Latent Structure Models for NLP

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deep-spin.github.io/tutorial

I. Introduction

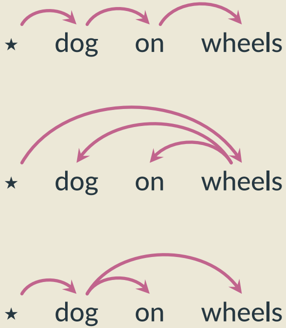
Structured prediction and NLP

- **Structured prediction**: a machine learning framework for predicting structured, constrained, and interdependent outputs
- **NLP** deals with *structured* and *ambiguous* textual data:
    - machine translation
    - speech recognition
    - syntactic parsing
    - semantic parsing
    - information extraction
    - ...

Examples of structure in NLP

Dependency parsing

. . .



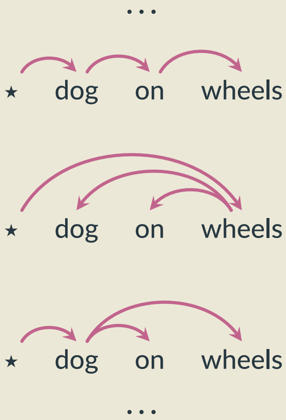
. . .

Examples of structure in NLP

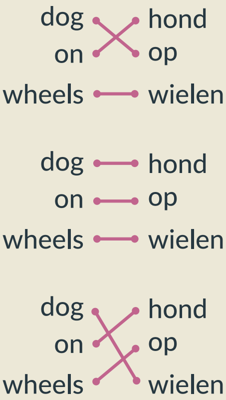
POS tagging

VERB	PREP	NOUN
dog	on	wheels
NOUN	PREP	NOUN
dog	on	wheels
NOUN	DET	NOUN
dog	on	wheels

Dependency parsing



Word alignments



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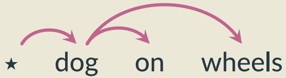
Exponentially many structures!



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NLP 5 years ago:

Structured prediction and pipelines

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- Big pipeline systems, connecting different structured predictors, trained separately
- **Advantages:** fast and simple to train, can rearrange pieces 😊
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- **Bigger disadvantage:** error propagates through the pipeline 💩

NLP today:

End-to-end training

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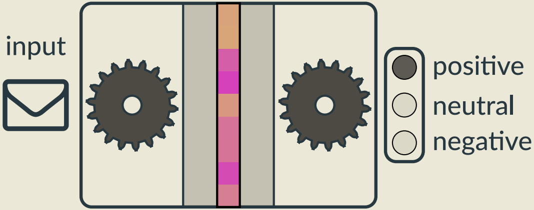
End-to-end training

- Forget pipelines—train everything from scratch!
- No more error propagation or linguistic annotations! 🎉
- Treat everything as *latent*! 🙌

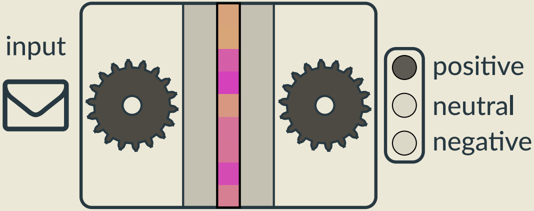
Representation learning

- Uncover hidden representations useful for the *downstream task*.
- Neural networks are well-suited for this: *deep computation graphs*.



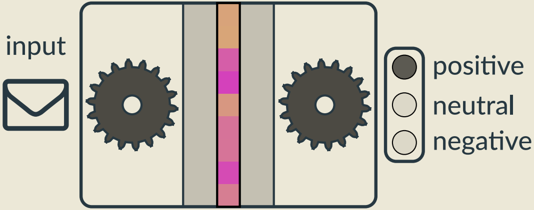
Representation learning

- Uncover hidden representations useful for the *downstream task*.
- Neural networks are well-suited for this: *deep computation graphs*.
- Neural representations are unstructured, inscrutable. Language data has underlying structure!



Latent structure models

- Seek *structured* hidden representations instead!



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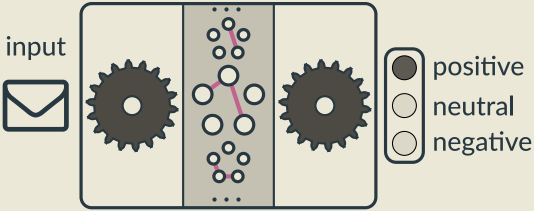

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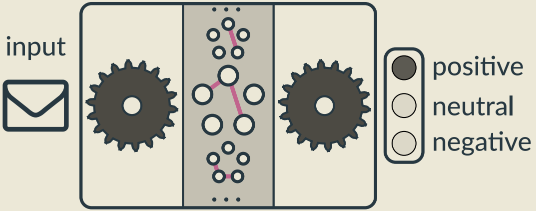
  - More interpretability;



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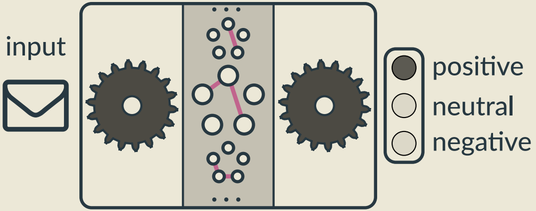
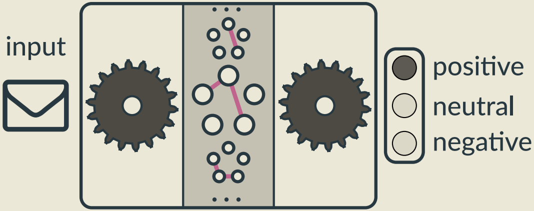


Latent structure models

- Seek *structured* hidden representations instead!
- They can bring us:
  - More interpretability;
  - Better inductive bias;
  - Hopefully: smaller models.



Latent structure models aren't so new!

- They have a very long history in NLP:
  - IBM Models for SMT (latent word alignments) [Brown et al., 1993]
  - HMMs [Rabiner, 1989]
  - CRFs with hidden variables [Quattoni et al., 2007]
  - Latent PCFGs [Petrov and Klein, 2008, Cohen et al., 2012]
- Trained with EM, spectral learning, method of moments, ...
- Often, very strict assumptions (e.g. strong factorizations)
- Today, neural networks opened up some new possibilities!

What this tutorial is about:

- Discrete, combinatorial latent structures
- Often the structure is inspired by some linguistic intuition
- We'll cover both:
  - RL methods (structure built incrementally, reward coming from downstream task)
  - ... vs end-to-end differentiable approaches (global optimization, marginalization)
  - stochastic computation graphs
  - ... vs deterministic graphs.
- All plugged in *discriminative* neural models.

This tutorial is *not* about:

- It’s not about continuous latent variables
- It’s not about deep generative learning
- We won’t cover GANs, VAEs, etc.
- There are (very good) recent tutorials on deep variational models for NLP:
  - “Variational Inference and Deep Generative Models” (Schulz and Aziz, ACL 2018)
  - “Deep Latent-Variable Models for Natural Language” (Kim, Wiseman, Rush, EMNLP 2018)

Background

Unstructured vs structured

- Simplest example of structure: Just a discrete choice among N categories.
- We call this *unstructured*.
- It will provide an important starting point.

The challenge of discrete choices

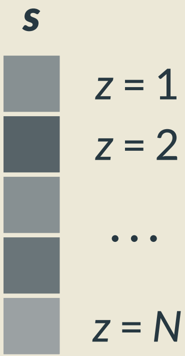
$$z = 1$$

$$z = 2$$

$$\dots$$

$$z = N$$

The challenge of discrete choices



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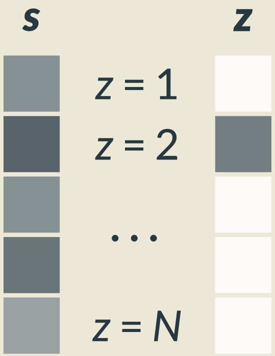

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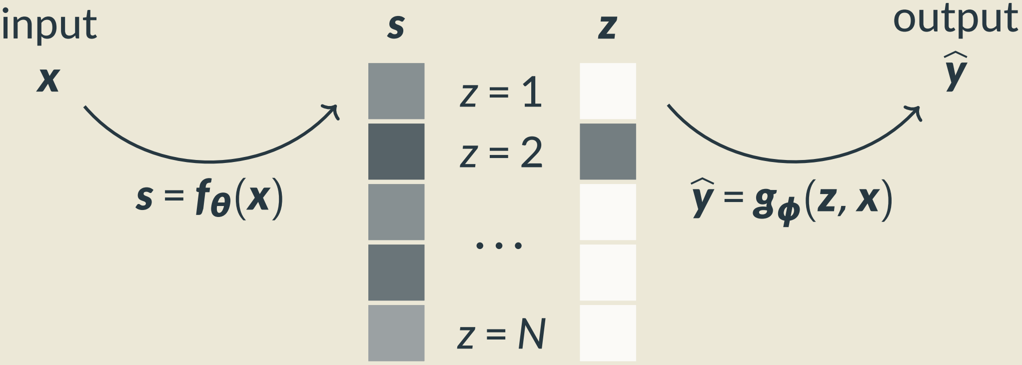

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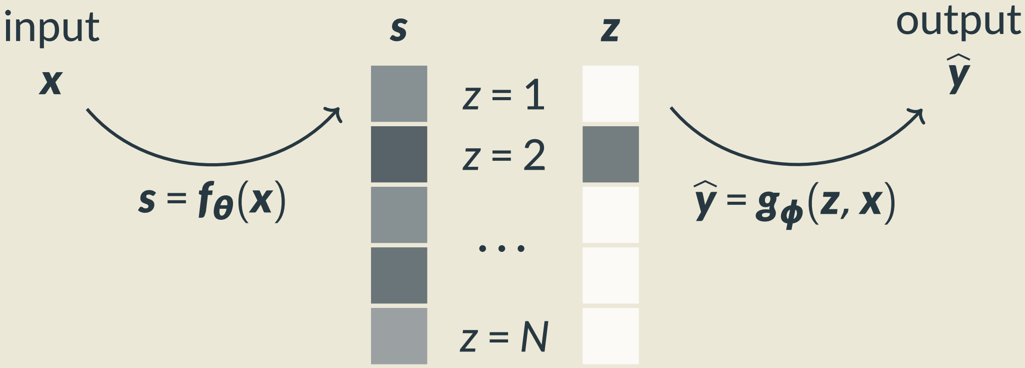
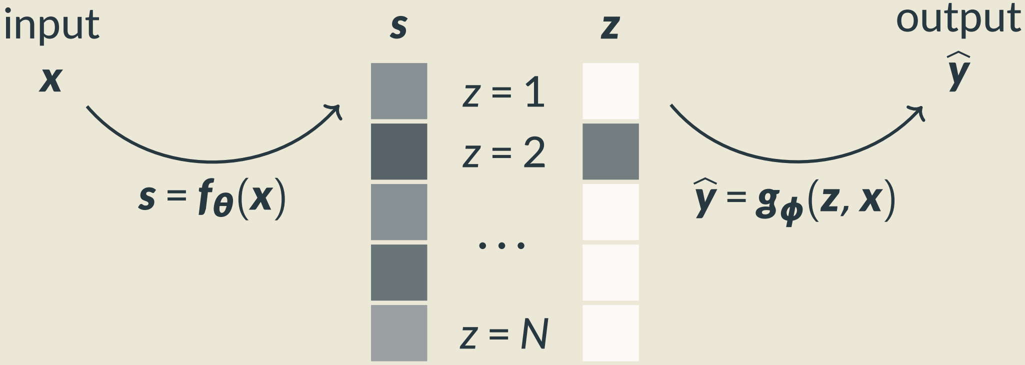


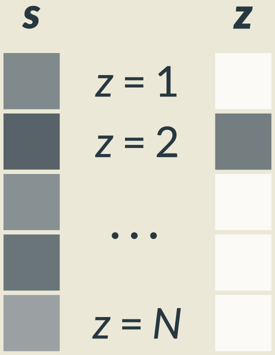
$$\frac{\partial L(\widehat{\boldsymbol{y}}, \boldsymbol{y})}{\partial \boldsymbol{w}} = ?$$

The challenge of discrete choices



$\frac{\partial L(\widehat{\boldsymbol{y}}, \boldsymbol{y})}{\partial \boldsymbol{w}} = ?$ or, essentially, $\frac{\partial \boldsymbol{z}}{\partial \boldsymbol{s}} = ?$

Discrete mappings are “flat”



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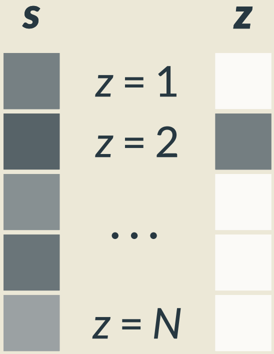

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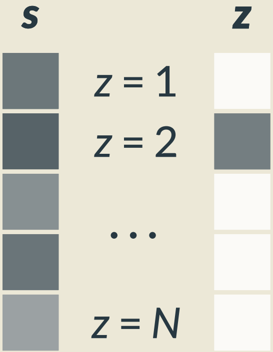

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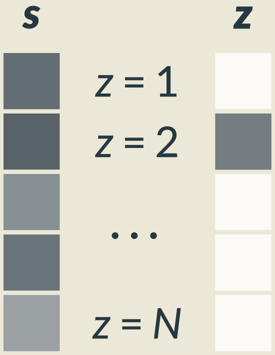

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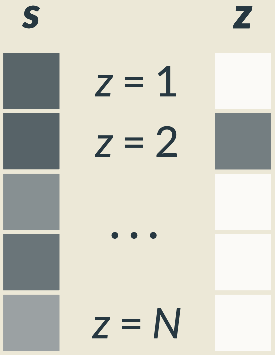

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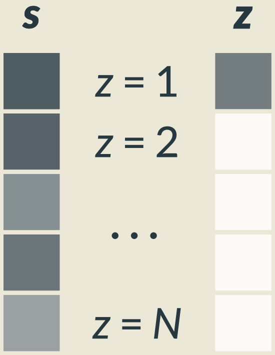

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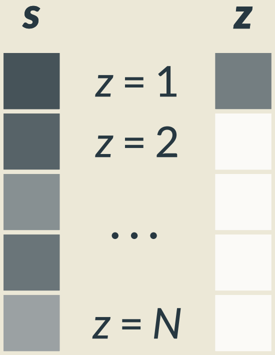

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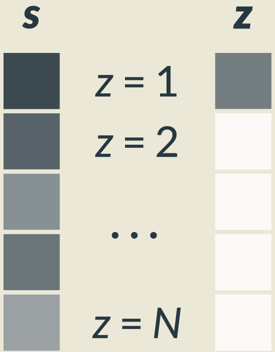

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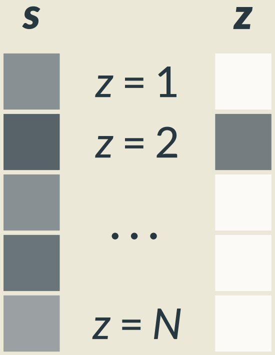
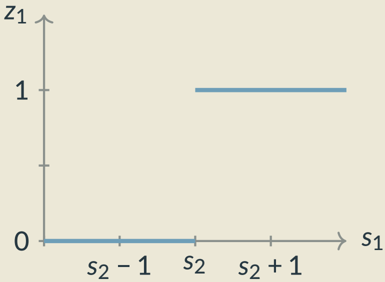
Argmax

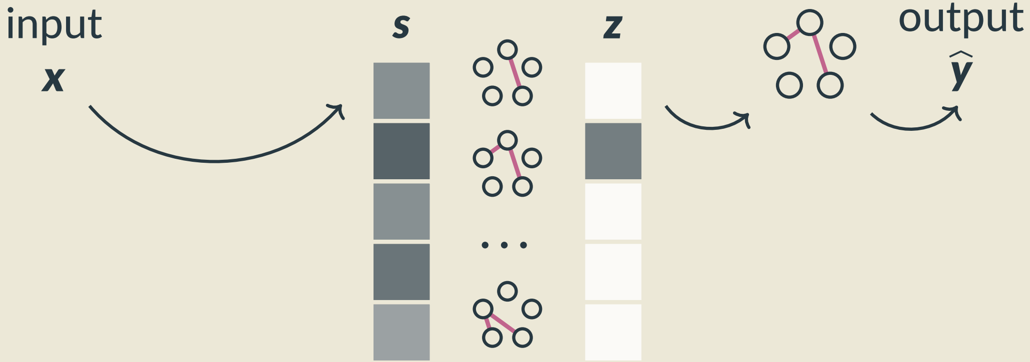




$$\frac{\partial \mathbf{z}}{\partial \mathbf{s}} = \mathbf{0}$$

Computing the most likely structure

is a very high-dimensional argmax



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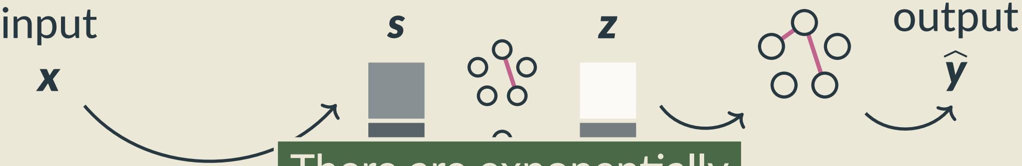
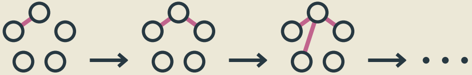


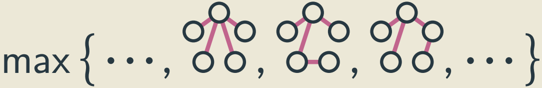
There are exponentially many structures

($\boldsymbol{s}$ cannot fit in memory;

we cannot "loop" over $\boldsymbol{s}$ nor $\boldsymbol{z}$)

Dealing with the combinatorial explosion





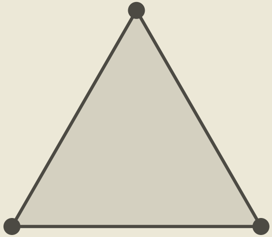
1. Incremental structures

- Build structure **greedily**, as sequence of discrete choices (e.g., shift-reduce).
- Scores (partial structure, action) tuples.
- **Advantages:** flexible, rich histories.
- **Disadvantages:** greedy, local decisions are suboptimal, error propagation.

2. Factorization into parts

- Optimizes **globally** (e.g. Viterbi, Chu-Liu-Edmonds, Kuhn-Munkres).
- Scores smaller parts.
- **Advantages:** optimal, elegant, can handle hard & global constraints.
- **Disadvantages:** strong assumptions.

The unstructured case: Probability simplex



$\triangle$

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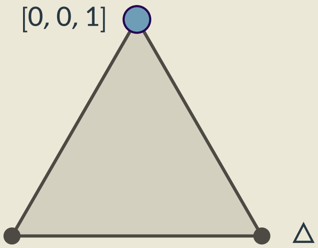

- Each vertex is an *indicator vector*, representing one class:

$$\boldsymbol{z}_c = [0, \ldots, 0, \underbrace{1}_{c^{\text{th}}\text{ position}}, 0, \ldots, 0].$$

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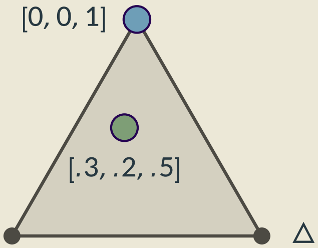

- Each vertex is an *indicator vector*, representing one class:

$$\boldsymbol{z}_c = [0, \dots, 0, \underbrace{1}_{c^{\text{th}}\text{ position}}, 0, \dots, 0].$$

- Points inside are *probability vectors*, a convex combination of classes:

$$\boldsymbol{p} \geq \mathbf{0}, \ \sum_c p_c = 1.$$

What's the analogous of △ for a structure?

- A structured object $\boldsymbol{z}$ can be represented as a *bit vector*.

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- Example:
  - a dependency tree can be represented a $O(L^2)$ vector indexed by arcs
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  - **structural constraints:** not all bit vectors represent valid trees!

