Bond Risk Premia with Machine Learning

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Outline

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- We selected the nine most representative macro variables, **introduced two other dimensional explanatory variables**, conducted regression tests using the data, and found that these factors significantly affect the in-sample excess return fit.
- We then used **a machine learning algorithm** to obtain a model that could predict excess returns better by adjusting the parameters and constructed a simple strategy that, after back-testing, achieved positive excess returns.

Literature Review

- Fama and Bliss(1987)

$$r(x-1:t+1)-r(x-1:t)=-a_2+(1-b_2)[f(x,x-1:t)-r(1:t)]-u_2(t+1) \quad (1)$$

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$$rx_{t+1}^{(n)}=\beta_0^{(n)}+\beta_1^{(n)}y_t^{(1)}+\beta_2^{(n)}f_t^{(2)}+\ldots+\beta_5^{(n)}f_t^{(5)}+\varepsilon_{t+1}^{(n)} \quad (2)$$

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- Ludvigson and Ng (2009)

$$rx_{t+1}^{(n)}=\beta_0+\beta_1^{\prime}\widehat{F}_t+\beta_2 CP_t+\varepsilon_{t+1} \quad (3)$$

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- Robin Greenwood and Dimitri Va (2014) proved that the supply and maturity structure of government debt are positively correlated with bond yields and expected returns.
- Additionally, holding the short rate constant,these effects should be stronger for longer-maturity bonds and during times when arbitrageurs are more risk-averse.

Model Theory

- Our theory mainly builds on the model of Ludvigson and Ng (2009) and Cochrane and Piazzesi (2005). Variables on the supply of government debt and investor sentiment to optimize were added according to our empirical principles of economics to improve the classical model's predictive ability.

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- We believe that **the linear combination of forward rates can predict the future inflation rate well.**

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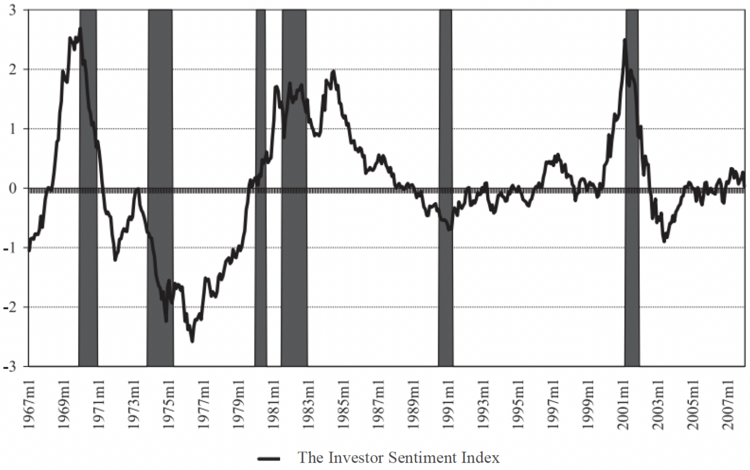
- In addition, we choose the variable of exchange rate(Ex rate) to reflect the level of interest rates and CPI as the inflation factor.
- We also consider the role of sentiment factor and the bond supply with statistical power to explain variation on the bond risk premium.

Model Theory



Definitions of Variables

Table 1: Definitions of Main Variables

PI	Personal income
Hw	Index of help-wanted advertising
Cps total	Civilian labor force: total
Emp	Employees on nonfarm pay rolls
Avg hrs	Average working hours
Starts	Housing starts
Inst	Ratio: Installment credit/PI
Ex rate	Exchange rate
CPI	CPI
Sent	Investors sentiment
Mwd	the maturity-weighted-debt-to-GDP ratio
CP	Cochrane and Piazzesi(2005) factor

Basic Data Parameters

Table 2: Summary Statistics

Variables	(1) Mean	(2) Min	(3) Max
PI	66671.409	2562.5	11595.7
Hw	79.963	63.82	91.64
Cps total	77.302	40	115.13
Emp	657.978	498	1177.4
Avg hrs	40.158	37.2	41.3
Starts	1485.403	478	2494
Inst	2.243	-41	62.3
Ex rate	96.125	76.01	132.56
CPI	63.813	18	97.6
Sent	-0.002	-2.19	3.21
Mwd	2.275	0.671	4.28
CP	0.789	-8.08	1.78

