Latent Structure Models for NLP

André Martins — Instituto de Telecomunicações & IST & Unbabel
Tsvetomila Mihaylova — Instituto de Telecomunicações
Nikita Nangia — NYU
Vlad Niculae — Instituto de Telecomunicações

deep-spin.github.io/tutorial

I. Introduction

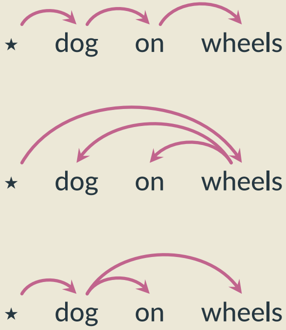
Structured prediction and NLP

- **Structured prediction**: a machine learning framework for predicting structured, constrained, and interdependent outputs
- **NLP** deals with *structured* and *ambiguous* textual data:
  - machine translation
  - speech recognition
  - syntactic parsing
  - semantic parsing
  - information extraction
  - ...

Examples of structure in NLP

Dependency parsing

...



...

Examples of structure in NLP

Dependency parsing

…

⋆ dog on wheels

Exponentially many parse trees!

Cannot enumerate.

⋆ dog on wheels

…

Examples of structure in NLP

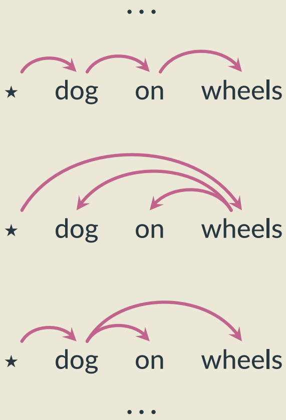
POS tagging

VERB PREP NOUN
dog on wheels

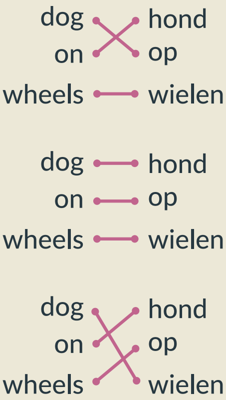
NOUN PREP NOUN
dog on wheels

NOUN DET NOUN
dog on wheels

Dependency parsing



Word alignments



NLP 5 years ago:

Structured prediction and pipelines

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- **Advantages:** fast and simple to train, can rearrange pieces 😊
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- **Bigger disadvantage:** error propagates through the pipeline 💩

NLP today:

End-to-end training

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- Forget pipelines—train everything from scratch!
- No more error propagation or linguistic annotations! 🎉

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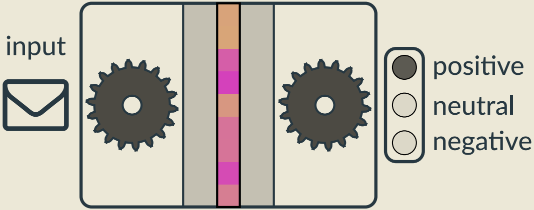
End-to-end training

- Forget pipelines—train everything from scratch!
- No more error propagation or linguistic annotations! 🎉
- Treat everything as *latent*! 🙌

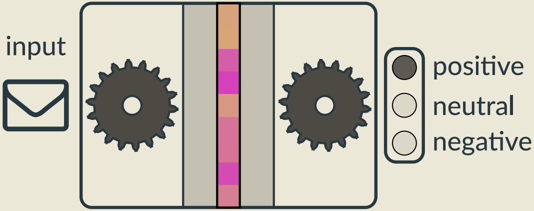
Representation learning

- Uncover hidden representations useful for the *downstream task*.
- Neural networks are well-suited for this: *deep computation graphs*.



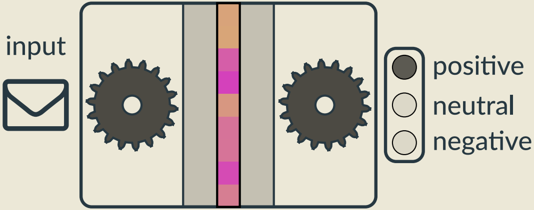
Representation learning

- Uncover hidden representations useful for the *downstream task*.
- Neural networks are well-suited for this: *deep computation graphs*.
- Neural representations are unstructured, inscrutable. Language data has underlying structure!



Latent structure models

- Seek *structured* hidden representations instead!



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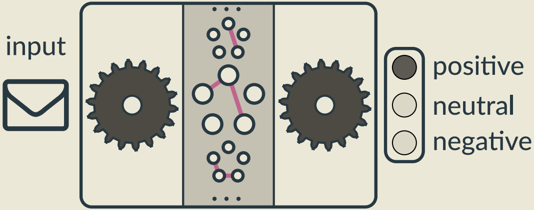

Latent structure models aren't so new!

- They have a very long history in NLP:
  - IBM Models for SMT (latent word alignments) [Brown et al., 1993]
  - HMMs [Rabiner, 1989]
  - CRFs with hidden variables [Quattoni et al., 2007]
  - Latent PCFGs [Petrov and Klein, 2008, Cohen et al., 2012]
- Trained with EM, spectral learning, method of moments, ...
- Often, very strict assumptions (e.g. strong factorizations)
- Today, neural networks opened up some new possibilities!

Why do we love latent structure models?

- The inferred latent variables can bring us some **interpretability**
- They offer a way of injecting prior knowledge as a **structured bias**
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  - smaller carbon footprint!

What this tutorial is about:

- Discrete, combinatorial latent structures
- Often the structure is inspired by some linguistic intuition
- We'll cover both:
  - RL methods (structure built incrementally, reward coming from downstream task)
  - ... vs end-to-end differentiable approaches (global optimization, marginalization)
  - stochastic computation graphs
  - ... vs deterministic graphs.
- All plugged in *discriminative* neural models.

This tutorial is *not* about:

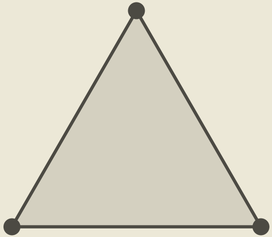
- It's not about continuous latent variables
- It's not about deep generative learning
- We won't cover GANs, VAEs, etc.
- There are (very good) recent tutorials on deep variational models for NLP:
  - "Variational Inference and Deep Generative Models" (Schulz and Aziz, ACL 2018)
  - "Deep Latent-Variable Models for Natural Language" (Kim, Wiseman, Rush, EMNLP 2018)

Background

Unstructured vs structured

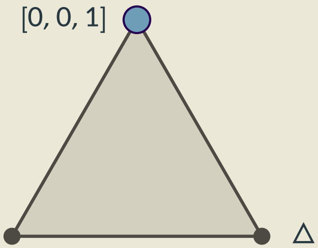
- To better explain the math, we'll often backtrack to *unstructured* models (where the latent variable is a categorical) before jumping to the *structured* ones

The unstructured case: Probability simplex



$\triangle$

The unstructured case: Probability simplex



- Each vertex is an *indicator vector*, representing one class:

$$\boldsymbol{z}_c = [0, \ldots, 0, \underbrace{1}_{c^{\text{th}} \text{ position}}, 0, \ldots, 0].$$

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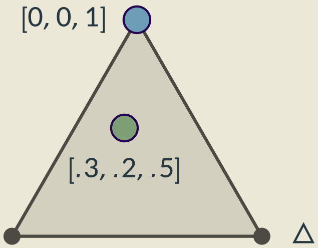

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$$\boldsymbol{z}_c = [0, \ldots, 0, \underbrace{1}_{c^{\text{th}} \text{ position}}, 0, \ldots, 0].$$

- Points inside are *probability vectors*, a convex combination of classes:

$$\boldsymbol{p} \geq \mathbf{0}, \ \sum_c p_c = 1.$$

What's the analogous of $\triangle$ for a structure?

- A structured object $\boldsymbol{z}$ can be represented as a *bit vector*.

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  - a dependency tree can be represented a $O(L^2)$ vector indexed by arcs
  - each entry is 1 iff the arc belongs to the tree
  - **structural constraints:** not all bit vectors represent valid trees!

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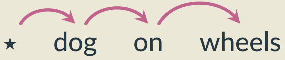
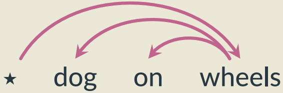
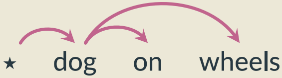
$\boldsymbol{z}_1 = [1, 0, 0, 0, 1, 0, 0, 0, 1]$



$\boldsymbol{z}_2 = [0, 0, 1, 0, 0, 1, 1, 0, 0]$

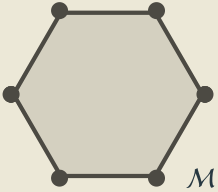


$\boldsymbol{z}_3 = [1, 0, 0, 0, 1, 0, 0, 1, 0]$



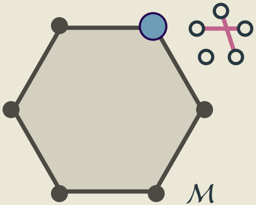
[Wainwright and Jordan, 2008]

The structured case: Marginal polytope



The structured case: Marginal polytope

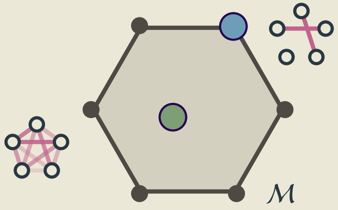
- Each vertex corresponds to one such *bit vector* $\boldsymbol{z}$



[Wainwright and Jordan, 2008]

The structured case: Marginal polytope

- Each vertex corresponds to one such *bit vector* $\boldsymbol{z}$
- Points inside correspond to *marginal distributions*: convex combinations of structured objects

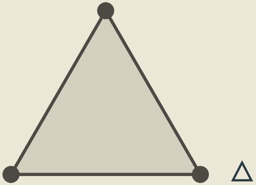


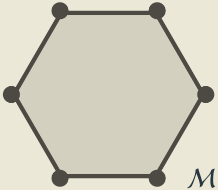
$$\boldsymbol{\mu} = \underbrace{p_1\boldsymbol{z}_1 + \ldots + p_N\boldsymbol{z}_N}_{\text{exponentially many terms}}, \quad \boldsymbol{p} \in \Delta.$$

$$\begin{aligned} p_1 &= 0.2, & \boldsymbol{z}_1 &= [1,0,0,0,1,0,0,0,1] \\ p_2 &= 0.7, & \boldsymbol{z}_2 &= [0,0,1,0,0,1,1,0,0] \\ p_3 &= 0.1, & \boldsymbol{z}_3 &= [1,0,0,0,1,0,0,1,0] \end{aligned} \quad \Rightarrow \quad \boldsymbol{\mu} = [.3,0,.7,0,.3,.7,.7,.1,.2].$$

Unstructured vs Structured

- Unstructured case: simplex Δ
- Structured case: marginal polytope $\mathcal{M}$





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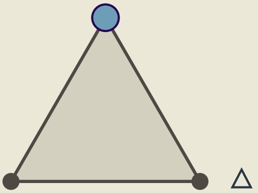

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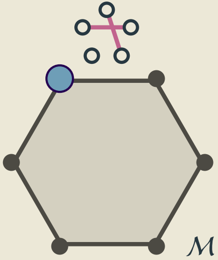
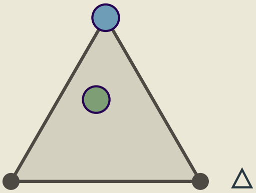
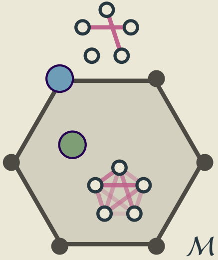


Unstructured vs Structured

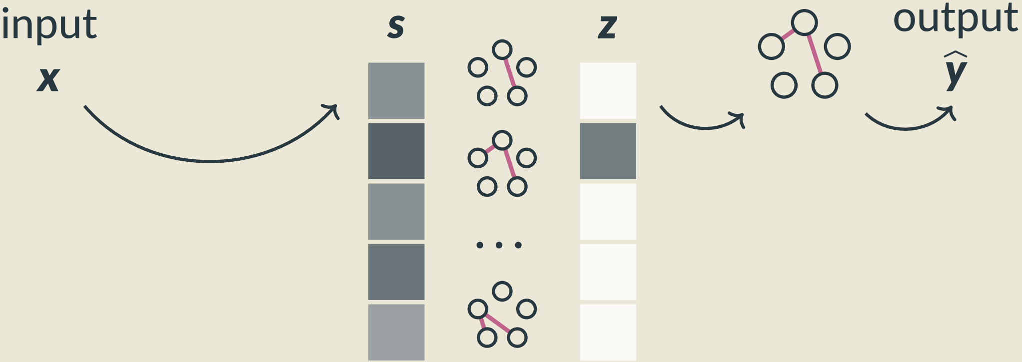
- Unstructured case: simplex $\triangle$
- Structured case: marginal polytope $\mathcal{M}$





Computing the most likely structure

is a very high-dimensional argmax



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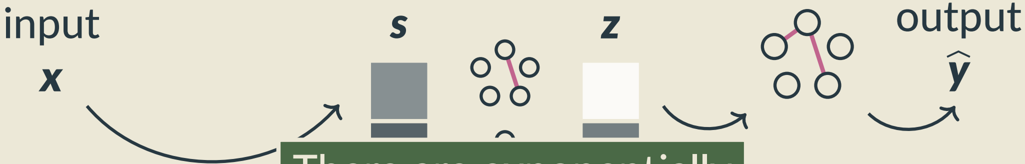
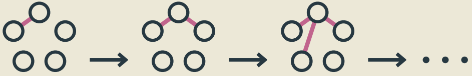


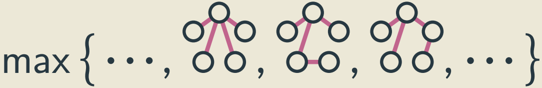
There are exponentially many structures

(*s* cannot fit in memory;

we cannot "loop" over *s* nor *z*)

Dealing with the combinatorial explosion





1. Incremental structures

- Build structure **greedily**, as sequence of discrete choices (e.g., shift-reduce).
- Scores (partial structure, action) tuples.
- **Advantages:** flexible, rich histories.
- **Disadvantages:** greedy, local decisions are suboptimal, error propagation.

2. Factorization into parts

- Optimizes **globally** (e.g. Viterbi, Chu-Liu-Edmonds, Kuhn-Munkres).
- Scores smaller parts.
- **Advantages:** optimal, elegant, can handle hard & global constraints.
- **Disadvantages:** strong assumptions.

The challenge of discrete choices.

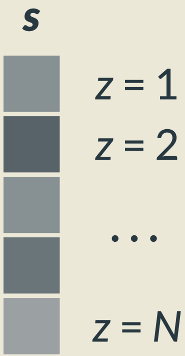
$$z = 1$$

$$z = 2$$

$$\dots$$

$$z = N$$

The challenge of discrete choices.