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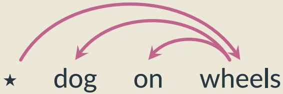
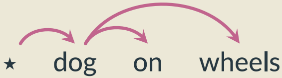
$\boldsymbol{z}_1 = [1, 0, 0, 0, 1, 0, 0, 0, 1]$

$\boldsymbol{z}_2 = [0, 0, 1, 0, 0, 1, 1, 0, 0]$

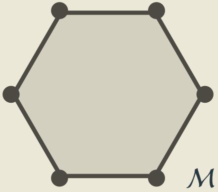


$\boldsymbol{z}_3 = [1, 0, 0, 0, 1, 0, 0, 1, 0]$



[Wainwright and Jordan, 2008]

The structured case: Marginal polytope



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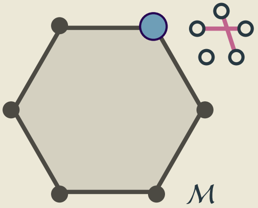

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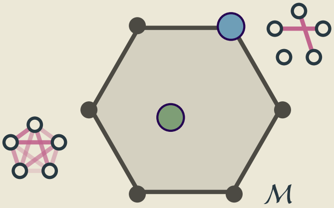
The structured case: Marginal polytope

- Each vertex corresponds to one such *bit vector* $\boldsymbol{z}$
- Points inside correspond to *marginal distributions*: convex combinations of structured objects

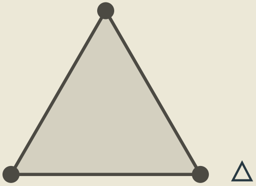


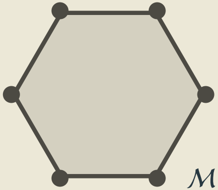
$$\boldsymbol{\mu} = \underbrace{p_1\boldsymbol{z}_1 + \ldots + p_N\boldsymbol{z}_N}_{\text{exponentially many terms}}, \quad \boldsymbol{p} \in \Delta.$$

$$\begin{array}{ll} p_1 = 0.2, & \boldsymbol{z}_1 = [1, 0, 0, 0, 1, 0, 0, 0, 1] \\ p_2 = 0.7, & \boldsymbol{z}_2 = [0, 0, 1, 0, 0, 1, 1, 0, 0] \\ p_3 = 0.1, & \boldsymbol{z}_3 = [1, 0, 0, 0, 1, 0, 0, 1, 0] \end{array} \quad \Rightarrow \quad \boldsymbol{\mu} = [.3, 0, .7, 0, .3, .7, .7, .1, .2].$$

Unstructured vs Structured

- Unstructured case: simplex $\triangle$
- Structured case: marginal polytope $\mathcal{M}$





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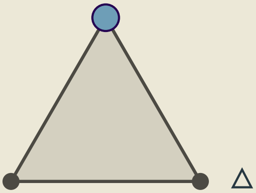

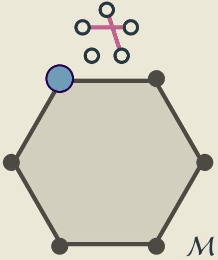

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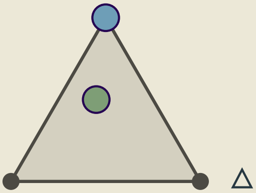

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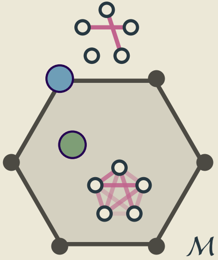
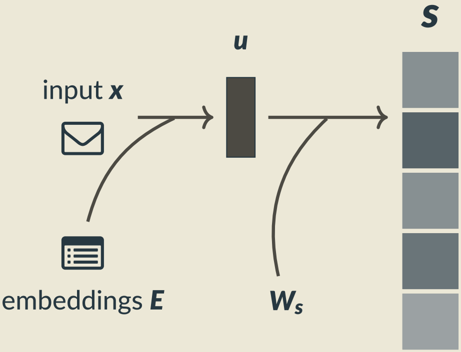


Example: Regression with latent categorization



$\boldsymbol{u} = \frac{1}{|\boldsymbol{x}|} \sum_j \boldsymbol{E}_{x_j}$

$\boldsymbol{s} = \boldsymbol{W_s} \boldsymbol{u}$

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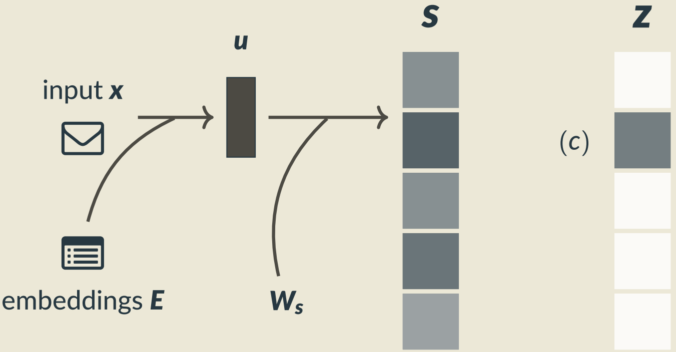

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predict topic c ($\boldsymbol{z} = \boldsymbol{e}_c$)

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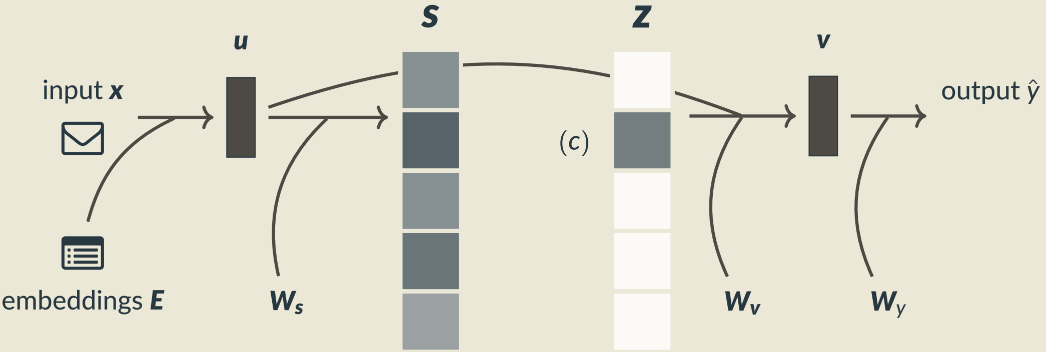

$\boldsymbol{u} = \frac{1}{|\boldsymbol{x}|}\sum_j \boldsymbol{E}_{x_j}$ $\quad \boldsymbol{s} = \boldsymbol{W_s u}$ $\quad \boldsymbol{v} = \tanh\left(\boldsymbol{W_v}[\boldsymbol{u}, \boldsymbol{z}]\right)$ $\quad \hat{y} = \boldsymbol{W_y v}$ $\quad L = (\hat{y} - y)^2$

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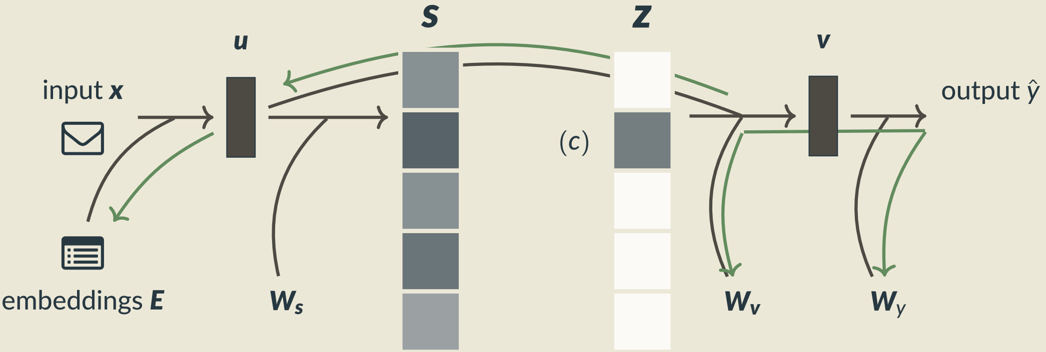
Example: Regression with latent categorization



$\boldsymbol{u} = \frac{1}{|\boldsymbol{x}|} \sum_j \boldsymbol{E}_{x_j}$ $\quad \boldsymbol{s} = \boldsymbol{W_s u}$ $\quad \boldsymbol{v} = \tanh\left(\boldsymbol{W_v}[\boldsymbol{u}, \boldsymbol{z}]\right)$ $\quad \hat{y} = \boldsymbol{W}_y \boldsymbol{v}$ $\quad L = (\hat{y} - y)^2$

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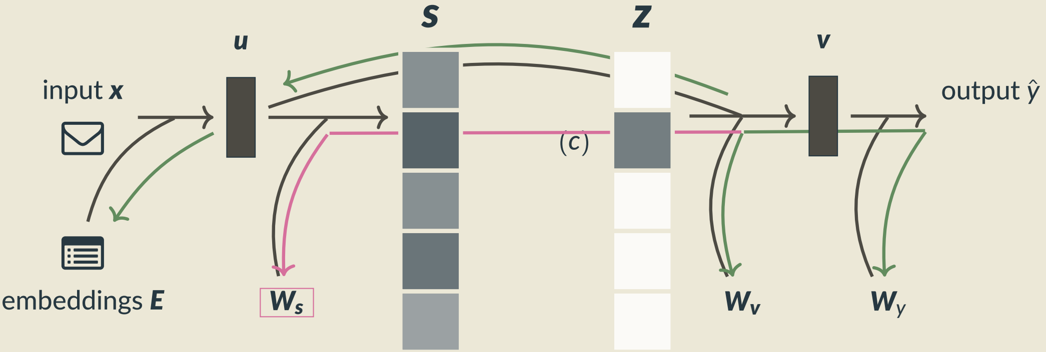

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$$\frac{\partial L}{\partial \boldsymbol{W_s}} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial \boldsymbol{v}} \frac{\partial \boldsymbol{v}}{\partial \boldsymbol{z}} \overbrace{\frac{\partial \boldsymbol{z}}{\partial \boldsymbol{s}}}^{\equiv 0} \frac{\partial \boldsymbol{s}}{\partial \boldsymbol{W_s}}$$

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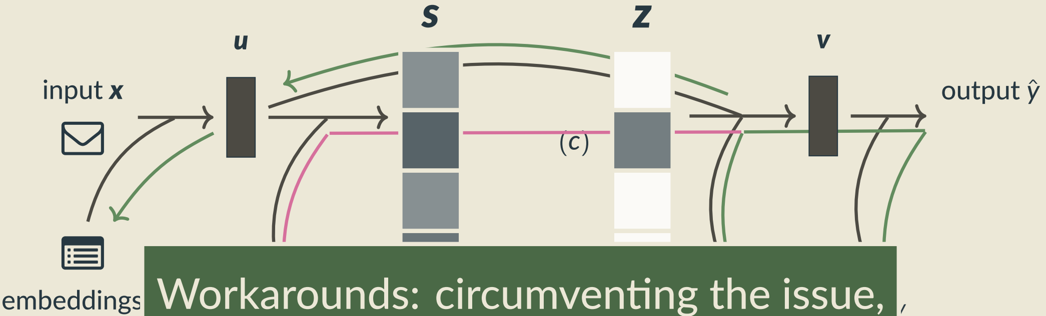
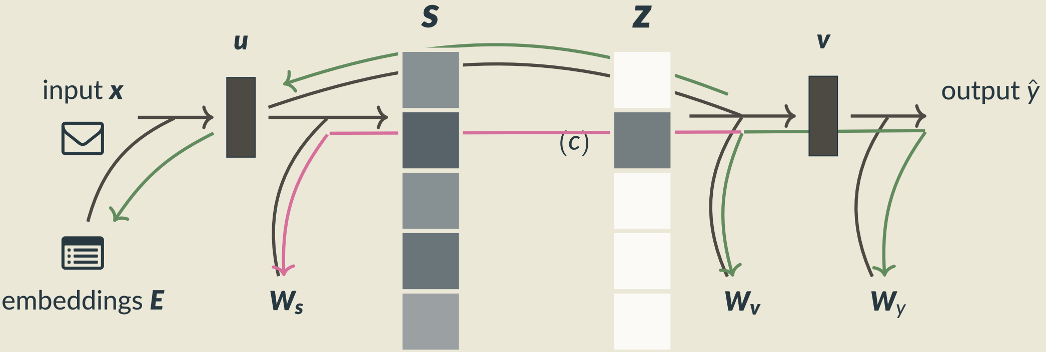


Workarounds: circumventing the issue, bypassing discrete variables

$\boldsymbol{u} = \frac{1}{|\boldsymbol{x}|}\sum_j \boldsymbol{E}_{x}$ ... $- y)^2$

Example: Regression with latent categorization



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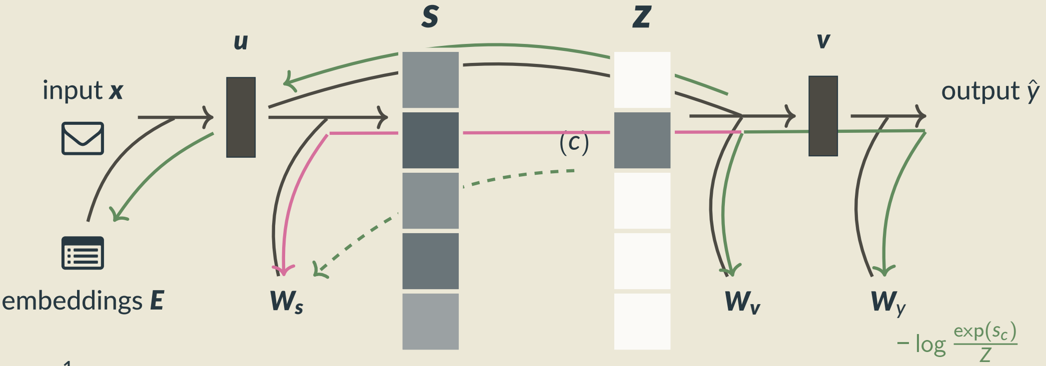
Option 1. Pretrain latent classifier $\boldsymbol{W_s}$

Example: Regression with latent categorization



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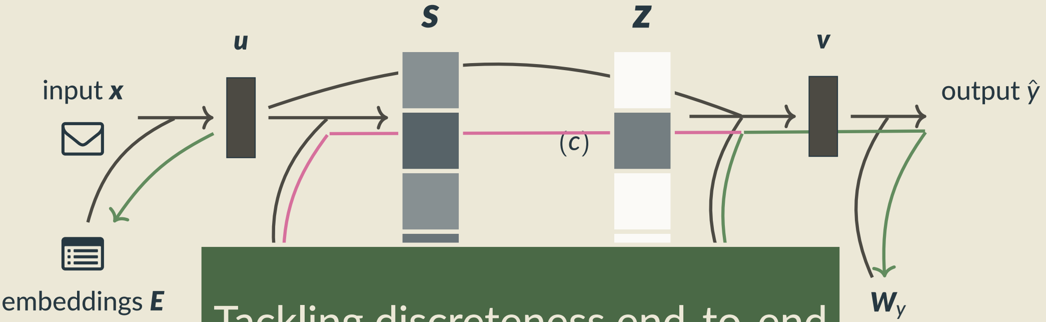
Option 2. Multi-task learning

Example: Regression with latent categorization

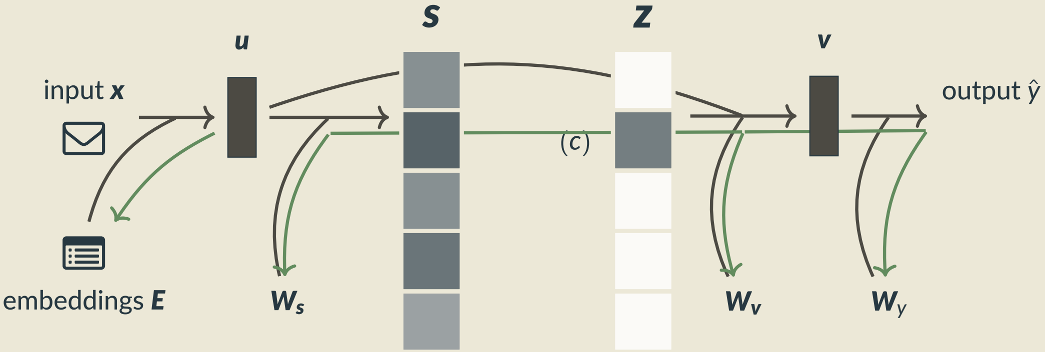


$\boldsymbol{u} = \frac{1}{|\boldsymbol{x}|} \sum_j \boldsymbol{E}_{x_j}$

Tackling discreteness end-to-end

$. = (\hat{y} - y)^2$

Example: Regression with latent categorization



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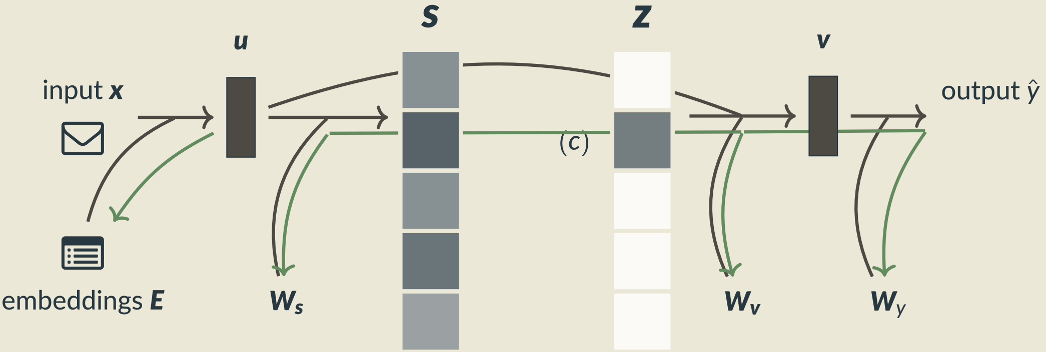
Option 3. Stochasticity! $\frac{\partial \mathbb{E}_{\boldsymbol{z}}(\hat{y}(\boldsymbol{z}) - y)^2}{\partial \boldsymbol{W}_s} \neq \boldsymbol{0}$

Example: Regression with latent categorization



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Option 4. Gradient surrogates (*e.g.* straight-through, $\frac{\partial \boldsymbol{z}}{\partial \boldsymbol{s}} \leftarrow \boldsymbol{I}$)

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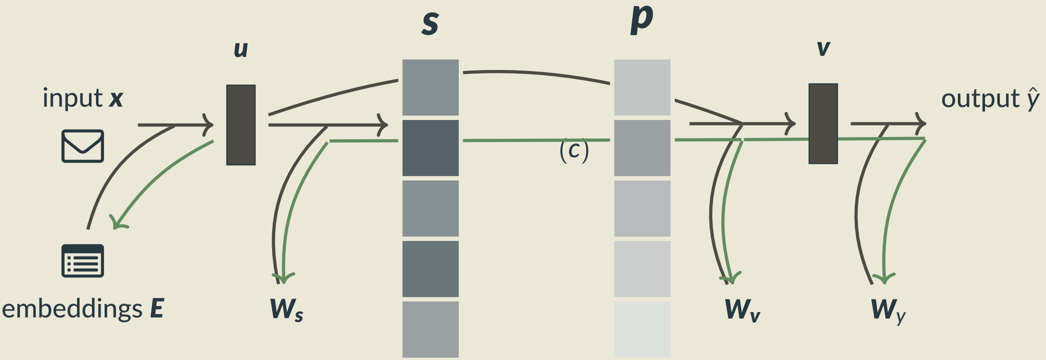

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Option 5. Continuous relaxation (*e.g.* softmax)

Dealing with discrete latent variables

1. Pre-train external classifier
2. Multi-task learning
3. Stochastic latent variables
4. Gradient surrogates
5. Continuous relaxation

Dealing with discrete latent variables

1. Pre-train external classifier
2. Multi-task learning
3. Stochastic latent variables (Part 2)
4. Gradient surrogates (Part 3)
5. Continuous relaxation (Part 4)

Roadmap of the tutorial

- Part 1: Introduction ✓
- Part 2: Reinforcement learning

Coffee Break

- Part 3: Gradient surrogates
- Part 4: End-to-end differentiable models (1/2)

Coffee Break

- Part 4: End-to-end differentiable models (2/2)
- Part 5: Conclusions

II. Reinforcement Learning Methods

Latent structure via marginalization

- Given a sentence-label pair (x, y) and its **known** parse tree $\mathbf{z}$,

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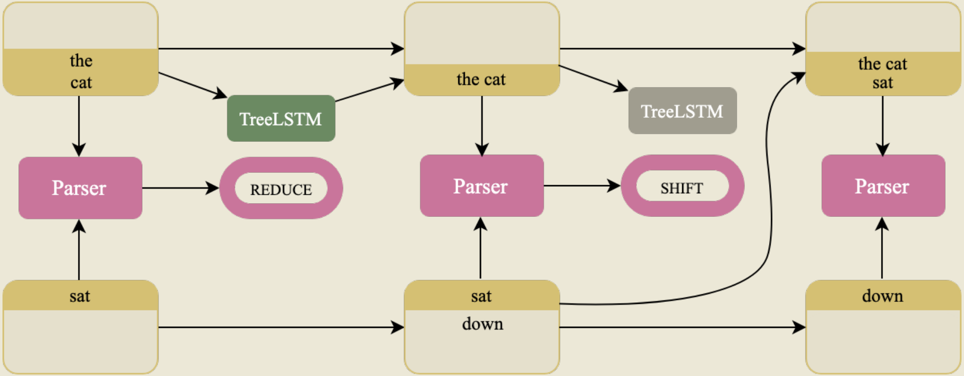
- But we don't know $\boldsymbol{z}$!
- In this section:
  we jointly learn a structured prediction model $\pi_{\boldsymbol{\theta}}(\boldsymbol{z} \mid x)$
  by optimizing the **expected loss**,

$$\mathbb{E}_{\pi_{\boldsymbol{\theta}}(\boldsymbol{z}|x)}\big[L(\boldsymbol{z})\big]$$

But first, supervised SPINN

[Bowman et al., 2016]

Stack-augmented Parser-Interpreter Neural-Network



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- **TreeLSTM** combines top two elements of the stack when the parser chooses the $\textsc{reduce}$ action.

