HUB IQ

Inteligencia Artificial & Machine Learning

Applicaciones en movilidad

Dr. Iván S. Razo Zapata

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Reducción de Dimensionalidad

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Problemas con la alta dimensionalidad de los datos

- En algunos dominios donde aplicamos algoritmos de aprendizaje de máquina nos enfrentamos a conjuntos de datos de alta dimensionalidad
  - Ejemplo: observaciones con decenas variables
- Costo de procesamiento y almacenamiento
- Atributos relevantes e irrelevantes
- Dificultad para evaluar distancias

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Técnicas comunes para reducción de dimensionalidad

- Selección de atributos
  - Seleccionar un subconjunto de atributos de los atributos originales
- Generación de atributos
  - Mapear los atributos originales a un nuevo espacio de menor dimensión

Selección de atributos

- Seleccionar un **subconjunto** de atributos
  - Que **no afecte significativamente** a la tarea de regresión/clasificación
- Un atributo es **irrelevante** si no afecta a la tarea
- Un atributo es **redundante** si no añade nada nuevo
- Un atributo se considera **relevante** si no es irrelevante o redundante

Selección de atributos

- Regularización
  - Ridge
  - Lasso

Regularización – Idea general

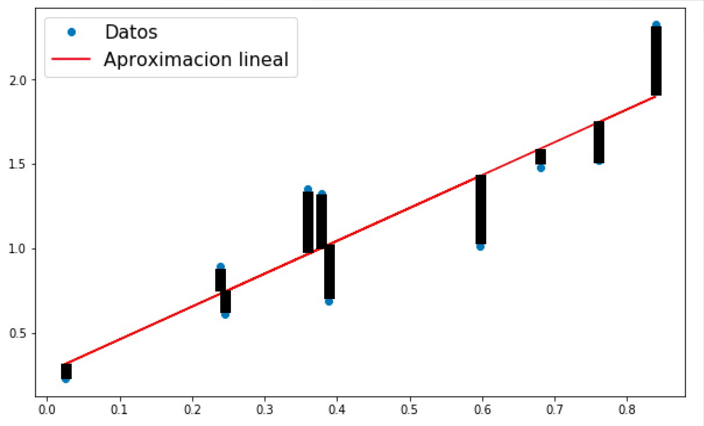
$$y = \theta_0 + \theta_1 x + \theta_2 x^2$$

$$E(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(\hat{y}_\theta(x) - y)^2$$

$$\theta_0 = \theta_0 - \alpha\frac{1}{m}\sum_{i=1}^{m}(\theta_0 + \theta_1 x + \theta_2 x^2 - y)$$

$$\theta_1 = \theta_1 - \alpha\frac{1}{m}\sum_{i=1}^{m}(\theta_0 + \theta_1 x + \theta_2 x^2 - y)x$$

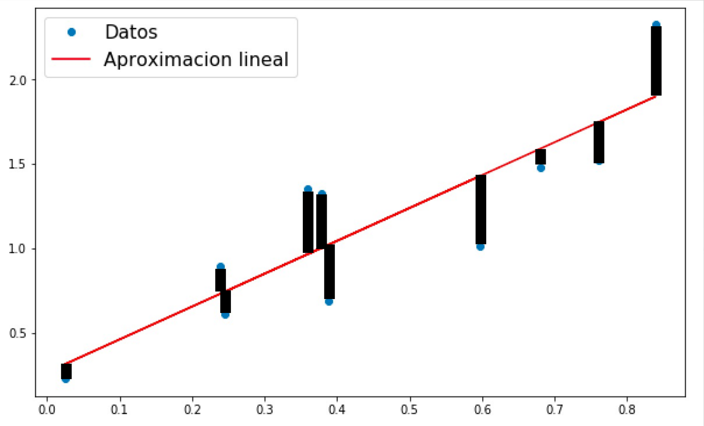
$$\theta_2 = \theta_2 - \alpha\frac{1}{m}\sum_{i=1}^{m}(\theta_0 + \theta_1 x + \theta_2 x^2 - y)x^2$$



Regularización – Idea general

$$y = \theta_0 + \theta_1 x + \theta_2 x^2$$

$$E(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(\hat{y}_\theta(x) - y)^2$$



$$\theta_j \leftarrow \theta_j - \alpha \times \sum_{i=1}^{m}(\hat{y}_\theta(x) - y)x$$

Regularización – Ridge

$$y = \theta_0 + \theta_1 x + \theta_2 x^2$$

$$E(\theta) = \frac{1}{2m}\left[\sum_{i=1}^{m}(\hat{y}_\theta(x) - y)^2 + \gamma \sum_{i=1}^{m} \theta^2\right]$$

Penalización L2

$$\theta_j \leftarrow \theta_j - \alpha \times \frac{E(\theta)}{\partial \theta_j}$$

Ridge – para regresión lineal

$$y = \theta_0 + \theta_1 x + \theta_2 x^2$$

$$E(\theta) = \frac{1}{2m}\left[\sum_{i=1}^{m}(\hat{y}_\theta(x) - y)^2 + \gamma \sum_{i=1}^{m} \theta^2\right]$$

$$\theta_1 \leftarrow \theta_1 - \alpha \times \frac{E(\theta)}{\partial \theta_1}$$

$$\frac{E(\theta)}{\partial \theta_1} = \frac{\frac{1}{2m}\left[\sum_{i=1}^{m}(\hat{y}_\theta(x) - y)^2 + \gamma \sum_{i=1}^{m} \theta^2\right]}{\partial \theta_1} = \frac{\partial}{\partial \theta_1}\frac{1}{2m}\sum_{i=1}^{m}(\hat{y}_\theta(x) - y)^2 + \frac{\partial}{\partial \theta_1}\frac{1}{2m}\gamma \sum_{i=1}^{m} \theta^2$$

$$\frac{E(\theta)}{\partial \theta_1} = \frac{2}{2m}\sum_{i=1}^{m}(\hat{y}_\theta(x) - y)\frac{\partial}{\partial \theta_1}(\hat{y}_\theta(x) - y) + \frac{2}{2m}\gamma\theta_1$$

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Ridge – para regresión lineal

$$y = \theta_0 + \theta_1 x + \theta_2 x^2$$

$$E(\theta) = \frac{1}{2m}\left[\sum_{i=1}^{m}(\hat{y}_\theta(x) - y)^2 + \gamma\sum_{i=1}^{m}\theta^2\right]$$

$$\theta_1 \leftarrow \theta_1 - \alpha \times \frac{E(\theta)}{\partial\theta_1}$$

$$\frac{E(\theta)}{\partial\theta_1} = \frac{2}{2m}\sum_{i=1}^{m}(\hat{y}_\theta(x) - y)\frac{\partial}{\partial\theta_1}(\hat{y}_\theta(x) - y) + \frac{\gamma}{m}\theta_1$$

$$\frac{E(\theta)}{\partial\theta_1} = \frac{1}{m}\sum_{i=1}^{m}(\hat{y}_\theta(x) - y)\frac{\partial}{\partial\theta_1}(\theta_0 + \theta_1 x + \theta_2 x^2 - y) + \frac{\gamma}{m}\theta_1$$

$$\frac{E(\theta)}{\partial\theta_1} = \frac{1}{m}\sum_{i=1}^{m}(\hat{y}_\theta(x) - y)x + \frac{\gamma}{m}\theta_1$$

Ridge – para regresión lineal

$$y = \theta_0 + \theta_1 x + \theta_2 x^2$$

$$E(\theta) = \frac{1}{2m}\left[\sum_{i=1}^{m}(\hat{y}_\theta(x) - y)^2 + \gamma \sum_{i=1}^{m}\theta^2\right]$$

$$\theta_1 \leftarrow \theta_1 - \alpha \times \frac{E(\theta)}{\partial\theta_1}$$

$$\frac{E(\theta)}{\partial\theta_1} = \frac{1}{m}\sum_{i=1}^{m}(\hat{y}_\theta(x) - y)x + \frac{\gamma}{m}\theta_1$$

$$\theta_1 \leftarrow \theta_1 - \alpha \times \left(\frac{1}{m}\sum_{i=1}^{m}(\hat{y}_\theta(x) - y)x + \frac{\gamma}{m}\theta_1\right)$$

$$\theta_1 \leftarrow \theta_1 - \alpha \times \frac{1}{m}\sum_{i=1}^{m}(\hat{y}_\theta(x) - y)x + \frac{\alpha\gamma}{m}\theta_1$$

$$\theta_1 \leftarrow \theta_1\left(1 - \frac{\alpha\gamma}{m}\right) - \alpha \times \frac{1}{m}\sum_{i=1}^{m}(\hat{y}_\theta(x) - y)x$$

Ridge – para regresión lineal

$$E(\theta) = \frac{1}{2m}\left[\sum_{i=1}^{m}(\hat{y}_\theta(x) - y)^2 + \gamma\sum_{i=1}^{m}\theta^2\right]$$

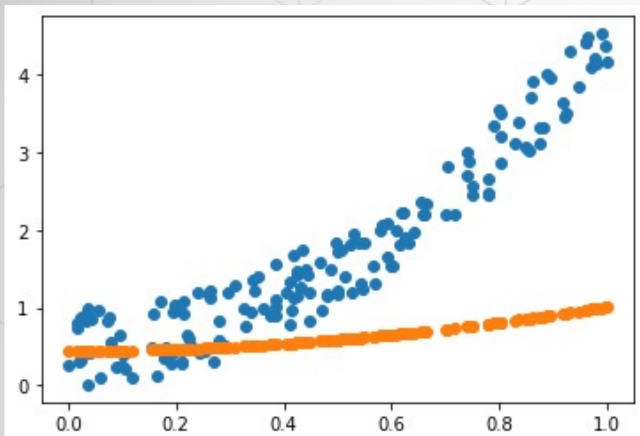
$$\theta_0 \leftarrow \theta_0 - \alpha\times\frac{1}{m}\sum_{i=1}^{m}(\hat{y}_\theta(x) - y)x$$

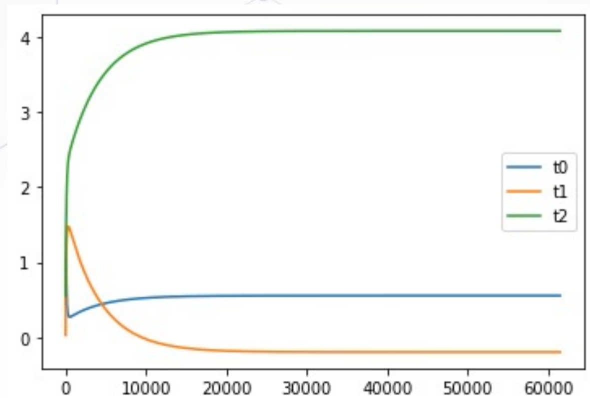
$$\theta_j \leftarrow \theta_j\left(1 - \frac{\alpha\gamma}{m}\right) - \alpha\times\frac{1}{m}\sum_{i=1}^{m}(\hat{y}_\theta(x) - y)x$$