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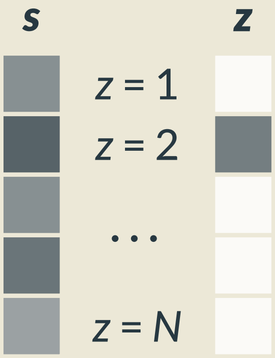

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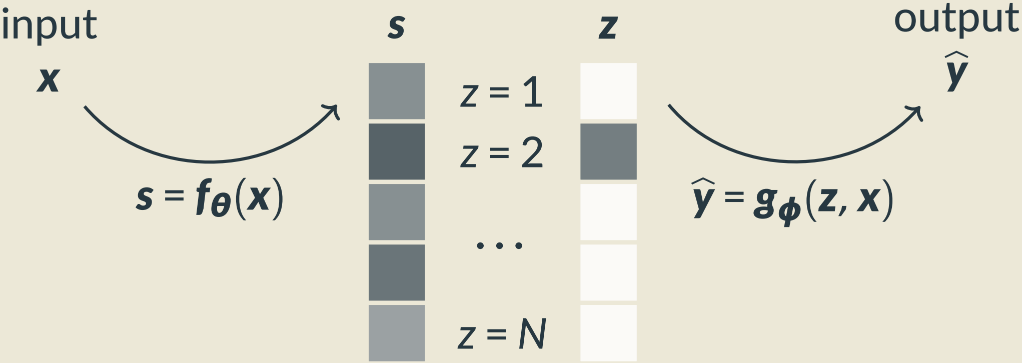

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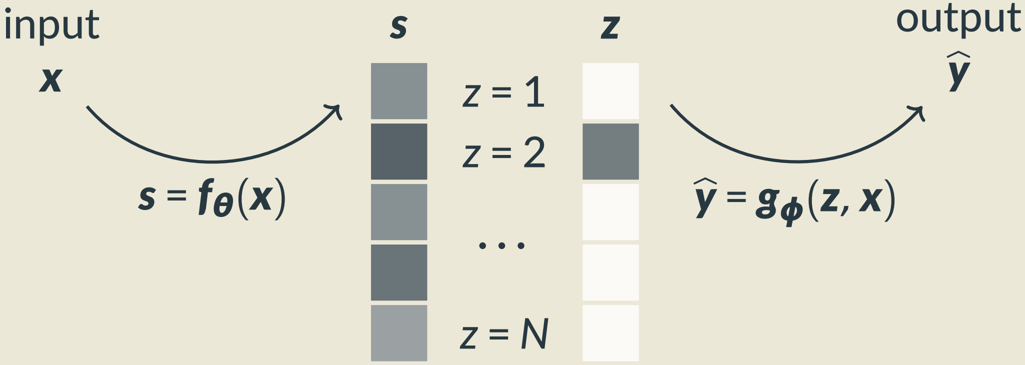
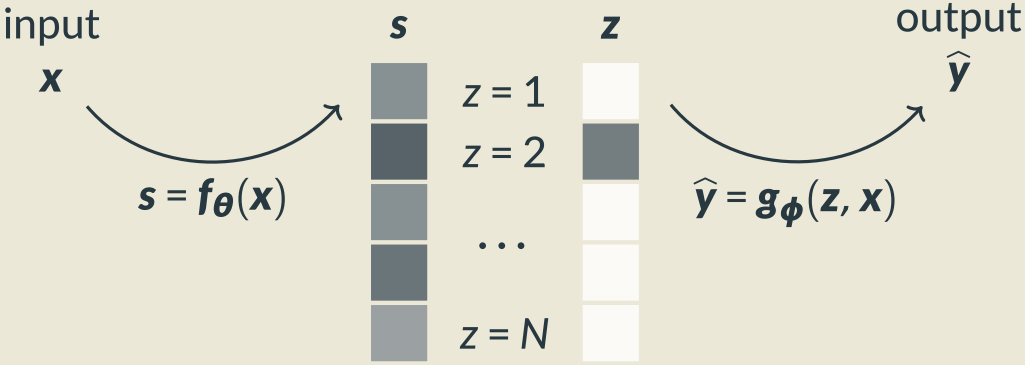


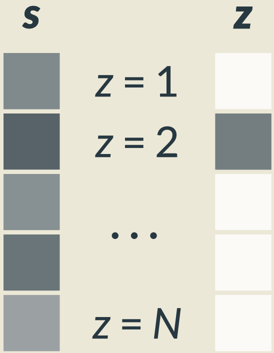
$$\frac{\partial L(\widehat{\boldsymbol{y}}, \boldsymbol{y})}{\partial \boldsymbol{w}} = ?$$

The challenge of discrete choices.



$\frac{\partial L(\widehat{\boldsymbol{y}}, \boldsymbol{y})}{\partial \boldsymbol{w}} = ?$ or, essentially, $\frac{\partial \boldsymbol{z}}{\partial \boldsymbol{s}} = ?$

Discrete mappings are “flat”



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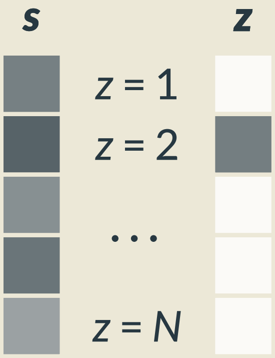

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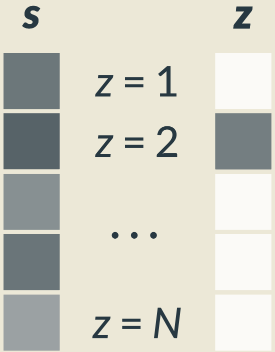

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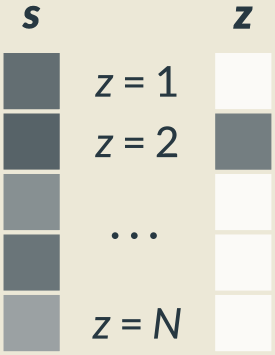

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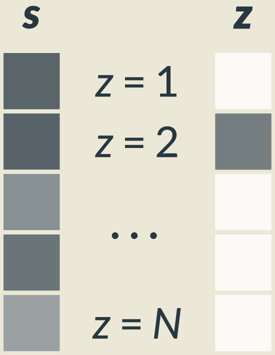

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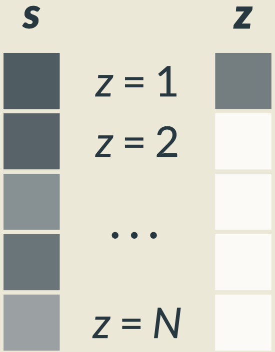

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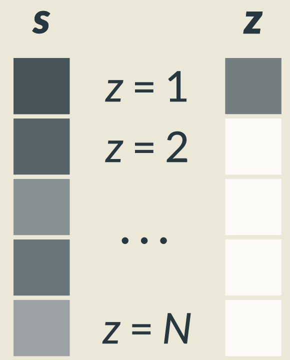

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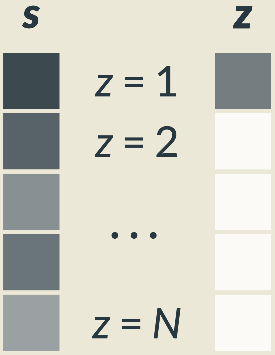

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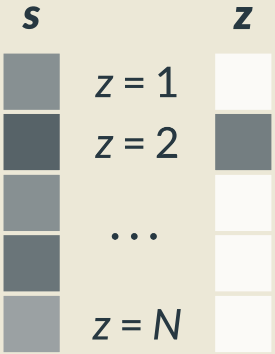
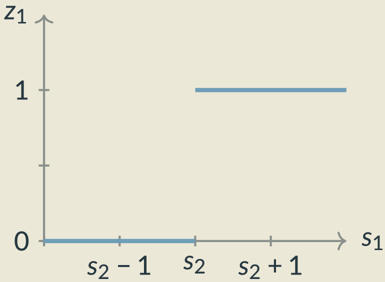
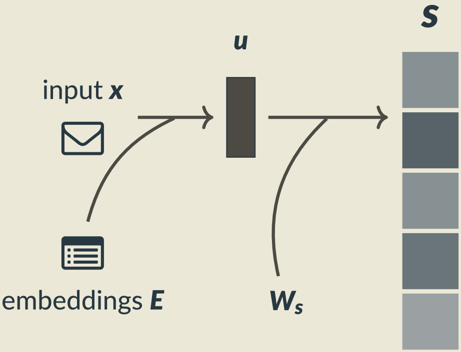
Argmax





$$\frac{\partial \boldsymbol{z}}{\partial \boldsymbol{s}} = \mathbf{0}$$

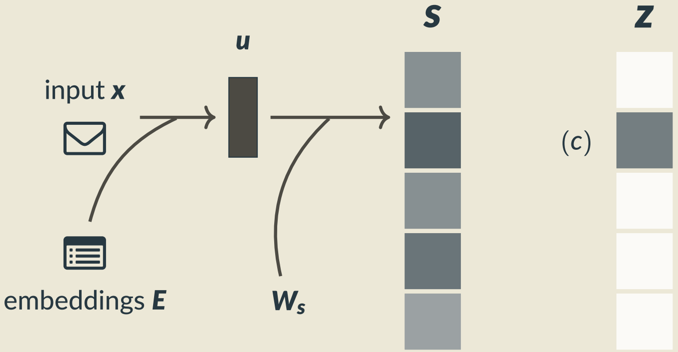
Example: Regression with latent categorization



$\boldsymbol{u} = \frac{1}{|\boldsymbol{x}|} \sum_j \boldsymbol{E}_{x_j}$

$\boldsymbol{s} = \boldsymbol{W_s u}$

Example: Regression with latent categorization



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predict topic c ($\boldsymbol{z} = \boldsymbol{e}_c$)

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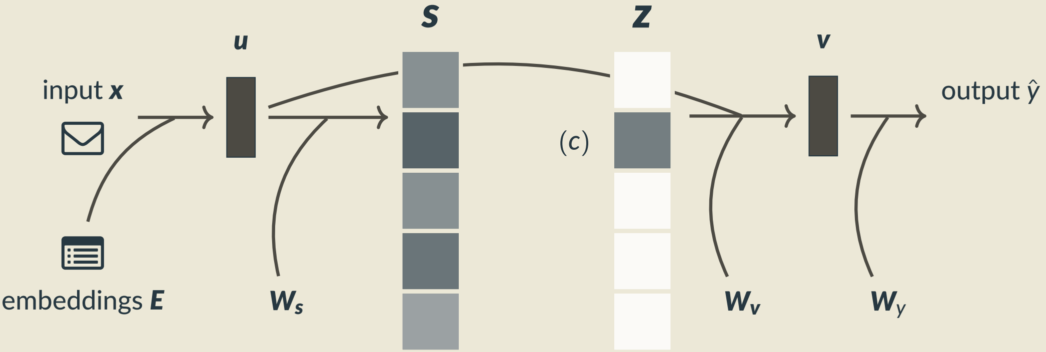

$\boldsymbol{u} = \frac{1}{|\boldsymbol{x}|}\sum_j \boldsymbol{E}_{x_j}$ $\quad$ $\boldsymbol{s} = \boldsymbol{W_s}\boldsymbol{u}$ $\quad$ $\boldsymbol{v} = \tanh\left(\boldsymbol{W_v}[\boldsymbol{u}, \boldsymbol{z}]\right)$ $\quad$ $\hat{y} = \boldsymbol{W}_y\boldsymbol{v}$ $\quad$ $L = (\hat{y} - y)^2$

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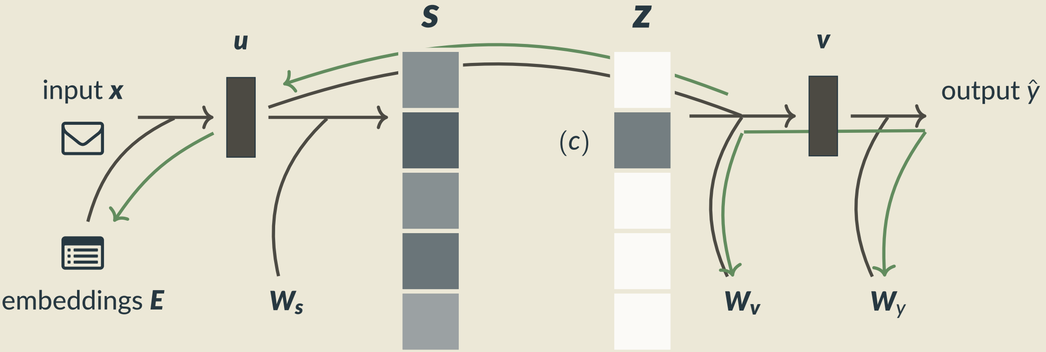
Example: Regression with latent categorization



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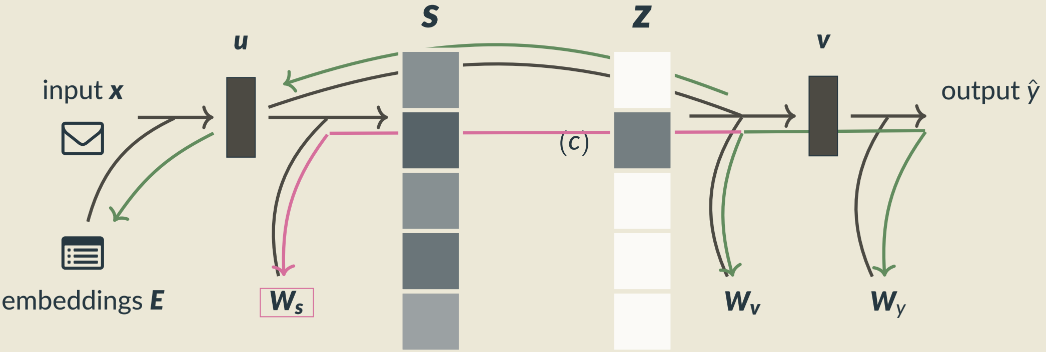

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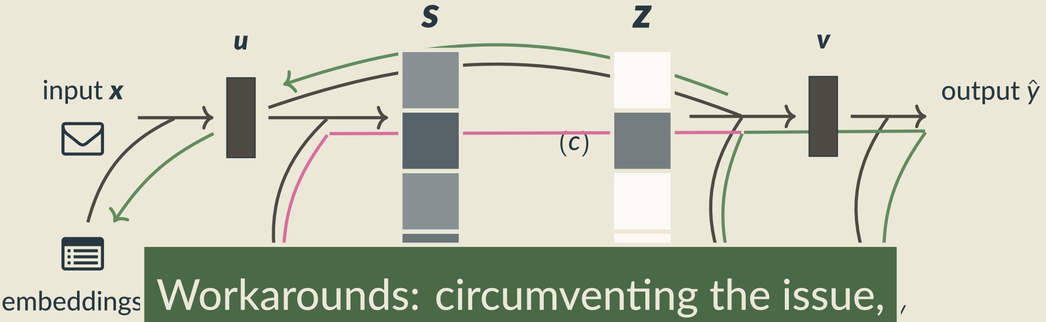
$$\frac{\partial L}{\partial \boldsymbol{W_s}} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial \boldsymbol{v}} \frac{\partial \boldsymbol{v}}{\partial \boldsymbol{z}} \overbrace{\frac{\partial \boldsymbol{z}}{\partial \boldsymbol{s}}}^{\equiv 0} \frac{\partial \boldsymbol{s}}{\partial \boldsymbol{W_s}}$$

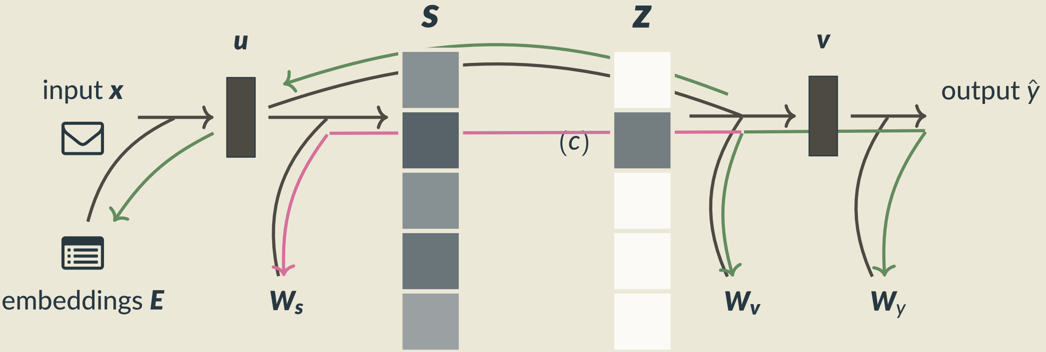
Example: Regression with latent categorization