[Bowman et al., 2016]

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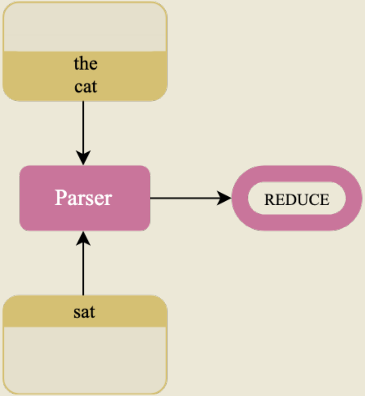

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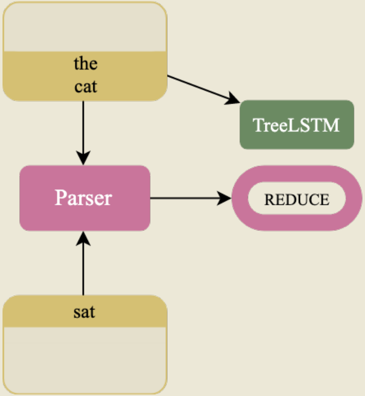

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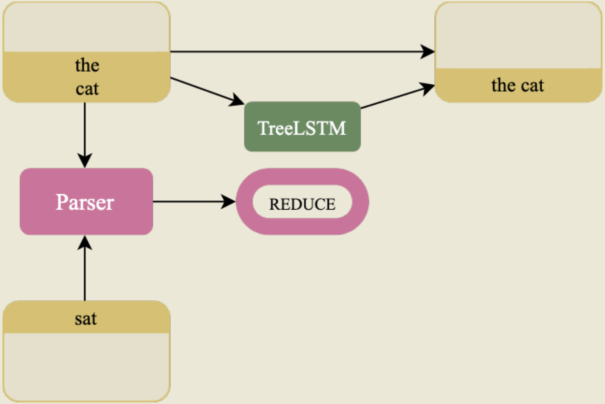

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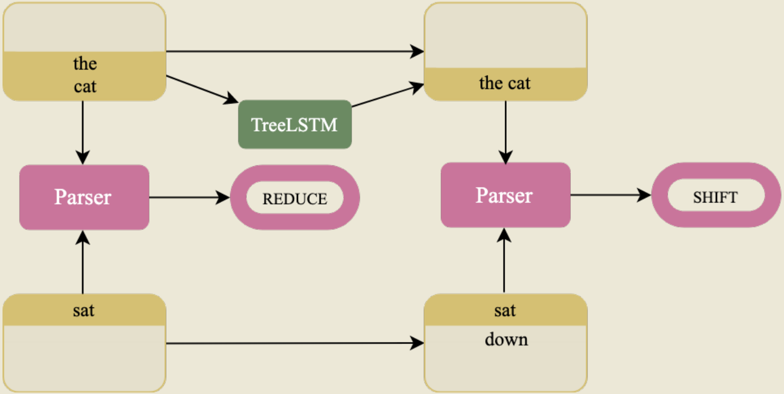

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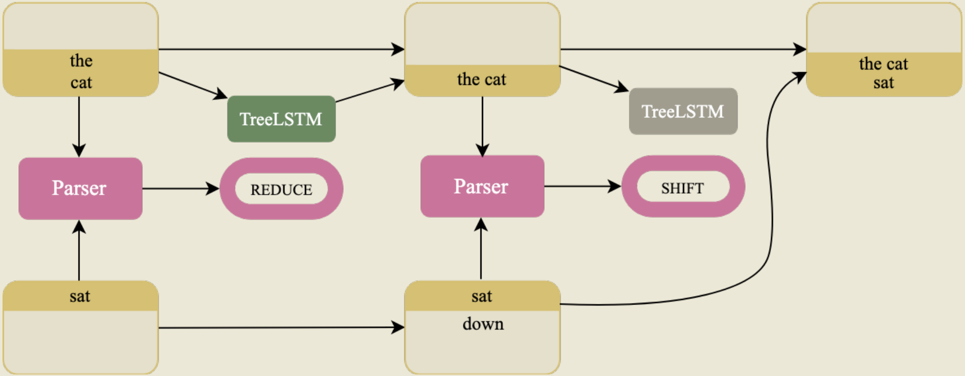

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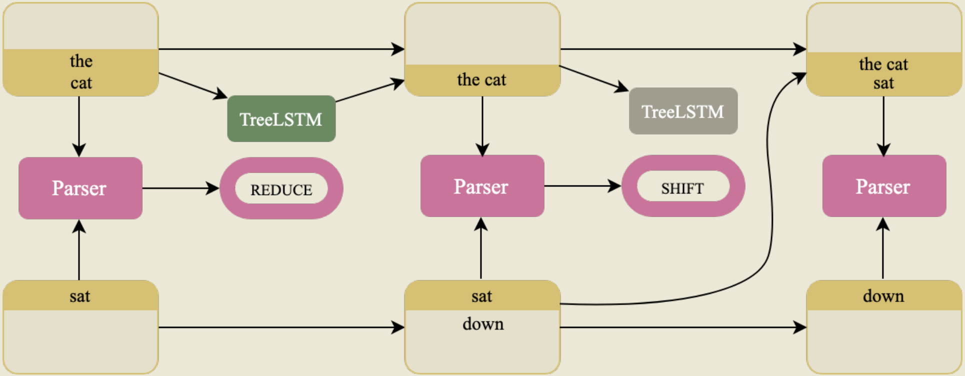


Shift-Reduce parsing

We can write a shift-reduce style parse as a sequence of Bernoulli random variables,

$$\boldsymbol{z} = \{z_1, \ldots, z_{2L-1}\}$$

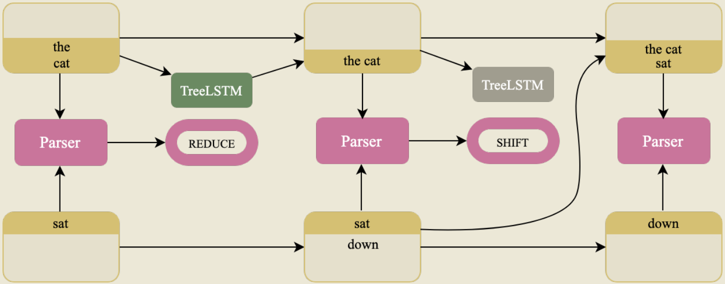
where, $z_j \in \{0, 1\}\ \forall j \in [1, 2L - 1]$

Shift-Reduce parsing

A sequence of Bernoulli trials but with conditional dependence,

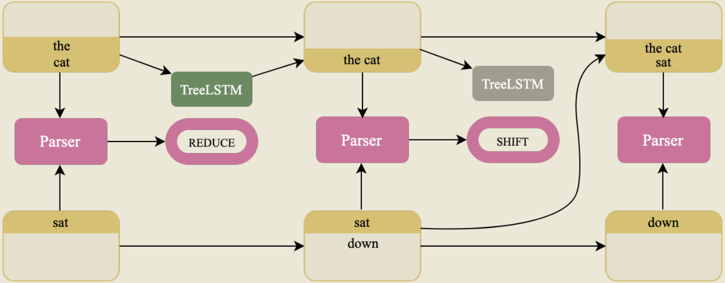
$$p(z_1, z_2, \dots, z_{2L-1}) = \prod_{j=1}^{2L-1} p(z_j \mid z_{<j})$$

Latent structure learning with SPINN



Latent structure learning with SPINN

- But now, remove syntactic supervision from SPINN.



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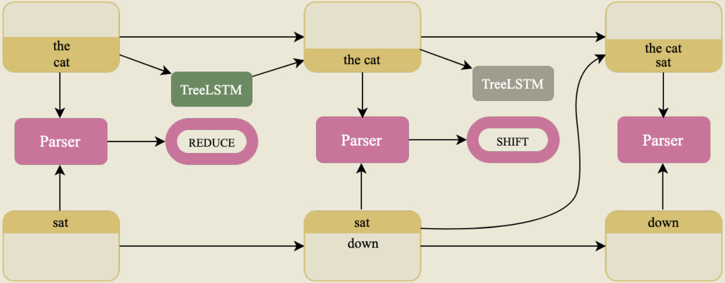

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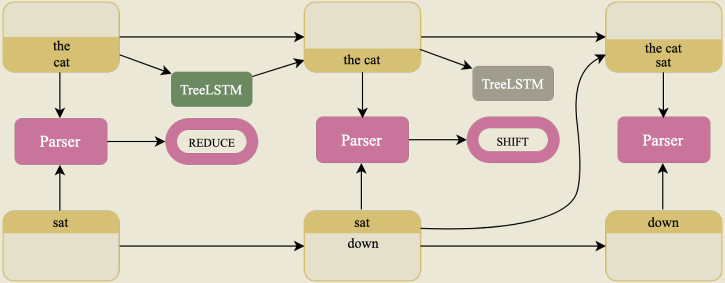

- We model the parse, $\boldsymbol{z}$, as a latent variable scored by $f_{\boldsymbol{\theta}}(x)$.
- With shift-reduce parsing, we're making discrete decisions
  $\Rightarrow$ REINFORCE as a "natural" solution.

Unsupervised SPINN

Unsupervised SPINN

No syntactic supervision.

Only reward is from the downstream task.

We only get this reward after parsing the full sentence.

[Williams, 1992]

SPINN with REINFORCE

Some basic terminology,

- The action space is $z_j \in \{\textsc{shift}, \textsc{reduce}\}$, and $\boldsymbol{z}$ is a sequence of actions.

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- Lear
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NOTE: Only a single reward at the end of parsing.

Through the looking glass of REINFORCE

$$\nabla_{\boldsymbol{\theta}} \mathbb{E}_{\boldsymbol{z} \sim \pi_{\boldsymbol{\theta}}(\boldsymbol{z}|x)}[L(\boldsymbol{z})]$$

Through the looking glass of REINFORCE

$$\nabla_{\boldsymbol{\theta}} \mathbb{E}_{\boldsymbol{z} \sim \pi_{\boldsymbol{\theta}}(\boldsymbol{z}|x)}[L(\boldsymbol{z})] = \nabla_{\boldsymbol{\theta}} \left[\sum_{\boldsymbol{z}} L(\boldsymbol{z}) \pi_{\boldsymbol{\theta}}(\boldsymbol{z} \mid x) \right]$$

(Need to turn it into $\mathbb{E}[\cdot]$ so we can MC-estimate)

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$$\nabla \log f = \frac{\nabla f}{f}, \text{ so } \nabla f = f \, \nabla \log f.$$

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SPINN with REINFORCE, aka RL-SPINN

Yogatama et al. [2017] uses REINFORCE to train SPINN!

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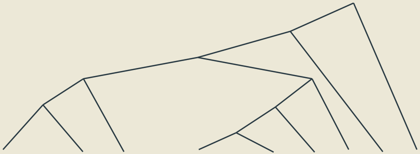
However, this vanilla implementation isn't very effective at learning syntax.

This model fails to solve a simple toy problem.

Toy problem: ListOps



[max 2 9 [min 4 7] 0]

Toy problem: ListOps

Model	Accuracy $\mu(\sigma)$	Accuracy max	Self F1
LSTM	71.5 (1.5)	**74.4**	-
RL-SPINN	60.7 (2.6)	64.8	30.8
Random Trees	-	-	30.1

Model	F1 wrt. LB	F1 wrt. RB	F1 wrt. GT	Avg. Depth
48D RL-SPINN	**64.5**	**16.0**	32.1	**14.6**
128D RL-SPINN	43.5	13.0	**71.1**	10.4
GT Trees	41.6	8.8	100.0	9.6
Random Trees	24.0	24.0	24.2	5.2

[Nangia and Bowman, 2018]

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But why?

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RL-SPINN’s Troubles

This system faces at least two big problems,

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This system faces at least two big problems,

1. High variance of gradients
2. Coadaptation

High variance

- We have a single reward at the end of parsing.

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  Catalan number of binary trees.

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$$
\begin{aligned}
3 \text{ tokens} &\Rightarrow 5 \text{ trees} \\
5 \text{ tokens} &\Rightarrow 42 \text{ trees} \\
10 \text{ tokens} &\Rightarrow 16796 \text{ trees}
\end{aligned}
$$

High variance

- We have a single reward at the end of parsing.
- We are sampling parses from very large search space!
  Catalan number of binary trees.
- And the policy is stochastic.

High variance

So, sometimes the policy lands in a “rewarding state”:

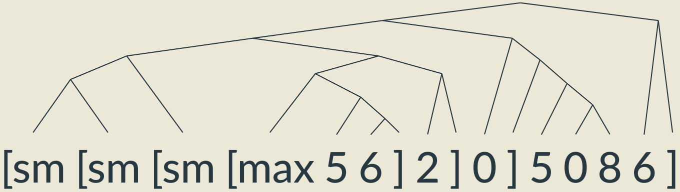

Figure: Truth: 7; Pred: 7

High variance

Sometimes it doesn't:

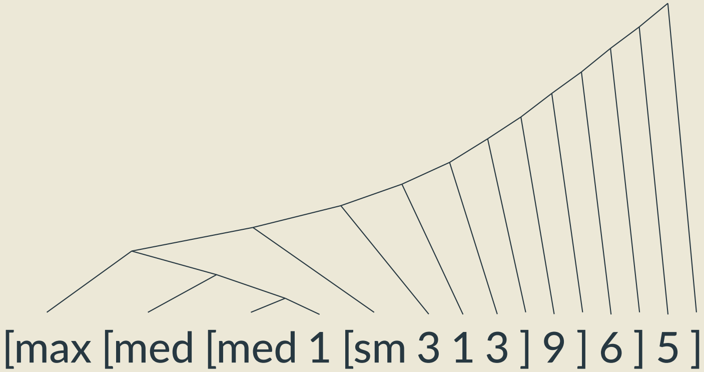

Figure: Truth: 6; Pred: 5

High variance

Catalan number of parses means we need many many samples to lower variance!

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Possible solutions,

1. Gradient normalization
2. Control variates, aka baselines

Control variates

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  So,

$$\nabla \mathbb{E}_{\boldsymbol{z} \sim \pi(\boldsymbol{z})} = \mathbb{E}_{\boldsymbol{z} \sim \pi(\boldsymbol{z})} [(L(\boldsymbol{z}) - b(\boldsymbol{x})) \nabla \pi(\boldsymbol{z})]$$

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- A simple control variate: moving average of recent rewards
- Parameters are updated using the advantage which is the difference between the reward, $\mathcal{R}$, and the baseline prediction.

  So,

$$\nabla \mathbb{E}_{\boldsymbol{z} \sim \pi(\boldsymbol{z})} = \mathbb{E}_{\boldsymbol{z} \sim \pi(\boldsymbol{z})} [(L(\boldsymbol{z}) - b(\boldsymbol{x})) \nabla \pi(\boldsymbol{z})]$$

  Which we can do because,

$$\sum_{z} b(\boldsymbol{x}) \nabla \pi(\boldsymbol{z}) = b(\boldsymbol{x}) \sum_{z} \nabla \pi(\boldsymbol{z}) = b(\boldsymbol{x}) \nabla 1 = 0$$

Issues with SPINN with REINFORCE

This system faces two big problems,

1. High variance of gradients
2. Coadaptation

Coadaptation problem

Learning composition function parameters $\boldsymbol{\phi}$ with backpropagation,
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Difference in variance of two gradient estimates.

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Possible solution:
Proximal Policy Optimization (Schulman et al., 2017)

Making REINFORCE+SPINN work

Havrylov et al. [2019] use,

1. Input dependent control variate
2. Gradient normalization
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However, does not learn English grammars.

Should I? Shouldn't I?

- Unbiased!

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- High variance 😟
- Has not yet been very effective at learning English syntax.

Roadmap of the tutorial

- Part 1: Introduction ✓
- Part 2: Reinforcement learning ✓

Coffee Break

- Part 3: Gradient surrogates
- Part 4: End-to-end differentiable models (1/2)

Coffee Break

- Part 4: End-to-end differentiable models (2/2)
- Part 5: Conclusions

III. Gradient Surrogates

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$$\mathbb{E}_{\pi_{\boldsymbol{\theta}}(\boldsymbol{z}|x)}\left[L(\boldsymbol{z})\right]$$

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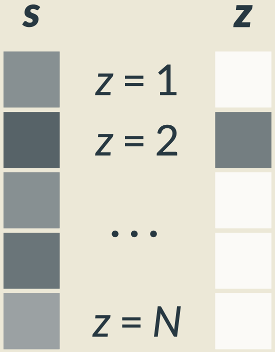
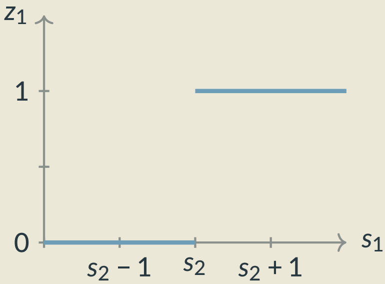
- 3A: try to optimize the deterministic loss directly
- 3B: use this strategy to reduce variance in the stochastic model.