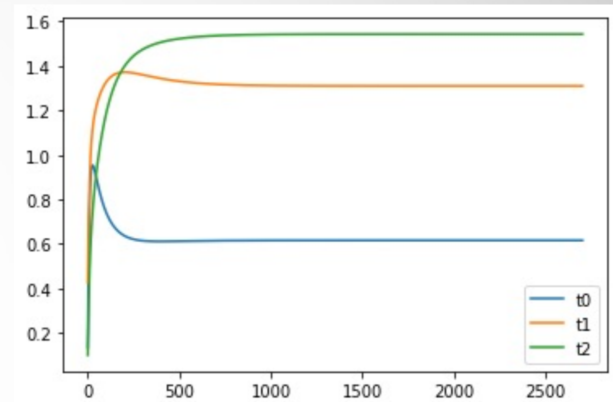
$$j \geq 1$$

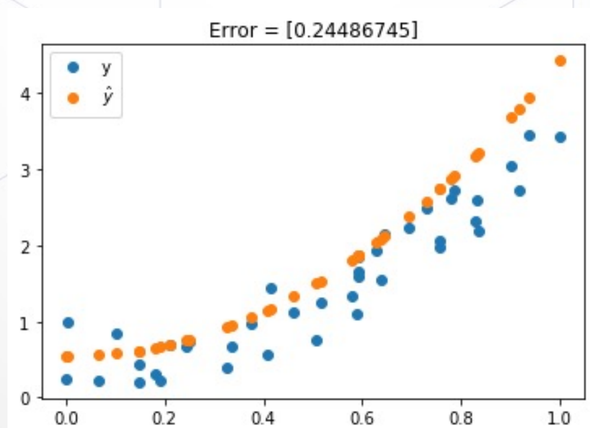
Ridge – para regresión lineal

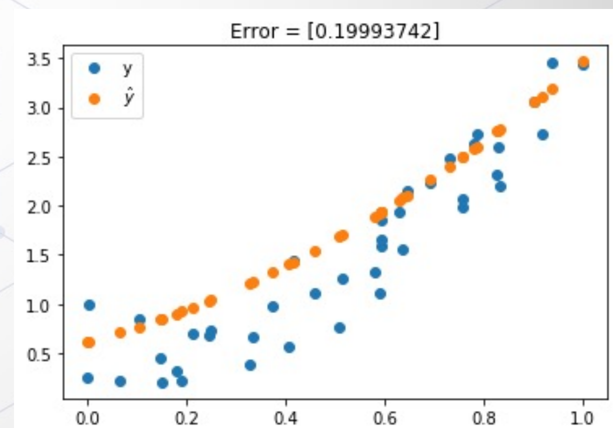
$$\gamma = 10$$





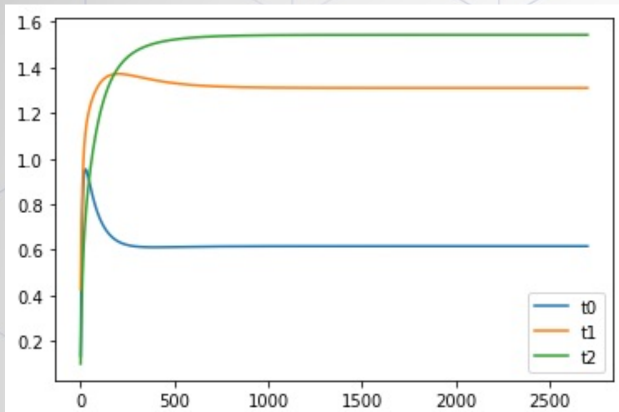




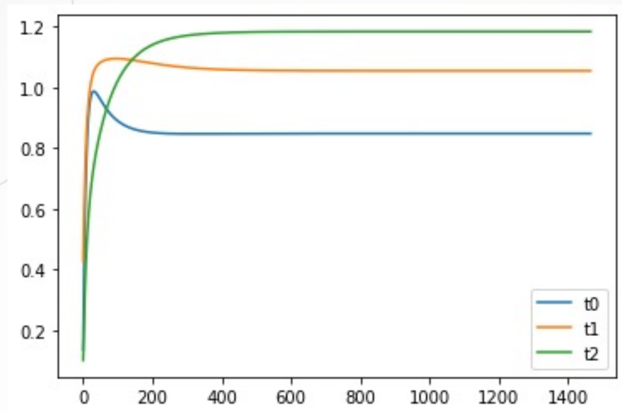


Ridge – para regresión lineal

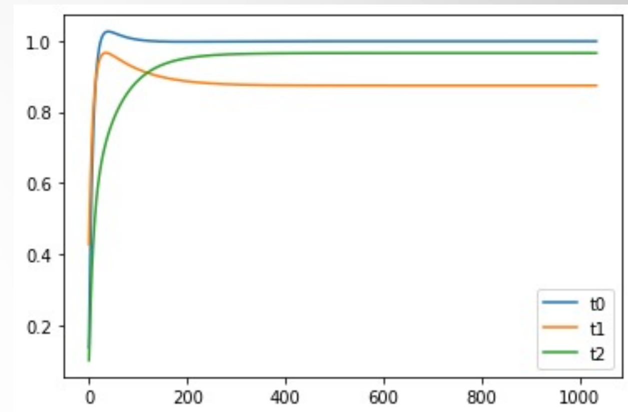
$\gamma = 10$

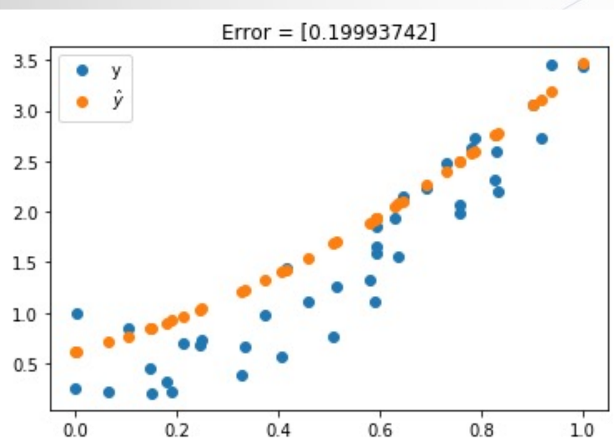


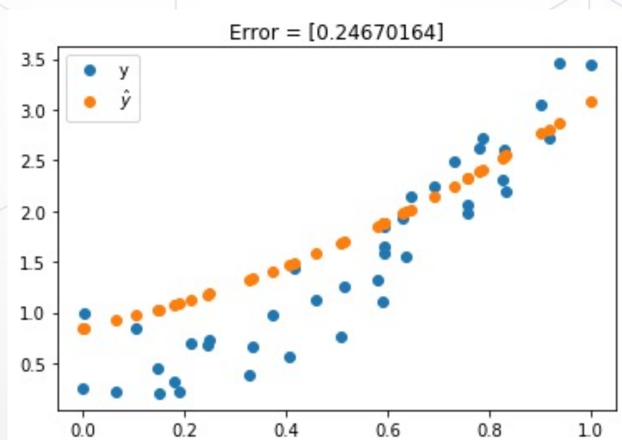
$\gamma = 20$

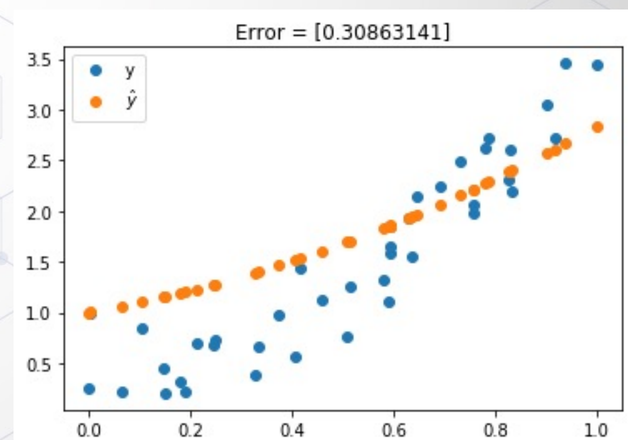


$\gamma = 30$

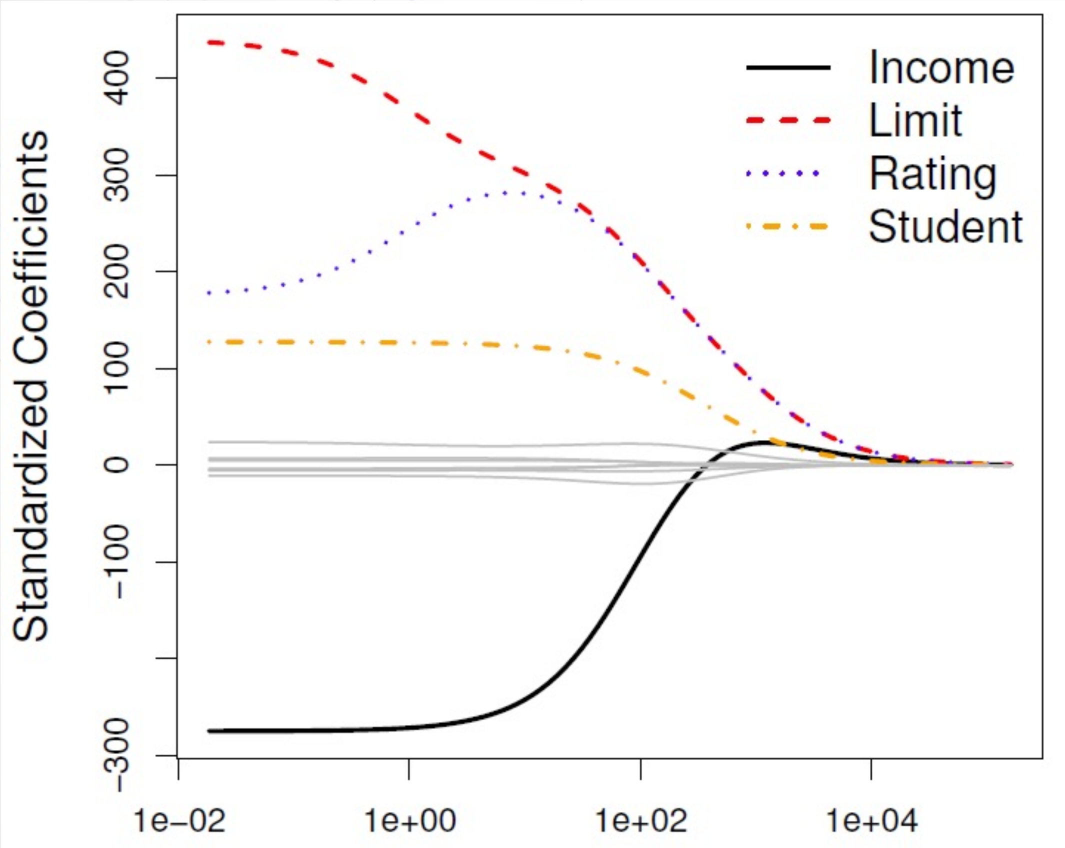








Ridge – para regresión lineal



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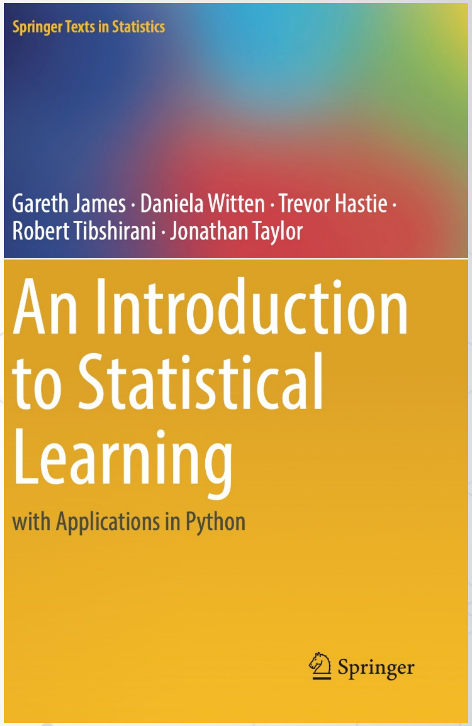
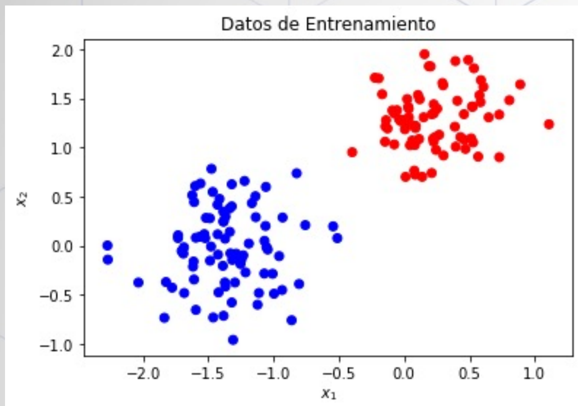
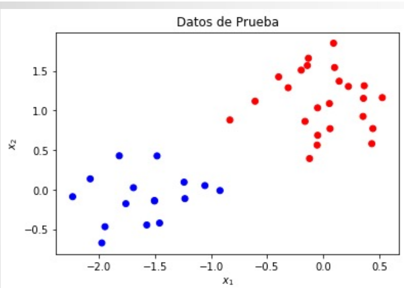