$u = \frac{1}{|x|} \sum_j E_{x_j}$ … $-y)^2$

Workarounds: circumventing the issue, bypassing discrete variables

Example: Regression with latent categorization



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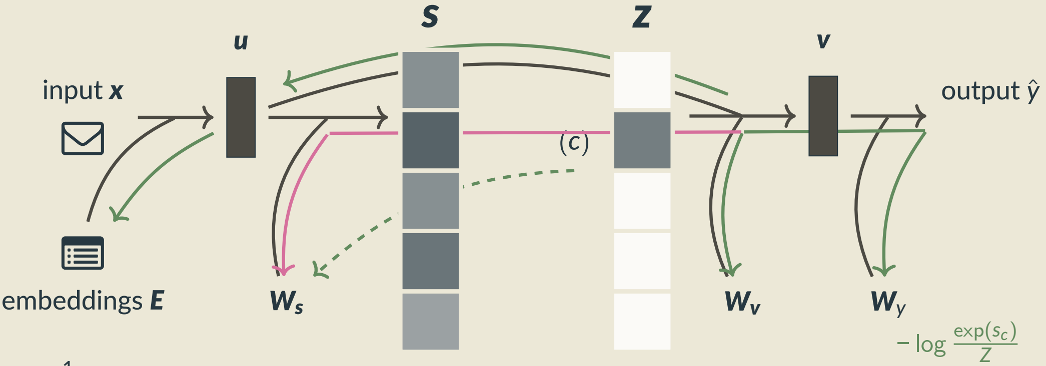
Option 1. Pretrain latent classifier $\boldsymbol{W_s}$

Example: Regression with latent categorization



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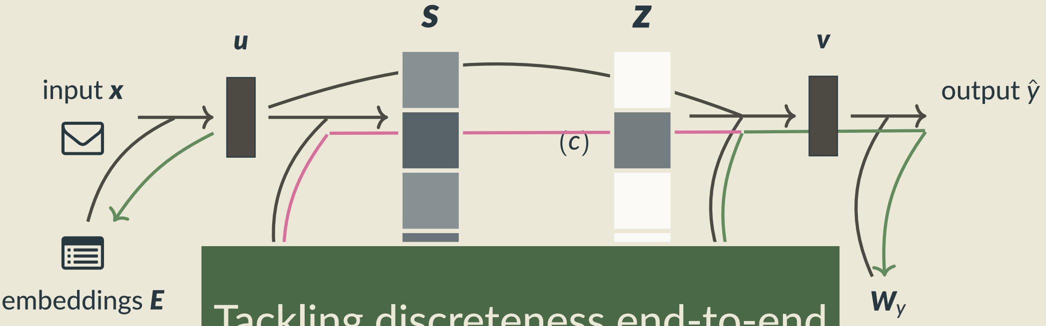
Option 2. Multi-task learning

Example: Regression with latent categorization



Tackling discreteness end-to-end

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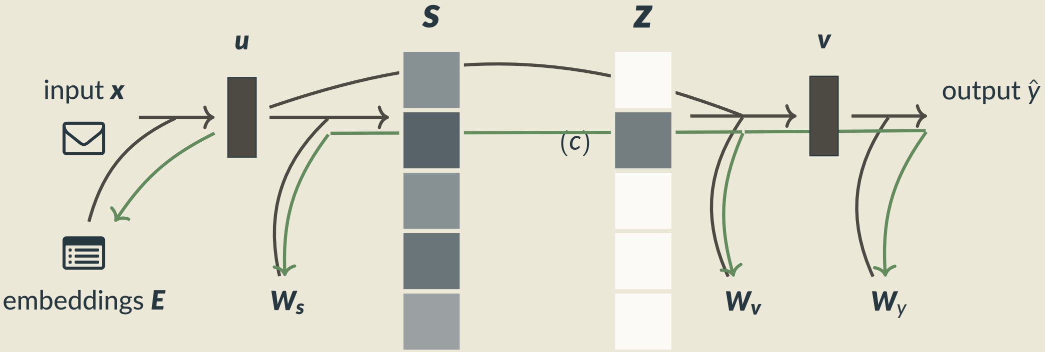
Example: Regression with latent categorization



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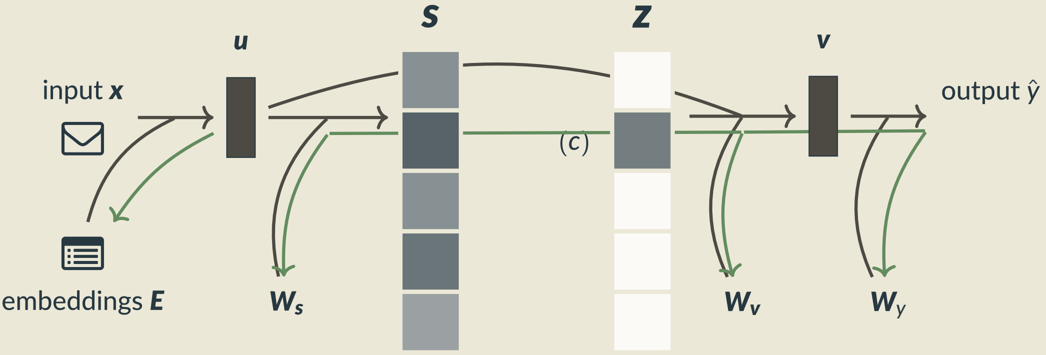
Option 3. Stochasticity! $\frac{\partial \mathbb{E}_{\boldsymbol{z}}(\hat{y}(\boldsymbol{z}) - y)^2}{\partial \boldsymbol{W}_s} \neq \boldsymbol{0}$

Example: Regression with latent categorization



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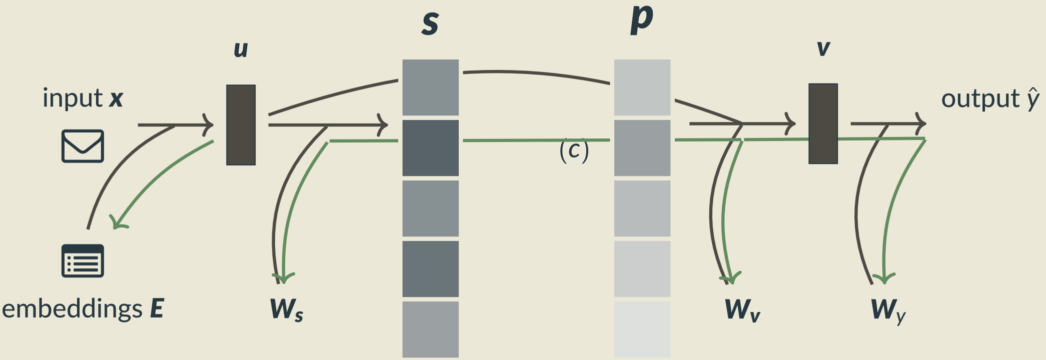
Option 4. Gradient surrogates (*e.g.* straight-through, $\frac{\partial \boldsymbol{z}}{\partial \boldsymbol{s}} \leftarrow \boldsymbol{I}$)

Example: Regression with latent categorization



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Option 5. Continuous relaxation (*e.g.* softmax)

Dealing with discrete latent variables

1. Pre-train external classifier
2. Multi-task learning
3. Stochastic latent variables
4. Gradient surrogates
5. Continuous relaxation

Dealing with discrete latent variables

1. Pre-train external classifier
2. Multi-task learning
3. Stochastic latent variables (Part 2)
4. Gradient surrogates (Part 3)
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Roadmap of the tutorial

- Part 1: Introduction ✓
- Part 2: Reinforcement learning
- Part 3: Gradient surrogates

Coffee Break

- Part 4: End-to-end differentiable models
- Part 5: Conclusions

II. Reinforcement Learning Methods

Latent structure via marginalization

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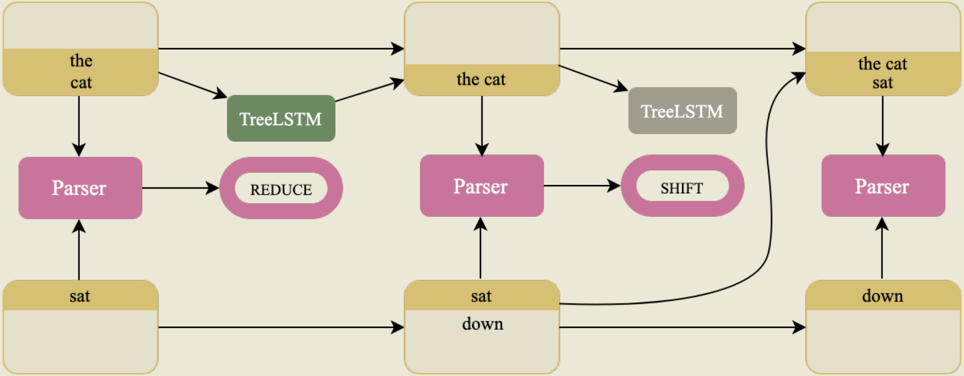
  by optimizing the **expected loss**,

$$\mathbb{E}_{\pi_{\boldsymbol{\theta}}(\boldsymbol{z}|x)}\left[L(\boldsymbol{z})\right]$$

But first, supervised SPINN

[Bowman et al., 2016]

Stack-augmented Parser-Interpreter Neural-Network



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- **TreeLSTM** combines top two elements of the stack when the parser choses the REDUCE action.

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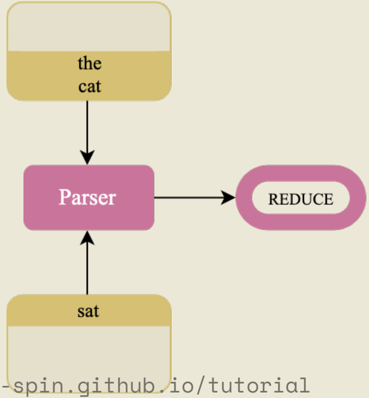

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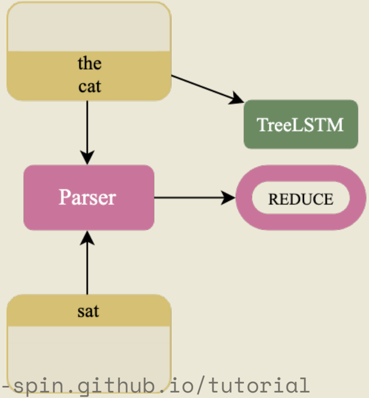

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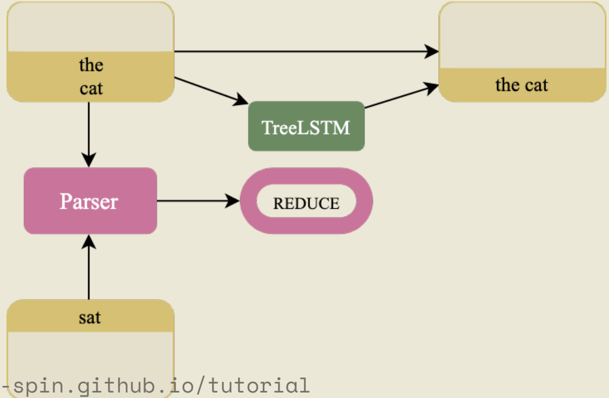

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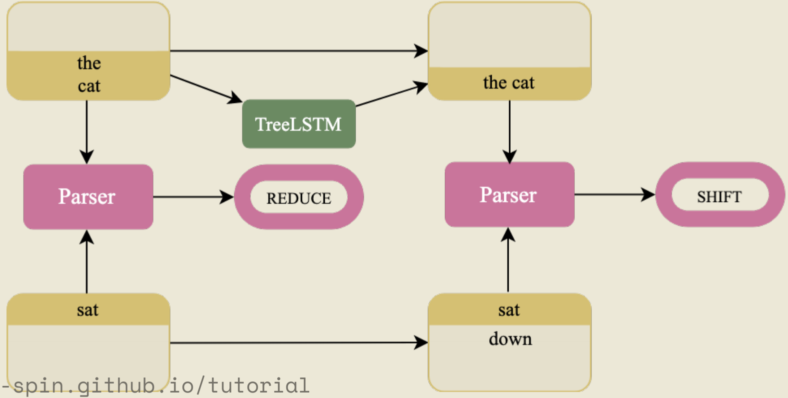

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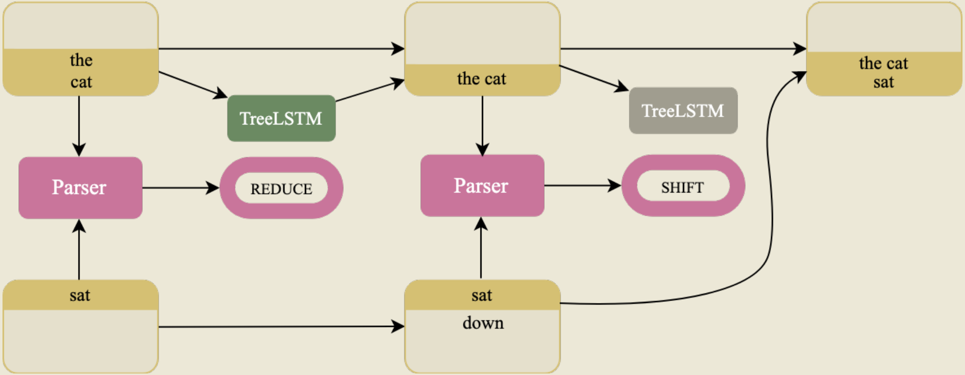

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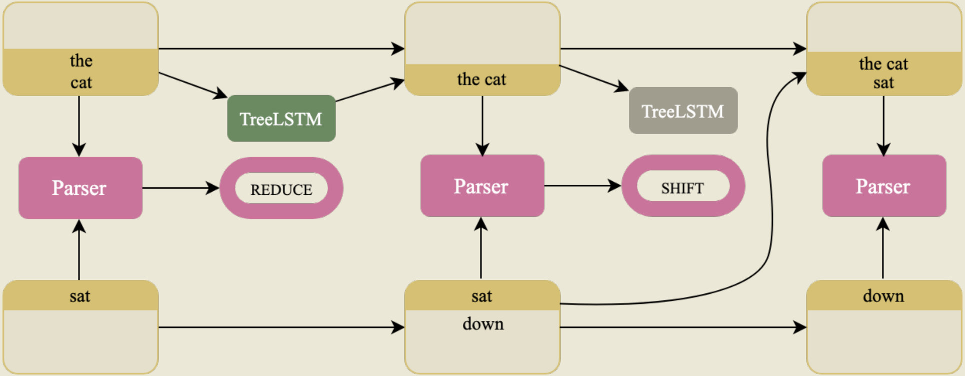


Shift-Reduce parsing

We can write a shift-reduce style parse as a sequence of Bernoulli random variables,

$$\boldsymbol{z} = \{z_1, \ldots, z_{2L-1}\}$$

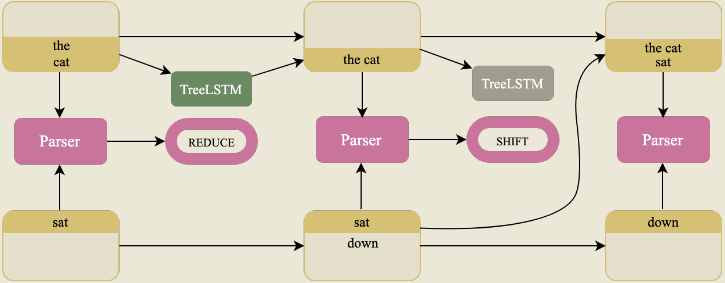
where, $z_j \in \{0, 1\} \ \forall j \in [1, 2L-1]$

Shift-Reduce parsing

A sequence of Bernoulli trials but with conditional dependence,

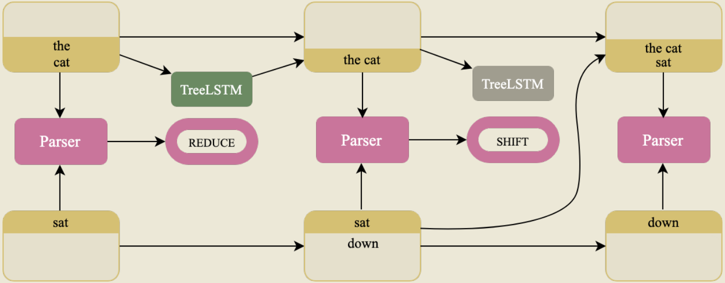
$$p(z_1, z_2, \ldots, z_{2L-1}) = \prod_{j=1}^{2L-1} p(z_j \mid z_{<j})$$

Latent structure learning with SPINN



Latent structure learning with SPINN

- But now, remove syntactic supervision from SPINN.