Recap: The argmax problem

$$\boldsymbol{z} = \arg\max(\boldsymbol{s})$$

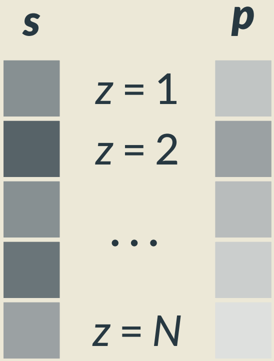


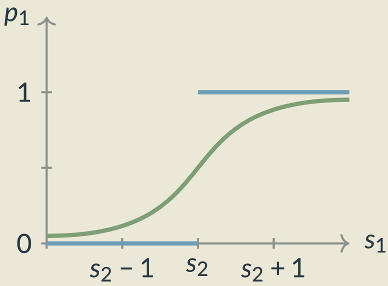


$$\frac{\partial \boldsymbol{z}}{\partial \boldsymbol{s}} = \mathbf{0}$$

Softmax

$$p_j = \exp(s_j)/Z$$





$$\frac{\partial \boldsymbol{p}}{\partial \boldsymbol{s}} = \operatorname{diag}(\boldsymbol{p}) - \boldsymbol{p}\boldsymbol{p}^\top$$

[Hinton, 2012, Bengio et al., 2013]

Straight-Through Estimator

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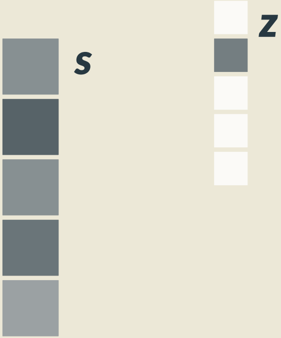
- Forward: $\boldsymbol{z} = \arg\max(\boldsymbol{s})$



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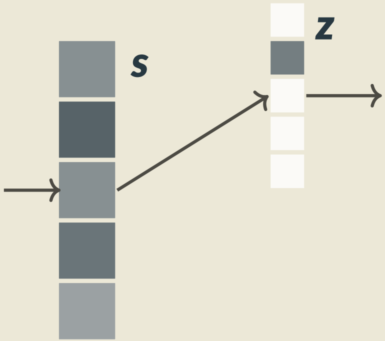

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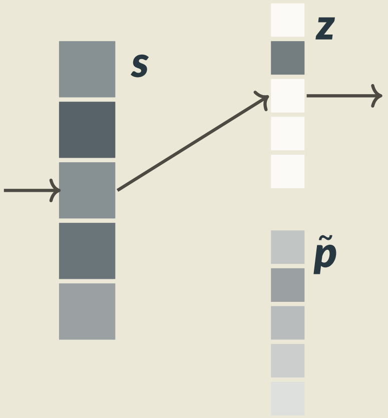

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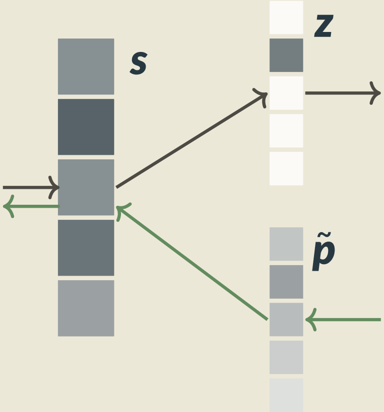

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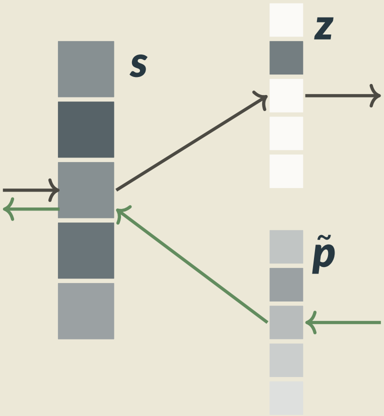

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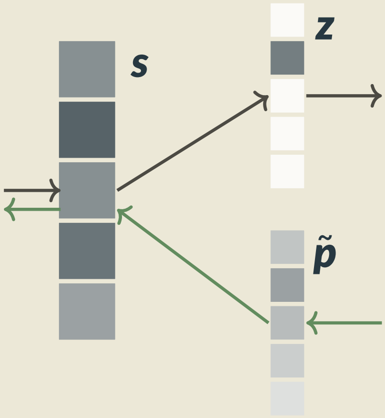

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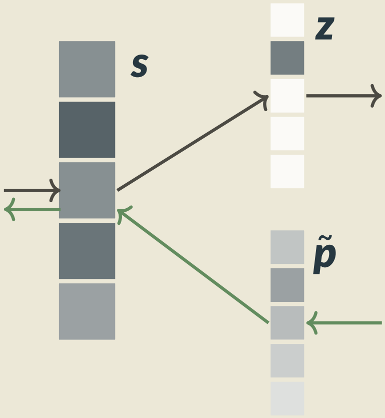
- More explanation in a while



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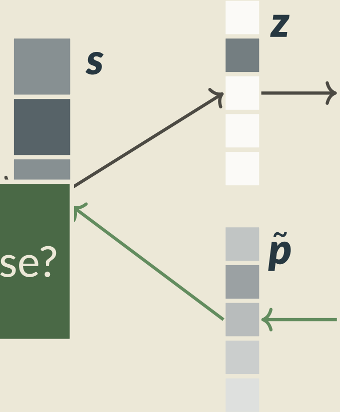
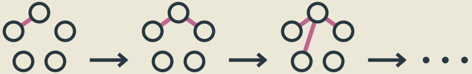
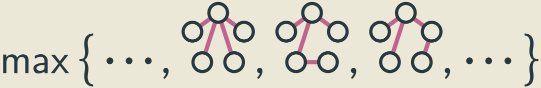
What about the structured case?



Dealing with the combinatorial explosion





1. Incremental structures

- Build structure **greedily**, as sequence of discrete choices (e.g., shift-reduce).
- Scores (partial structure, action) tuples.
- **Advantages:** flexible, rich histories.
- **Disadvantages:** greedy, local decisions are suboptimal, error propagation.

2. Factorization into parts

- Optimizes **globally** (e.g. Viterbi, Chu-Liu-Edmonds, Kuhn-Munkres).
- Scores smaller parts.
- **Advantages:** optimal, elegant, can handle hard & global constraints.
- **Disadvantages:** strong assumptions.

STE for incremental structures

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Example: Latent Tree Learning with Differentiable Parsers: Shift-Reduce Parsing and Chart Parsing [Maillard and Clark, 2018] (STE through beam search).

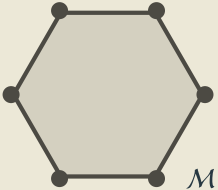
STE for the factorized approach

Requires a bit more work:

- Recap: marginal polytope
- Predicting structures globally: Maximum A Posteriori (MAP)
- Deriving Straight-Through and SPIGOT

[Wainwright and Jordan, 2008]

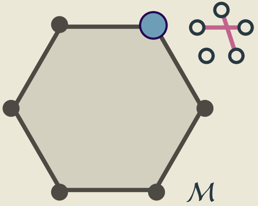
The structured case: Marginal polytope



[Wainwright and Jordan, 2008]

The structured case: Marginal polytope

- Each vertex corresponds to one such *bit vector* $\boldsymbol{z}$



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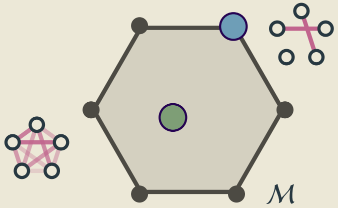
The structured case: Marginal polytope

- Each vertex corresponds to one such *bit vector* $\boldsymbol{z}$
- Points inside correspond to *marginal distributions*: convex combinations of structured objects

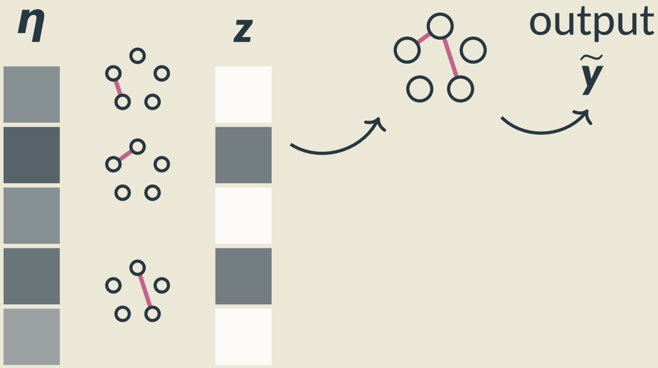


$$\boldsymbol{\mu} = \underbrace{p_1\boldsymbol{z}_1 + \ldots + p_N\boldsymbol{z}_N}_{\text{exponentially many terms}}, \quad \boldsymbol{p} \in \Delta.$$

$$\begin{aligned} p_1 &= 0.2, & \boldsymbol{z}_1 &= [1, 0, 0, 0, 1, 0, 0, 0, 1] \\ p_2 &= 0.7, & \boldsymbol{z}_2 &= [0, 0, 1, 0, 0, 1, 1, 0, 0] \\ p_3 &= 0.1, & \boldsymbol{z}_3 &= [1, 0, 0, 0, 1, 0, 0, 1, 0] \end{aligned} \quad \Rightarrow \quad \boldsymbol{\mu} = [.3, 0, .7, 0, .3, .7, .7, .1, .2].$$

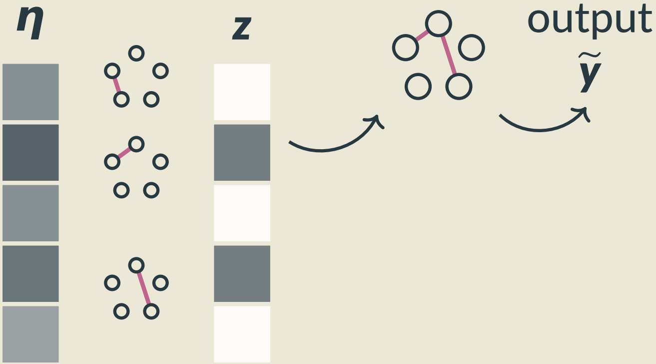
Predicting structures from scores of parts

- $\eta(i \rightarrow j)$: score of arc $i \rightarrow j$
- $z(i \rightarrow j)$: is arc $i \rightarrow j$ selected?



Predicting structures from scores of parts

- $\eta(i \to j)$: score of arc $i \to j$
- $z(i \to j)$: is arc $i \to j$ selected?
- Task-specific algorithm for the highest-scoring structure.



Algorithms for specific structures

	Best structure (MAP)
Sequence tagging	Viterbi [Rabiner, 1989]
Constituent trees	CKY [Kasami, 1966, Younger, 1967] [Cocke and Schwartz, 1970]
Temporal alignments	DTW [Sakoe and Chiba, 1978]
Dependency trees	Max. Spanning Arborescence [Chu and Liu, 1965, Edmonds, 1967]
Assignments	Kuhn-Munkres [Kuhn, 1955, Jonker and Volgenant, 1987]

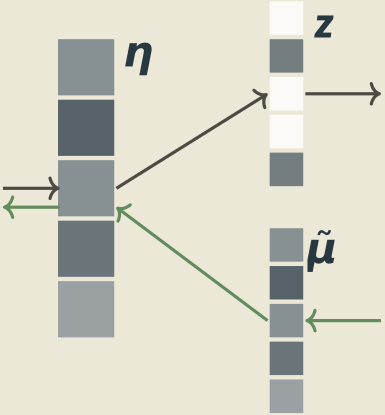
Structured Straight-Through

- Forward pass:

  Find highest-scoring structure:

  $$\boldsymbol{z} = \underset{\boldsymbol{z}\in\mathcal{Z}}{\arg\max}\, \boldsymbol{\eta}^\top \boldsymbol{z}$$
- Backward pass:

  pretend we used $\tilde{\boldsymbol{\mu}} = \boldsymbol{\eta}$.



[Martins and Niculae, 2019]

Straight-Through Estimator

Revisited

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- In the forward pass, $\boldsymbol{z} = \arg\max(\boldsymbol{s})$.
- if we had labels (multi-task learning), $L_{\text{MTL}} = L\big(\hat{y}(\boldsymbol{z}), y\big) + L_{\text{hid}}\big(\boldsymbol{s}, \boldsymbol{z}^{\text{true}}\big)$

[Martins and Niculae, 2019]

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- One choice: perceptron loss $L_{\text{hid}}(\boldsymbol{s}, \boldsymbol{z}^{\text{true}}) = \boldsymbol{s}^\top \boldsymbol{z} - \boldsymbol{s}^\top \boldsymbol{z}^{\text{true}}$, $\quad \frac{\partial L_{\text{hid}}}{\partial \boldsymbol{s}} = \boldsymbol{z} - \boldsymbol{z}^{\text{true}}$.

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[Martins and Niculae, 2019]

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- We don't have labels! Induce labels by "pulling back" the downstream target:
  the "best" (unconstrained) latent value would be: $\arg\min_{\tilde{\boldsymbol{z}} \in \mathbb{R}^D} L\big(\hat{y}(\tilde{\boldsymbol{z}}), y\big)$

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$$\frac{\partial L_{\mathrm{MTL}}}{\partial \boldsymbol{s}} = \underbrace{\frac{\partial L}{\partial \boldsymbol{s}}}_{=0} + \frac{\partial L_{\mathrm{hid}}}{\partial \boldsymbol{s}} = \boldsymbol{z} - \left(\boldsymbol{z} - \frac{\partial L}{\partial \boldsymbol{z}}\right) = \frac{\partial L}{\partial \boldsymbol{z}}$$

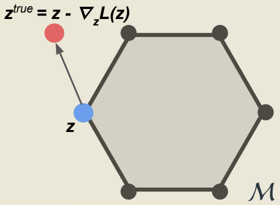
[Peng et al., 2018, Martins and Niculae, 2019]

Straight-Through in the structured case

- Structured STE: perceptron update with induced annotation

$$\arg\min_{\boldsymbol{\mu}\in\mathbb{R}^D} L(\hat{y}(\boldsymbol{\mu}), y) \qquad \approx \boldsymbol{z} - \nabla_{\boldsymbol{z}} L(\boldsymbol{z}) \to \boldsymbol{z}^{\text{true}}$$

(one step of gradient descent)



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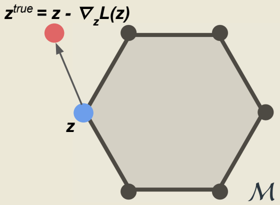
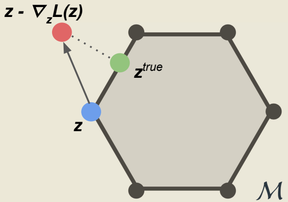


- SPIGOT takes into account the constraints; uses the induced annotation

$$\arg\min_{\boldsymbol{\mu}\in\mathcal{M}} L(\hat{y}(\boldsymbol{\mu}), y) \quad \approx \mathrm{Proj}_{\mathcal{M}}\left(\boldsymbol{z} - \nabla_{\boldsymbol{z}} L(\boldsymbol{z})\right) \rightarrow \boldsymbol{z}^{\text{true}}$$

(one step of *projected* gradient descent!)



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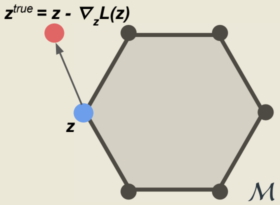

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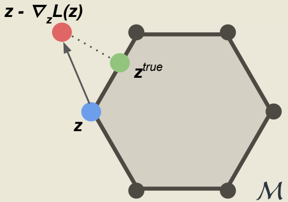

- We discuss a generic way to compute the projection in part 4.

Summary: Straight-Through Estimator

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Now we will see how to apply STE for stochastic graphs, as an alternative approach of REINFORCE.

Stochastic node in the computation graph

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Recall the stochastic objective:

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- An alternative is using the *reparameterization trick* [Kingma and Welling, 2014].

Categorical reparameterization

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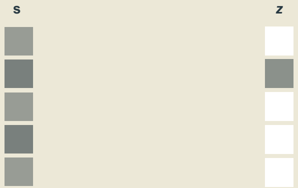
- Sampling from a categorical value in the middle of the computation graph.
  $\boldsymbol{z} \sim \pi_{\boldsymbol{\theta}}(\boldsymbol{z} \mid x) \propto \exp \boldsymbol{s}_{\boldsymbol{\theta}}(\boldsymbol{z} \mid x)$



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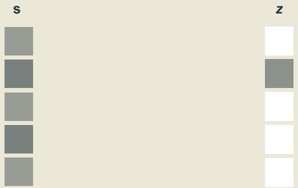

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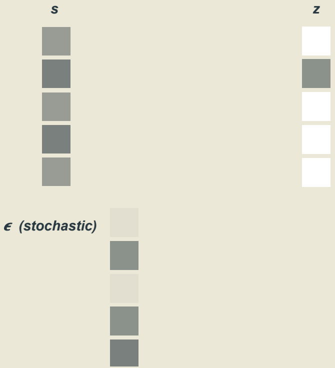

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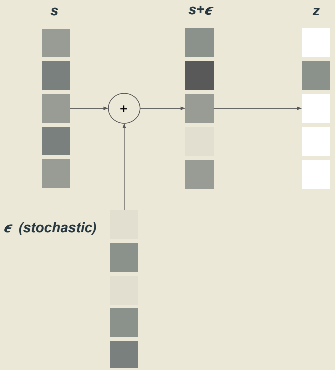

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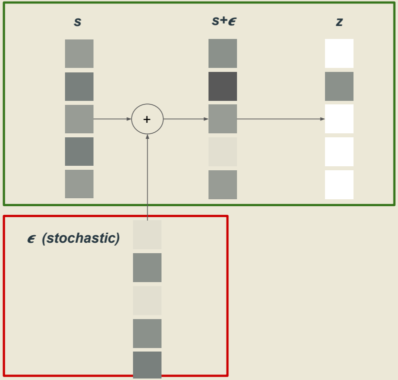

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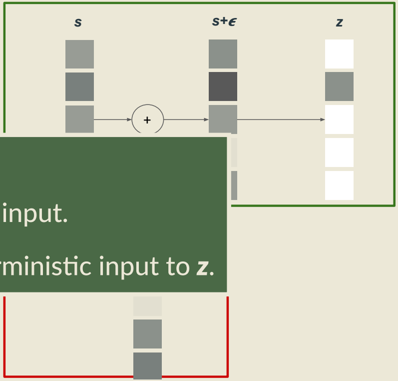


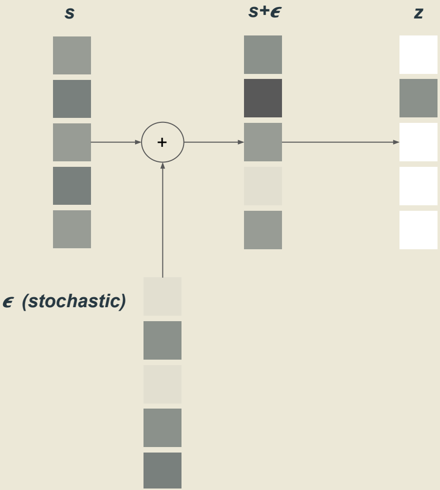
<u>As a result:</u>

Stochasticity is moved as an input.

We can backpropagate through the deterministic input to $\boldsymbol{z}$.

[Jang et al., 2017, Maddison et al., 2016]

Categorical reparameterization



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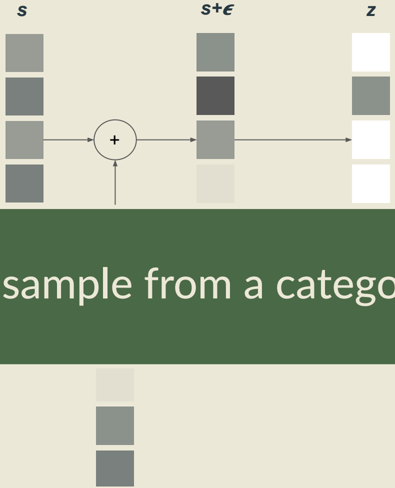

How do we sample from a categorical variable?

Sampling from a categorical variable

We want to sample from a categorical variable with scores $\boldsymbol{s}$ (class i has a score s_i)

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Derivation & more info: *[Adams, 2013, Vieira, 2014]*

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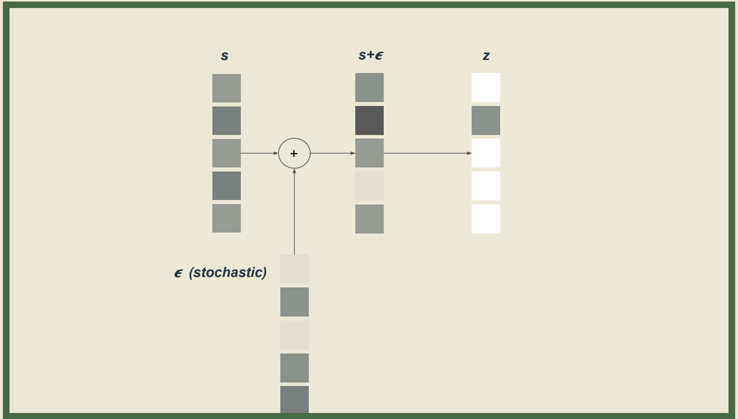

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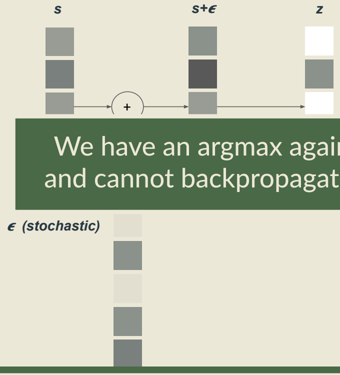

We have an argmax again and cannot backpropagate!

The t ... *now.)*

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[Jang et al., 2017, Maddison et al., 2016]

Straight-Through Gumbel Estimator

Apply a variant of the Straight-Through Estimator to Gumbel-Max!