Empirical Results

Table 3: $rx^{(n)} = \beta_0 + \beta_1 X + \beta_2 CP + \varepsilon$

	Panel A $rx^{(2)}$	Panel B $rx^{(3)}$	Panel C $rx^{(4)}$	Panel D $rx^{(5)}$
PI	-0.001***	-0.001***	-0.002***	-0.002***
	(-7.63)	(-7.53)	(-7.79)	(-7.63)
Hw	-0.207***	-0.294***	-0.390***	-0.479***
	(-6.69)	(-4.96)	(-4.61)	(-4.46)
Cpstotal	-0.01	-0.030***	-0.038**	-0.042*
	(-1.60)	(-2.38)	(-2.12)	(-1.85)
Emp	-0.003***	-0.006***	-0.008***	-0.009***
	(-5.55)	(-5.83)	(-5.22)	(-4.82)
Avghrs	-0.479**	-1.419***	-2.210***	-2.637***
	(-2.44)	(-3.77)	(-4.11)	(-3.86)
Starts	0.002***	0.003***	0.006***	0.007***
	(-10.17)	(-10.99)	(-11.42)	(-11.39)
Inst	-0.045***	-0.080***	-0.105***	-0.116**
	(-3.13)	(-2.92)	(-2.69)	(-2.34)
Exrate	0.059***	0.107***	0.151***	0.186***
	(-6.39)	(-6.03)	(-5.98)	(-5.8)
PPI	-0.374	1.104	2.283	2.56
	(-0.39)	(-0.6)	(-0.87)	(-0.77)
CPI	-0.029***	-0.058***	-0.085***	-0.108***
	(-5.49)	(-5.77)	(-5.87)	(-5.91)
Sent	-0.391***	-0.723***	-1.090***	-1.338***
	(-6.85)	(-6.61)	(-6.99)	(-6.76)
Mwd	1.090***	2.121***	3.239***	3.981***
	(-11.92)	(-12.1)	(-12.96)	(-12.54)
CP	-0.007	-0.134**	-0.355***	-0.535***
	(-0.24)	(-2.42)	(-4.49)	(-5.33)
Observations	461	461	461	461
R^2-ajusted	0.61	0.57	0.53	0.5

Note: The table reports estimates from OLS regressions of excess bonds returns on the lagged variables named in table 1. The t-statistics are reported in the parenthesis. *, **, ***represented the coefficients are statistically significant at the 10%, 5% and 1% separately. A constant is always included in the regression even though its estimate is not reported in the table.

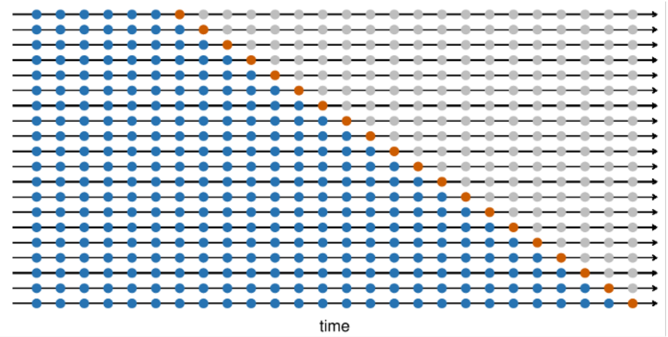
Model Optimization

Theorem

The algorithm is to initialize multiple weak learners and iteratively train them to finally obtain the final learner, thus solving the overfitting problem generated by a single learner fitting too strongly to the sample.

Model Optimization

- The figure below illustrates a series of training and testing sets, where **the blue observations constitute the training set and the orange observations constitute the testing set.**



Robustness Check

- In the multiple linear regression model, the in-sample determinable coefficients are still good, but the out-of-sample determinable coefficients perform poorly.

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- It can also be found that the out-of-sample R-squared is even negative for two-year and three-year bonds, while the out-of-sample prediction improves as maturity rises, which we explain by the fact that the noise cancels each other out as the time period increases.

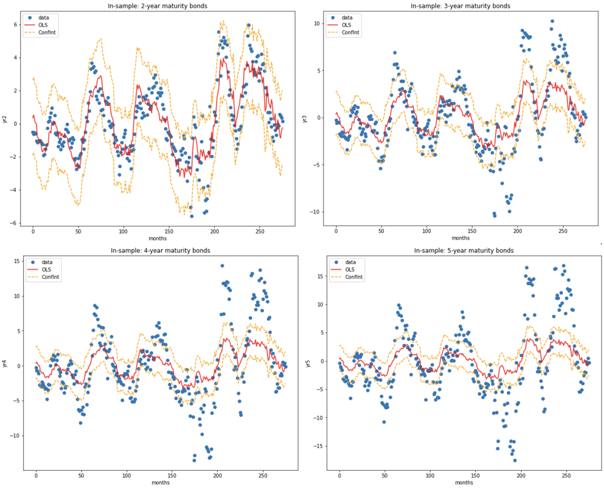
Robustness Check

Table 4: OLS: Result of Robustness Check

	Panel A $rx^{(2)}$	Panel B $rx^{(3)}$	Panel C $rx^{(4)}$	Panel D $rx^{(5)}$
$In-Sample-R^2$	0.746	0.746	0.746	0.746
$Out-Sample-R^2$	-0.5070	-0.328	0.135	0.180