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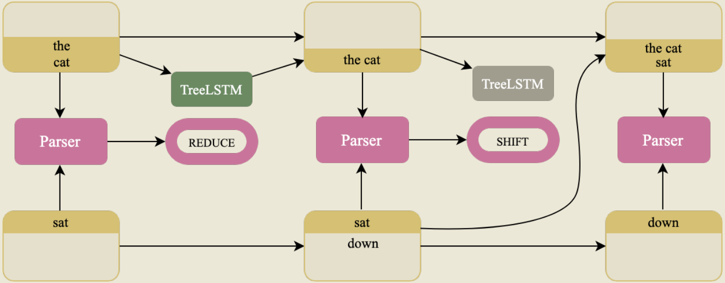

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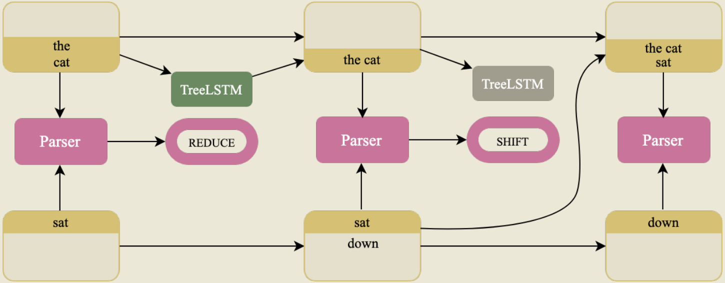

- We model the parse, $\boldsymbol{z}$, as a latent variable with our parser as the score function estimator, $f_{\boldsymbol{\theta}}(x)$.
- With shift-reduce parsing, we're making discrete decisions $\Rightarrow$ REINFORCE as a "natural" solution.

Unsupervised SPINN

Unsupervised SPINN

No syntactic supervision.

Only reward is from the downstream task.

We only get this reward after parsing the full sentence.

SPINN with REINFORCE

Some basic terminology,

- The action space is $z_j \in \{\textsc{shift}, \textsc{reduce}\}$, and $\boldsymbol{z}$ is a sequence of actions.

[Williams, 1992]

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- Maximize the reward, where $\mathcal{R}$ is performance on the downstream task like sentence classification.

[Williams, 1992]

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NOTE: Only a single reward at the end of parsing.

Through the looking glass of REINFORCE

$$\nabla_{\boldsymbol{\theta}} \mathbb{E}_{\boldsymbol{z} \sim \pi_{\boldsymbol{\theta}}(\boldsymbol{z}|x)}[L(\boldsymbol{z})]$$

Through the looking glass of REINFORCE

$$\nabla_{\boldsymbol{\theta}} \mathbb{E}_{\boldsymbol{z} \sim \pi_{\boldsymbol{\theta}}(\boldsymbol{z}|x)}[L(\boldsymbol{z})] = \nabla_{\boldsymbol{\theta}} \left[\sum_{\boldsymbol{z}} L(\boldsymbol{z}) \pi_{\boldsymbol{\theta}}(\boldsymbol{z} \mid x) \right]$$

(By definition of expectation. How to evaluate?)

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(By Leibniz integral rule for $\log$*)*

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SPINN with REINFORCE, aka RL-SPINN

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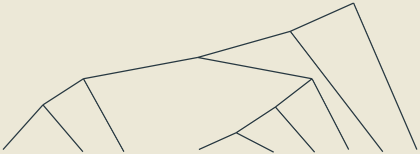
This model fails to solve a simple toy problem.

[Nangia and Bowman, 2018]

Toy problem: ListOps



[max 2 9 [min 4 7] 0]

Toy problem: ListOps

Model	Accuracy $\mu(\sigma)$	Accuracy max	Self F1
LSTM	71.5 (1.5)	**74.4**	-
RL-SPINN	60.7 (2.6)	64.8	30.8
Random Trees	-	-	30.1

Model	F1 wrt. LB	F1 wrt. RB	F1 wrt. GT	Avg. Depth
48D RL-SPINN	**64.5**	**16.0**	32.1	**14.6**
128D RL-SPINN	43.5	13.0	**71.1**	10.4
GT Trees	41.6	8.8	100.0	9.6
Random Trees	24.0	24.0	24.2	5.2

[Nangia and Bowman, 2018]

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But why?

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1. High variance of gradients
2. Coadaptation

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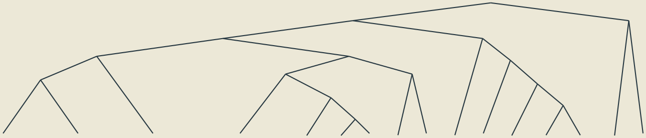
$$\begin{aligned} 3 \text{ tokens} &\Rightarrow 5 \text{ trees} \\ 5 \text{ tokens} &\Rightarrow 42 \text{ trees} \\ 10 \text{ tokens} &\Rightarrow 16796 \text{ trees} \end{aligned}$$

High variance

- We have a single reward at the end of parsing.
- We are sampling parses from very large search space!
  Catalan number of binary trees.
- And the policy is stochastic.

High variance

So, sometimes the policy lands in a “rewarding state”:



[sm [sm [sm [max 5 6] 2] 0] 5 0 8 6]

Figure: Truth: 7; Pred: 7

High variance

Sometimes it doesn't:

[max [med [med 1 [sm 3 1 3] 9] 6] 5]

Figure: Truth: 6; Pred: 5

High variance

Catalan number of parses means we need many many samples to lower variance!

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Possible solutions,

1. Gradient normalization
2. Control variates, aka baselines

Control variates

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  Which we can do because,

$$\sum_{z} b(\boldsymbol{x}) \nabla \pi(\boldsymbol{z}) = b(\boldsymbol{x}) \sum_{z} \nabla \pi(\boldsymbol{z}) = b(\boldsymbol{x}) \nabla 1 = 0$$

Issues with SPINN with REINFORCE

This system faces two big problems,

1. High variance of gradients
2. Coadaptation

Coadaptation problem

Learning composition function parameters $\boldsymbol{\phi}$ with backpropagation, and parser parameters $\boldsymbol{\theta}$ with REINFORCE.

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Difference in variance of two gradient estimates.

Coadaptation problem

Learning composition function parameters $\boldsymbol{\phi}$ with backpropagation, and parser parameters $\boldsymbol{\theta}$ with REINFORCE.

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Possible solution:
Proximal Policy Optimization (Schulman et al., 2017)

Making REINFORCE+SPINN work

Havrylov et al. [2019] use,

1. Input dependent control variate
2. Gradient normalization
3. Proximal Policy Optmization

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III. Gradient Surrogates

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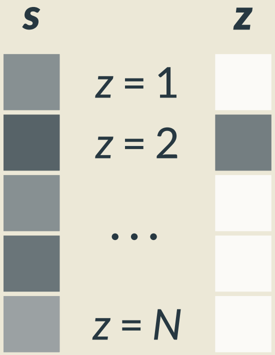
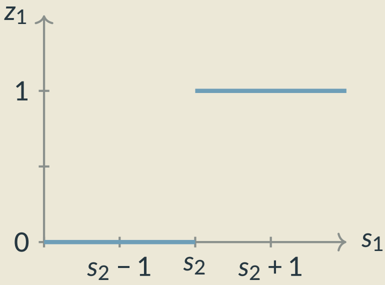
- 3A: try to optimize the deterministic loss directly
- 3B: use this strategy to reduce variance in the stochastic model.

Recap: The argmax problem

$$\boldsymbol{z} = \arg\max(\boldsymbol{s})$$

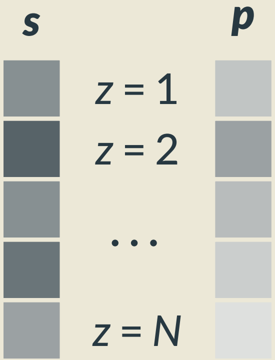


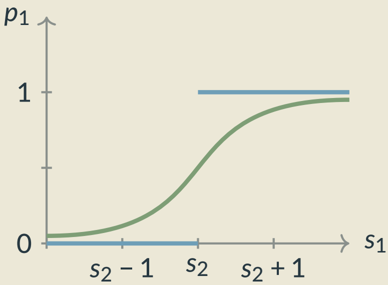


$$\frac{\partial \boldsymbol{z}}{\partial \boldsymbol{s}} = \boldsymbol{0}$$

Softmax

$$p_j = \exp(s_j)/Z$$





$$\frac{\partial \boldsymbol{p}}{\partial \boldsymbol{s}} = \operatorname{diag}(\boldsymbol{p}) - \boldsymbol{p}\boldsymbol{p}^\top$$

Straight-Through Estimator

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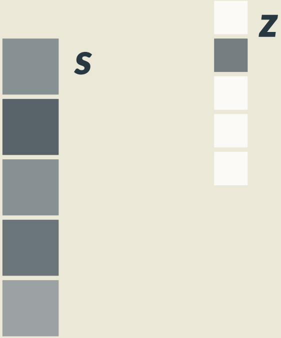

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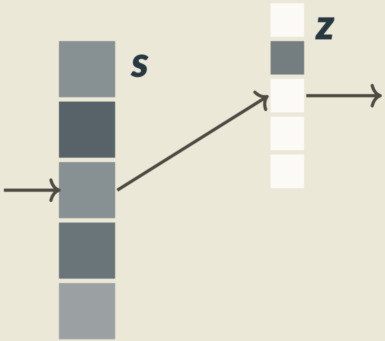

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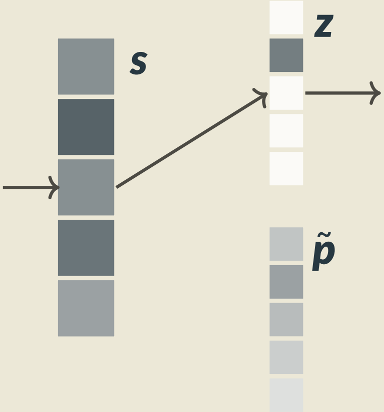

[Hinton, 2012, Bengio et al., 2013]

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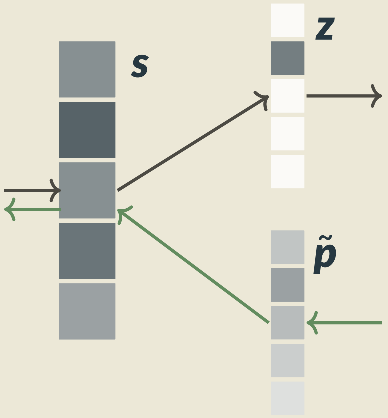

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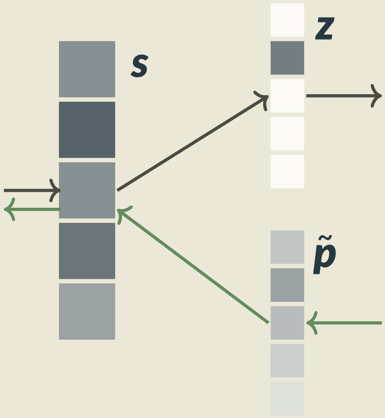

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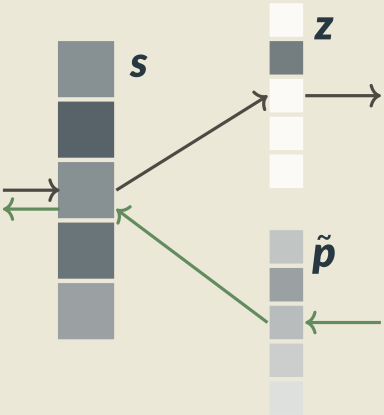

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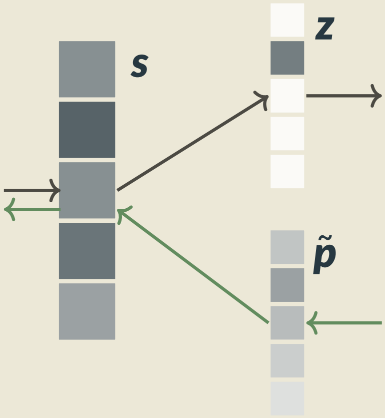

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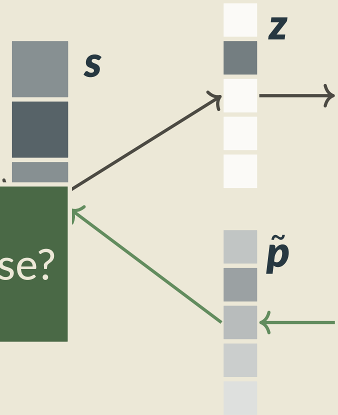
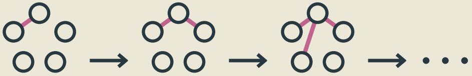
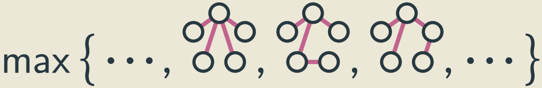
- More explanati

What about the structured case?



Dealing with the combinatorial explosion





1. Incremental structures

- Build structure **greedily**, as sequence of discrete choices (e.g., shift-reduce).
- Scores (partial structure, action) tuples.
- **Advantages:** flexible, rich histories.
- **Disadvantages:** greedy, local decisions are suboptimal, error propagation.

2. Factorization into parts

- Optimizes **globally** (e.g. Viterbi, Chu-Liu-Edmonds, Kuhn-Munkres).
- Scores smaller parts.
- **Advantages:** optimal, elegant, can handle hard & global constraints.
- **Disadvantages:** strong assumptions.

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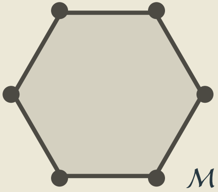
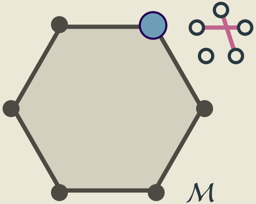
  Example: Latent Tree Learning with Differentiable Parsers: Shift-Reduce Parsing and Chart Parsing [Maillard and Clark, 2018] (STE through beam search).

The structured case: Marginal polytope



The structured case: Marginal polytope

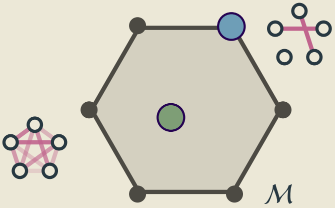
- Each vertex corresponds to one such *bit vector* $\boldsymbol{z}$



[Wainwright and Jordan, 2008]

The structured case: Marginal polytope

- Each vertex corresponds to one such *bit vector* $\boldsymbol{z}$
- Points inside correspond to *marginal distributions*: convex combinations of structured objects

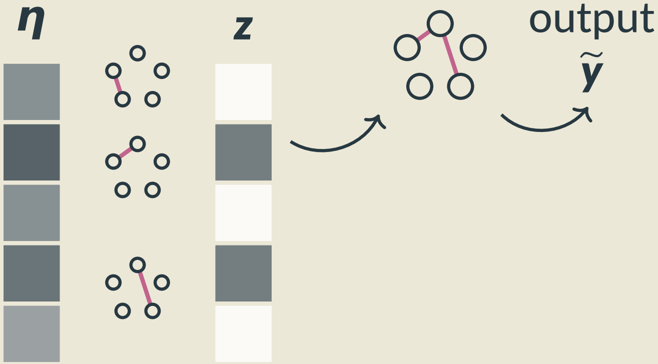


$$\boldsymbol{\mu} = \underbrace{p_1\boldsymbol{z}_1 + \ldots + p_N\boldsymbol{z}_N}_{\text{exponentially many terms}}, \ \boldsymbol{p} \in \Delta.$$

$$\begin{aligned} p_1 &= 0.2, & \boldsymbol{z}_1 &= [1, 0, 0, 0, 1, 0, 0, 0, 1] \\ p_2 &= 0.7, & \boldsymbol{z}_2 &= [0, 0, 1, 0, 0, 1, 1, 0, 0] \\ p_3 &= 0.1, & \boldsymbol{z}_3 &= [1, 0, 0, 0, 1, 0, 0, 1, 0] \end{aligned} \quad \Rightarrow \quad \boldsymbol{\mu} = [.3, 0, .7, 0, .3, .7, .7, .1, .2].$$

STE for factorization into parts

- $\eta(i \to j)$: score of arc $i \to j$
- $z(i \to j)$: is arc $i \to j$ selected?