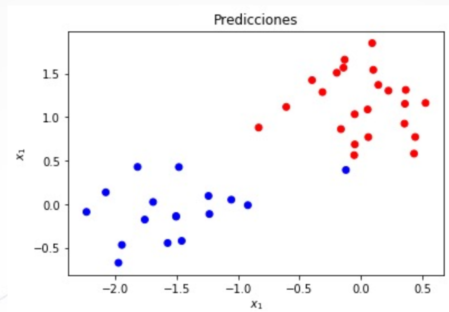
https://www.statlearning.com/

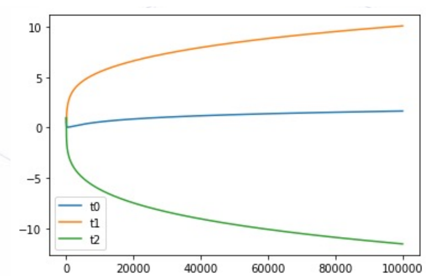
Ridge - para regresión logística

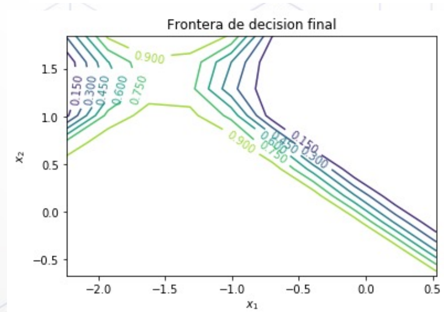
$\gamma = 10$

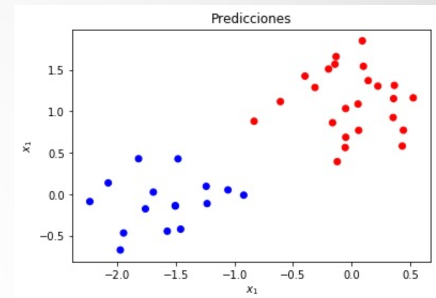


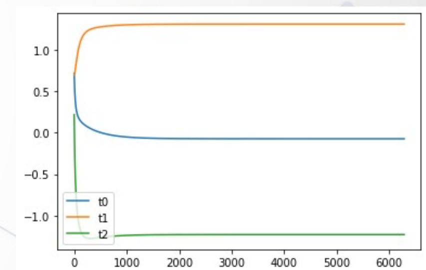


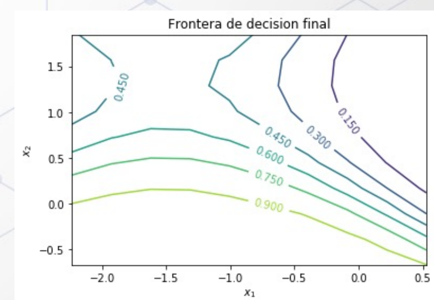






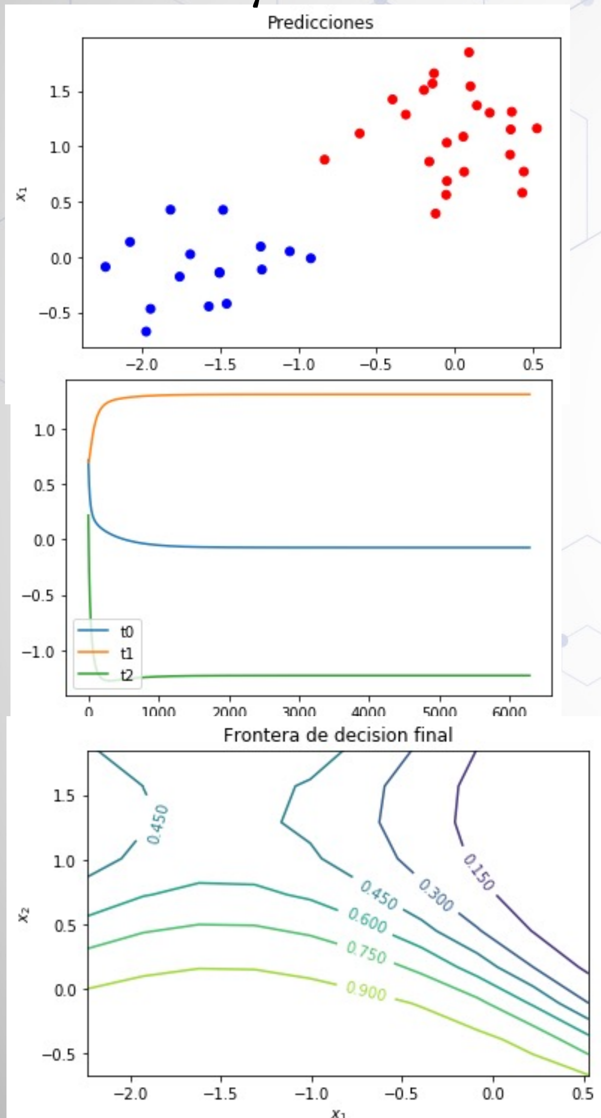




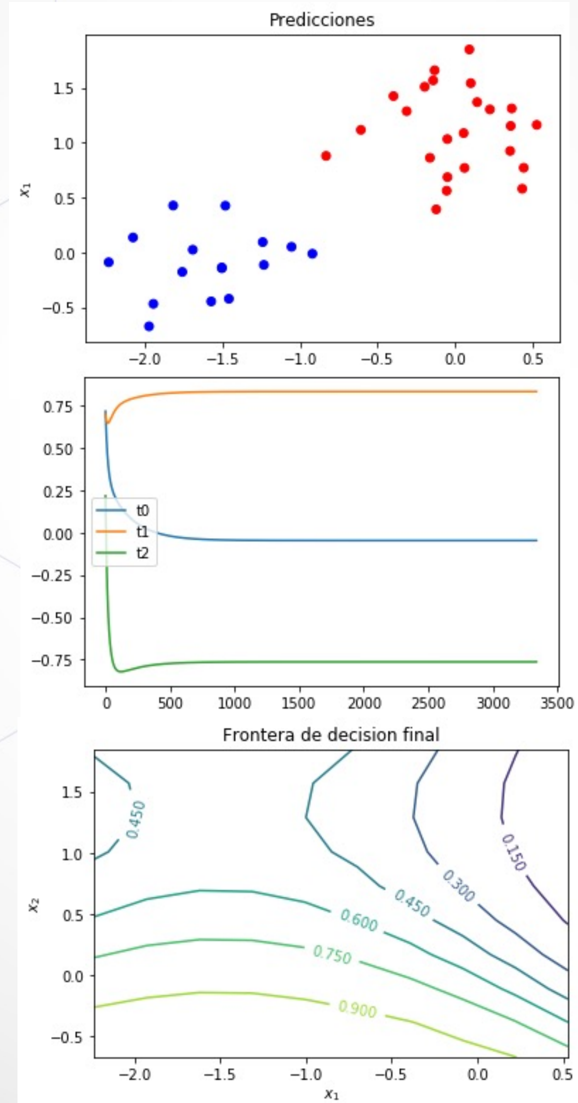


Ridge - para regresión logística

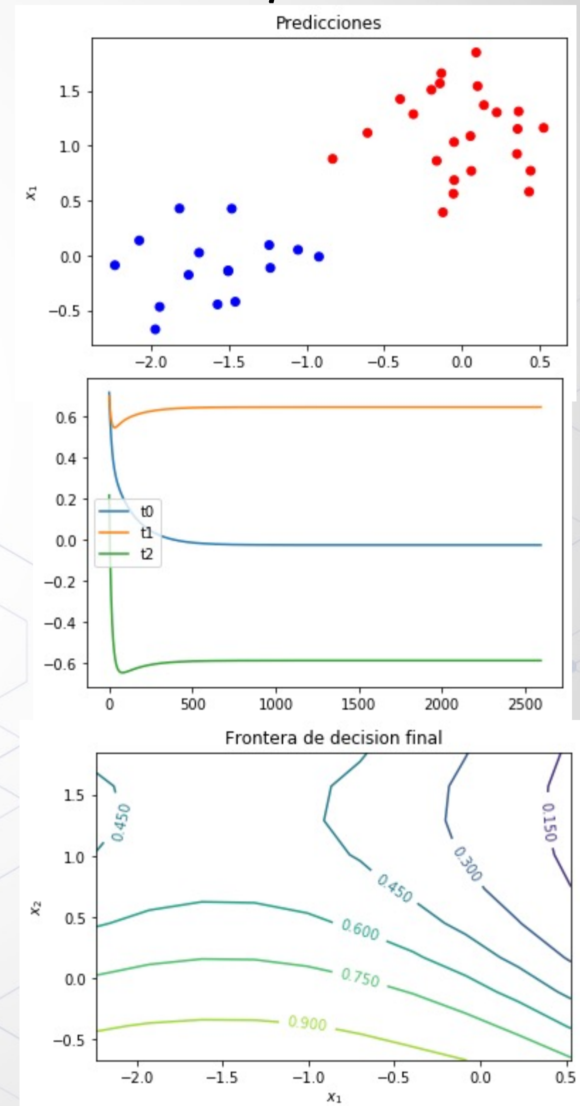
$\gamma = 10$



$\gamma = 30$



$\gamma = 50$



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Regularización – Lasso

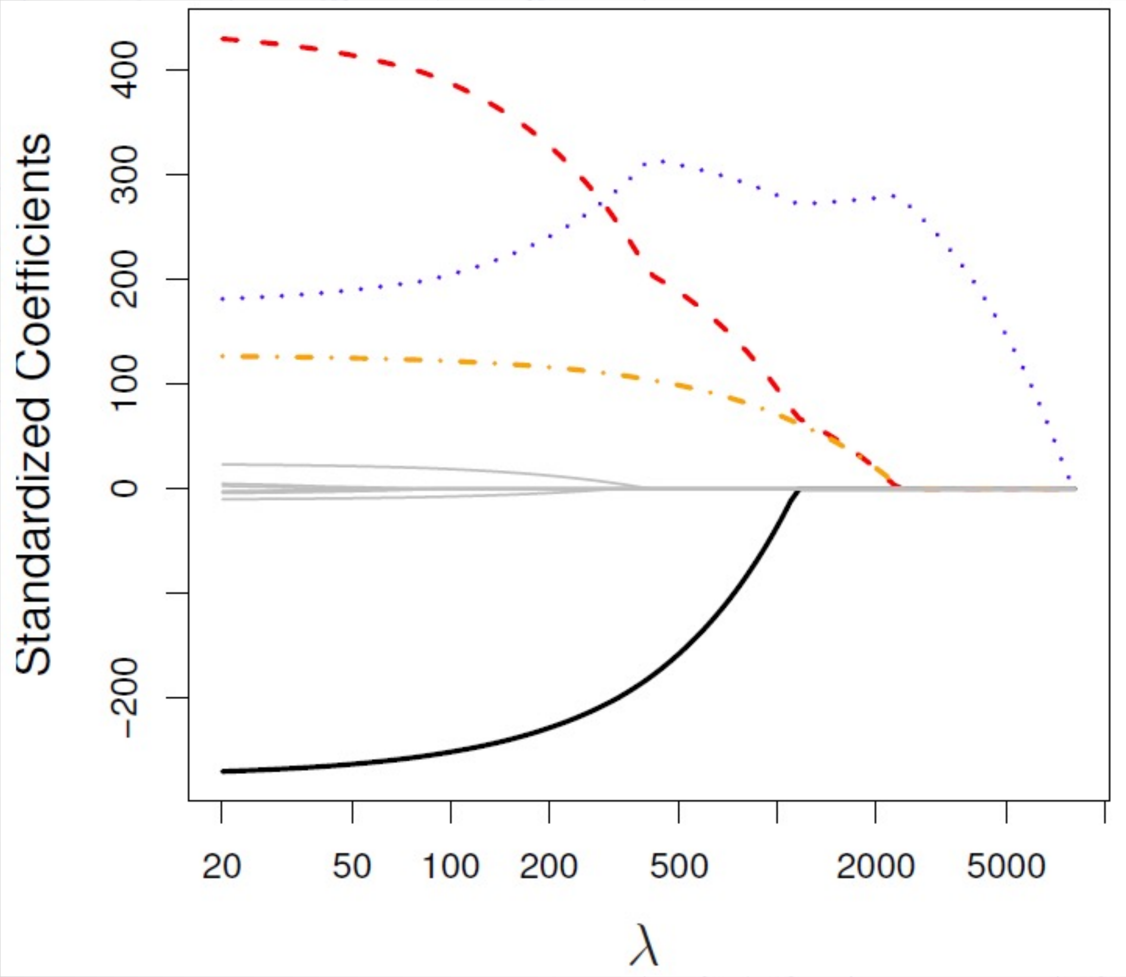
$$y = \theta_0 + \theta_1 x + \theta_2 x^2$$

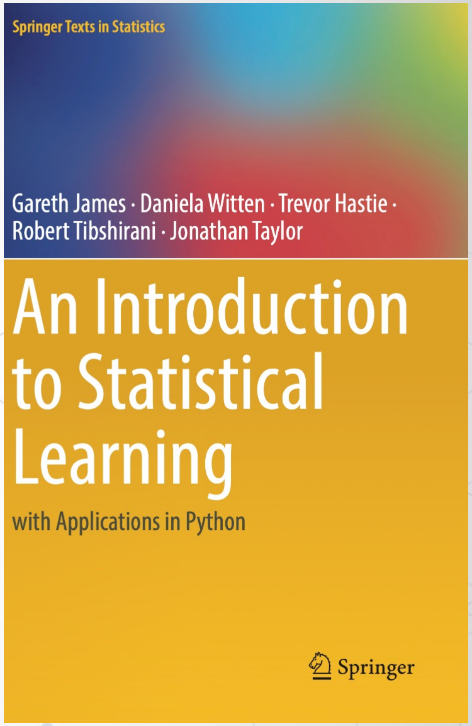
$$E(\theta) = \frac{1}{2m}\left[\sum_{i=1}^{m}(\hat{y}_\theta(x) - y)^2 + \gamma\sum_{i=1}^{m}|\theta|\right]$$