Note: Out-of-Sample $R^2 = 1 - \frac{\sum_{j=1}^{T}(y_j - \hat{y}_j)^2}{\sum_{j=1}^{T}(y_j - \bar{y}_j)^2}$

Robustness Check



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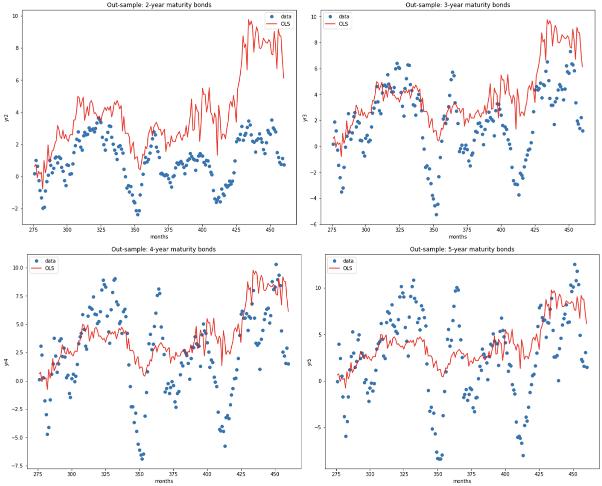

Robustness Check

- As for machine learning, after cross-validation to adjust the hyperparameters and bringing in out-of-sample data regression, the model achieves good explanatory power in the training group, cross-test group and out-of-sample prediction group.

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- As for machine learning, after cross-validation to adjust the hyperparameters and bringing in out-of-sample data regression, the model achieves good explanatory power in the training group, cross-test group and out-of-sample prediction group.
- Meanwhile, a simple strategy is constructed, i.e., buying bonds when the excess return is predicted to be positive, otherwise no operation is performed, and the yield is simulated in the out-of-sample data, and positive returns are achieved in all four groups of bonds with different maturities.

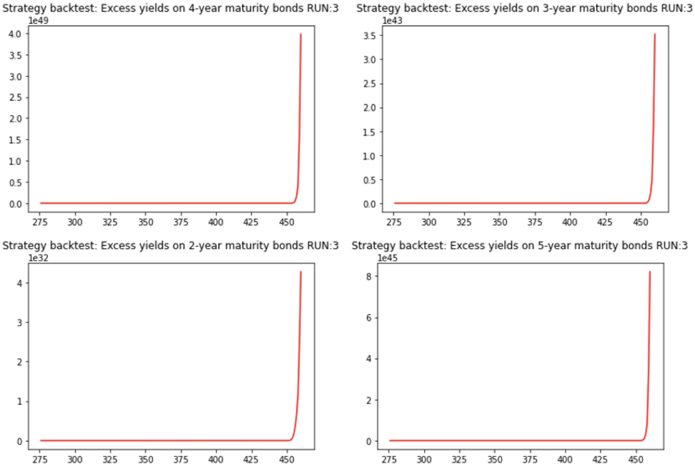
Robustness Check

Table 5: XGBoost: Result of Robustness Check

	Panel A $rx^{(2)}$	Panel B $rx^{(3)}$	Panel C $rx^{(4)}$	Panel D $rx^{(5)}$
TrainingSet, accuracy	0.97	0.97	0.96	0.97
TrainingSet, AUC	1.00	1.00	0.99	0.99
CvSet, accuracy	0.44	0.36	0.36	0.40
CvSet, AUC	0.74	0.74	0.77	0.84
OutSample, accuracy	0.58	0.59	0.60	0.58
OutSample, AUC	0.85	0.82	0.80	0.78

Note:
1 Accuracy, calculated using the '$accuracy_score$' function in sklearn, is the accuracy of the comparison between the predicted label and the real label;
2 AUC (Area under the Curve of ROC), calculates the area of the curve ROC, $0.5 < AUC < 1$, which is better than random guessing, that is, this classifier (model) can have predictive value if the threshold is properly set.

Robustness Check



Conclusion

- Our main contribution is the comprehensive analysis through the addition of multiple factors, that is above and beyond the information contained in the term structure of bonds and macroeconomic factors.

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- Our main contribution is the comprehensive analysis through the addition of multiple factors, that is above and beyond the information contained in the term structure of bonds and macroeconomic factors.
- Also, we back-tested our model with a simple trading strategy and confirmed that it is possible to learn these variables using the XGBoost model to achieve positive returns.

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