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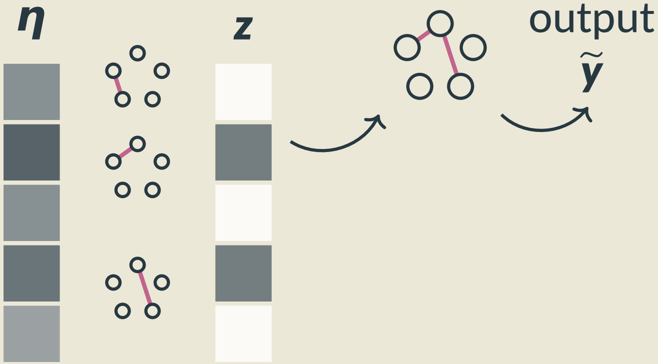

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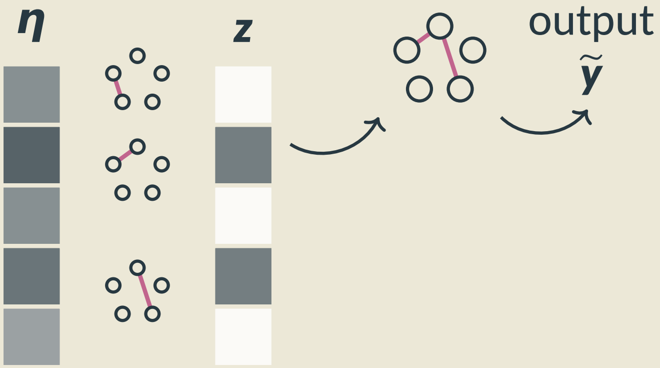

Algorithms for specific structures

	Best structure (MAP)
Sequence tagging	Viterbi [Rabiner, 1989]
Constituent trees	CKY [Kasami, 1966, Younger, 1967] [Cocke and Schwartz, 1970]
Temporal alignments	DTW [Sakoe and Chiba, 1978]
Dependency trees	Max. Spanning Arborescence [Chu and Liu, 1965, Edmonds, 1967]
Assignments	Kuhn-Munkres [Kuhn, 1955, Jonker and Volgenant, 1987]

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[Martins and Niculae]

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[Peng et al., 2018, Martins and Niculae]

Straight Through in the structured case

- Structured STE: perceptron update with fake annotation

$$\arg\min_{\boldsymbol{\mu}\in\mathbb{R}^d} L(\hat{y}(\boldsymbol{\mu}), y) \qquad \approx \boldsymbol{z} - \nabla_{\boldsymbol{z}} L(\boldsymbol{z}) \rightarrow \boldsymbol{z}^{\text{true}}$$

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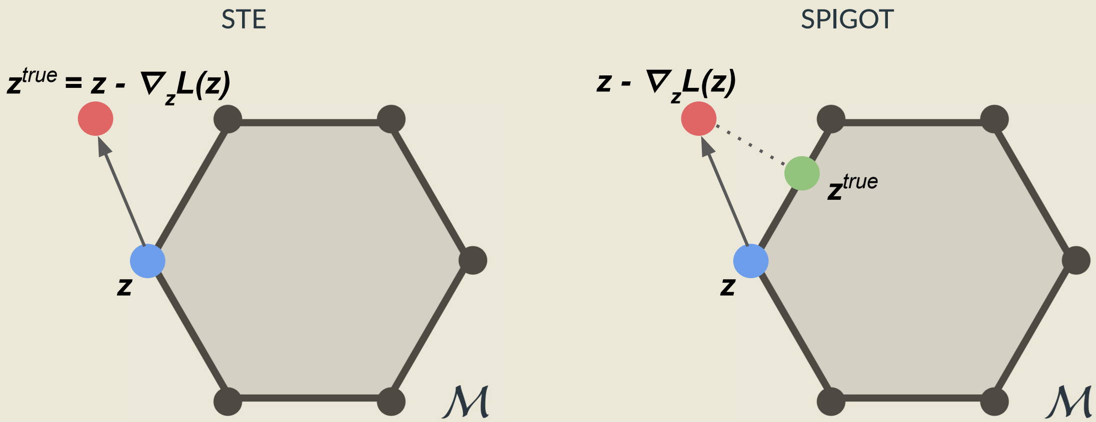
- We discuss a generic way to compute the projection in part 3.

SPIGOT vs STE



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Now we will see how to apply STE for stochastic graphs, as an alternative approach of the score-function estimators.

Stochastic node in the computation graph

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- An alternative is using the *reparameterization trick* [Kingma and Welling, 2014].

Categorical reparameterization

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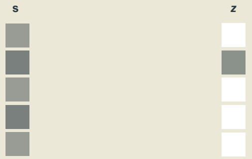
- Sampling from a categorical value in the middle of the computation graph.
  $\boldsymbol{z} \sim \pi_{\boldsymbol{\theta}}(\boldsymbol{z} \mid x) \propto \exp \boldsymbol{s}_{\boldsymbol{\theta}}(\boldsymbol{z} \mid x)$



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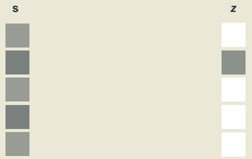

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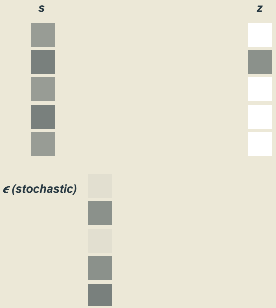

[Jang et al., 2017, Maddison et al., 2016]

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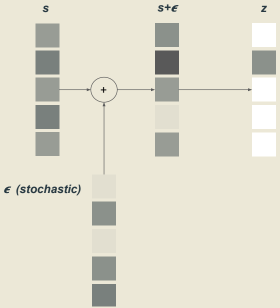

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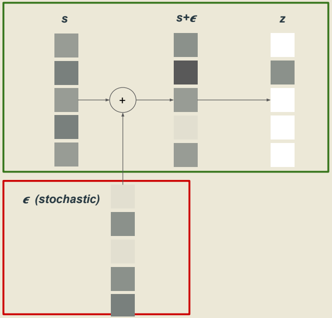

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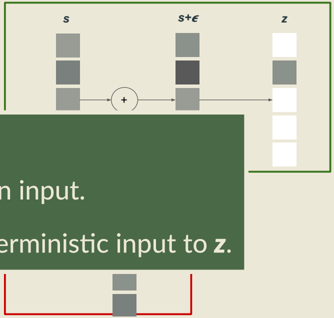


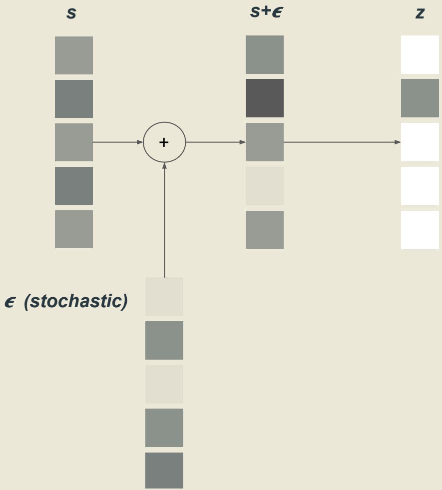
As a result:

Stochasticity is moved as an input.

We can backpropagate through the deterministic input to $\boldsymbol{z}$.

[Jang et al., 2017, Maddison et al., 2016]

Categorical reparameterization



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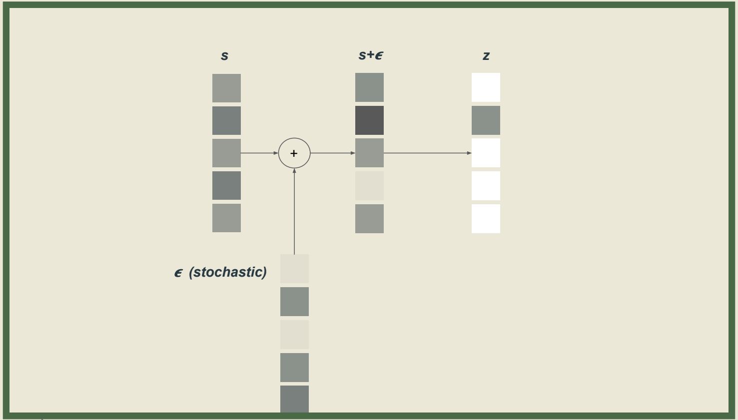

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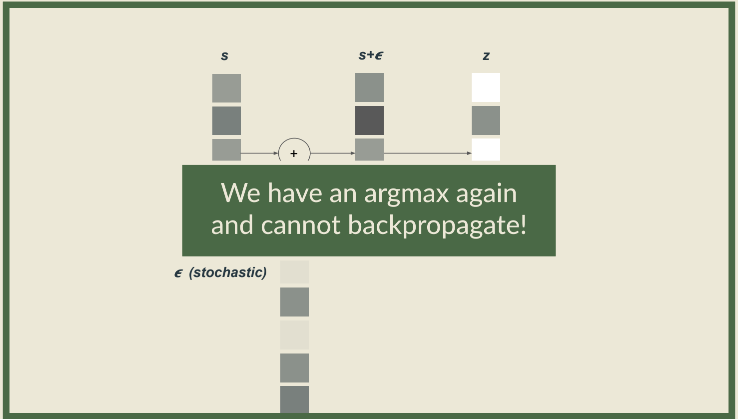

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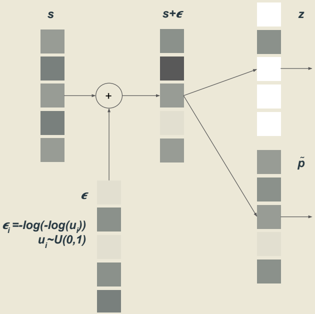

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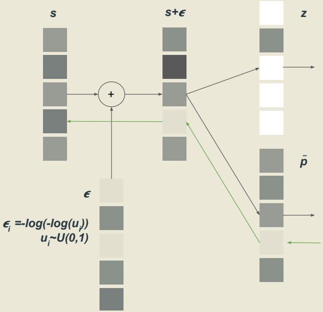

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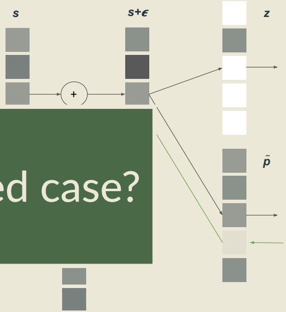
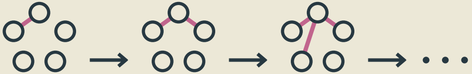
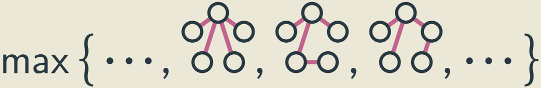
What about the structured case?



Dealing with the combinatorial explosion





1. Incremental structures

- Build structure **greedily**, as sequence of discrete choices (e.g., shift-reduce).
- Scores (partial structure, action) tuples.
- **Advantages:** flexible, rich histories.
- **Disadvantages:** greedy, local decisions are suboptimal, error propagation.

2. Factorization into parts

- Optimizes **globally** (e.g. Viterbi, Chu-Liu-Edmonds, Kuhn-Munkres).
- Scores smaller parts.
- **Advantages:** optimal, elegant, can handle hard & global constraints.
- **Disadvantages:** strong assumptions.

Sampling from incremental structures

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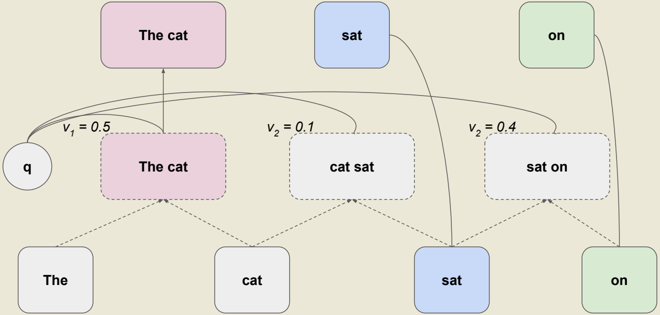
Example: Gumbel Tree-LSTM [Choi et al., 2018].

Example: Gumbel Tree-LSTM

- Building task-specific tree structures.
- Straight-Through Gumbel-Softmax at each step to select one arc.



Sampling from factorized models

[Papandreou and Yuille, 2011, Corro and Titov, 2019a,b]

Perturb-and-MAP

Reparameterize by **perturbing the arc scores.** (inexact!)

- Sample from the normal Gumbel distribution.
- Perturb the arc scores with the Gumbel noise.
- Compute MAP (task-specific algorithm).

- $\boldsymbol{\epsilon} \sim G(0, 1)$
- $\tilde{\boldsymbol{\eta}} = \boldsymbol{\eta} + \boldsymbol{\epsilon}$
- $\arg\max_{\boldsymbol{z} \in \mathcal{Z}} \tilde{\boldsymbol{\eta}}^\top \boldsymbol{z}$

Summary: Gradient surrogates

- Based on the **Straight-Through Estimator**.
- Can be used for stochastic or deterministic computation graphs.
- **Forward pass**: Get an argmax (might be structured).
- **Backpropagation**: use a function, which we hope is close to argmax.
- Examples:
  - Argmax for iterative structures and factorization into parts
  - Sampling from iterative structures and factorization into parts

Gradient surrogates: Pros and cons

Pros

- Do not suffer from the high variance problem of REINFORCE.
- Allow for flexibility to select or sample a latent structured in the middle of the computation graph.
- Efficient computation.

Cons

- The Gumbel sampling with Perturb-and-MAP is an approximation.
- Bias, due to function mismatch on the backpropagation
  (next section will address this problem.)