Penalización L1

$$\theta_j \leftarrow \theta_j - \alpha \times \frac{E(\theta)}{\partial\theta_j}$$

Regularización – Lasso





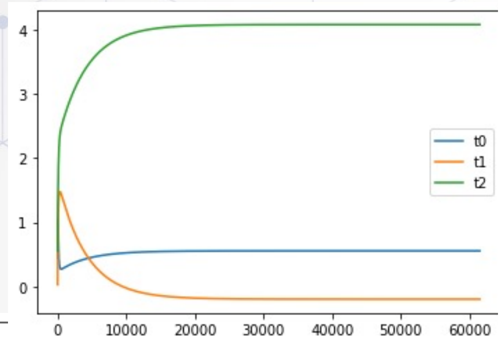
https://www.statlearning.com/

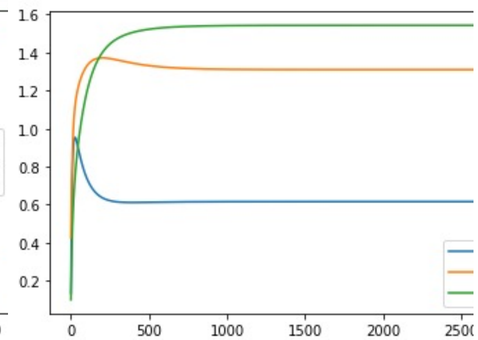
Lasso – para regresión lineal

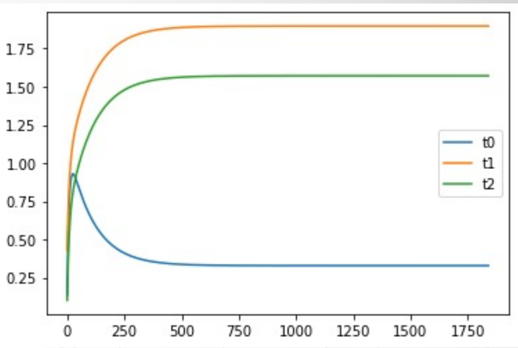
$\gamma = 10$

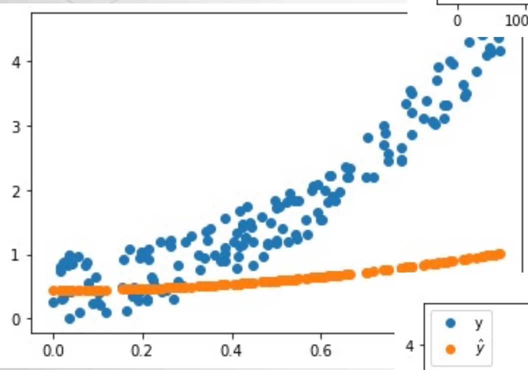
Ridge

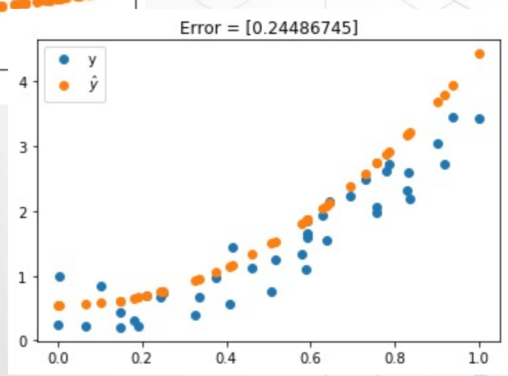
Lasso

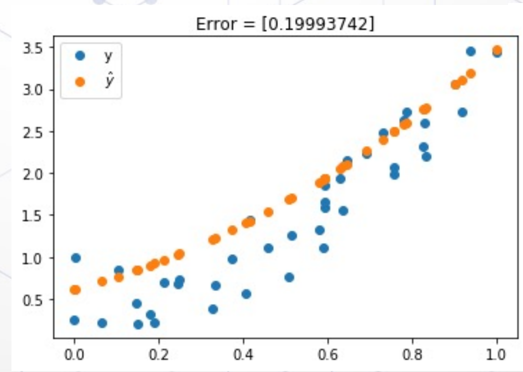


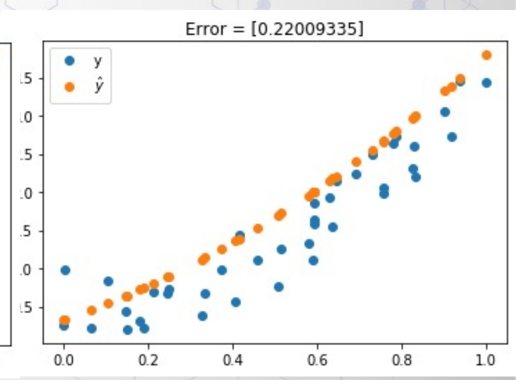






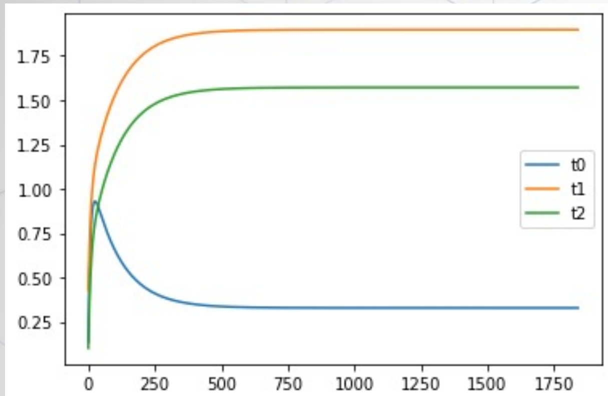


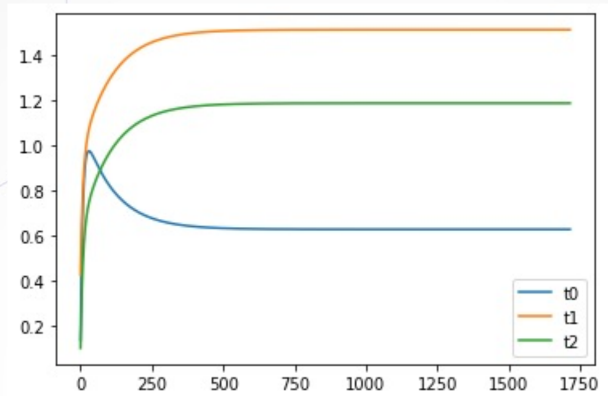


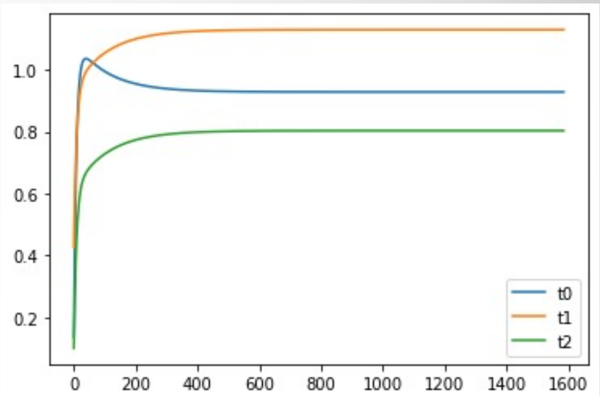


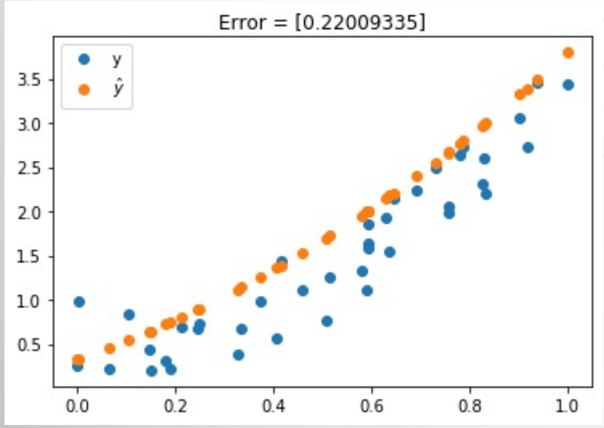
Lasso – para regresión lineal

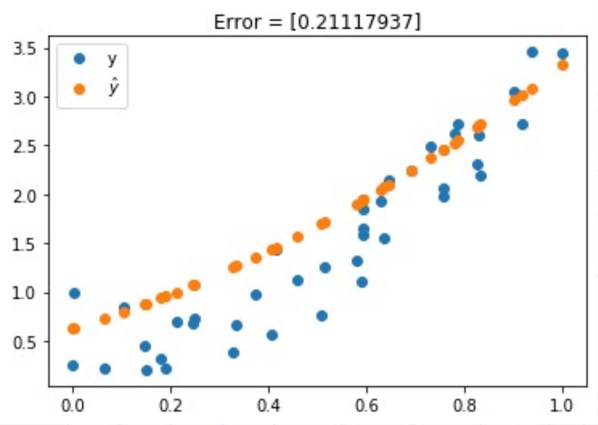
$\gamma = 10$ | $\gamma = 30$ | $\gamma = 50$

