1 The problem :

Ω is a open bounded domain with a regulier boundary. A is a tensor which is uniformly bounded

$$\exists C > 0, \forall (x,y) \in \Omega, \forall i,j, A_{i,j}(x,y) \leq C$$

and satisfies the hypothesis of uniform coerciveness.

$$\exists c > 0, \forall (x,y) \in \Omega, \forall \xi \in \mathbb{R}^2, A(x,y)\xi \cdot \xi \geq |\xi|^2$$

and $f \in L^2(\Omega)$. Ω is a square.

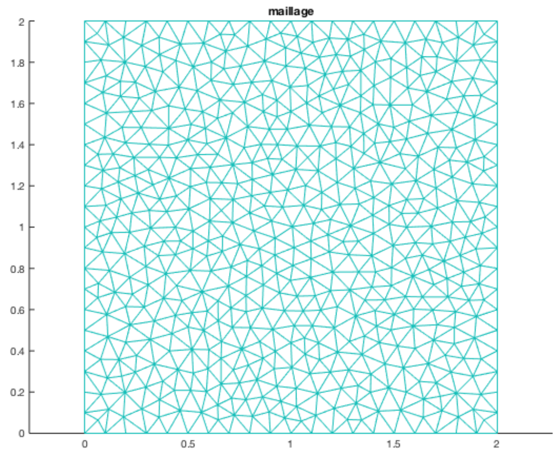

FIGURE 1 – mesh

1.1 With Neumann boundary condition :

Find $u \in H^1(\Omega)$ for :

$$\begin{cases} u - \nabla \cdot (A(x,y)\nabla u) = f \quad dans \quad \Omega \\ A(x,y)\nabla u \cdot n = 0 \quad sur \quad \partial\Omega \end{cases} \tag{1}$$

Variational formulation of the problem :

$$\begin{aligned} \int_\Omega uv - \int_\Omega \nabla \cdot (A(x,y)\nabla u) v &= \int_\Omega f v \\ \int_\Omega uv + \int_\Omega A(x,y)\nabla u \nabla v - \int_{\partial\Omega} (A(x,y)\nabla u \cdot n) v &= \int_\Omega f v \\ \int_\Omega uv + \int_\Omega A(x,y)\nabla u \nabla v &= \int_\Omega f v \end{aligned} \tag{2}$$

1.2 With Drichelet boundary condition :

Find $u \in H^1(\Omega)$ for :

$$\begin{cases} u - \nabla \cdot (A(x,y)\nabla u) = f \quad dans \quad \Omega \\ u = 0 \quad sur \quad \partial\Omega \end{cases} \tag{3}$$

Variational formulation of the problem :

$$
\begin{aligned}
\int_\Omega uv - \int_\Omega \nabla\cdot(A(x,y)\nabla u)\,v &= \int_\Omega fv \quad dans\Omega \\
\int_\Omega uv + \int_\Omega A(x,y)\nabla u\nabla v - \int_{\partial\Omega}(A(x,y)\nabla u\cdot n)\,v &= \int_\Omega fv \\
\int_\Omega uv + \int_\Omega A(x,y)\nabla u\nabla v &= \int_\Omega fv
\end{aligned}
\tag{4}
$$

1.3 With periodic boundary condition :

Find $u \in H^1(\Omega)$ for :

$$
\begin{cases}
u - \nabla\cdot(A(x,y)\nabla u) = f \quad dans \quad \Omega \\
u|_{x=0} = u|_{x=L} \quad et \quad u|_{y=0} = u|_{y=L} \\
A(x,y)\nabla u\cdot e_x|_{x=0} = A(x,y)\nabla u\cdot e_x|_{x=L} \quad et \quad A(x,y)\nabla u\cdot e_y|_{y=0} = A(x,y)\nabla u\cdot e_y|_{y=L}
\end{cases}
\tag{5}
$$

Variational formulation of the problem :

$$
\begin{aligned}
\int_\Omega uv - \int_\Omega \nabla\cdot(A(x,y)\nabla u)\,v &= \int_\Omega fv \\
\int_\Omega uv + \int_\Omega A(x,y)\nabla u\nabla v - \int_{\partial\Omega}(A(x,y)\nabla u\cdot n)\,v &= \int_\Omega fv \\
\int_\Omega uv + \int_\Omega A(x,y)\nabla u\nabla v &= \int_\Omega fv
\end{aligned}
\tag{6}
$$

more details for the proof of well-posedness, the discretization in the report of french version.