Overview

$$\mathbb{E}_{\pi_{\boldsymbol{\theta}}(\boldsymbol{z}|x)}\Big[L(\boldsymbol{z})\Big] \qquad L\Big(\arg\max_{\boldsymbol{z}} \pi_{\boldsymbol{\theta}}(\boldsymbol{z} \mid x)\Big)$$

- REINFORCE
- Straight Through–Gumbel (Perturb & MAP)

- Straight Through
- SPIGOT

IV. End-to-end differentiable methods

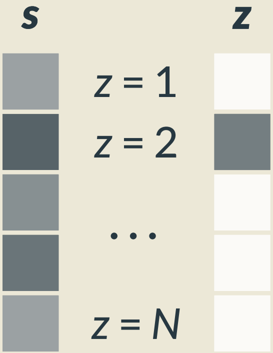
End-to-end differentiable methods

1. Digging into softmax
2. Alternatives to softmax
3. Generalizing to structured prediction
4. Stochasticity and global structures

Recall: Discrete choices & differentiability

input $\boldsymbol{x}$ → $\boldsymbol{s}$ → $\boldsymbol{z}$ → output $\widehat{\boldsymbol{y}}$

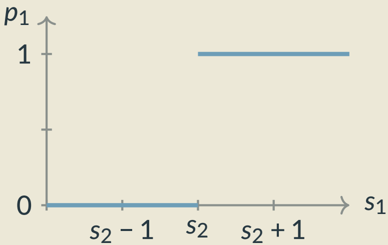
$\boldsymbol{s} = \boldsymbol{f_\theta}(\boldsymbol{x})$



$y = \boldsymbol{g_\phi}(\boldsymbol{z}, \boldsymbol{x})$

$\frac{\partial \boldsymbol{z}}{\partial \boldsymbol{s}} = \mathbf{0}$ or n/a

(argmax)



One solution: smooth relaxation

input $\boldsymbol{x}$ $\longrightarrow$ $\boldsymbol{s} = \boldsymbol{f_\theta}(\boldsymbol{x})$ $\longrightarrow$ $\boldsymbol{s}$ | $z = 1$, $z = 2$, $\dots$, $z = N$ | $\boldsymbol{p}$ $\longrightarrow$ $y = \boldsymbol{g_\phi}(\boldsymbol{z}, \boldsymbol{x})$ $\longrightarrow$ output $\widehat{\boldsymbol{y}}$

$\boldsymbol{p} = \mathrm{softmax}(\boldsymbol{s}) = \mathbb{E}[\boldsymbol{z}]$, i.e.
replace $\mathbb{E}\big[f(\boldsymbol{z})\big]$ with $f\big(\mathbb{E}[\boldsymbol{z}]\big)$

$\dfrac{\partial \boldsymbol{p}}{\partial \boldsymbol{s}} =$ 😊

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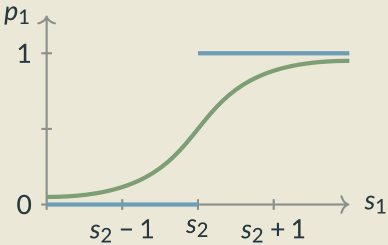

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$\boldsymbol{s}$ $\boldsymbol{p}$

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😟

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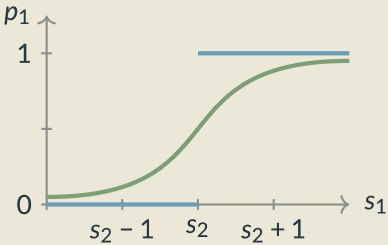

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$\mathbb{E}_{\pi_{\boldsymbol{\theta}}(\mathbf{z}|x)}\left[L(\mathbf{z})\right]$

- REINFORCE
- Straight Through–Gumbel (Perturb & MAP)
- SparseMAP

$L\left(\arg\max_{z} \pi_{\boldsymbol{\theta}}(\mathbf{z} \mid x)\right)$

- Straight Through
- SPIGOT

$L\left(\mathbb{E}_{\pi_{\boldsymbol{\theta}}(\mathbf{z}|x)}[\mathbf{z}]\right)$

- Structured Attn. Nets
- SparseMAP

Model restrictions:

- $\operatorname{dom} L$ may be only $\mathcal{Z}$,
- $\nabla_{\mathbf{z}} L$ need not exist!

- $L(\mathbf{z})$ with $\mathbf{z} \in \mathcal{Z}$ in forward
- needs (relaxed) $\nabla_{\mathbf{z}} L$ in backward.

- $L(\mathbf{z})$ must be relaxed and differentiable.
- (sparsity gets us closer to $\mathcal{Z}$).

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Often defined via $p_i = \dfrac{\exp s_i}{\sum_j \exp s_j}$, but where does it come from?

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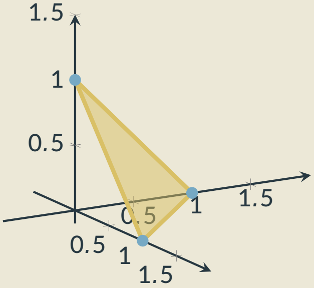

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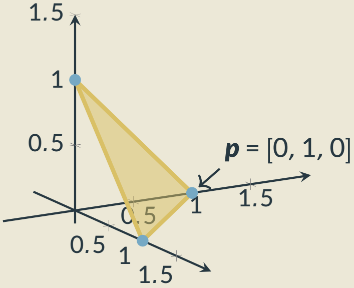

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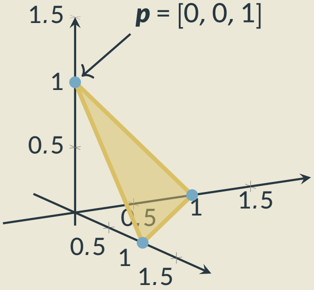

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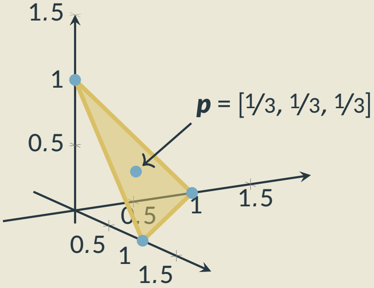

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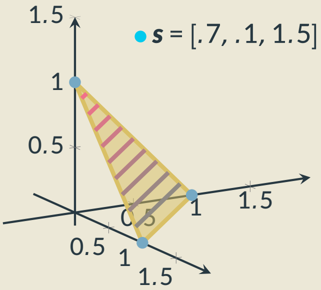

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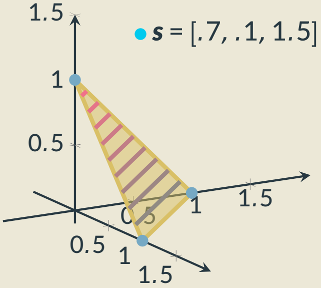
argmax



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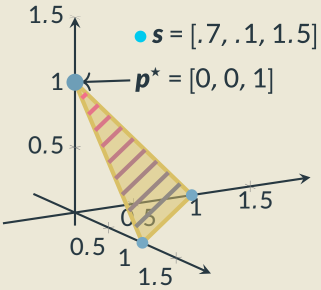

$$\arg\max_{\boldsymbol{p}\in\triangle} \boldsymbol{p}^\top \boldsymbol{s}$$

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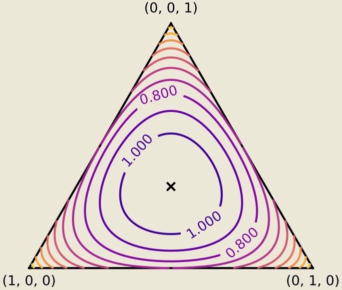

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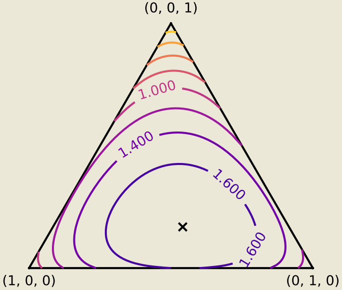
softmax maximizes **expected score** + **entropy**:



$$\underset{\boldsymbol{p} \in \triangle}{\arg\max}\, \boldsymbol{p}^\top \boldsymbol{s} + \mathrm{H}(\boldsymbol{p})$$

Variational form of softmax

Proposition. The unique solution to $\underset{\boldsymbol{p}\in\triangle}{\arg\max}\, \boldsymbol{p}^\top \boldsymbol{s} + \mathrm{H}(\boldsymbol{p})$ is given by $p_j = \frac{\exp s_j}{\sum_i \exp s_i}$.

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$$\begin{aligned} 0 = \nabla_{p_i} \mathcal{L}(\boldsymbol{p}, \boldsymbol{\nu}, \tau) &= -s_i + \log p_i + 1 - \nu_i + \tau \\ \boldsymbol{p}^\top \boldsymbol{\nu} &= 0 \\ \boldsymbol{p} &\in \triangle \\ \boldsymbol{\nu} &\geq \mathbf{0} \end{aligned}$$

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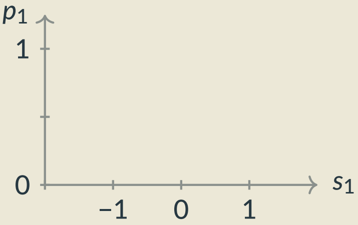
Answer: $Z = \sum_j \exp(s_j)$

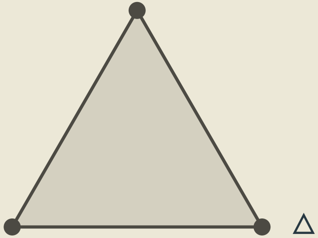
So, $p_i = \dfrac{\exp(s_i)}{\sum_j \exp(s_j)}$.

Classic result, e.g., [Boyd and Vandenberghe, 2004, Wainwright and Jordan, 2008]

Generalizing softmax: Smoothed argmaxes

$$\hat{\boldsymbol{p}}_\Omega(\boldsymbol{s}) = \arg\max_{\boldsymbol{p}\in\triangle} \boldsymbol{p}^\top \boldsymbol{s} - \Omega(\boldsymbol{p})$$





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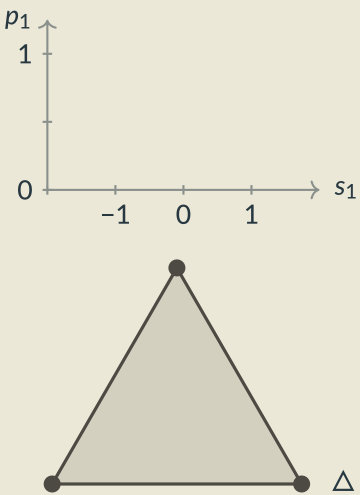

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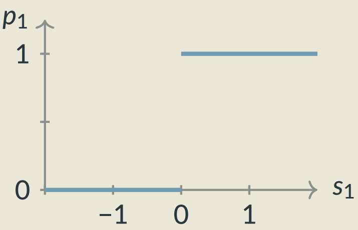

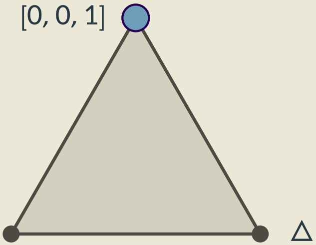

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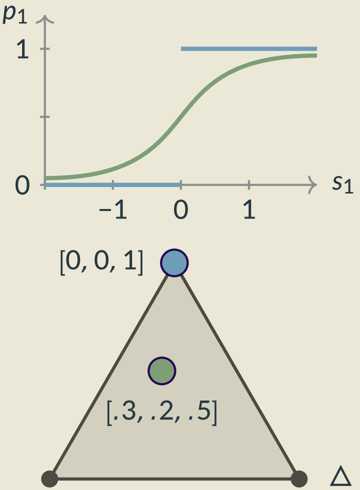

[Niculae and Blondel, 2017, Martins and Astudillo, 2016]

Generalizing softmax: Smoothed argmaxes

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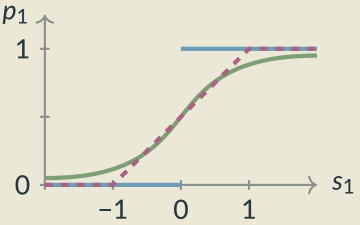
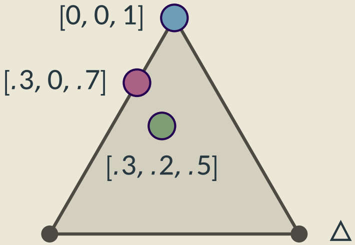
- sparsemax: $\Omega(\boldsymbol{p}) = 1/2\|\boldsymbol{p}\|_2^2$





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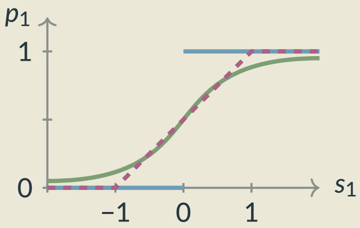

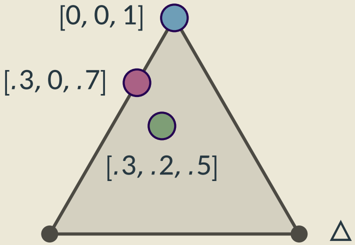

Generalized entropy interpolates in between [Tsallis, 1988]
Used in Sparse Seq2Seq: [Peters et al., 2019]
(Mon 13:50, poster session 2D)

[Niculae and Blondel, 2017, Malaviya et al., 2018]

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fusedmax: $\Omega(\boldsymbol{p}) = 1/2 \|\boldsymbol{p}\|_2^2 + \sum_j |p_j - p_{j-1}|$

csparsemax: $\Omega(\boldsymbol{p}) = 1/2 \|\boldsymbol{p}\|_2^2 + \iota(\boldsymbol{a} \le \boldsymbol{p} \le \boldsymbol{b})$

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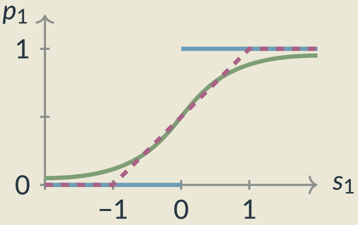
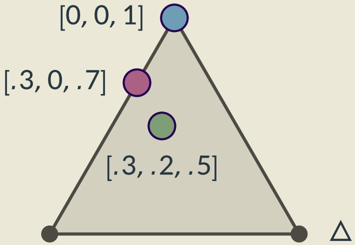
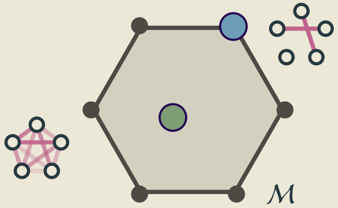




[Wainwright and Jordan, 2008]

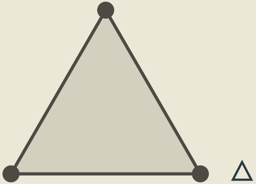
The structured case: Marginal polytope

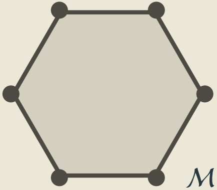
- Each vertex corresponds to one such *bit vector* $\boldsymbol{z}$
- Points inside correspond to *marginal distributions*: convex combinations of structured objects



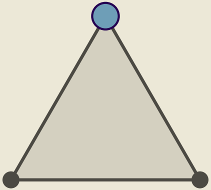
$$\boldsymbol{\mu} = \underbrace{p_1\boldsymbol{z}_1 + \ldots + p_N\boldsymbol{z}_N}_{\text{exponentially many terms}}, \quad \boldsymbol{p} \in \Delta.$$

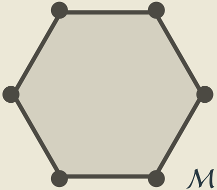
$$\begin{array}{ll} p_1 = 0.2, & \boldsymbol{z}_1 = [1, 0, 0, 0, 1, 0, 0, 0, 1] \\ p_2 = 0.7, & \boldsymbol{z}_2 = [0, 0, 1, 0, 0, 1, 1, 0, 0] \\ p_3 = 0.1, & \boldsymbol{z}_3 = [1, 0, 0, 0, 1, 0, 0, 1, 0] \end{array} \Rightarrow \quad \boldsymbol{\mu} = [.3, 0, .7, 0, .3, .7, .7, .1, .2].$$


$\triangle$


$\mathcal{M}$

- **argmax** $\arg\max_{\boldsymbol{p} \in \triangle} \boldsymbol{p}^\top \boldsymbol{s}$





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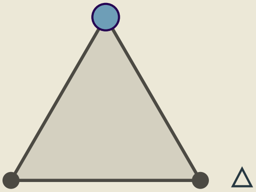
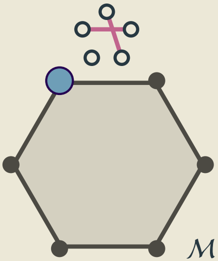
MAP $\arg\max_{\boldsymbol{\mu} \in \mathcal{M}} \boldsymbol{\mu}^\top \boldsymbol{\eta}$





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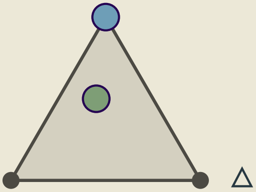

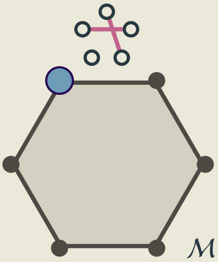

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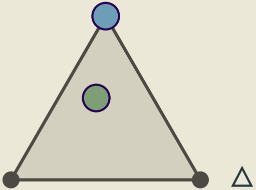

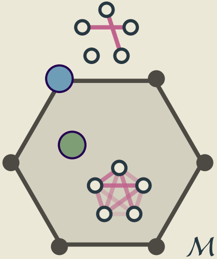

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Just like softmax relaxes argmax,
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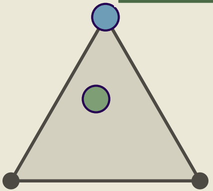

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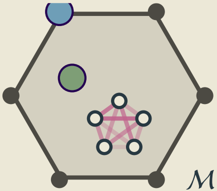

$\mathcal{M}$

argmax $\arg\max_{\boldsymbol{p}\in\triangle} \boldsymbol{p}^\top \boldsymbol{s}$

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Just like softmax relaxes argmax, marginals relax MAP **differentiably**!