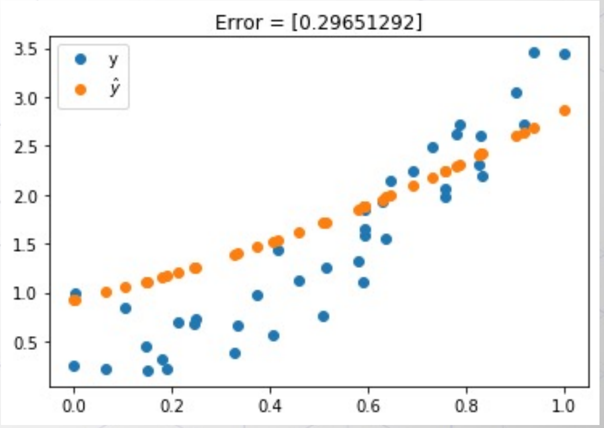






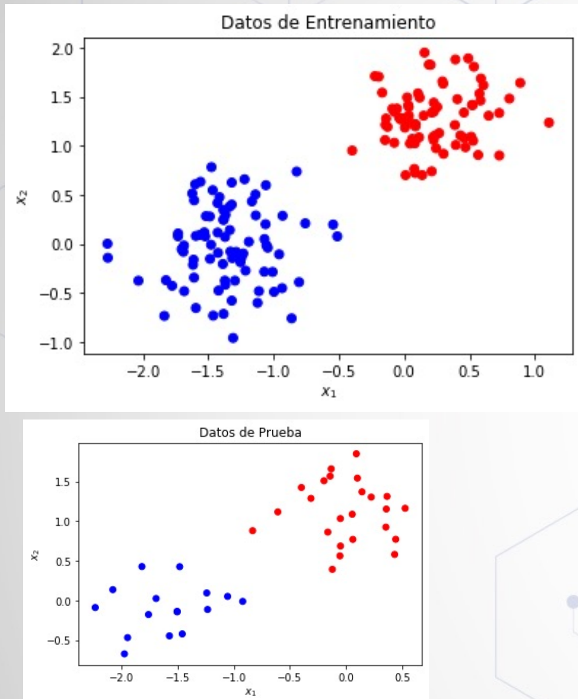


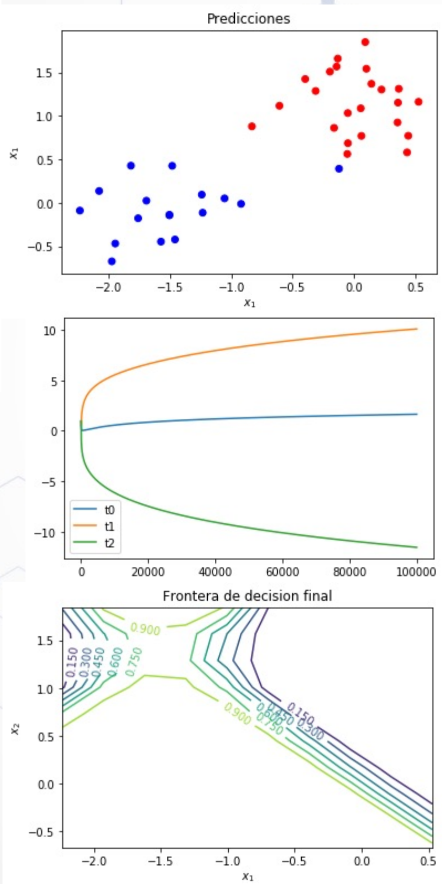




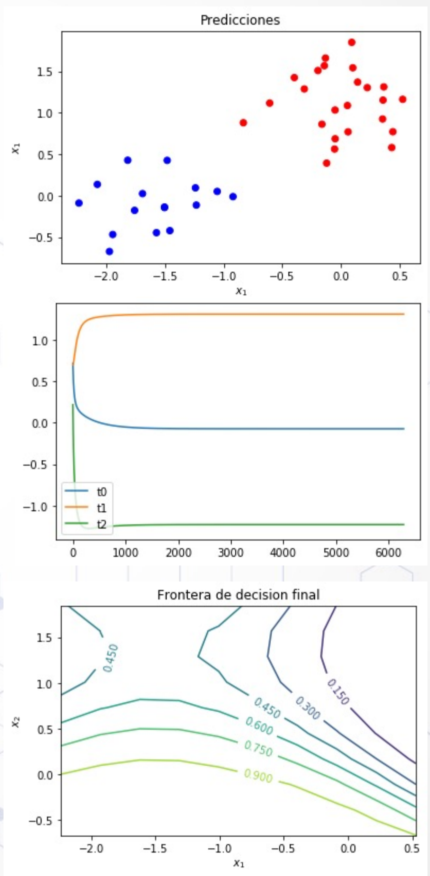
Lasso - para regresión logística

$\gamma = 10$

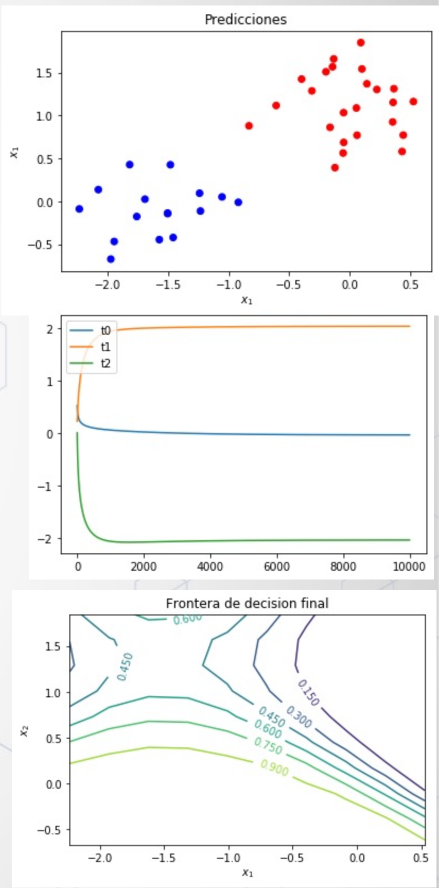




Ridge

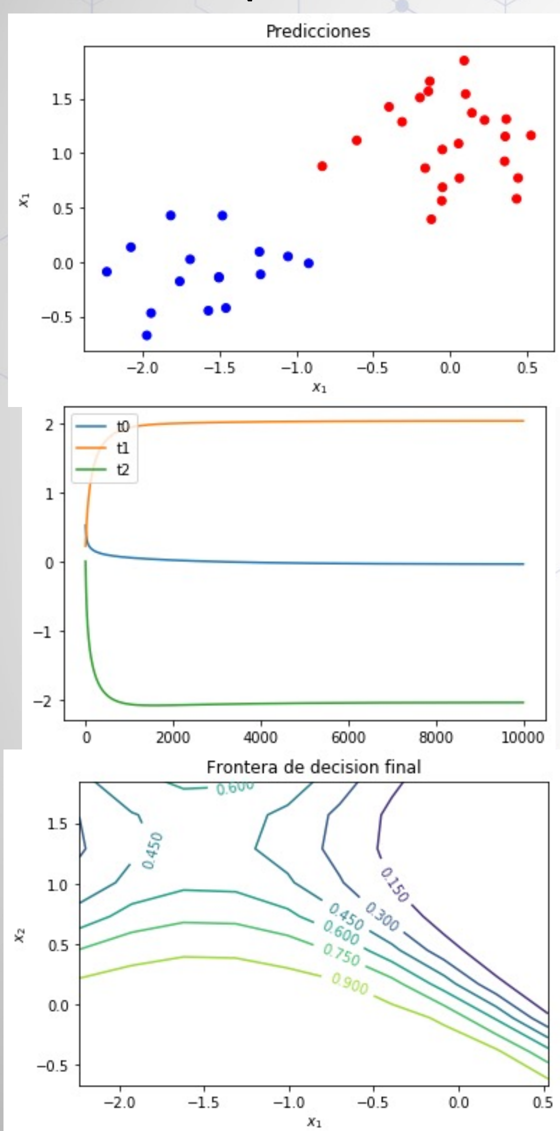


Lasso

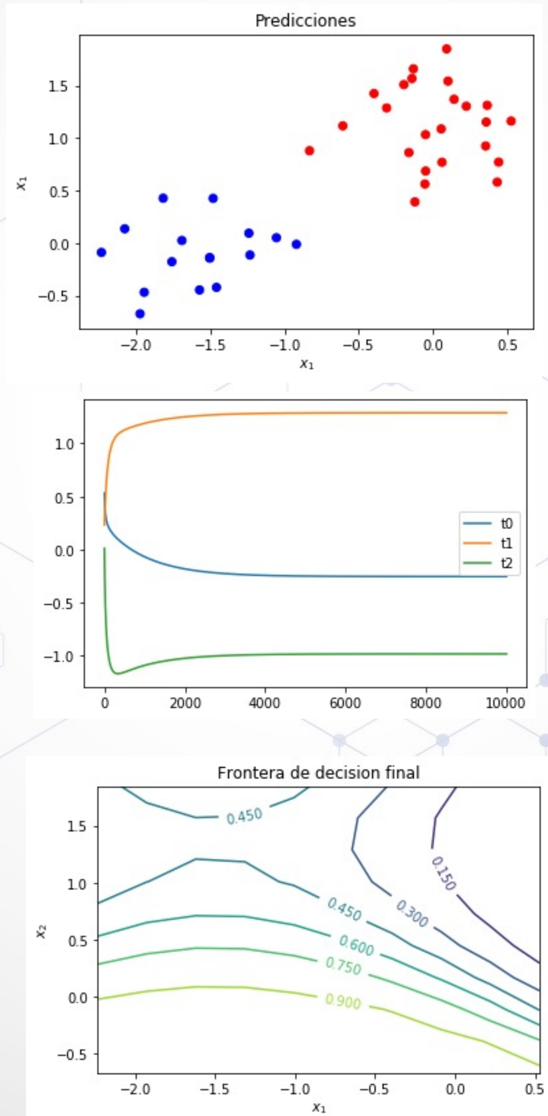


Lasso - para regresión logística

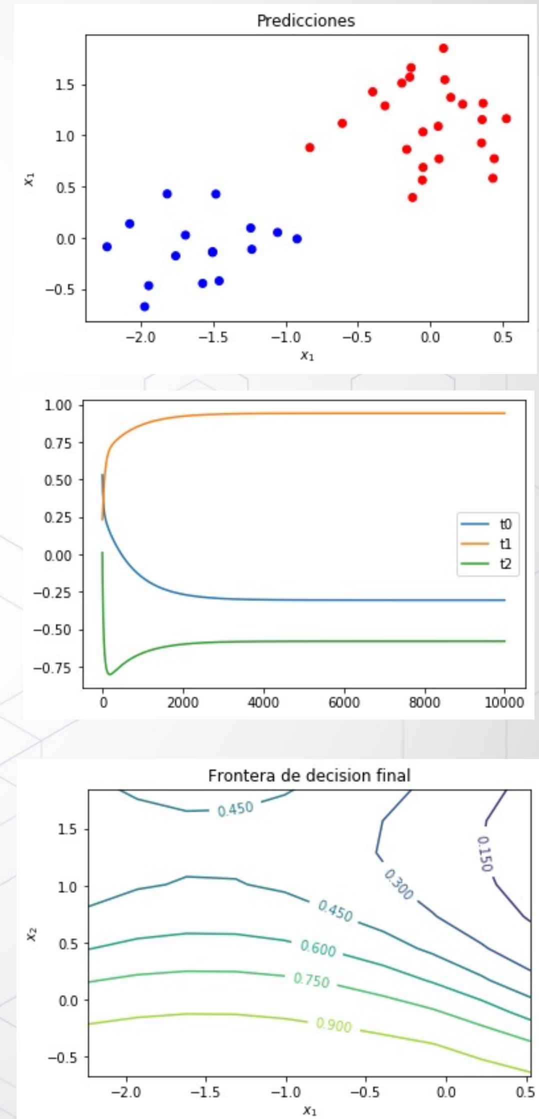
$\gamma = 10$



$\gamma = 30$



$\gamma = 50$



Código

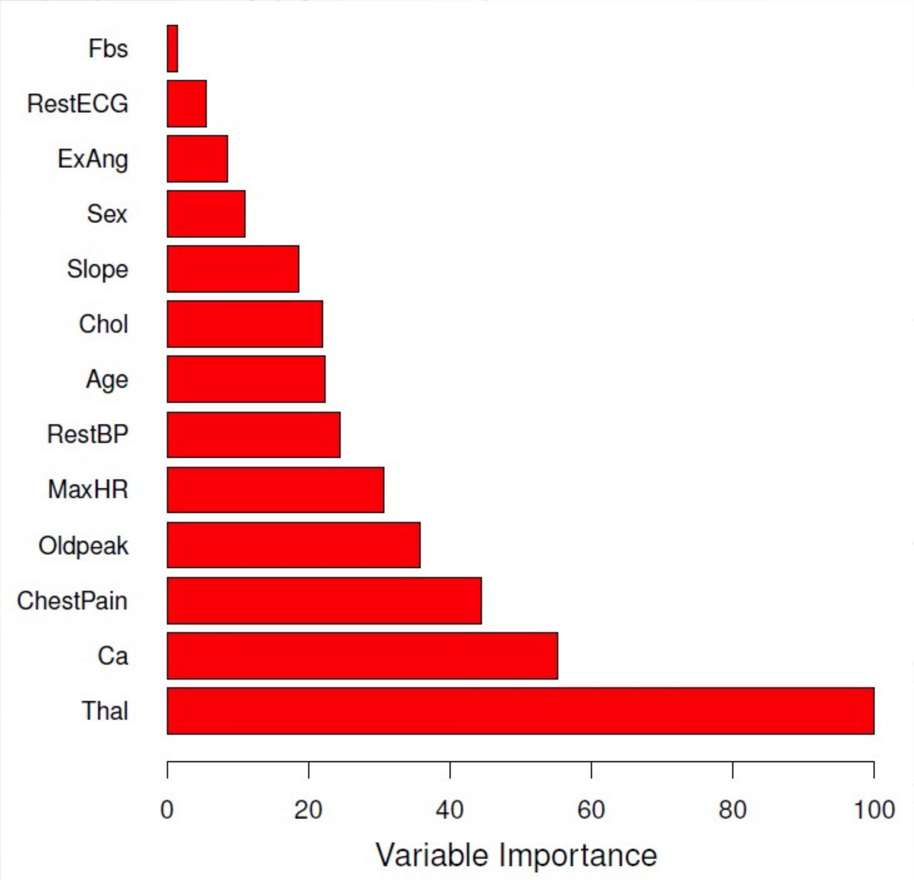
Ejercicio