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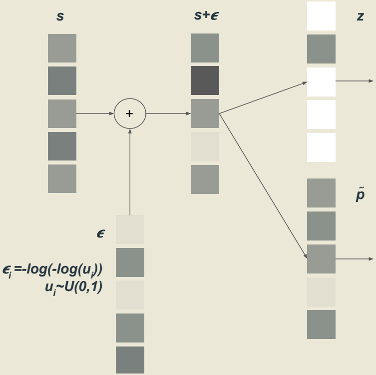

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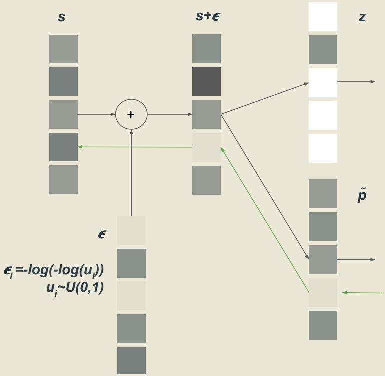

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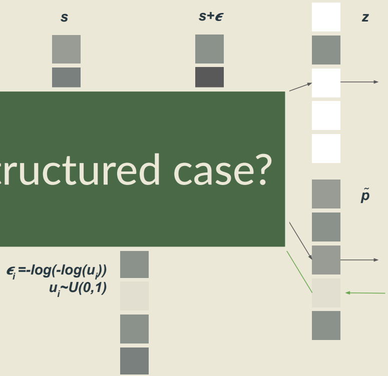
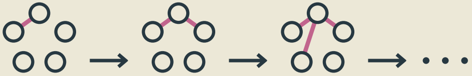
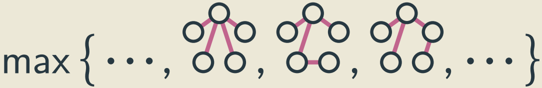


What about the structured case?

Dealing with the combinatorial explosion





1. Incremental structures

- Build structure **greedily**, as sequence of discrete choices (e.g., shift-reduce).
- Scores (partial structure, action) tuples.
- **Advantages:** flexible, rich histories.
- **Disadvantages:** greedy, local decisions are suboptimal, error propagation.

2. Factorization into parts

- Optimizes **globally** (e.g. Viterbi, Chu-Liu-Edmonds, Kuhn-Munkres).
- Scores smaller parts.
- **Advantages:** optimal, elegant, can handle hard & global constraints.
- **Disadvantages:** strong assumptions.

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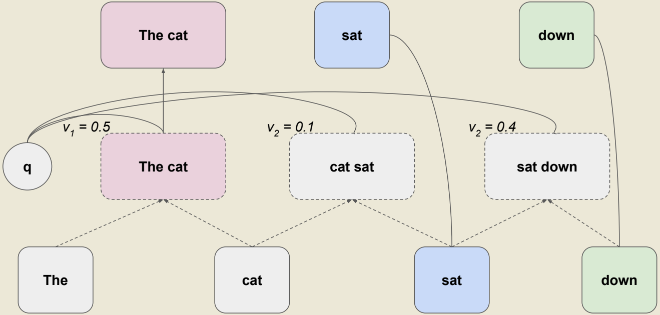
Example: Gumbel Tree-LSTM [Choi et al., 2018].

[Choi et al., 2018]

Example: Gumbel Tree-LSTM

- Building task-specific tree structures.
- Straight-Through Gumbel-Softmax at each step to select one arc.



[Papandreou and Yuille, 2011, Corro and Titov, 2019a,b]

Sampling from factorized models

Perturb-and-MAP

Reparameterize by **perturbing the arc scores.** (inexact!)

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- Sample from the normal Gumbel distribution.
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- Perturb the arc scores with the Gumbel noise.

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- Compute MAP (task-specific algorithm).
- Backward: we could use Straight-Through with Identity.

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Summary: Gradient surrogates

- Based on the **Straight-Through Estimator**.
- Can be used for stochastic or deterministic computation graphs.
- **Forward pass**: Get an argmax (might be structured).
- **Backpropagation**: use a function, which we hope is close to argmax.
- Examples:
  - Argmax for iterative structures and factorization into parts
  - Sampling from iterative structures and factorization into parts

Gradient surrogates: Pros and cons

Pros

- Do not suffer from the high variance problem of REINFORCE.
- Allow for flexibility to select or sample a latent structured in the middle of the computation graph.
- Efficient computation.

Cons

- The Gumbel sampling with Perturb-and-MAP is an approximation.
- Bias, due to function mismatch on the backpropagation
  (next section will address this problem.)

Overview

$$\mathbb{E}_{\pi_{\boldsymbol{\theta}}(\boldsymbol{z}|x)}\Big[L(\boldsymbol{z})\Big] \qquad L\Big(\arg\max_z \pi_{\boldsymbol{\theta}}(\boldsymbol{z} \mid x)\Big)$$

- REINFORCE
- Straight-Through Gumbel
  (Perturb & MAP)

- Straight-Through
- SPIGOT

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$$L\left(\mathbb{E}_{\pi_{\boldsymbol{\theta}}(\boldsymbol{z}|x)}[\boldsymbol{z}]\right)$$

- Structured Attn. Nets
- SparseMAP

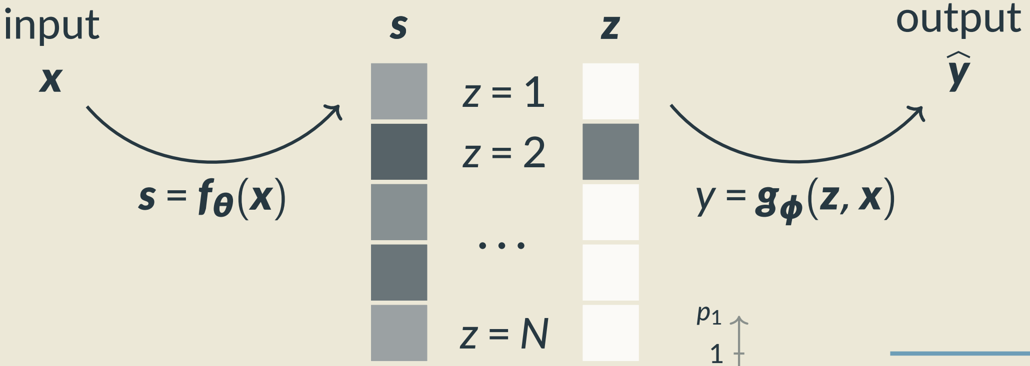
And more, in the next section!

IV. End-to-end Differentiable Relaxations

End-to-end differentiable relaxations

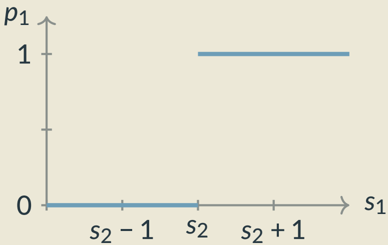
1. Digging into softmax
2. Alternatives to softmax
3. Generalizing to structured prediction
4. Stochasticity and global structures

Recall: Discrete choices & differentiability



$$\frac{\partial \boldsymbol{z}}{\partial \boldsymbol{s}} = \boldsymbol{0} \text{ or n/a}$$

(argmax)



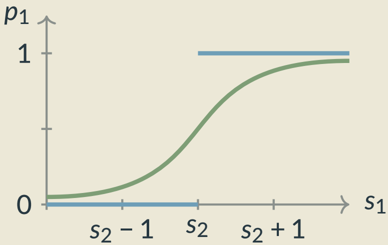
One solution: smooth relaxation

input $\boldsymbol{x}$ $\quad \boldsymbol{s} = \boldsymbol{f_\theta}(\boldsymbol{x}) \quad$ $\boldsymbol{s}$ $\quad$ $z = 1$, $z = 2$, $\ldots$, $z = N$ $\quad$ $\boldsymbol{p}$ $\quad y = \boldsymbol{g_\phi}(\boldsymbol{z}, \boldsymbol{x})$ $\quad$ output $\widehat{\boldsymbol{y}}$

$\boldsymbol{p} = \mathrm{softmax}(\boldsymbol{s}) = \mathbb{E}[\boldsymbol{z}]$, i.e.
replace $\mathbb{E}\big[f(\boldsymbol{z})\big]$ with $f\big(\mathbb{E}[\boldsymbol{z}]\big)$

$\dfrac{\partial \boldsymbol{p}}{\partial \boldsymbol{s}} =$ 😊

(softmax)



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input $\boldsymbol{x}$ → $\boldsymbol{s}$ → $\boldsymbol{p}$ → output $\widehat{\boldsymbol{y}}$

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$\cdots$

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😟

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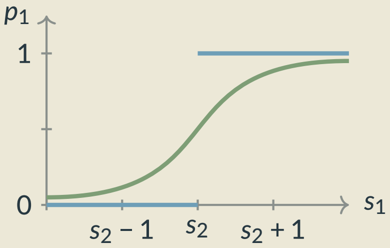

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What is softmax?

Often defined via $p_i = \dfrac{\exp s_i}{\sum_j \exp s_j}$, but where does it come from?

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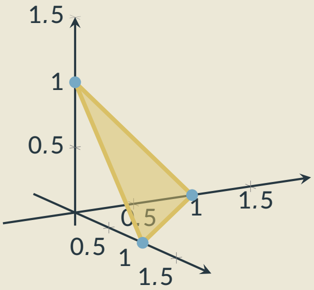

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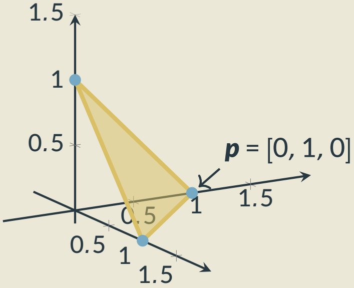

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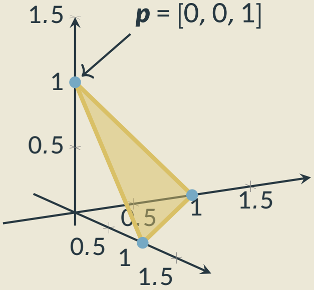

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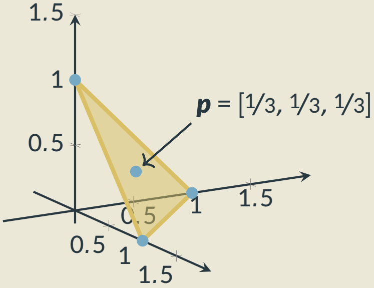

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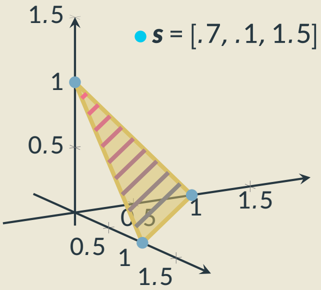

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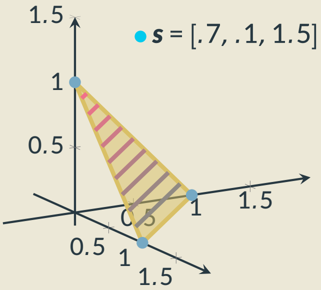
argmax



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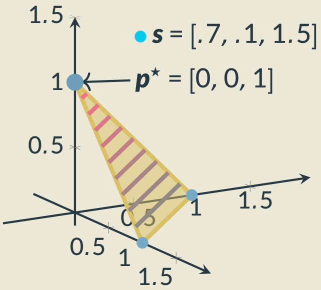

$$\arg\max_{\boldsymbol{p} \in \triangle} \boldsymbol{p}^\top \boldsymbol{s}$$

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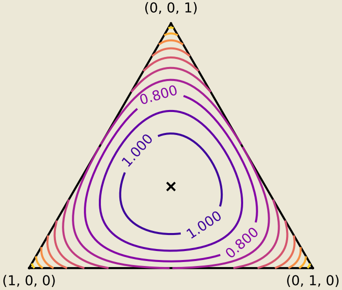

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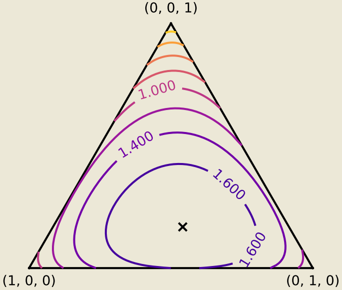
softmax maximizes **expected score** + **entropy**:



$$\arg\max_{\boldsymbol{p} \in \triangle} \boldsymbol{p}^\top \boldsymbol{s} + \mathrm{H}(\boldsymbol{p})$$

Variational form of softmax

Proposition. The unique solution to $\underset{\boldsymbol{p}\in\triangle}{\arg\max}\, \boldsymbol{p}^\top \boldsymbol{s} + \mathrm{H}(\boldsymbol{p})$ is given by $p_j = \frac{\exp s_j}{\sum_i \exp s_i}$.

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Explicit form of the optimization problem:

$$
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\text{maximize} \quad & \textstyle\sum_j p_j s_j - p_j \log p_j \\
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$$\mathcal{L}(\boldsymbol{p}, \boldsymbol{\nu}, \tau) = -\textstyle\sum_j p_j s_j - p_j \log p_j - \boldsymbol{p}^\top \boldsymbol{\nu} + \tau(\boldsymbol{p}^\top \mathbf{1} - 1)$$

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Optimality conditions (KKT):

$$\begin{aligned} 0 = \nabla_{p_i} \mathcal{L}(\boldsymbol{p}, \boldsymbol{\nu}, \tau) &= -s_i + \log p_i + 1 - \nu_i + \tau \\ \boldsymbol{p}^\top \boldsymbol{\nu} &= 0 \\ \boldsymbol{p} &\in \triangle \\ \boldsymbol{\nu} &\geq \mathbf{0} \end{aligned}$$

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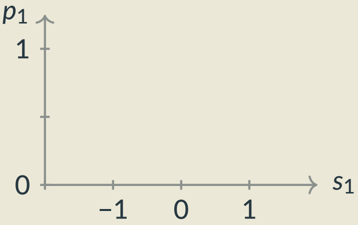
Must find Z such that $\sum_j p_j = 1$.
Answer: $Z = \sum_j \exp(s_j)$

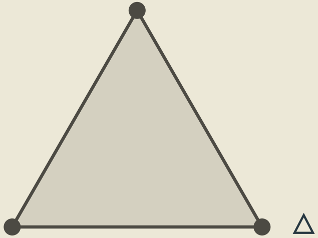
So, $p_i = \dfrac{\exp(s_i)}{\sum_j \exp(s_j)}$.

Classic result, e.g., [Boyd and Vandenberghe, 2004, Wainwright and Jordan, 2008]

Generalizing softmax: Smoothed argmaxes

$$\hat{\boldsymbol{p}}_\Omega(\boldsymbol{s}) = \underset{\boldsymbol{p}\in\triangle}{\arg\max}\, \boldsymbol{p}^\top \boldsymbol{s} - \Omega(\boldsymbol{p})$$





[Niculae and Blondel, 2017]

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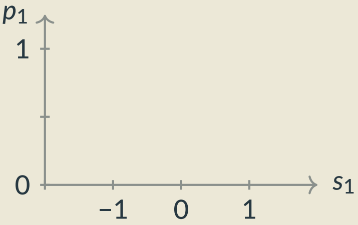

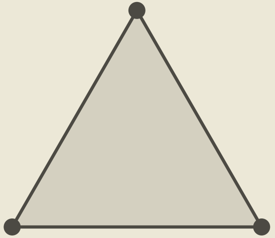

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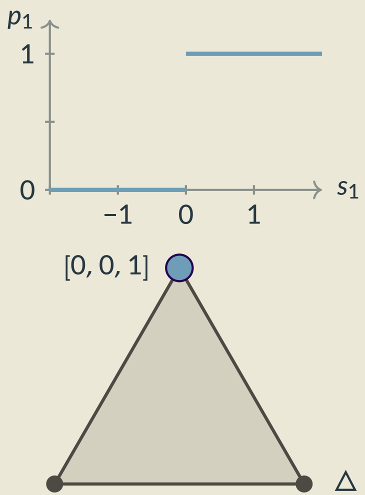

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Generalizing softmax: Smoothed argmaxes

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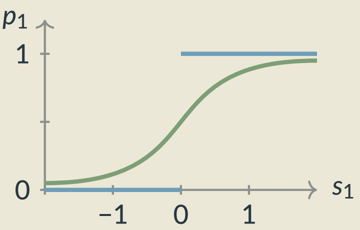

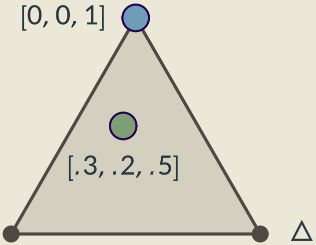

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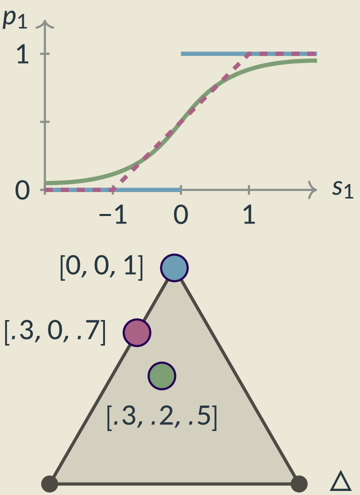
- sparsemax: $\Omega(\boldsymbol{p}) = 1/2 \|\boldsymbol{p}\|_2^2$



[Niculae and Blondel, 2017, Blondel et al., 2019]

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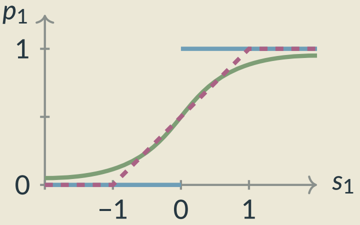
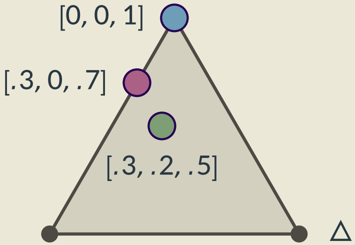
- sparsemax: $\Omega(\boldsymbol{p}) = 1/2 \|\boldsymbol{p}\|_2^2$
- α-entmax: $\Omega(\boldsymbol{p}) = 1/\alpha(\alpha-1) \sum_j p_j^\alpha$





Generalized entropy interpolates in between [Tsallis, 1988]
Used in Sparse Seq2Seq: [Peters et al., 2019] and Adaptively Sparse Transformers [Correia et al., 2019]

Generalizing softmax: Smoothed argmaxes

$$\hat{\boldsymbol{p}}_\Omega(\boldsymbol{s}) = \arg\max_{\boldsymbol{p}\in\triangle} \boldsymbol{p}^\top \boldsymbol{s} - \Omega(\boldsymbol{p})$$

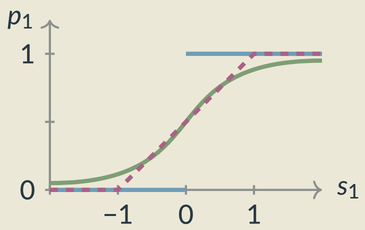
- argmax: $\Omega(\boldsymbol{p}) = 0$
- softmax: $\Omega(\boldsymbol{p}) = \sum_j p_j \log p_j$
- sparsemax: $\Omega(\boldsymbol{p}) = 1/2\|\boldsymbol{p}\|_2^2$

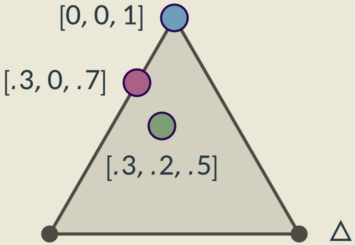
α-entmax: $\Omega(\boldsymbol{p}) = 1/\alpha(\alpha-1) \sum_j p_i^\alpha$

fusedmax: $\Omega(\boldsymbol{p}) = 1/2\|\boldsymbol{p}\|_2^2 + \sum_j |p_j - p_{j-1}|$

csparsemax: $\Omega(\boldsymbol{p}) = 1/2\|\boldsymbol{p}\|_2^2 + \iota(\boldsymbol{a} \le \boldsymbol{p} \le \boldsymbol{b})$

csoftmax: $\Omega(\boldsymbol{p}) = \sum_j p_j \log p_j + \iota(\boldsymbol{a} \le \boldsymbol{p} \le \boldsymbol{b})$

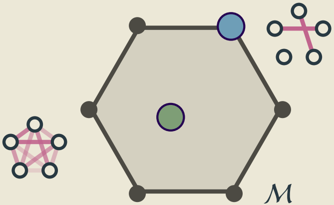




[Wainwright and Jordan, 2008]