Unlike argmax/softmax, computation is not obvious!

$\triangle$

$\mathcal{M}$

Algorithms for specific structures

	Best structure (MAP)	Marginals
Sequence tagging	Viterbi [Rabiner, 1989]	Forward-Backward [Rabiner, 1989]
Constituent trees	CKY [Kasami, 1966, Younger, 1967] [Cocke and Schwartz, 1970]	Inside-Outside [Baker, 1979]
Temporal alignments	DTW [Sakoe and Chiba, 1978]	Soft-DTW [Cuturi and Blondel, 2017]
Dependency trees	Max. Spanning Arborescence [Chu and Liu, 1965, Edmonds, 1967]	Matrix-Tree [Kirchhoff, 1847]
Assignments	Kuhn-Munkres [Kuhn, 1955, Jonker and Volgenant, 1987]	#P-complete [Valiant, 1979, Taskar, 2004]

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Derivatives of marginals 1: DP

Dynamic programming: marginals by **Forward-Backward**, **Inside-Outside**, etc.

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Marginals in a sequence tagging model.

---

input: d tags, n tokens, $\boldsymbol{\eta}_U \in \mathbb{R}^{n\times d}$, $\boldsymbol{\eta}_V \in \mathbb{R}^{d\times d}$

initialize $\boldsymbol{\alpha}_1 = \mathbf{0}$, $\boldsymbol{\beta}_n = \mathbf{0}$

for $i \in 2, \ldots, n$ **do** *# forward log-probabilities*

$\quad \alpha_{i,k} = \log \sum_{k'} \exp\left(\alpha_{i-1,k'} + (\boldsymbol{\eta}_U)_{i,k} + (\boldsymbol{\eta}_V)_{k',k}\right)$ for all k

for $i \in n-1, \ldots, 1$ **do** *# backward log-probabilities*

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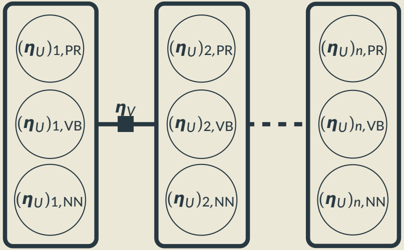
$Z = \sum_k \exp \alpha_{n,k}$ *# partition function*

return $\boldsymbol{\mu} = \exp\left(\boldsymbol{\alpha} + \boldsymbol{\beta} - \log Z\right)$ *# marginals*



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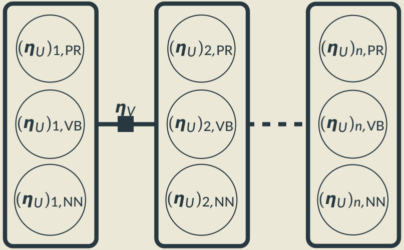

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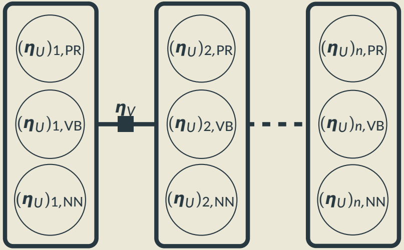

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- With circular dependencies, this breaks! Can get an approximation Stoyanov et al. [2011]

Marginals in a sequence tagging model.

---

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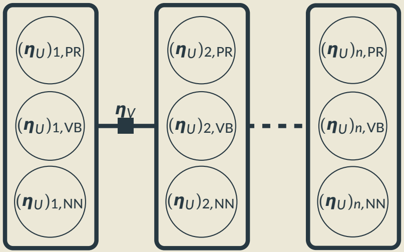


Derivatives of marginals 2: Matrix-Tree

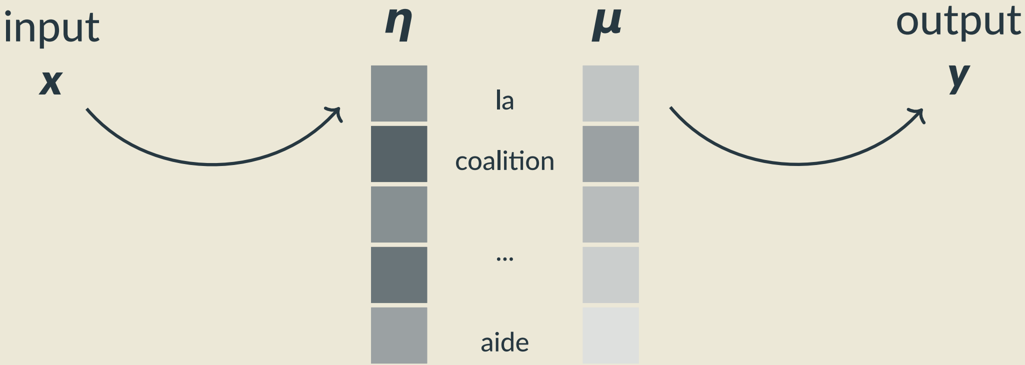
$\boldsymbol{L}(\boldsymbol{s})$: Laplacian of the edge score graph

$$Z = \det \boldsymbol{L}(\boldsymbol{s})$$

$$\boldsymbol{\mu} = \boldsymbol{L}(\boldsymbol{s})^{-1}$$

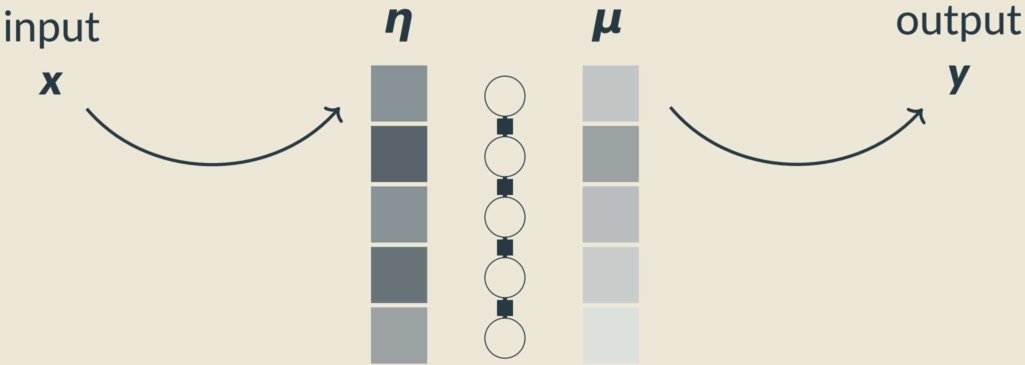
$$\nabla\boldsymbol{\mu} = \nabla\boldsymbol{L}^{-1} = \boldsymbol{L}^{-1}\left(\frac{\partial \boldsymbol{L}}{\partial \boldsymbol{\eta}}\right)\boldsymbol{L}^{-1}$$

Structured Attention Networks



Structured Attention Networks

[Kim et al., 2017]



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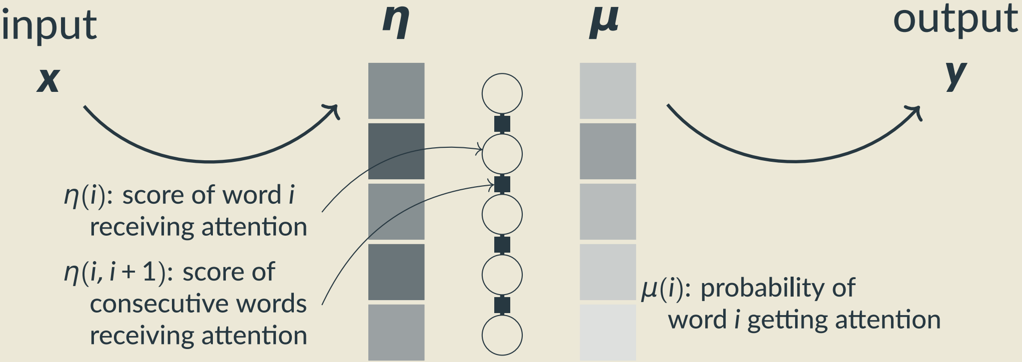

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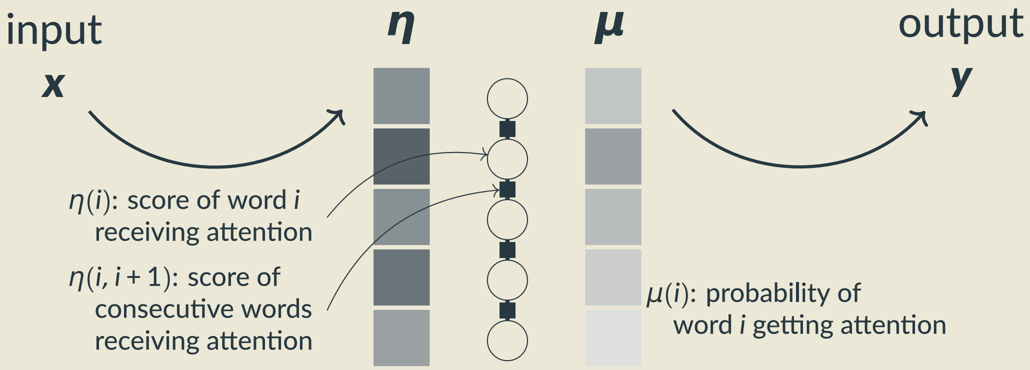

CRF marginals (from *forward–backward*) give attention weights $\in (0, 1)$

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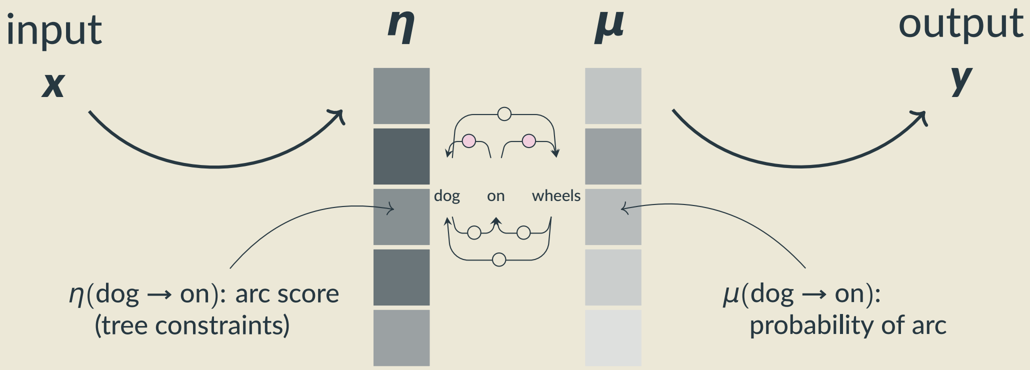

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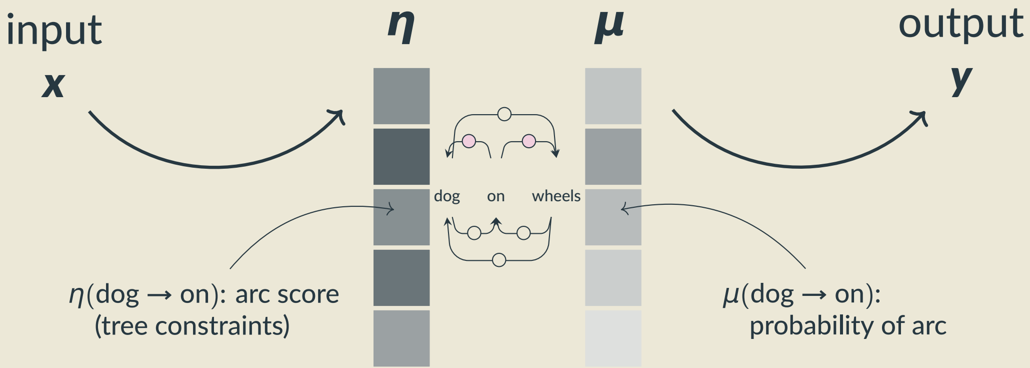

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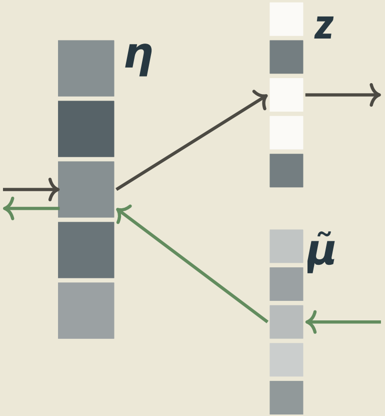
Similar idea for projective dependency trees with *inside–outside* and non-projective with the Matrix-Tree theorem [Liu and Lapata, 2018].

Differentiable Perturb & Parse

Extending Gumbel-Softmax to structured stochastic models

- Forward pass:
  sample structure $\boldsymbol{z}$ (approximately)
  $$\boldsymbol{z} = \arg\max_{\boldsymbol{z} \in \mathcal{Z}} (\boldsymbol{\eta} + \boldsymbol{\epsilon})^\top \boldsymbol{z}$$
- Backward pass:
  pretend we did marginal inference
  $$\tilde{\boldsymbol{\mu}} = \arg\max_{\boldsymbol{\mu} \in \mathcal{M}} (\boldsymbol{\eta} + \boldsymbol{\epsilon})^\top \boldsymbol{z} + \tilde{\mathrm{H}}(\boldsymbol{\mu})$$
  (or some similar relaxation)



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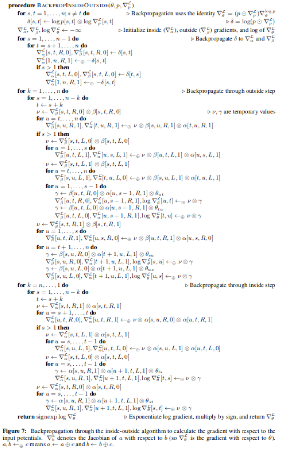

```
procedure BackpropInsideOutside(θ, p, ∇^L_p)
  for s, t = 1, ..., n; s ≠ t do        ▷ Backpropagation uses the identity ∇^L_θ = (p ⊙ ∇^L_p)∇^{log p}_θ
    δ[s, t] ← log p[s, t] ⊗ log ∇^L_p[s, t]        ▷ δ = log(p ⊙ ∇^L_p)
  ∇^L_α, ∇^L_β, log ∇^L_θ ← −∞        ▷ Initialize inside (∇^L_α), outside (∇^L_β) gradients, and log of ∇^L_θ
  for s = 1, ..., n − 1 do        ▷ Backpropagate δ to ∇^L_α and ∇^L_β
    for t = s + 1, ..., n do
      ∇^L_α[s, t, R, 0], ∇^L_β[s, t, R, 0] ← δ[s, t]
      ∇^L_α[1, n, R, 1] ←_⊕ −δ[s, t]
      if s > 1 then
        ∇^L_α[s, t, L, 0], ∇^L_β[s, t, L, 0] ← δ[t, s]
        ∇^L_α[1, n, R, 1] ←_⊕ −δ[s, t]
  for k = 1, ..., n do        ▷ Backpropagate through outside step
    for s = 1, ..., n − k do
      t ← s + k
      ν ← ∇^L_β[s, t, R, 0] ⊗ β[s, t, R, 0]        ▷ ν, γ are temporary values
      for u = t, ..., n do
        ∇^L_β[s, u, R, 1], ∇^L_α[t, u, R, 1] ←_⊕ ν ⊗ β[s, u, R, 1] ⊗ α[t, u, R, 1]
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          ∇^L_β[u, t, L, 1], ∇^L_α[u, s, L, 1] ←_⊕ ν ⊗ β[u, t, L, 1] ⊗ α[u, s, L, 1]
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        for u = 1, ..., s − 1 do
          γ ← β[u, t, R, 0] ⊗ α[u, s − 1, R, 1] ⊗ θ_ut
          ∇^L_β[u, t, R, 0], ∇^L_α[u, s − 1, R, 1], log ∇^L_θ[u, t] ←_⊕ ν ⊗ γ
          γ ← β[u, t, L, 0] ⊗ α[u, s − 1, R, 1] ⊗ θ_tu
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        ∇^L_β[s, u, L, 0], ∇^L_α[t + 1, u, L, 1], log ∇^L_θ[u, s] ←_⊕ ν ⊗ γ
  for k = n, ..., 1 do        ▷ Backpropagate through inside step
    for s = 1, ..., n − k do
      t ← s + k
      ν ← ∇^L_α[s, t, R, 1] ⊗ α[s, t, R, 1]
      for u = s + 1, ..., t do
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  return signexp log ∇^L_θ        ▷ Exponentiate log gradient, multiply by sign, and return ∇^L_θ
```

Figure 7: Backpropagation through the inside-outside algorithm to calculate the gradient with respect to the input potentials. ∇^a_b denotes the Jacobian of a with respect to b (so $\nabla^{\mathcal{L}}_\theta$ is the gradient with respect to θ). $a, b \leftarrow_\oplus c$ means $a \leftarrow a \oplus c$ and $b \leftarrow b \oplus c$.