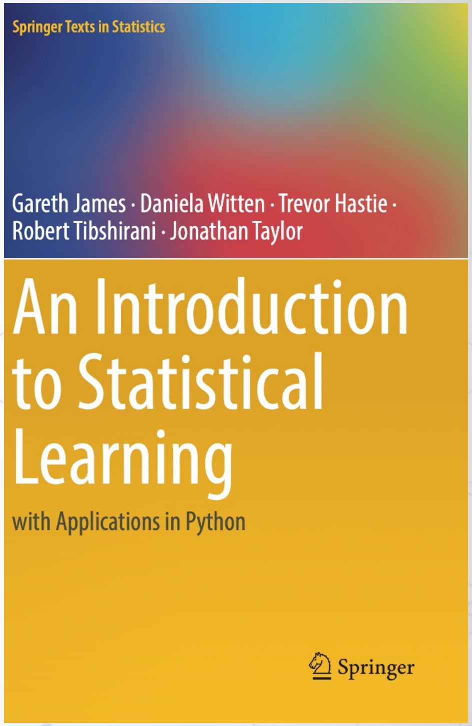
- Implementar regularización Ridge
- Regresión lineal
- Regresión logística

$$\theta_j \leftarrow \theta_j \left(1 - \frac{\alpha\gamma}{m}\right) - \alpha \times \frac{1}{m} \sum_{i=1}^{m} (\hat{y}_\theta(x) - y)x$$

$$j \geq 1$$

Selección de atributos – Random Forest





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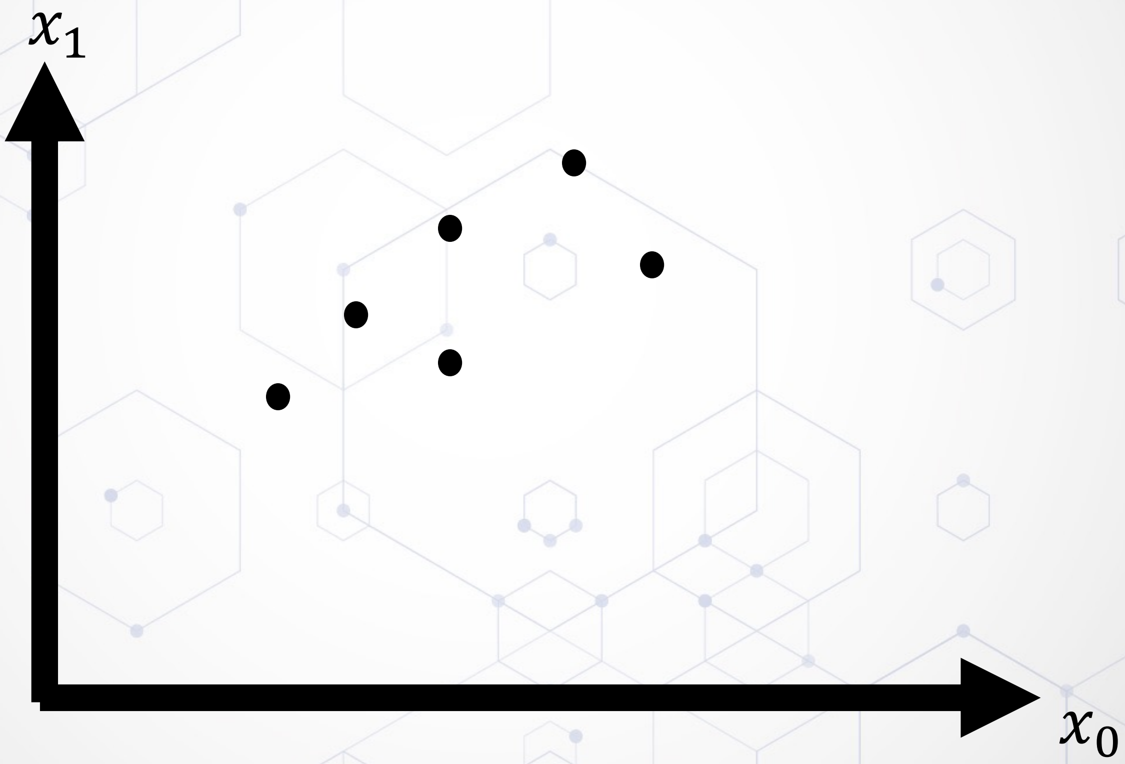
Generación de atributos

- Mapear los atributos originales a un nuevo espacio de menor dimensión

Principal Component Analysis (PCA)