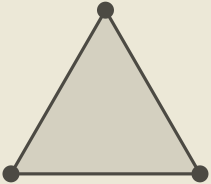
The structured case: Marginal polytope

- Each vertex corresponds to one such *bit vector* $\boldsymbol{z}$
- Points inside correspond to *marginal distributions*: convex combinations of structured objects



$$\boldsymbol{\mu} = \underbrace{p_1 \boldsymbol{z}_1 + \ldots + p_N \boldsymbol{z}_N}_{\text{exponentially many terms}}, \quad \boldsymbol{p} \in \Delta.$$

$$\begin{aligned} p_1 &= 0.2, \quad \boldsymbol{z}_1 = [1, 0, 0, 0, 1, 0, 0, 0, 1] \\ p_2 &= 0.7, \quad \boldsymbol{z}_2 = [0, 0, 1, 0, 0, 1, 1, 0, 0] \\ p_3 &= 0.1, \quad \boldsymbol{z}_3 = [1, 0, 0, 0, 1, 0, 0, 1, 0] \end{aligned} \quad \Rightarrow \quad \boldsymbol{\mu} = [.3, 0, .7, 0, .3, .7, .7, .1, .2].$$



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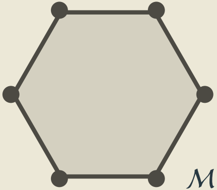
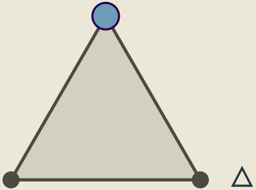
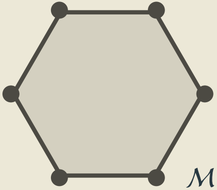


$\mathcal{M}$

- **argmax** $\arg\max_{\boldsymbol{p}\in\triangle} \boldsymbol{p}^\top \boldsymbol{s}$





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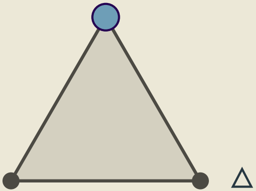

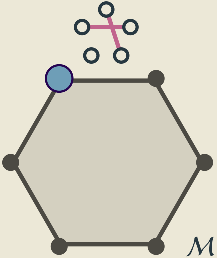

argmax $\arg\max_{\boldsymbol{p}\in\triangle} \boldsymbol{p}^\top \boldsymbol{s}$

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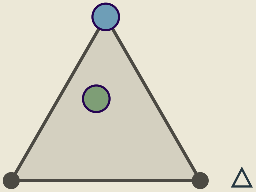

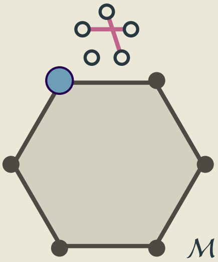

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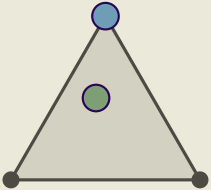

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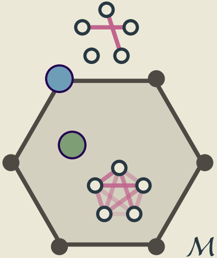

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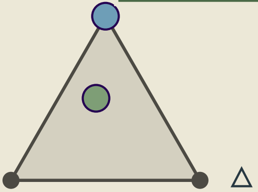

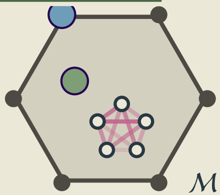

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Unlike argmax/softmax, computation is not obvious!

$\triangle$

$\mathcal{M}$

Algorithms for specific structures

	Best structure (MAP)	Marginals
Sequence tagging	Viterbi [Rabiner, 1989]	Forward-Backward [Rabiner, 1989]
Constituent trees	CKY [Kasami, 1966, Younger, 1967] [Cocke and Schwartz, 1970]	Inside-Outside [Baker, 1979]
Temporal alignments	DTW [Sakoe and Chiba, 1978]	Soft-DTW [Cuturi and Blondel, 2017]
Dependency trees	Max. Spanning Arborescence [Chu and Liu, 1965, Edmonds, 1967]	Matrix-Tree [Kirchhoff, 1847]
Assignments	Kuhn-Munkres [Kuhn, 1955, Jonker and Volgenant, 1987]	#P-complete [Valiant, 1979, Taskar, 2004]

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Derivatives of marginals 1: DP

Dynamic programming: marginals by **Forward-Backward**, **Inside-Outside**, etc.

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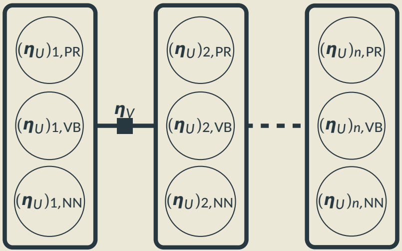
Marginals in a sequence tagging model.

input: d tags, n tokens, $\boldsymbol{\eta}_U \in \mathbb{R}^{n \times d}$, $\boldsymbol{\eta}_V \in \mathbb{R}^{d \times d}$
initialize $\boldsymbol{\alpha}_1 = \mathbf{0}$, $\boldsymbol{\beta}_n = \mathbf{0}$
for $i \in 2, \ldots, n$ **do** *# forward log-probabilities*
  $\alpha_{i,k} = \log \sum_{k'} \exp\left(\alpha_{i-1,k'} + (\boldsymbol{\eta}_U)_{i,k} + (\boldsymbol{\eta}_V)_{k',k}\right)$ for all k
for $i \in n-1, \ldots, 1$ **do** *# backward log-probabilities*
  $\beta_{i,k} = \log \sum_{k'} \exp\left(\beta_{i+1,k'} + (\boldsymbol{\eta}_U)_{i+1,k'} + (\boldsymbol{\eta}_V)_{k,k'}\right)$ for all k
$Z = \sum_k \exp \alpha_{n,k}$ *# partition function*
return $\boldsymbol{\mu} = \exp\left(\boldsymbol{\alpha} + \boldsymbol{\beta} - \log Z\right)$ *# marginals*



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- Alg. consists of differentiable ops: PyTorch autograd can handle it! (v. bad idea)

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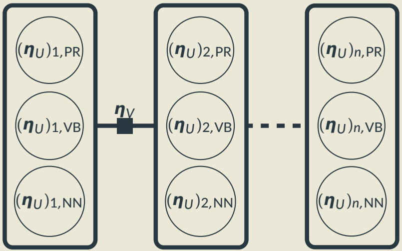

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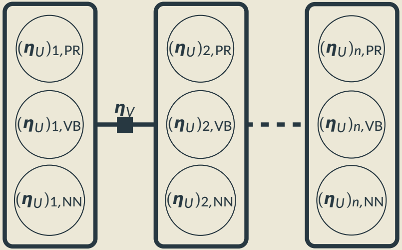

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- Alg. consists of differentiable ops: PyTorch autograd can handle it! (v. bad idea)
- Better book-keeping: Li and Eisner [2009], Mensch and Blondel [2018]
- With circular dependencies, this breaks! Can get an approximation [Stoyanov et al., 2011]

Marginals in a sequence tagging model.

---

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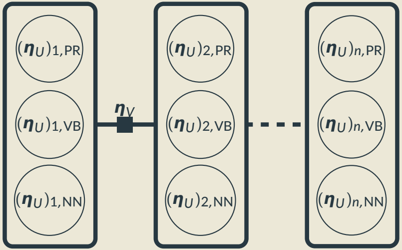


Derivatives of marginals 2: Matrix-Tree

$\boldsymbol{L}(\boldsymbol{s})$: Laplacian of the edge score graph

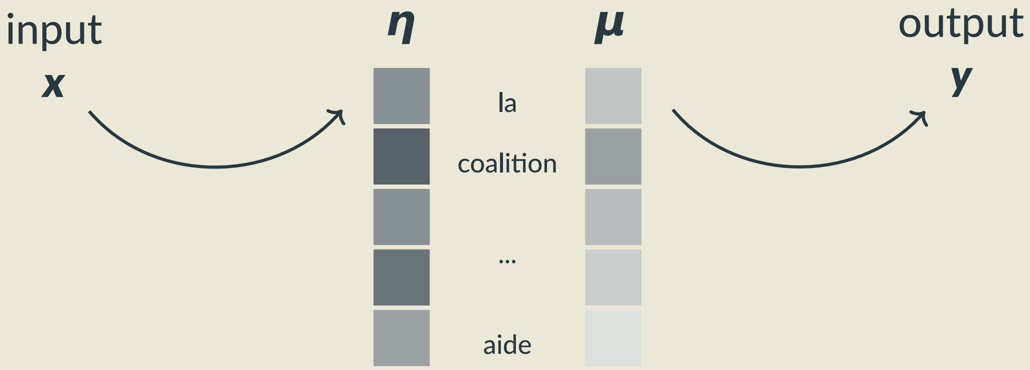
$$Z = \det \boldsymbol{L}(\boldsymbol{s})$$

$$\boldsymbol{\mu} = \boldsymbol{L}(\boldsymbol{s})^{-1}$$

$$\nabla \boldsymbol{\mu} = \nabla \boldsymbol{L}^{-1} = \boldsymbol{L}^{-1}\left(\frac{\partial \boldsymbol{L}}{\partial \boldsymbol{\eta}}\right)\boldsymbol{L}^{-1}$$

[Kim et al., 2017]

Structured Attention Networks



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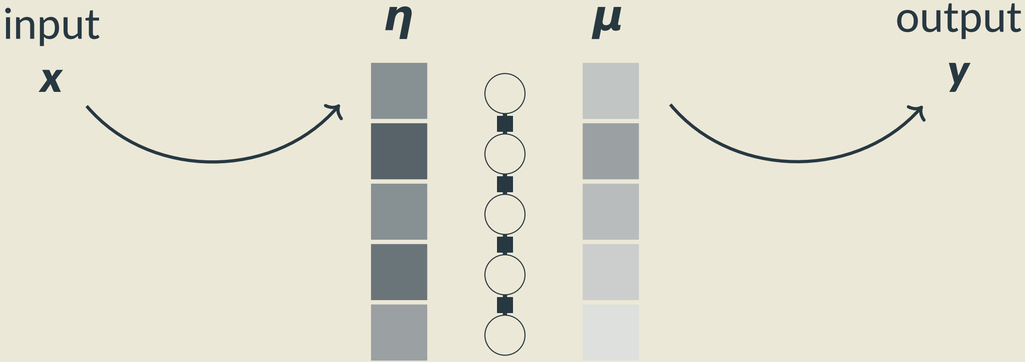

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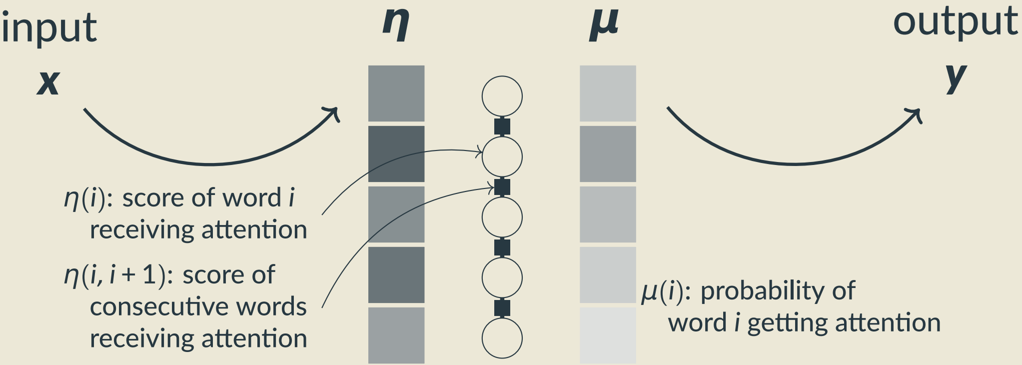

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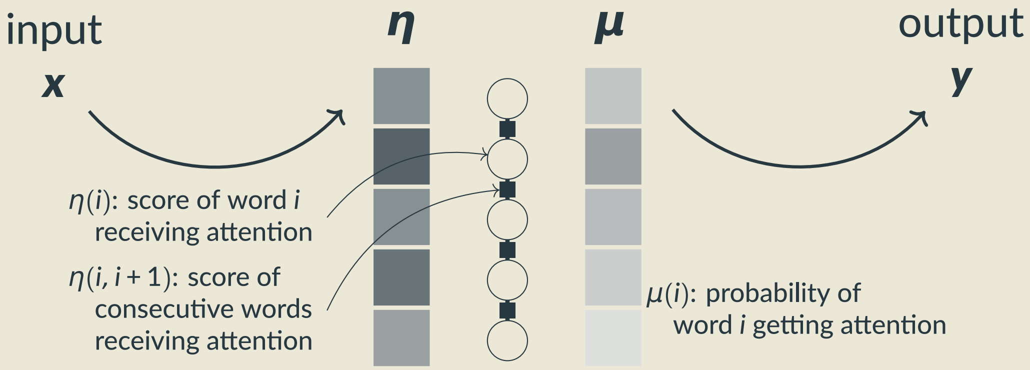

CRF marginals (from *forward–backward*) give attention weights $\in (0, 1)$

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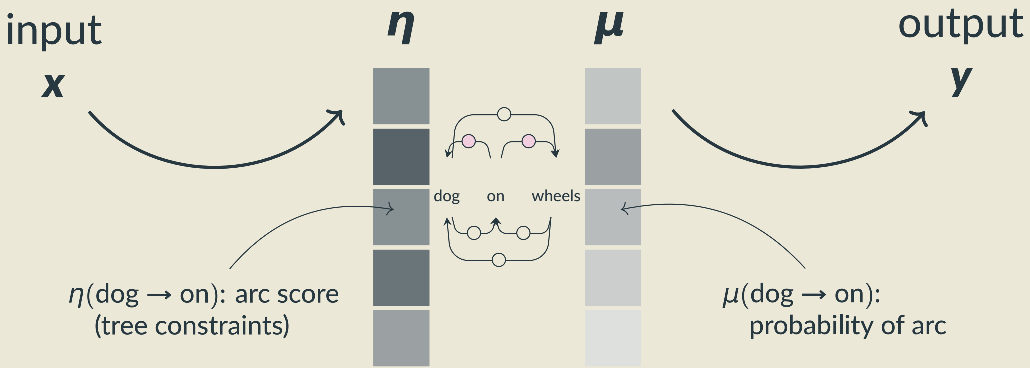

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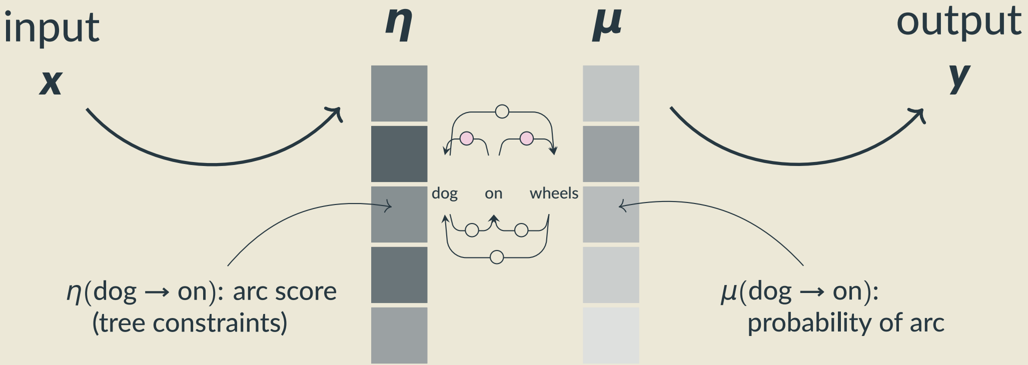

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Similar idea for projective dependency trees with *inside–outside*
and non-projective with the Matrix-Tree theorem [Liu and Lapata, 2018].

Differentiable Perturb & Parse

Extending Gumbel-Softmax to structured stochastic models

- Forward pass:

  sample structure $\boldsymbol{z}$ (approximately)

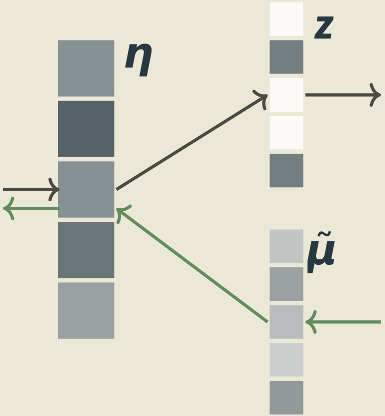
  $$\boldsymbol{z} = \arg\max_{\boldsymbol{z}\in\mathcal{Z}} (\boldsymbol{\eta} + \boldsymbol{\epsilon})^\top \boldsymbol{z}$$

- Backward pass:

  pretend we did marginal inference

  $$\tilde{\boldsymbol{\mu}} = \arg\max_{\boldsymbol{\mu}\in\mathcal{M}} (\boldsymbol{\eta} + \boldsymbol{\epsilon})^\top \boldsymbol{z} + \tilde{\mathrm{H}}(\boldsymbol{\mu})$$

  (or some similar relaxation)



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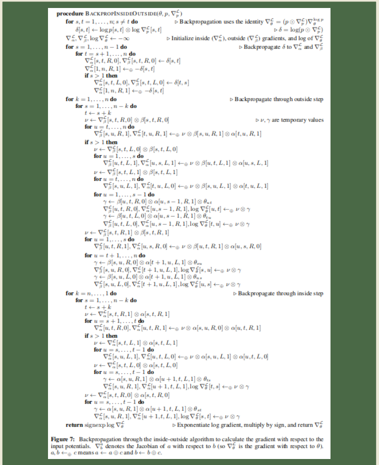


