A1

Yuhan

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Q1

a.

$D_m{}^{-1} A_m{}^T \hat{Z} A_n D_n{}^{-1} = D_m{}^{-1} A_m{}^T A_m Z A_n{}^T A_n D_n{}^{-1}$

since $A_n{}^T A_n = D_n$

the above notation$= D_m{}^{-1} D_m Z D_n D_n{}^{-1}$

D_n is diagonal given

$D_m{}^{-1} D_m = D_n D_n{}^{-1} = I$

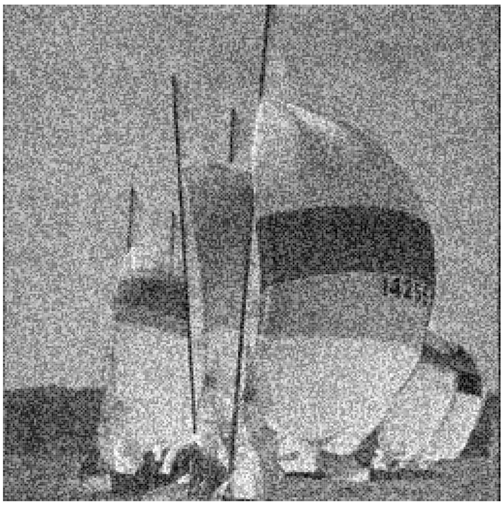
so $D_m{}^{-1} A_m{}^T \hat{Z} A_n D_n{}^{-1} = D_m{}^{-1} D_m Z D_n D_n{}^{-1} = IZI = Z$

b.

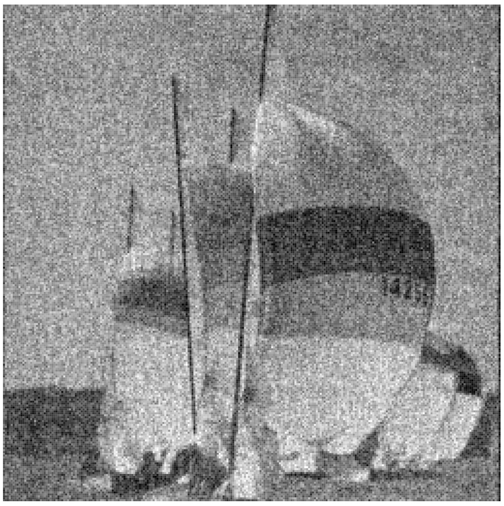
```r
###########Given function,labelled as denoise1()
denoise1 <- function(dctmat,lambda) {
  # if lambda is missing, set it to the 0.8 quantile of abs(dctmat)
  if(missing(lambda)) lambda <- quantile(abs(dct),0.8)
  # hard-thresholding
  a <- dctmat[1,1]
  dctmat1 <- ifelse(abs(dctmat)>lambda,dctmat,0)
  dctmat1[1,1] <- a
  # inverse DCT to obtain denoised image "clean"
  clean <- mvdct(dctmat1,inverted=T)
  clean <- ifelse(clean<0,0,clean)
  clean <- ifelse(clean>1,1,clean)
  clean
}

##########Modified Function,,labelled as denoise2()
denoise2 <- function(dctmat,lambda) {
  # if lambda is missing, set it to the 0.8 quantile of abs(dctmat)
  if(missing(lambda)) lambda <- quantile(abs(dct),0.8)
  # soft-thresholding
  a <- dctmat[1,1]
  dctmat1 <- sign(dctmat)*pmax(abs(dctmat)-lambda,0)
  dctmat1[1,1] <- a
  # inverse DCT to obtain denoised image "clean"
  clean <- mvdct(dctmat1,inverted=T)
  clean <- ifelse(clean<0,0,clean)
  clean <- ifelse(clean>1,1,clean)
  clean
}
```

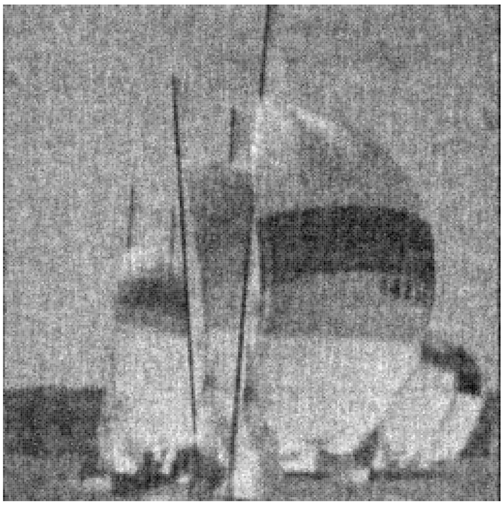
c.