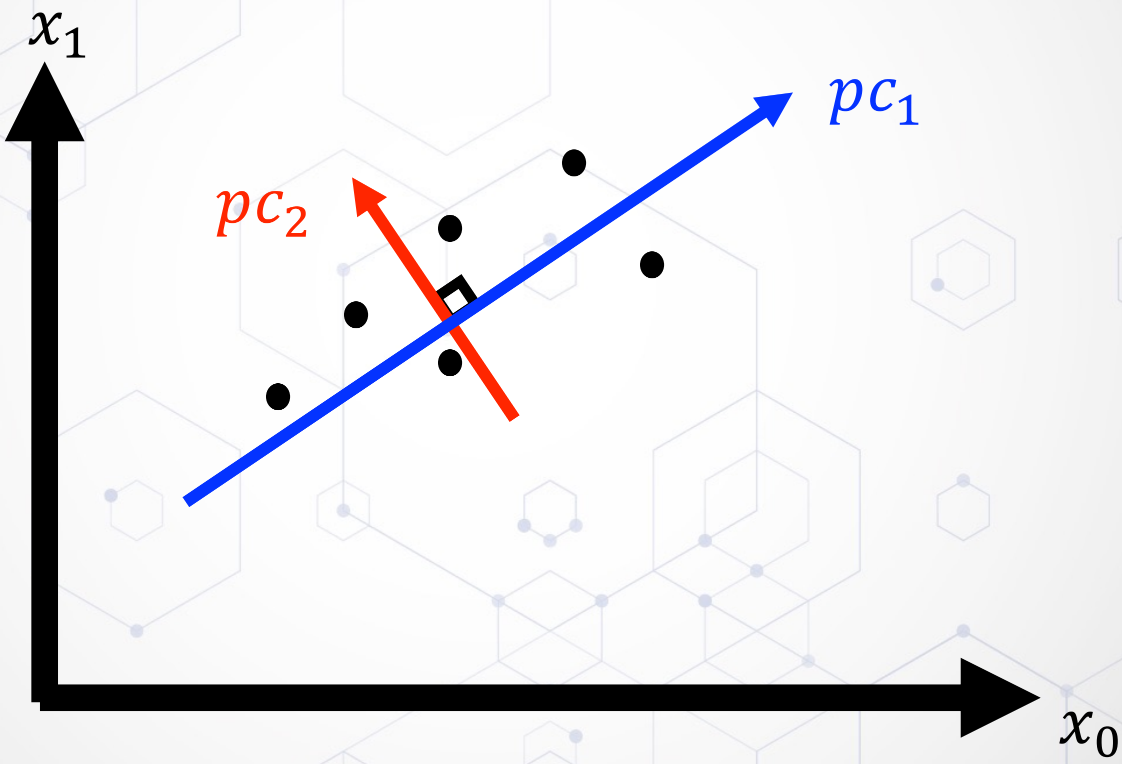
Idea General

- Datos en dos dimensiones



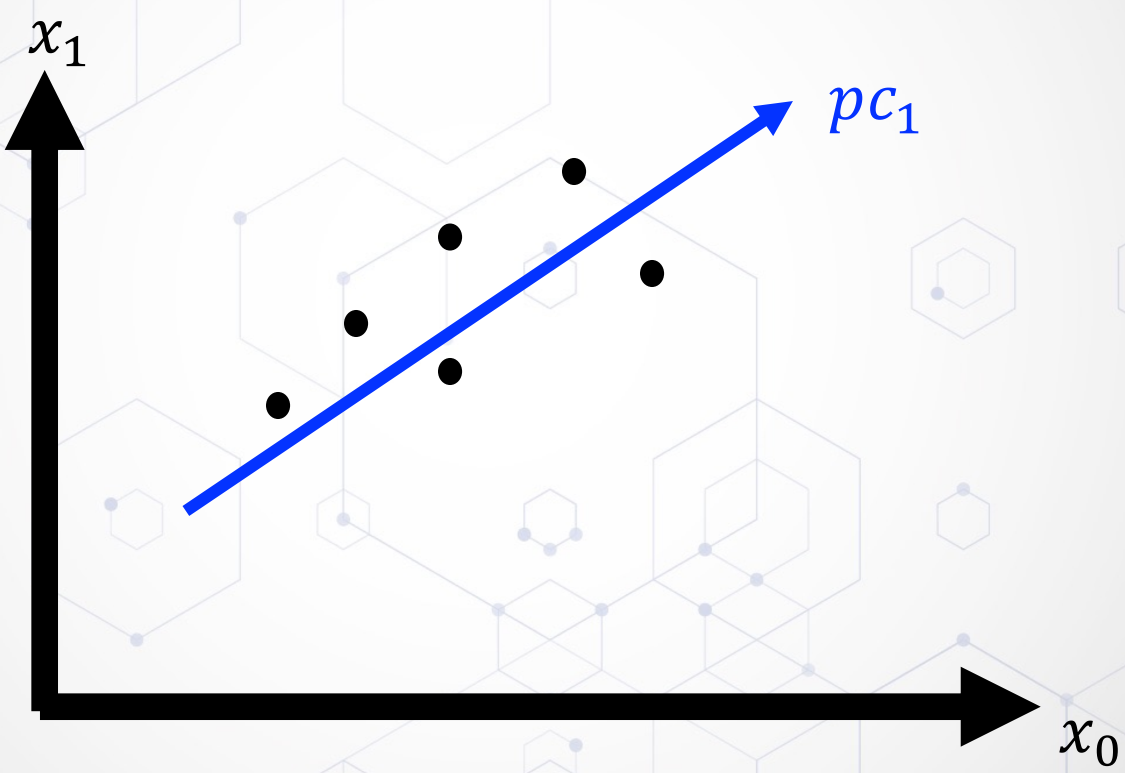
Idea General

- Bases en otro espacio de dimensión -> principal components



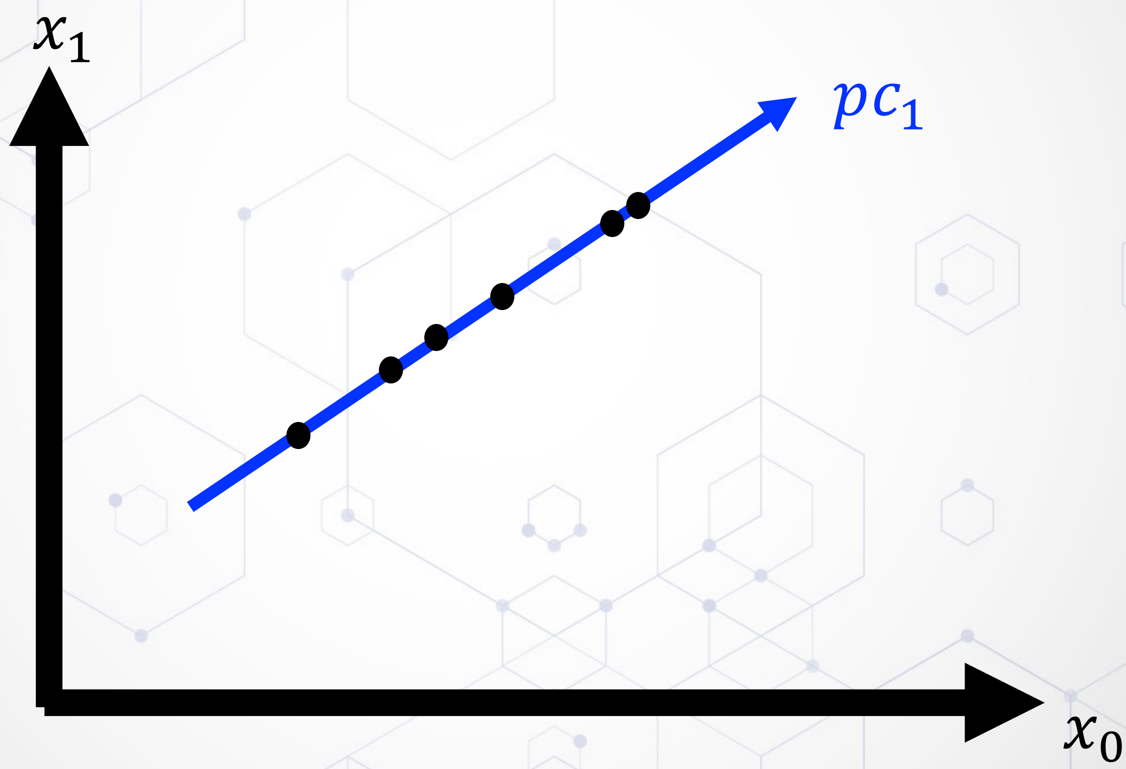
Idea General

- Una dimensión / una base



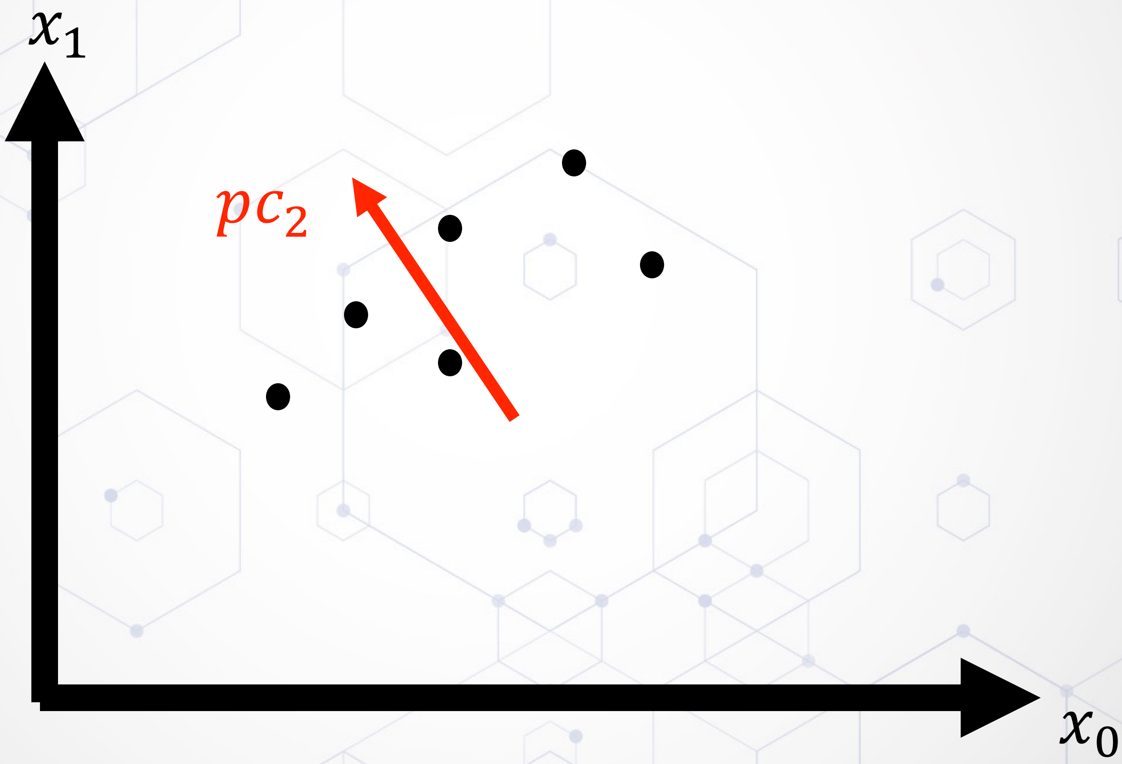
Idea General

- Proyección en una base



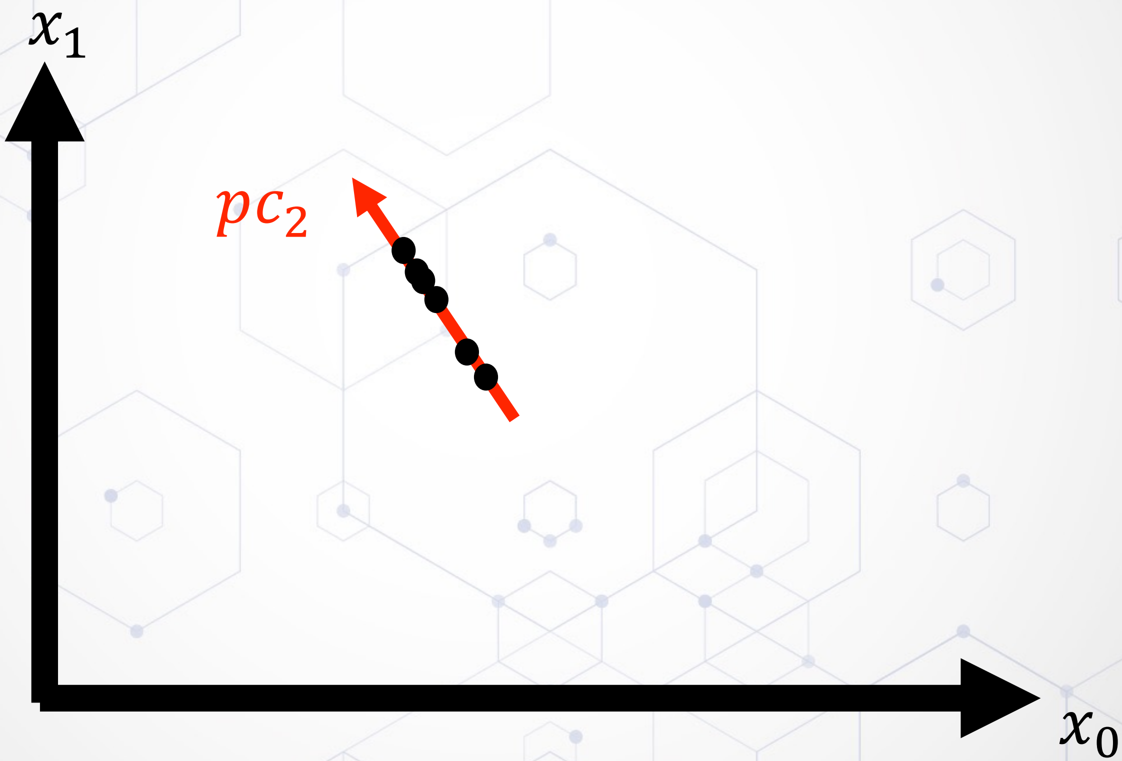
Idea General

- Segunda base



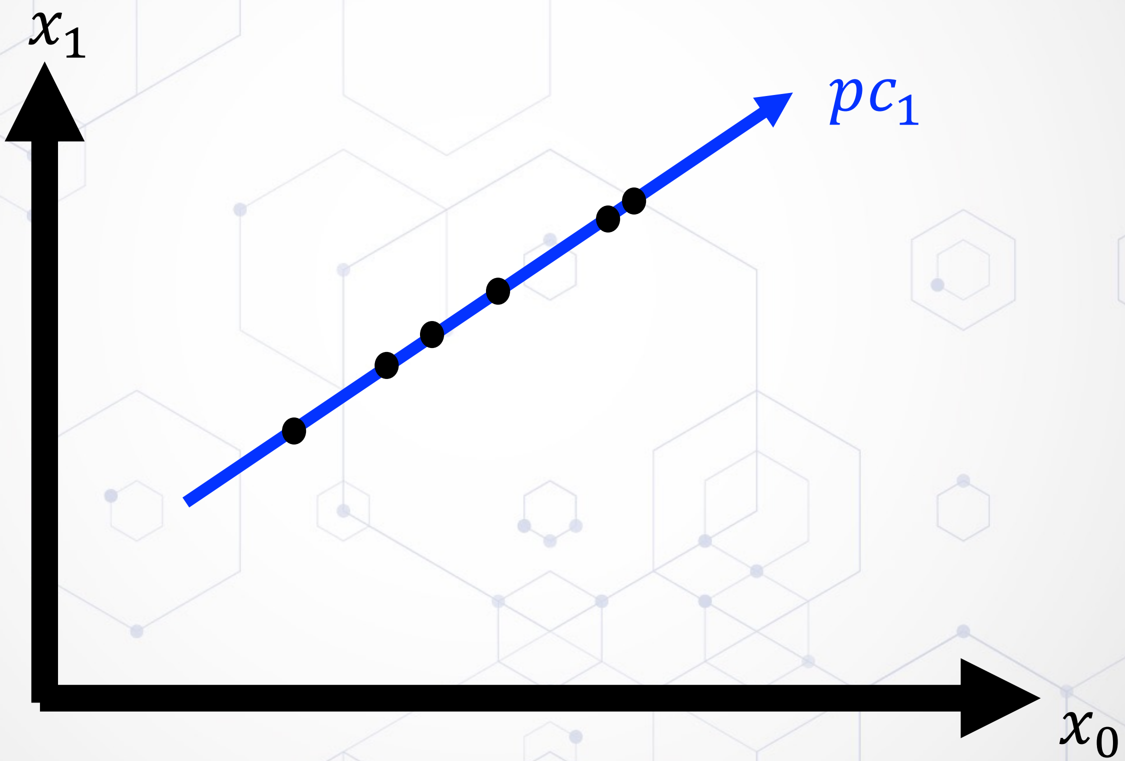
Idea General

- Proyección en la segunda base



Idea General

- "Mejor" base / componente principal

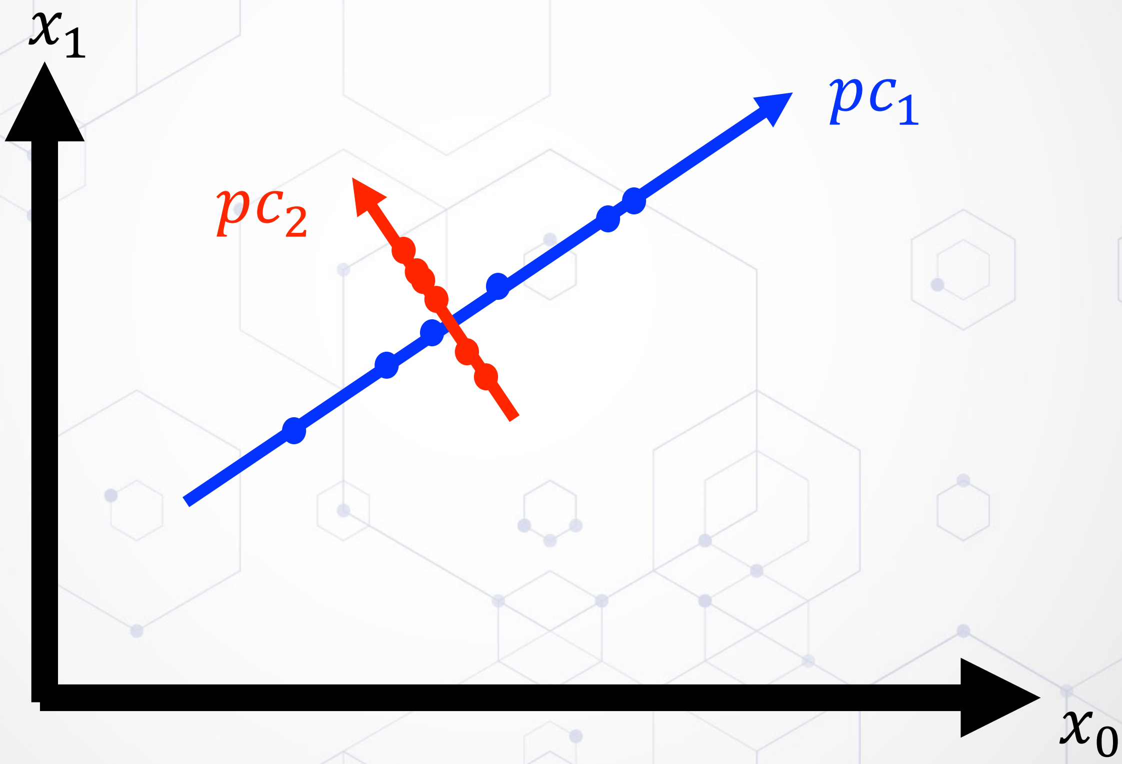


Idea General

- 3D -> 2D