procedure BACKPROPINSIDEOUTSIDE($\theta, p, \nabla_p^{\mathcal{L}}$)
for $s, t = 1, \dots, n; s \neq t$ **do** ▷ Backpropagation uses the identity $\nabla_\theta^{\mathcal{L}} = (p \odot \nabla_p^{\mathcal{L}})\nabla_\theta^{\log p}$
$\delta[s,t] \leftarrow \log p[s,t] \otimes \log \nabla_p^{\mathcal{L}}[s,t]$ ▷ $\delta = \log(p \odot \nabla_p^{\mathcal{L}})$
$\nabla_\alpha^{\mathcal{L}}, \nabla_\beta^{\mathcal{L}}, \log \nabla_\theta^{\mathcal{L}} \leftarrow -\infty$ ▷ Initialize inside ($\nabla_\alpha^{\mathcal{L}}$), outside ($\nabla_\beta^{\mathcal{L}}$) gradients, and log of $\nabla_\theta^{\mathcal{L}}$
for $s = 1, \dots, n-1$ **do** ▷ Backpropagate δ to $\nabla_\alpha^{\mathcal{L}}$ and $\nabla_\beta^{\mathcal{L}}$
for $t = s+1, \dots, n$ **do**
$\nabla_\alpha^{\mathcal{L}}[s,t,R,0], \nabla_\beta^{\mathcal{L}}[s,t,R,0] \leftarrow \delta[s,t]$
$\nabla_\alpha^{\mathcal{L}}[1,n,R,1] \leftarrow_\oplus -\delta[s,t]$
if $s > 1$ **then**
$\nabla_\alpha^{\mathcal{L}}[s,t,L,0], \nabla_\beta^{\mathcal{L}}[s,t,L,0] \leftarrow \delta[t,s]$
$\nabla_\alpha^{\mathcal{L}}[1,n,R,1] \leftarrow_\oplus -\delta[s,t]$
for $k = 1, \dots, n$ **do** ▷ Backpropagate through outside step
for $s = 1, \dots, n-k$ **do**
$t \leftarrow s+k$
$\nu \leftarrow \nabla_\beta^{\mathcal{L}}[s,t,R,0] \otimes \beta[s,t,R,0]$ ▷ ν, γ are temporary values
for $u = t, \dots, n$ **do**
$\nabla_\beta^{\mathcal{L}}[s,u,R,1], \nabla_\alpha^{\mathcal{L}}[t,u,R,1] \leftarrow_\oplus \nu \otimes \beta[s,u,R,1] \otimes \alpha[t,u,R,1]$
if $s > 1$ **then**
$\nu \leftarrow \nabla_\beta^{\mathcal{L}}[s,t,L,0] \otimes \beta[s,t,L,0]$
for $u = 1, \dots, s$ **do**
$\nabla_\beta^{\mathcal{L}}[u,t,L,1], \nabla_\alpha^{\mathcal{L}}[u,s,L,1] \leftarrow_\oplus \nu \otimes \beta[u,t,L,1] \otimes \alpha[u,s,L,1]$
$\nu \leftarrow \nabla_\beta^{\mathcal{L}}[s,t,L,1] \otimes \beta[s,t,L,1]$
for $u = t, \dots, n$ **do**
$\nabla_\beta^{\mathcal{L}}[s,u,L,1], \nabla_\alpha^{\mathcal{L}}[t,u,L,0] \leftarrow_\oplus \nu \otimes \beta[s,u,L,1] \otimes \alpha[t,u,L,1]$
for $u = 1, \dots, s-1$ **do**
$\gamma \leftarrow \beta[u,t,R,0] \otimes \alpha[u,s-1,R,1] \otimes \theta_{ut}$
$\nabla_\beta^{\mathcal{L}}[u,t,R,0], \nabla_\alpha^{\mathcal{L}}[u,s-1,R,1], \log \nabla_\theta^{\mathcal{L}}[u,t] \leftarrow_\oplus \nu \otimes \gamma$
$\gamma \leftarrow \beta[u,t,L,0] \otimes \alpha[u,s-1,R,1] \otimes \theta_{tu}$
$\nabla_\beta^{\mathcal{L}}[u,t,L,0], \nabla_\alpha^{\mathcal{L}}[u,s-1,R,1], \log \nabla_\theta^{\mathcal{L}}[t,u] \leftarrow_\oplus \nu \otimes \gamma$
$\nu \leftarrow \nabla_\beta^{\mathcal{L}}[s,t,R,1] \otimes \beta[s,t,R,1]$
for $u = 1, \dots, s$ **do**
$\nabla_\beta^{\mathcal{L}}[u,t,R,1], \nabla_\alpha^{\mathcal{L}}[u,s,R,0] \leftarrow_\oplus \nu \otimes \beta[u,t,R,1] \otimes \alpha[u,s,R,0]$
for $u = t+1, \dots, n$ **do**
$\gamma \leftarrow \beta[s,u,R,0] \otimes \alpha[t+1,u,L,1] \otimes \theta_{su}$
$\nabla_\beta^{\mathcal{L}}[s,u,R,0], \nabla_\alpha^{\mathcal{L}}[t+1,u,L,1], \log \nabla_\theta^{\mathcal{L}}[s,u] \leftarrow_\oplus \nu \otimes \gamma$
$\gamma \leftarrow \beta[s,u,L,0] \otimes \alpha[t+1,u,L,1] \otimes \theta_{us}$
$\nabla_\beta^{\mathcal{L}}[s,u,L,0], \nabla_\alpha^{\mathcal{L}}[t+1,u,L,1], \log \nabla_\theta^{\mathcal{L}}[u,s] \leftarrow_\oplus \nu \otimes \gamma$
for $k = n, \dots, 1$ **do** ▷ Backpropagate through inside step
for $s = 1, \dots, n-k$ **do**
$t \leftarrow s+k$
$\nu \leftarrow \nabla_\alpha^{\mathcal{L}}[s,t,R,1] \otimes \alpha[s,t,R,1]$
for $u = s+1, \dots, t$ **do**
$\nabla_\alpha^{\mathcal{L}}[u,t,R,0], \nabla_\alpha^{\mathcal{L}}[u,t,R,1] \leftarrow_\oplus \nu \otimes \alpha[s,u,R,0] \otimes \alpha[u,t,R,1]$
if $s > 1$ **then**
$\nu \leftarrow \nabla_\alpha^{\mathcal{L}}[s,t,L,1] \otimes \alpha[s,t,L,1]$
for $u = s, \dots, t-1$ **do**
$\nabla_\alpha^{\mathcal{L}}[s,u,L,1], \nabla_\alpha^{\mathcal{L}}[u,t,L,0] \leftarrow_\oplus \nu \otimes \alpha[s,u,L,1] \otimes \alpha[u,t,L,0]$
$\nu \leftarrow \nabla_\alpha^{\mathcal{L}}[s,t,L,0] \otimes \alpha[s,t,L,0]$
for $u = s, \dots, t-1$ **do**
$\gamma \leftarrow \alpha[s,u,R,1] \otimes \alpha[u+1,t,L,1] \otimes \theta_{ts}$
$\nabla_\alpha^{\mathcal{L}}[s,u,R,1], \nabla_\alpha^{\mathcal{L}}[u+1,t,L,1], \log \nabla_\theta^{\mathcal{L}}[t,s] \leftarrow_\oplus \nu \otimes \gamma$
$\nu \leftarrow \nabla_\alpha^{\mathcal{L}}[s,t,R,0] \otimes \alpha[s,t,R,0]$
for $u = s, \dots, t-1$ **do**
$\gamma \leftarrow \alpha[s,u,R,1] \otimes \alpha[u+1,t,L,1] \otimes \theta_{st}$
$\nabla_\alpha^{\mathcal{L}}[s,u,R,1], \nabla_\alpha^{\mathcal{L}}[u+1,t,L,1], \log \nabla_\theta^{\mathcal{L}}[s,t] \leftarrow_\oplus \nu \otimes \gamma$
return signexp $\log \nabla_\theta^{\mathcal{L}}$ ▷ Exponentiate log gradient, multiply by sign, and return $\nabla_\theta^{\mathcal{L}}$

Figure 7: Backpropagation through the inside-outside algorithm to calculate the gradient with respect to the input potentials. ∇_b^a denotes the Jacobian of a with respect to b (so $\nabla_\theta^{\mathcal{L}}$ is the gradient with respect to θ). $a, b \leftarrow_\oplus c$ means $a \leftarrow a \oplus c$ and $b \leftarrow b \oplus c$.

Back-propagating through marginals

Pros:

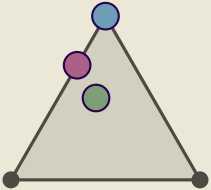
- Familiar algorithms for NLPers,
- (Structured Attention Networks:) All computations exact.

Cons:

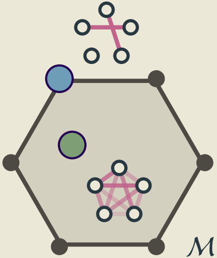
- (Structured Attention Networks:) forward pass marginals are dense;
  (fixed by Perturb & MAP, at cost of rough approximation)
- Efficient & numerically stable back-propagation through DPs is tricky;
  (somewhat alleviated by Mensch and Blondel [2018])
- Not applicable when marginals are unavailable.
- Case-by-case algorithms required, can get tedious.

- **argmax** $\arg\max_{\boldsymbol{p}\in\triangle} \boldsymbol{p}^\top \boldsymbol{s}$
- **softmax** $\arg\max_{\boldsymbol{p}\in\triangle} \boldsymbol{p}^\top \boldsymbol{s} + \mathrm{H}(\boldsymbol{p})$
- **sparsemax** $\arg\max_{\boldsymbol{p}\in\triangle} \boldsymbol{p}^\top \boldsymbol{s} - 1/2\|\boldsymbol{p}\|^2$

- **MAP** $\arg\max_{\boldsymbol{\mu}\in\mathcal{M}} \boldsymbol{\mu}^\top \boldsymbol{\eta}$
- **marginals** $\arg\max_{\boldsymbol{\mu}\in\mathcal{M}} \boldsymbol{\mu}^\top \boldsymbol{\eta} + \widetilde{\mathrm{H}}(\boldsymbol{\mu})$



$\triangle$

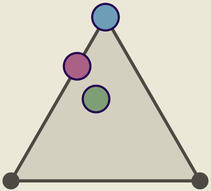


$\mathcal{M}$

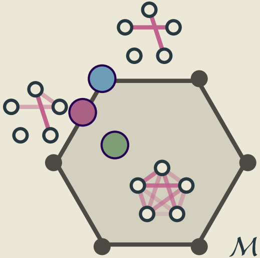
[Niculae et al., 2018a]

- **argmax** $\arg\max_{\boldsymbol{p}\in\triangle} \boldsymbol{p}^\top \boldsymbol{s}$
- **softmax** $\arg\max_{\boldsymbol{p}\in\triangle} \boldsymbol{p}^\top \boldsymbol{s} + \mathrm{H}(\boldsymbol{p})$
- **sparsemax** $\arg\max_{\boldsymbol{p}\in\triangle} \boldsymbol{p}^\top \boldsymbol{s} - 1/2\|\boldsymbol{p}\|^2$

- **MAP** $\arg\max_{\boldsymbol{\mu}\in\mathcal{M}} \boldsymbol{\mu}^\top \boldsymbol{\eta}$
- **marginals** $\arg\max_{\boldsymbol{\mu}\in\mathcal{M}} \boldsymbol{\mu}^\top \boldsymbol{\eta} + \widetilde{\mathrm{H}}(\boldsymbol{\mu})$
- **SparseMAP** $\arg\max_{\boldsymbol{\mu}\in\mathcal{M}} \boldsymbol{\mu}^\top \boldsymbol{\eta} - 1/2\|\boldsymbol{\mu}\|^2$



$\triangle$



$\mathcal{M}$

SparseMAP solution

$$\boldsymbol{\mu}^\star = \operatorname*{arg\,max}_{\boldsymbol{\mu}\in\mathcal{M}} \boldsymbol{\mu}^\top \boldsymbol{\eta} - 1/2\|\boldsymbol{\mu}\|^2$$

=  = .6  + .4 

($\boldsymbol{\mu}^\star$ is unique, but may have multiple decompositions $\boldsymbol{p}$. Active Set recovers a sparse one.)

Algorithms for SparseMAP

$$\boldsymbol{\mu}^\star = \arg\max_{\boldsymbol{\mu}\in\mathcal{M}} \boldsymbol{\mu}^\top\boldsymbol{\eta} - 1/2\|\boldsymbol{\mu}\|^2$$

This is also $\mathrm{proj}_{\mathcal{M}}$ required by SPIGOT!

Algorithms for SparseMAP

$$\boldsymbol{\mu}^\star = \underset{\boldsymbol{\mu} \in \mathcal{M}}{\arg\max}\, \boldsymbol{\mu}^\top \boldsymbol{\eta} - 1/2 \|\boldsymbol{\mu}\|^2$$

linear constraints
(alas, exponentially many!)

quadratic objective

Algorithms for SparseMAP

$$\boldsymbol{\mu}^\star = \arg\max_{\boldsymbol{\mu} \in \mathcal{M}} \boldsymbol{\mu}^\top \boldsymbol{\eta} - 1/2 \|\boldsymbol{\mu}\|^2$$

linear constraints
(alas, exponentially many!)

quadratic objective

Conditional Gradient

[Frank and Wolfe, 1956, Lacoste-Julien and Jaggi, 2015]

Algorithms for SparseMAP

$$\boldsymbol{\mu}^{\star} = \underset{\boldsymbol{\mu} \in \mathcal{M}}{\arg\max}\, \boldsymbol{\mu}^{\top}\boldsymbol{\eta} - 1/2\|\boldsymbol{\mu}\|^2$$

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Conditional Gradient

[Frank and Wolfe, 1956, Lacoste-Julien and Jaggi, 2015]

- select a new corner of $\mathcal{M}$

Algorithms for SparseMAP

$$\boldsymbol{\mu}^\star = \underset{\boldsymbol{\mu}\in\mathcal{M}}{\arg\max}\, \boldsymbol{\mu}^\top \boldsymbol{\eta} - 1/2\|\boldsymbol{\mu}\|^2$$

linear constraints
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quadratic objective

Conditional Gradient

[Frank and Wolfe, 1956, Lacoste-Julien and Jaggi, 2015]

- select a new corner of $\mathcal{M}$

$$\underset{\boldsymbol{\mu}\in\mathcal{M}}{\arg\max}\, \boldsymbol{\mu}^\top \underbrace{\left(\boldsymbol{\eta} - \boldsymbol{\mu}^{(t-1)}\right)}_{\tilde{\boldsymbol{\eta}}}$$

Algorithms for SparseMAP

$$\boldsymbol{\mu}^\star = \underset{\boldsymbol{\mu}\in\mathcal{M}}{\arg\max}\, \boldsymbol{\mu}^\top \boldsymbol{\eta} - 1/2\|\boldsymbol{\mu}\|^2$$

linear constraints
(alas, exponentially many!)

quadratic objective

Conditional Gradient

[Frank and Wolfe, 1956, Lacoste-Julien and Jaggi, 2015]

- select a new corner of $\mathcal{M}$
- update the (sparse) coefficients of $\boldsymbol{p}$
  - Update rules: vanilla, away-step, pairwise

Algorithms for SparseMAP

$$\boldsymbol{\mu}^\star = \arg\max_{\boldsymbol{\mu}\in\mathcal{M}} \boldsymbol{\mu}^\top \boldsymbol{\eta} - 1/2\|\boldsymbol{\mu}\|^2$$

linear constraints
(alas, exponentially many!)

quadratic objective

Conditional Gradient

[Frank and Wolfe, 1956, Lacoste-Julien and Jaggi, 2015]

- select a new corner of $\mathcal{M}$
- update the (sparse) coefficients of $\boldsymbol{p}$
  - Update rules: vanilla, away-step, pairwise
  - Quadratic objective: **Active Set**

    a.k.a. Min-Norm Point, [Wolfe, 1976]

    [Martins et al., 2015, Nocedal and Wright, 1999]

Algorithms for SparseMAP

$$\boldsymbol{\mu}^\star = \arg\max_{\boldsymbol{\mu}\in\mathcal{M}} \boldsymbol{\mu}^\top \boldsymbol{\eta} - 1/2\|\boldsymbol{\mu}\|^2$$

linear constraints
(alas, exponentially many!)

quadratic objective

Conditional Gradient

[Frank and Wolfe, 1956, Lacoste-Julien and Jaggi, 2015]

- select a new corne
- update the (sparse)
  - Update rules: vanilla
  - Quadratic objective:

  a.k.a. Min-Norm Point, [Wolfe, 1976]

  [Martins et al., 2015, Nocedal and Wright, 1999]

Active Set achieves
finite & **linear** convergence!

Algorithms for SparseMAP

$$\boldsymbol{\mu}^\star = \arg\max_{\boldsymbol{\mu}\in\mathcal{M}} \boldsymbol{\mu}^\top \boldsymbol{\eta} - 1/2\|\boldsymbol{\mu}\|^2$$

linear constraints
(alas, exponentially many!)

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Backward pass

$\frac{\partial \boldsymbol{\mu}}{\partial \boldsymbol{\eta}}$ is sparse

Algorithms for SparseMAP

$$\boldsymbol{\mu}^\star = \underset{\boldsymbol{\mu}\in\mathcal{M}}{\arg\max}\, \boldsymbol{\mu}^\top \boldsymbol{\eta} - 1/2\|\boldsymbol{\mu}\|^2$$

linear constraints
(alas, exponentially many!)

quadratic objective

Conditional Gradient

[Frank and Wolfe, 1956, Lacoste-Julien and Jaggi, 2015]

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    [Martins et al., 2015, Nocedal and Wright, 1999]

Backward pass

$\frac{\partial \boldsymbol{\mu}}{\partial \boldsymbol{\eta}}$ is sparse

computing $\left(\frac{\partial \boldsymbol{\mu}}{\partial \boldsymbol{\eta}}\right)^\top \boldsymbol{dy}$

takes $\mathcal{O}(\dim(\boldsymbol{\mu})\,\mathrm{nnz}(\boldsymbol{p}^\star))$

Algorithms for SparseMAP

$$\boldsymbol{\mu}^\star = \underset{\boldsymbol{\mu}\in\mathcal{M}}{\arg\max}\, \boldsymbol{\mu}^\top \boldsymbol{\eta} - 1/2\|\boldsymbol{\mu}\|^2$$

linear constraints
(alas, exponentially many!)

quadratic objective

Conditi

[Frank and Wolfe, 1956

- select a new c
- update the (sparse) coefficients of $\boldsymbol{p}$
  - Update rules: vanilla, away-step, pairwise
  - Quadratic objective: **Active Set**

    a.k.a. Min-Norm Point, [Wolfe, 1976]

    [Martins et al., 2015, Nocedal and Wright, 1999]

Completely modular: just add MAP

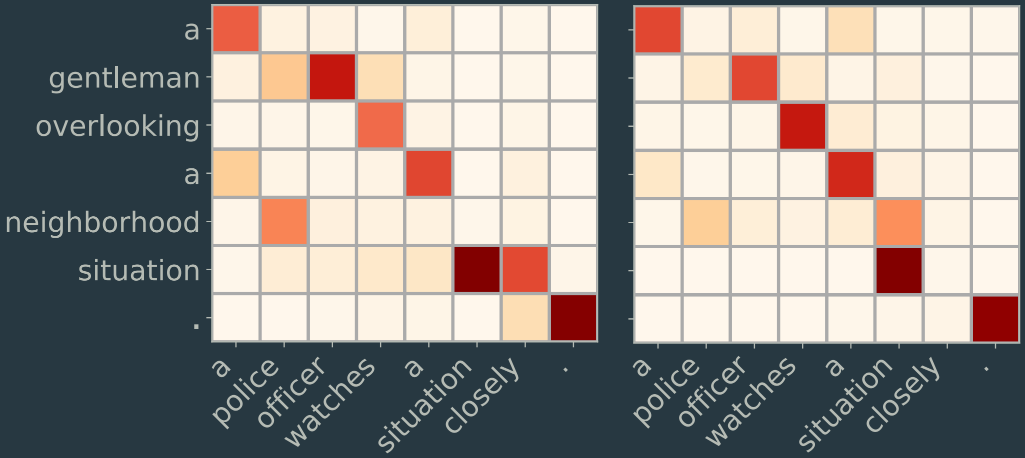
pass

rse

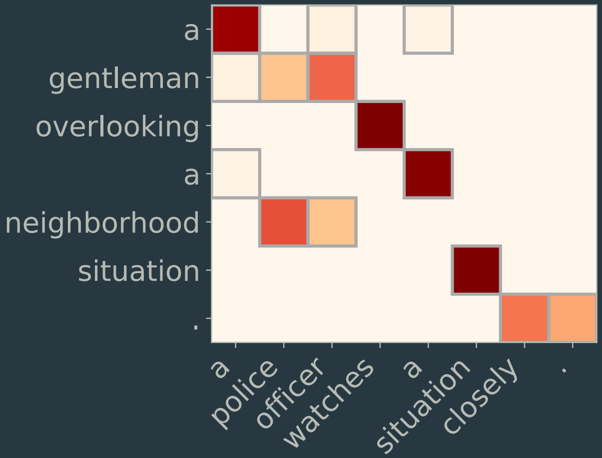
computing $\left(\frac{\partial \boldsymbol{\mu}}{\partial \boldsymbol{\eta}}\right)^\top \boldsymbol{dy}$

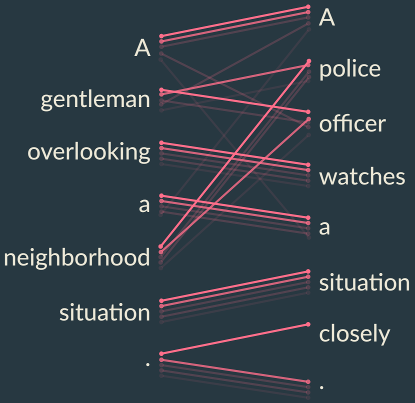
takes $\mathcal{O}(\dim(\boldsymbol{\mu})\, \mathrm{nnz}(\boldsymbol{p}^\star))$

[Chen et al., 2017]



[Niculae et al., 2018a]





Overview

$$\mathbb{E}_{\pi_{\boldsymbol{\theta}}(\boldsymbol{z}|x)}\left[L(\boldsymbol{z})\right] \qquad L\left(\arg\max_{z} \pi_{\boldsymbol{\theta}}(\boldsymbol{z} \mid x)\right) \qquad L\left(\mathbb{E}_{\pi_{\boldsymbol{\theta}}(\boldsymbol{z}|x)}[\boldsymbol{z}]\right)$$

- REINFORCE
- Straight-Through Gumbel (Perturb & MAP)

- Straight-Through
- SPIGOT

- Structured Attn. Nets
- SparseMAP

Structured latent variables without sampling

$$\mathbb{E}_{\boldsymbol{z}}\left[L(\boldsymbol{z})\right] = \sum_{\boldsymbol{z}\in\mathcal{Z}} L\left(\hat{y}\quad(\boldsymbol{z})\right)\pi\quad(\boldsymbol{z}\mid x)$$

Structured latent variables without sampling

$$\mathbb{E}_{\boldsymbol{z}}\big[L(\boldsymbol{z})\big] = \sum_{\boldsymbol{z} \in \mathcal{Z}} L\big(\hat{y}_{\boldsymbol{\phi}}(\boldsymbol{z})\big)\, \pi_{\boldsymbol{\theta}}(\boldsymbol{z} \mid x)$$

Structured latent variables without sampling

e.g., a TreeLSTM defined by $\boldsymbol{z}$

$$\mathbb{E}_{\boldsymbol{z}}\big[L(\boldsymbol{z})\big] = \sum_{\boldsymbol{z}\in\mathcal{Z}} L\big(\hat{y}_{\boldsymbol{\phi}}(\boldsymbol{z})\big)\ \pi_{\boldsymbol{\theta}}(\boldsymbol{z} \mid x)$$

Structured latent variables without sampling

e.g., a TreeLSTM defined by $\boldsymbol{z}$

$$\mathbb{E}_{\boldsymbol{z}}\left[L(\boldsymbol{z})\right] = \sum_{\boldsymbol{z} \in \mathcal{Z}} L\left(\hat{y}_{\boldsymbol{\phi}}(\boldsymbol{z})\right) \pi_{\boldsymbol{\theta}}(\boldsymbol{z} \mid x)$$

parsing model,
using some scorer $f_{\boldsymbol{\theta}}(\boldsymbol{z}; x)$

Structured latent variables without sampling

sum over
all possible trees

e.g., a TreeLSTM defined by $\boldsymbol{z}$

$$\mathbb{E}_{\boldsymbol{z}}\left[L(\boldsymbol{z})\right] = \sum_{\boldsymbol{z}\in\mathcal{Z}} L\left(\hat{y}_{\boldsymbol{\phi}}(\boldsymbol{z})\right)\ \pi_{\boldsymbol{\theta}}(\boldsymbol{z} \mid x)$$

parsing model,
using some scorer $f_{\boldsymbol{\theta}}(\boldsymbol{z}; x)$

Exponentially large sum!