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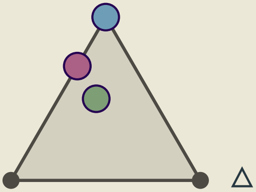
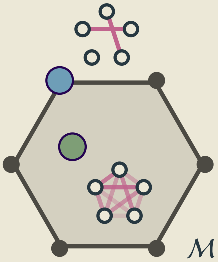
- **sparsemax** $\arg\max_{\boldsymbol{p}\in\triangle} \boldsymbol{p}^\top \boldsymbol{s} - 1/2\|\boldsymbol{p}\|^2$

- **MAP** $\arg\max_{\boldsymbol{\mu}\in\mathcal{M}} \boldsymbol{\mu}^\top \boldsymbol{\eta}$
- **marginals** $\arg\max_{\boldsymbol{\mu}\in\mathcal{M}} \boldsymbol{\mu}^\top \boldsymbol{\eta} + \widetilde{\mathrm{H}}(\boldsymbol{\mu})$





[Niculae et al., 2018a]

- **argmax** $\arg\max_{\boldsymbol{p}\in\triangle} \boldsymbol{p}^\top \boldsymbol{s}$
- **softmax** $\arg\max_{\boldsymbol{p}\in\triangle} \boldsymbol{p}^\top \boldsymbol{s} + \mathrm{H}(\boldsymbol{p})$
- **sparsemax** $\arg\max_{\boldsymbol{p}\in\triangle} \boldsymbol{p}^\top \boldsymbol{s} - 1/2\|\boldsymbol{p}\|^2$

- **MAP** $\arg\max_{\boldsymbol{\mu}\in\mathcal{M}} \boldsymbol{\mu}^\top \boldsymbol{\eta}$
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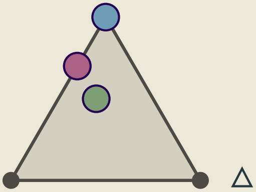
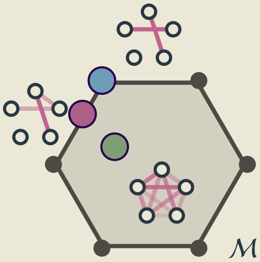
- **SparseMAP** $\arg\max_{\boldsymbol{\mu}\in\mathcal{M}} \boldsymbol{\mu}^\top \boldsymbol{\eta} - 1/2\|\boldsymbol{\mu}\|^2$





SparseMAP solution

$$\boldsymbol{\mu}^\star = \operatorname*{arg\,max}_{\boldsymbol{\mu}\in\mathcal{M}} \boldsymbol{\mu}^\top \boldsymbol{\eta} - 1/2\|\boldsymbol{\mu}\|^2$$

=  = .6  + .4 

($\boldsymbol{\mu}^\star$ is unique, but may have multiple decompositions $\boldsymbol{p}$. Active Set recovers a sparse one.)

Algorithms for SparseMAP

$$\boldsymbol{\mu}^\star = \arg\max_{\boldsymbol{\mu}\in\mathcal{M}} \boldsymbol{\mu}^\top\boldsymbol{\eta} - 1/2\|\boldsymbol{\mu}\|^2$$

Algorithms for SparseMAP

$$\boldsymbol{\mu}^\star = \underset{\boldsymbol{\mu}\in\mathcal{M}}{\arg\max}\, \boldsymbol{\mu}^\top \boldsymbol{\eta} - 1/2\|\boldsymbol{\mu}\|^2$$

linear constraints
(alas, exponentially many!)

quadratic objective

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Conditional Gradient

[Frank and Wolfe, 1956, Lacoste-Julien and Jaggi, 2015]

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$$\arg\max_{\boldsymbol{\mu}\in\mathcal{M}} \boldsymbol{\mu}^\top \underbrace{\left(\boldsymbol{\eta} - \boldsymbol{\mu}^{(t-1)}\right)}_{\tilde{\boldsymbol{\eta}}}$$

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    a.k.a. Min-Norm Point, [Wolfe, 1976]

    [Martins et al., 2015, Nocedal and Wright, 1999, Vinyes and Obozinski, 2017]

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Active Set achieves
finite & **linear** convergence!

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Backward pass

$\frac{\partial \boldsymbol{\mu}}{\partial \boldsymbol{\eta}}$ is sparse

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$\frac{\partial \boldsymbol{\mu}}{\partial \boldsymbol{\eta}}$ is sparse

computing $\left(\frac{\partial \boldsymbol{\mu}}{\partial \boldsymbol{\eta}}\right)^\top \boldsymbol{dy}$

takes $\mathcal{O}(\dim(\boldsymbol{\mu})\, \mathrm{nnz}(\boldsymbol{p}^\star))$

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Completely modular: just add MAP

pass

rse

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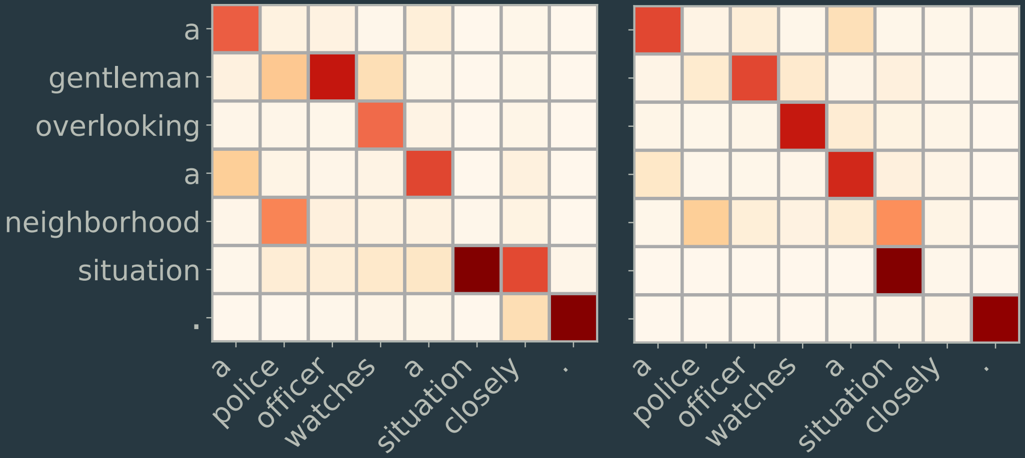
[Chen et al., 2017]



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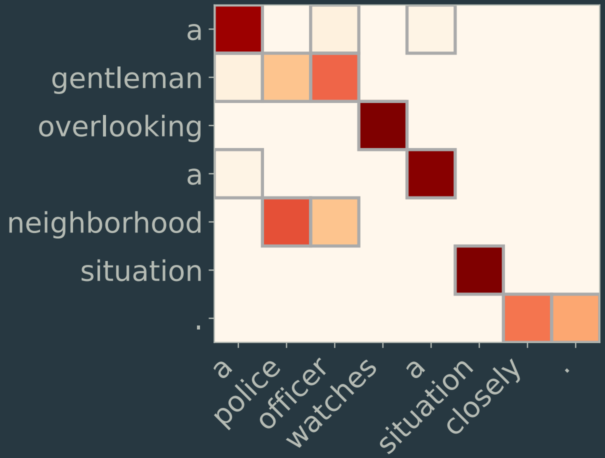

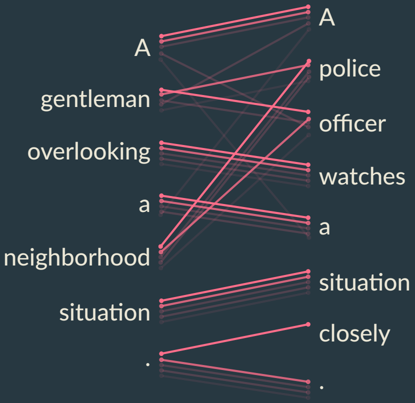

Overview

$$\mathbb{E}_{\pi_{\boldsymbol{\theta}}(\boldsymbol{z}|x)}\Big[L(\boldsymbol{z})\Big]$$

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$$L\Big(\arg\max_{\boldsymbol{z}} \pi_{\boldsymbol{\theta}}(\boldsymbol{z} \mid x)\Big)$$

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Structured latent variables without sampling

$$\mathbb{E}_{\boldsymbol{z}}\big[L(\boldsymbol{z})\big] = \sum_{\boldsymbol{z}\in\mathcal{Z}} L\big(\hat{y}\ \ (\boldsymbol{z})\big)\,\pi\ \ (\boldsymbol{z}\mid x)$$

Structured latent variables without sampling

$$\mathbb{E}_{\boldsymbol{z}}\left[L(\boldsymbol{z})\right] = \sum_{\boldsymbol{z}\in\mathcal{Z}} L\left(\hat{y}_{\boldsymbol{\phi}}(\boldsymbol{z})\right)\, \pi_{\boldsymbol{\theta}}(\boldsymbol{z} \mid x)$$

Structured latent variables without sampling

e.g., a TreeLSTM defined by $\boldsymbol{z}$

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parsing model,
using some scorer $f_{\boldsymbol{\theta}}(\boldsymbol{z}; x)$

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sum over
all possible trees

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Exponentially large sum!

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How to define $\pi_{\boldsymbol{\theta}}$?

idea 1

idea 2

idea 3

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How to define $\pi_{\boldsymbol{\theta}}$?

All methods we've seen require sampling; hard in general.

idea 2

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idea 3				

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STE / SPIGOT relax $\hat{y}$ in backward.

$$\frac{\partial \mathbb{E}\big[L(\boldsymbol{z})\big]}{\partial \boldsymbol{\theta}}$$

idea 1 $\pi_{\boldsymbol{\theta}}($ … 😊

idea 2 $\pi_{\boldsymbol{\theta}}(\boldsymbol{z}) = 1$ if $\boldsymbol{z} = \mathrm{MAP}(f_{\boldsymbol{\theta}}(\cdot))$ else 0 argmax 😊 😟

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Structured latent variables without sampling

 $= .7\times$  $+ .3\times$ 

recall our shorthand $L(\boldsymbol{z}) = L(\hat{y}_{\boldsymbol{\phi}}(\boldsymbol{z}), y)$

Structured latent variables without sampling

 $= .7\times$  $+ .3\times$  $+0\times$  $+ \ldots$

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Structured latent variables without sampling

 $= .7\times$  $+ .3\times$  $+0\times$  $+ \dots$

$\mathbb{E}[L(\boldsymbol{z})] = .7\times L($  $) + .3\times L($  $)$

recall our shorthand $L(\boldsymbol{z}) = L(\hat{y}_{\boldsymbol{\phi}}(\boldsymbol{z}), y)$

Stanford Sentiment (Accuracy)

Socher et al	
Bigram Naive Bayes	83.1
[Niculae et al., 2018b]	
TreeLSTM w/ CoreNLP	83.2
TreeLSTM w/ SparseMAP	84.7
[Corro and Titov, 2019b]	
GCN w/ CoreNLP	83.8
GCN w/ Perturb-and-MAP	84.6

Stanford Natural Language Inference (Accuracy)

[Kim et al., 2017]	
Simple Attention	86.2
Structured Attention	86.8
[Liu and Lapata, 2018]	
100D SAN -	86.8
Yogatama et al	
100D RL-SPINN	80.5
[Choi et al., 2018]	
100D ST Gumbel-Tree	82.6
300D -	85.6
600D -	86.0
[Corro and Titov, 2019b]	
Latent Tree + 1 GCN -	85.2
Latent Tree + 2 GCN -	86.2

V. Conclusions

Is it syntax?!

- Unlike e.g. unsupervised parsing, the structures we learn are guided by **a downstream objective** (typically discriminative).
- They don't typically resemble grammatical structure (yet) [Williams et al., 2018]
  (future work: more inductive biases and constraints?)

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- Unlike e.g. unsupervised parsing, the structures we learn are guided by **a downstream objective** (typically discriminative).
- They don't typically resemble grammatical structure (yet) [Williams et al., 2018]
  (future work: more inductive biases and constraints?)
- Common to compare latent structures with parser outputs.
  But is this always a meaningful comparison?

Syntax vs. Composition Order

CoreNLP parse, $p = 21.4\%$

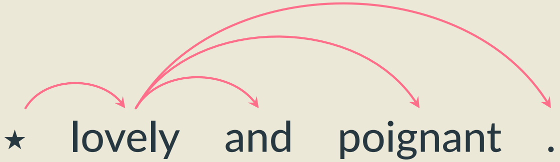
$\star$ lovely and poignant .

[Niculae et al., 2018b]

Syntax vs. Composition Order

$p = 22.6\%$

CoreNLP parse, $p = 21.4\%$



. . .

[Niculae et al., 2018b]

Syntax vs. Composition Order

$p = 22.6\%$

★ lovely and poignant .

CoreNLP parse, $p = 21.4\%$

★ lovely and poignant .

…

$p = 15.33\%$

★ a deep and meaningful film .

$p = 15.27\%$

★ a deep and meaningful film .

…

CoreNLP parse, $p = 0\%$

★ a deep and meaningful film .

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$$L\Big(\arg\max_{z} \pi_{\boldsymbol{\theta}}(\boldsymbol{z} \mid x)\Big)$$

- Straight Through[MAP,MRG]
- SPIGOT[MAP+]

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- Structured Attn. Nets[MRG]
- SparseMAP[MAP+]

Computation:

SPL: Sampling. (Simple in incremental/unstructured, hard for most global structures.)

MAP: Finding the highest-scoring structure.

MRG: Marginal inference.

Conclusions

- Latent structure models are desirable for interpretability, structural bias, and higher predictive power with fewer parameters.
- Stochastic latent variables can be dealt with RL or straight-through gradients.
- Deterministic argmax requires surrogate gradients (e.g. SPIGOT).
- Continuous relaxations of argmax include SANs and SparseMAP.
- Intuitively, some of these different methods are trying to do similar things or require the same building blocks (e.g. SPIGOT and SparseMAP).

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