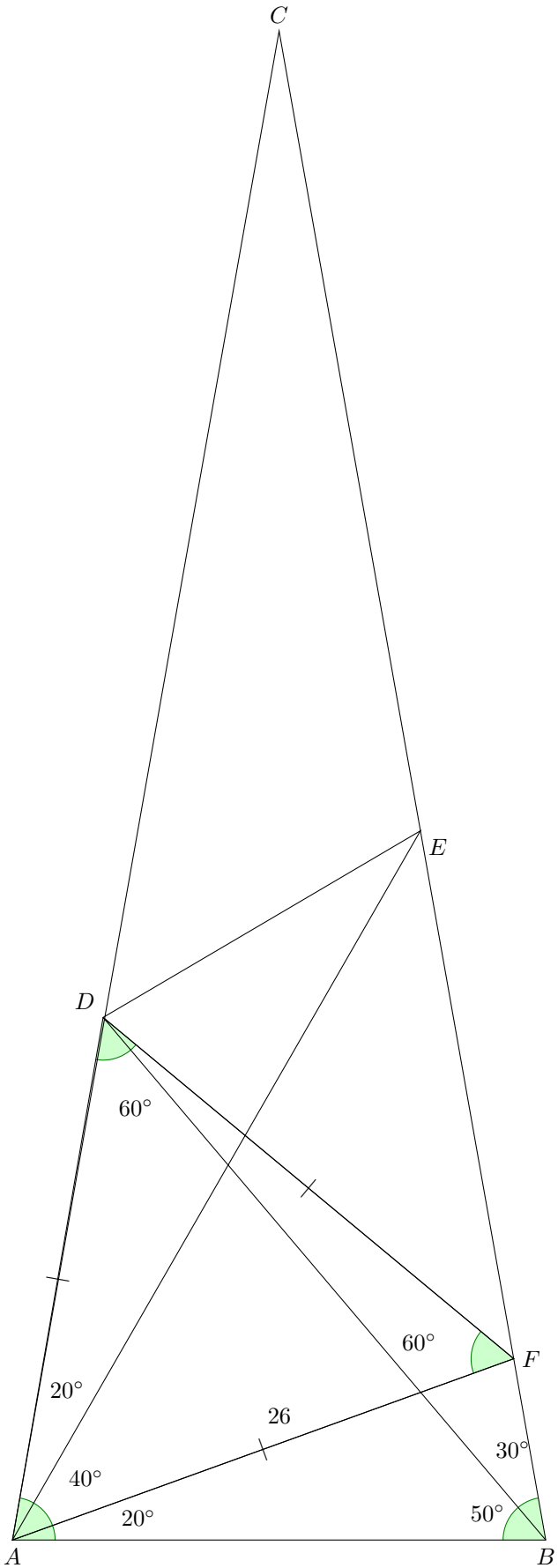

Figure 29: $\overline{DA} = \overline{AF} = \overline{FD}$

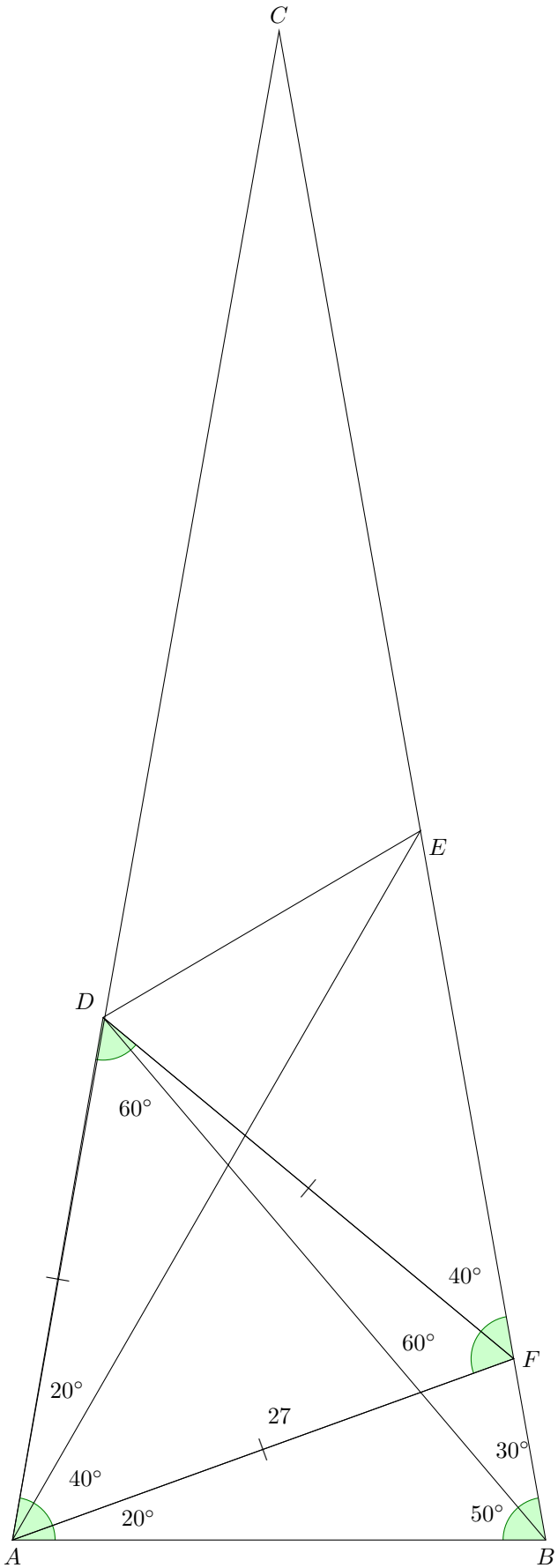

Figure 30: $\overline{DA} = \overline{AF} = \overline{FD} \therefore \angle D\hat{F}E = 40°$

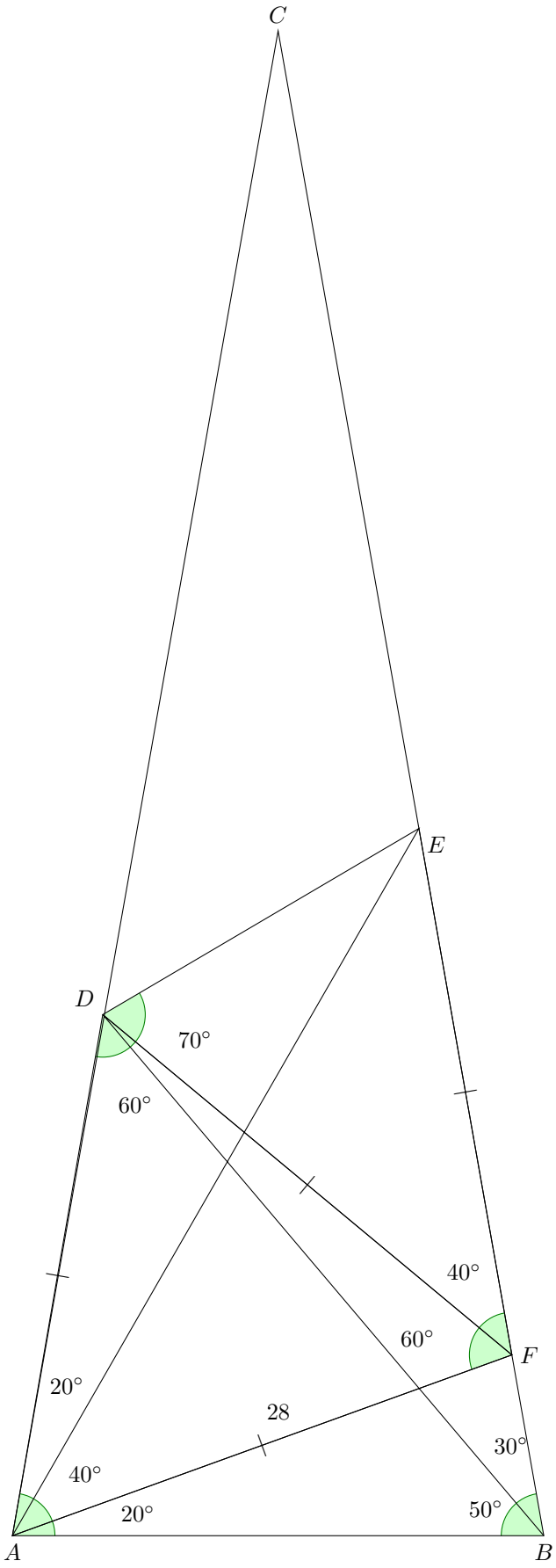

Figure 31: $\angle A\hat{F}E = 40° \therefore \overline{EF} = \overline{DF} \therefore \angle F\hat{D}E = \angle F\hat{E}D = 70°$