Structured latent variables without sampling

sum over
all possible trees

e.g., a TreeLSTM defined by $\boldsymbol{z}$

$$\mathbb{E}_{\boldsymbol{z}}\big[L(\boldsymbol{z})\big] = \sum_{\boldsymbol{z}\in\mathcal{Z}} L\big(\hat{y}_{\boldsymbol{\phi}}(\boldsymbol{z})\big)\,\pi_{\boldsymbol{\theta}}(\boldsymbol{z} \mid x)$$

parsing model,
using some scorer $f_{\boldsymbol{\theta}}(\boldsymbol{z}; x)$

How to define $\pi_{\boldsymbol{\theta}}$?

idea 1

idea 2

idea 3

Structured latent variables without sampling

sum over
all possible trees

e.g., a TreeLSTM defined by $\boldsymbol{z}$

$$\mathbb{E}_{\boldsymbol{z}}\big[L(\boldsymbol{z})\big] = \sum_{\boldsymbol{z}\in\mathcal{Z}} L\big(\hat{y}_{\boldsymbol{\phi}}(\boldsymbol{z})\big)\ \pi_{\boldsymbol{\theta}}(\boldsymbol{z} \mid x)$$

parsing model,
using some scorer $f_{\boldsymbol{\theta}}(\boldsymbol{z}; x)$

How to define $\pi_{\boldsymbol{\theta}}$?

$$\sum_{h\in\mathcal{H}}$$

idea 1

idea 2

idea 3

Structured latent variables without sampling

sum over
all possible trees

e.g., a TreeLSTM defined by $\mathbf{z}$

$$\mathbb{E}_{\mathbf{z}}\big[L(\mathbf{z})\big] = \sum_{\mathbf{z}\in\mathcal{Z}} L\big(\hat{y}_{\boldsymbol{\phi}}(\mathbf{z})\big)\ \pi_{\boldsymbol{\theta}}(\mathbf{z} \mid x)$$

parsing model,
using some scorer $f_{\boldsymbol{\theta}}(\mathbf{z}; x)$

How to define $\pi_{\boldsymbol{\theta}}$?

$$\sum_{h\in\mathcal{H}} \qquad \frac{\partial\mathbb{E}\big[L(\mathbf{z})\big]}{\partial\boldsymbol{\theta}}$$

idea 1

idea 2

idea 3

Structured latent variables without sampling

sum over
all possible trees

e.g., a TreeLSTM defined by $\boldsymbol{z}$

$$\mathbb{E}_{\boldsymbol{z}}\left[L(\boldsymbol{z})\right] = \sum_{\boldsymbol{z}\in\mathcal{Z}} L\left(\hat{y}_{\boldsymbol{\phi}}(\boldsymbol{z})\right) \pi_{\boldsymbol{\theta}}(\boldsymbol{z} \mid x)$$

parsing model,
using some scorer $f_{\boldsymbol{\theta}}(\boldsymbol{z}; x)$

How to define $\pi_{\boldsymbol{\theta}}$?

$$\sum_{h\in\mathcal{H}} \quad \frac{\partial \mathbb{E}\left[L(\boldsymbol{z})\right]}{\partial \boldsymbol{\theta}}$$

idea 1 $\pi_{\boldsymbol{\theta}}(\boldsymbol{z}) \propto \exp\left(f_{\boldsymbol{\theta}}(\boldsymbol{z})\right)$ softmax

idea 2

idea 3

Structured latent variables without sampling

sum over
all possible trees

e.g., a TreeLSTM defined by $\boldsymbol{z}$

$$\mathbb{E}_{\boldsymbol{z}}\left[L(\boldsymbol{z})\right] = \sum_{\boldsymbol{z} \in \mathcal{Z}} L\left(\hat{y}_{\boldsymbol{\phi}}(\boldsymbol{z})\right) \pi_{\boldsymbol{\theta}}(\boldsymbol{z} \mid x)$$

parsing model,
using some scorer $f_{\boldsymbol{\theta}}(\boldsymbol{z}; x)$

How to define $\pi_{\boldsymbol{\theta}}$?

$$\sum_{h \in \mathcal{H}} \quad \frac{\partial \mathbb{E}\left[L(\boldsymbol{z})\right]}{\partial \boldsymbol{\theta}}$$

idea 1 $\pi_{\boldsymbol{\theta}}(\boldsymbol{z}) \propto \exp\left(f_{\boldsymbol{\theta}}(\boldsymbol{z})\right)$ softmax 😊

idea 2

idea 3

Structured latent variables without sampling

sum over
all possible trees

e.g., a TreeLSTM defined by $\boldsymbol{z}$

$$\mathbb{E}_{\boldsymbol{z}}\left[L(\boldsymbol{z})\right] = \sum_{\boldsymbol{z}\in\mathcal{Z}} L\left(\hat{y}_{\boldsymbol{\phi}}(\boldsymbol{z})\right)\, \pi_{\boldsymbol{\theta}}(\boldsymbol{z} \mid x)$$

parsing model,
using some scorer $f_{\boldsymbol{\theta}}(\boldsymbol{z}; x)$

How to define $\pi_{\boldsymbol{\theta}}$?

			$\sum_{h\in\mathcal{H}}$	$\frac{\partial\mathbb{E}\left[L(\boldsymbol{z})\right]}{\partial\boldsymbol{\theta}}$
idea 1	$\pi_{\boldsymbol{\theta}}(\boldsymbol{z}) \propto \exp\left(f_{\boldsymbol{\theta}}(\boldsymbol{z})\right)$	softmax	😱	😊
idea 2				
idea 3				

Structured latent variables without sampling

sum over
all possible trees

e.g., a TreeLSTM defined by $\boldsymbol{z}$

$$\mathbb{E}_{\boldsymbol{z}}\big[L(\boldsymbol{z})\big] = \sum_{\boldsymbol{z}\in\mathcal{Z}} L\big(\hat{y}_{\boldsymbol{\phi}}(\boldsymbol{z})\big)\ \pi_{\boldsymbol{\theta}}(\boldsymbol{z} \mid x)$$

parsing model,
using some scorer $f_{\boldsymbol{\theta}}(\boldsymbol{z}; x)$

How to define $\pi_{\boldsymbol{\theta}}$?

All methods we've seen require sampling; hard in general.

idea 2

idea 3

Structured latent variables without sampling

sum over
all possible trees

e.g., a TreeLSTM defined by $\boldsymbol{z}$

$$\mathbb{E}_{\boldsymbol{z}}\big[L(\boldsymbol{z})\big] = \sum_{\boldsymbol{z}\in\mathcal{Z}} L\big(\hat{y}_{\boldsymbol{\phi}}(\boldsymbol{z})\big)\ \pi_{\boldsymbol{\theta}}(\boldsymbol{z} \mid x)$$

parsing model,
using some scorer $f_{\boldsymbol{\theta}}(\boldsymbol{z}; x)$

How to define $\pi_{\boldsymbol{\theta}}$?

$$\sum_{h\in\mathcal{H}} \qquad \frac{\partial \mathbb{E}\big[L(\boldsymbol{z})\big]}{\partial \boldsymbol{\theta}}$$

idea 1	$\pi_{\boldsymbol{\theta}}(\boldsymbol{z}) \propto \exp\big(f_{\boldsymbol{\theta}}(\boldsymbol{z})\big)$	softmax	😱	😊
idea 2	$\pi_{\boldsymbol{\theta}}(\boldsymbol{z}) = 1$ if $\boldsymbol{z} = \mathrm{MAP}(f_{\boldsymbol{\theta}}(\cdot))$ else 0	argmax		
idea 3				

Structured latent variables without sampling

sum over
all possible trees

e.g., a TreeLSTM defined by $\boldsymbol{z}$

$$\mathbb{E}_{\boldsymbol{z}}\big[L(\boldsymbol{z})\big] = \sum_{\boldsymbol{z}\in\mathcal{Z}} L\big(\hat{y}_{\boldsymbol{\phi}}(\boldsymbol{z})\big)\; \pi_{\boldsymbol{\theta}}(\boldsymbol{z} \mid x)$$

parsing model,
using some scorer $f_{\boldsymbol{\theta}}(\boldsymbol{z}; x)$

How to define $\pi_{\boldsymbol{\theta}}$?

			$\sum_{h\in\mathcal{H}}$	$\frac{\partial \mathbb{E}[L(\boldsymbol{z})]}{\partial \boldsymbol{\theta}}$
idea 1	$\pi_{\boldsymbol{\theta}}(\boldsymbol{z}) \propto \exp\big(f_{\boldsymbol{\theta}}(\boldsymbol{z})\big)$	softmax	😱	😊
idea 2	$\pi_{\boldsymbol{\theta}}(\boldsymbol{z}) = 1$ if $\boldsymbol{z} = \mathrm{MAP}(f_{\boldsymbol{\theta}}(\cdot))$ else 0	argmax	😊	
idea 3				

Structured latent variables without sampling

sum over
all possible trees

e.g., a TreeLSTM defined by $\boldsymbol{z}$

$$\mathbb{E}_{\boldsymbol{z}}\left[L(\boldsymbol{z})\right] = \sum_{\boldsymbol{z}\in\mathcal{Z}} L\left(\hat{y}_{\boldsymbol{\phi}}(\boldsymbol{z})\right)\, \pi_{\boldsymbol{\theta}}(\boldsymbol{z} \mid x)$$

parsing model,
using some scorer $f_{\boldsymbol{\theta}}(\boldsymbol{z}; x)$

How to define $\pi_{\boldsymbol{\theta}}$?

			$\sum_{h\in\mathcal{H}}$	$\frac{\partial\mathbb{E}[L(\boldsymbol{z})]}{\partial\boldsymbol{\theta}}$
idea 1	$\pi_{\boldsymbol{\theta}}(\boldsymbol{z}) \propto \exp\left(f_{\boldsymbol{\theta}}(\boldsymbol{z})\right)$	softmax	😱	😊
idea 2	$\pi_{\boldsymbol{\theta}}(\boldsymbol{z}) = 1$ if $\boldsymbol{z} = \mathrm{MAP}(f_{\boldsymbol{\theta}}(\cdot))$ else 0	argmax	😊	😟
idea 3				

Structured latent variables without sampling

sum over
all possible trees

e.g., a TreeLSTM defined by $\boldsymbol{z}$

$$\mathbb{E}_{\boldsymbol{z}}\left[L(\boldsymbol{z})\right] = \sum_{\boldsymbol{z}\in\mathcal{Z}} L\left(\hat{y}_{\boldsymbol{\phi}}(\boldsymbol{z})\right) \pi_{\boldsymbol{\theta}}(\boldsymbol{z} \mid x)$$

parsing model,
using some scorer $f_{\boldsymbol{\theta}}(\boldsymbol{z}; x)$

How to define $\pi_{\boldsymbol{\theta}}$?

			$\sum_{h\in\mathcal{H}}$	$\frac{\partial\mathbb{E}\left[L(\boldsymbol{z})\right]}{\partial\boldsymbol{\theta}}$
idea 1	$\pi_{\boldsymbol{\theta}}(\boldsymbol{z}) \propto \exp\left(f_{\boldsymbol{\theta}}(\boldsymbol{z})\right)$	softmax	😱	😊
idea 2	$\pi_{\boldsymbol{\theta}}(\boldsymbol{z}) = 1$ if $\boldsymbol{z} = \mathrm{MAP}(f_{\boldsymbol{\theta}}(\cdot))$ else 0	argmax	😊	😟
idea 3		SparseMAP	😊	😊

Structured latent variables without sampling

 $= .7\times$  $+ .3\times$ 

Structured latent variables without sampling

 = .7×  + .3×  +0×  + ...

Structured latent variables without sampling

 $= .7\times$  $+ .3\times$  $+ 0\times$  $+ \dots$

$\mathbb{E}[L(\boldsymbol{z})] = .7\times L($  $) + .3\times L($  $)$

recall our shorthand $L(\boldsymbol{z}) = L(\hat{y}_{\boldsymbol{\phi}}(\boldsymbol{z}), y)$

V. Conclusions

[Corro and Titov, 2019b, Havrylov et al., 2019]

Stanford Sentiment (Accuracy)

[Socher et al., 2013]	
Bigram Naive Bayes	83.1
[Niculae et al., 2018b]	
DepTreeLSTM w/ CoreNLP	83.2
DepTreeLSTM w/ SparseMAP	84.7
[Corro and Titov, 2019b]	
GCN w/ CoreNLP	83.8
GCN w/ Perturb-and-MAP	84.6
[Choi et al., 2018]	
ST Gumbel-Tree	90.7
[Havrylov et al., 2019]	
TreeLSTM + tricks	90.2

Stanford Natural Language Inference (Accuracy)

[Kim et al., 2017]	
Simple Attention	86.2
Structured Attention	86.8
[Liu and Lapata, 2018]	
100D Structured Attention	86.8
[Yogatama et al., 2017]	
100D RL-SPINN	80.5
[Choi et al., 2018]	
100D ST Gumbel-Tree	82.6
300D -	85.6
600D -	86.0
[Corro and Titov, 2019b]	
Latent Tree + 1 GCN -	85.2
Latent Tree + 2 GCN -	86.2
[Havrylov et al., 2019]	
100D TreeLSTM + tricks	84.3

Is it syntax?!

- Unlike e.g. unsupervised parsing, the structures we learn are guided by **a downstream objective** (typically discriminative).
- They don't typically resemble grammatical structure (yet) [Williams et al., 2018]

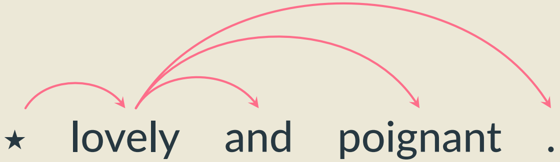
  (future work: more inductive biases and constraints?)

Is it syntax?!

- Unlike e.g. unsupervised parsing, the structures we learn are guided by **a downstream objective** (typically discriminative).
- They don't typically resemble grammatical structure (yet) [Williams et al., 2018]
  (future work: more inductive biases and constraints?)
- Common to compare latent structures with parser outputs.
  But is this always a meaningful comparison?

Syntax vs. Composition Order

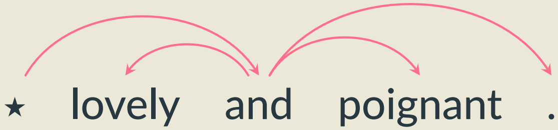
CoreNLP parse, $p = 21.4\%$



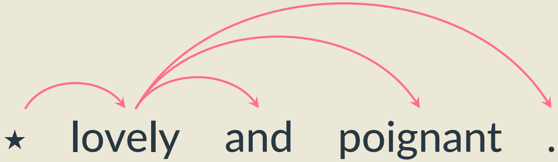
[Niculae et al., 2018b]

Syntax vs. Composition Order

$p = 22.6\%$



CoreNLP parse, $p = 21.4\%$



...

[Niculae et al., 2018b]

Syntax vs. Composition Order

$p = 22.6\%$

⋆ lovely and poignant .

CoreNLP parse, $p = 21.4\%$

⋆ lovely and poignant .

…

$p = 15.33\%$

⋆ a deep and meaningful film .

$p = 15.27\%$

⋆ a deep and meaningful film .

…

CoreNLP parse, $p = 0\%$

⋆ a deep and meaningful film .

Conclusions

- Latent structure models are desirable for interpretability, structural bias, and higher predictive power with fewer parameters.
- Stochastic latent variables can be dealt with RL or straight-through gradients.
- Deterministic argmax requires surrogate gradients (e.g. SPIGOT).
- Continuous relaxations of argmax include SANs and SparseMAP.
- Intuitively, some of these different methods are trying to do similar things or require the same building blocks (e.g. SPIGOT and SparseMAP).
- ... we didn't even get into deep *generative* models! These tools apply, but there are new challenges. [Corro and Titov, 2019a, Kim et al., 2019a,b, Kawakami et al., 2019]

Overview

$$\mathbb{E}_{\pi_{\boldsymbol{\theta}}(\boldsymbol{z}|x)}\left[L(\boldsymbol{z})\right]$$

- REINFORCE
- Straight-Through Gumbel (Perturb & MAP)
- SparseMAP

$$L\left(\arg\max_{z} \pi_{\boldsymbol{\theta}}(\boldsymbol{z} \mid x)\right)$$

- Straight-Through
- SPIGOT

$$L\left(\mathbb{E}_{\pi_{\boldsymbol{\theta}}(\boldsymbol{z}|x)}[\boldsymbol{z}]\right)$$

- Structured Attn. Nets
- SparseMAP

Overview

$$\mathbb{E}_{\pi_{\boldsymbol{\theta}}(\boldsymbol{z}|x)}\Big[L(\boldsymbol{z})\Big] \qquad L\Big(\arg\max_{z} \pi_{\boldsymbol{\theta}}(\boldsymbol{z} \mid x)\Big) \qquad L\Big(\mathbb{E}_{\pi_{\boldsymbol{\theta}}(\boldsymbol{z}|x)}[\boldsymbol{z}]\Big)$$

- REINFORCE[SPL]
- Straight-Through Gumbel (Perturb & MAP)[SPL,MRG]
- SparseMAP[MAP+]

- Straight-Through[MAP,MRG]
- SPIGOT[MAP+]

- Structured Attn. Nets[MRG]
- SparseMAP[MAP+]

Computation:

SPL: Sampling. (Simple in incremental/unstructured, hard for most global structures.)

MAP: Finding the highest-scoring structure.

MRG: Marginal inference.

Overview

Thank you!

$$\mathbb{E}_{\pi_{\boldsymbol{\theta}}(\boldsymbol{z}|x)}\Big[L(\boldsymbol{z})\Big] \qquad L\big(\arg\max_z \pi_{\boldsymbol{\theta}}(\boldsymbol{z} \mid x)\big) \qquad L\Big(\mathbb{E}_{\pi_{\boldsymbol{\theta}}(\boldsymbol{z}|x)}[\boldsymbol{z}]\Big)$$

- REINFORCE^SPL
- Straight-Through Gumbel (Perturb & MAP)^SPL,MRG
- SparseMAP^MAP+

- Straight-Through^MAP,MRG
- SPIGOT^MAP+

- Structured Attn. Nets^MRG
- SparseMAP^MAP+

Computation:

SPL: Sampling. (Simple in incremental/unstructured, hard for most global structures.)

MAP: Finding the highest-scoring structure.

MRG: Marginal inference.

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