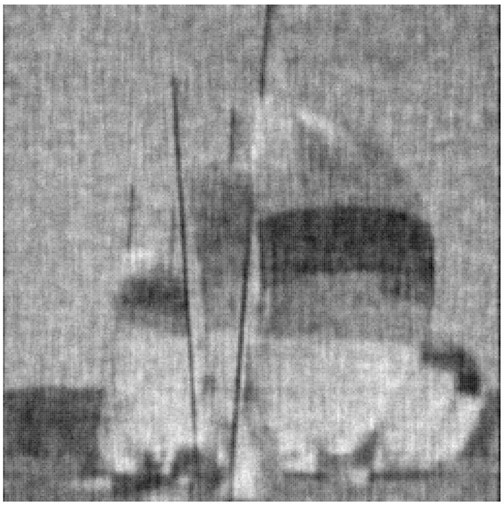
lambda = 10



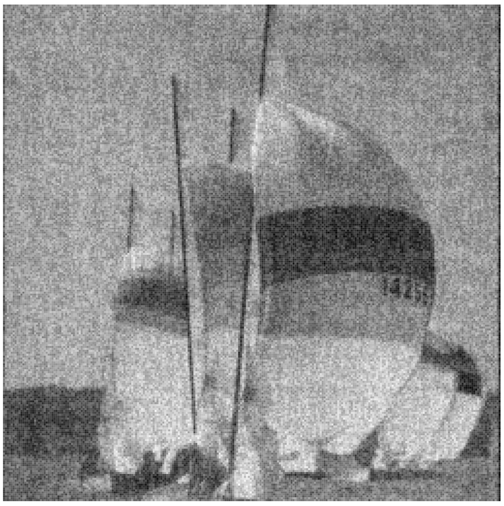
lambda = 20



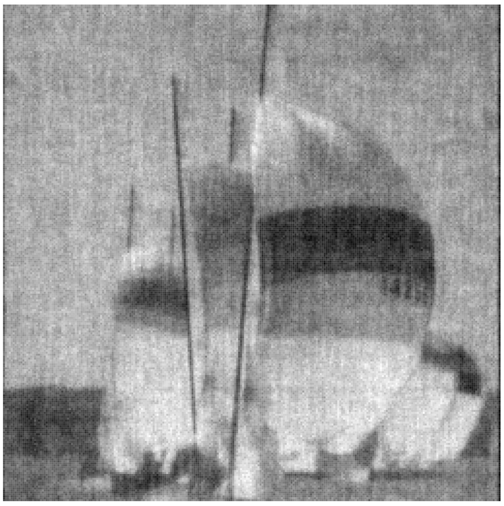
lambda = 30



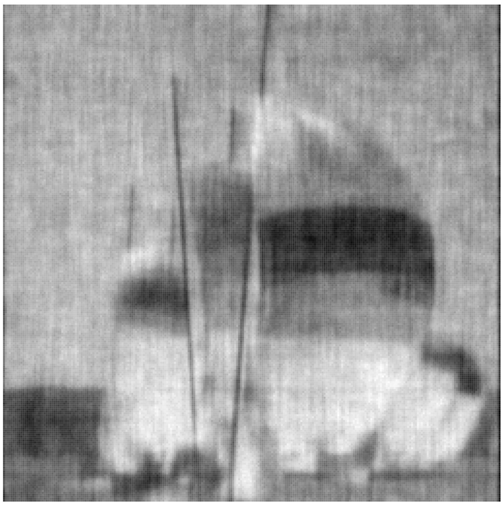
lambda = 40



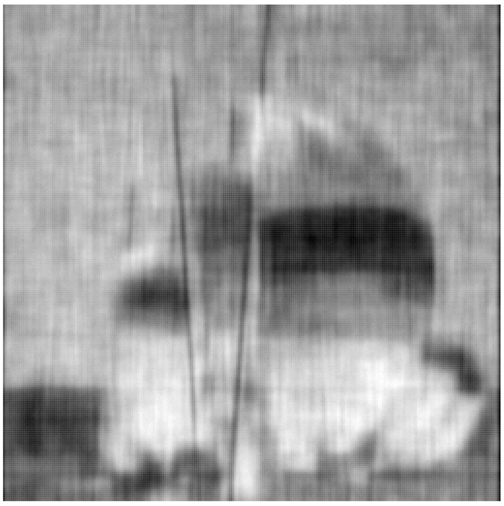
lambda = 10



lambda = 20



lambda = 30



lambda = 40

For both method, as value of lamba increases, contrast of picture decreases. In addition, contrast of hard-threshold approach looks better to me.

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a.

$E_X(s^X) = E_{U+2V}(s^{U+2V}) = E_U(s^U E_{2V}(s^{2V})) = E_U(s^X) * E_{2V}(s^{2V})$

$E(s^U) = E(s^{Poisson(\lambda_u)}) = exp(\lambda_u(s-1))$

$E(s^{2V}) = exp(\lambda_v(s^2-1))$

$E_X(s^X) = E_U(s^X) * E_{2V}(s^{2V}) = exp(\lambda_u(s-1))exp(\lambda_v(s^2-1)) = exp(\lambda_u(s-1) + \lambda_v(s^2-1))$

b.

###c.