- nD -> mD
  - Tal que n >> m

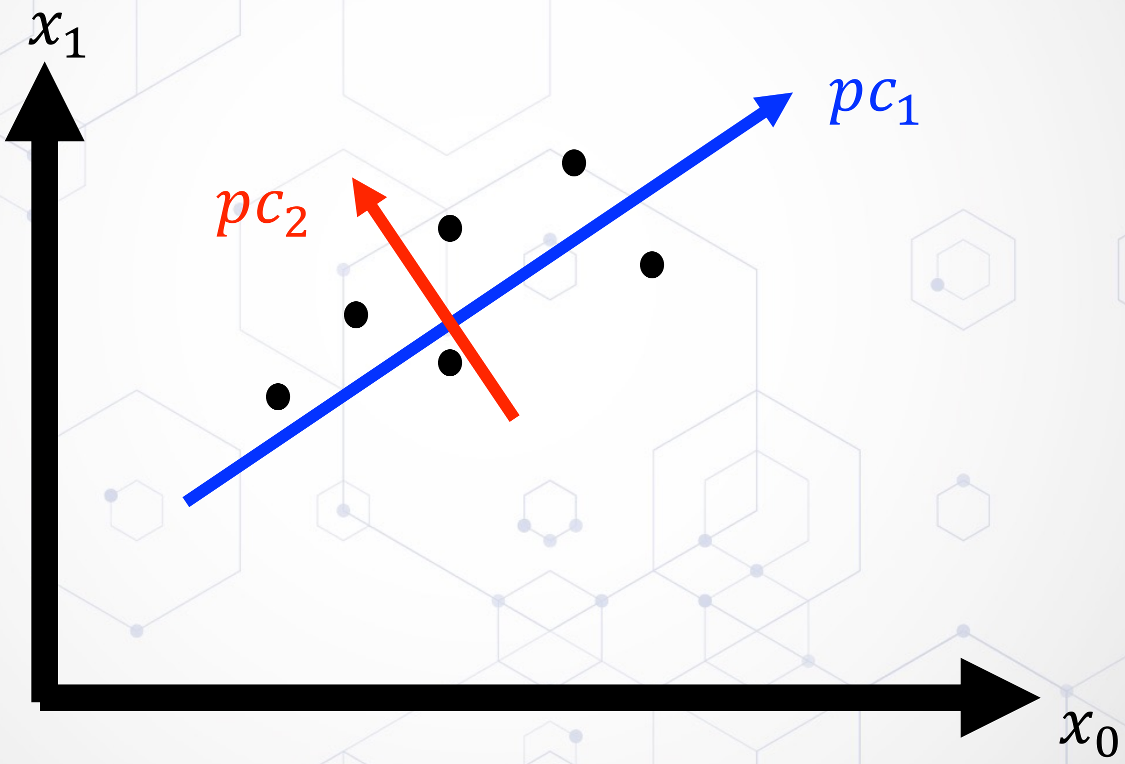
Idea General

- Recuperación de la información



Idea General

- Recuperación de la información



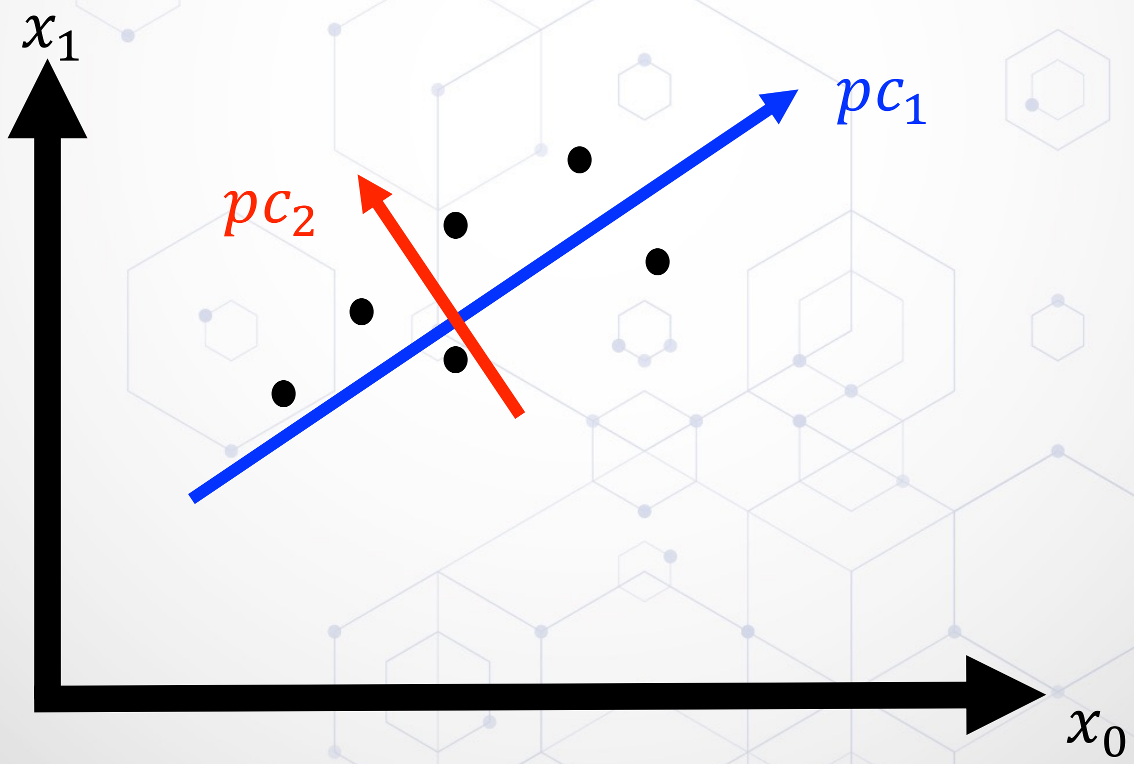
¿Cómo?

Idea General

- PCA es un método que encuentra un conjunto de **bases** que:
  - maximizan la varianza de los datos originales
    - encuentran las direcciones de mayor variación
  - son ortogonales entre sí
- Las bases o componentes principales (Principal components) se obtienen de los **vectores propios (eigenvectors)** de la matriz de covarianza … i.e. proporcionan la "dirección" con mayor varianza

Principal Component Analysis

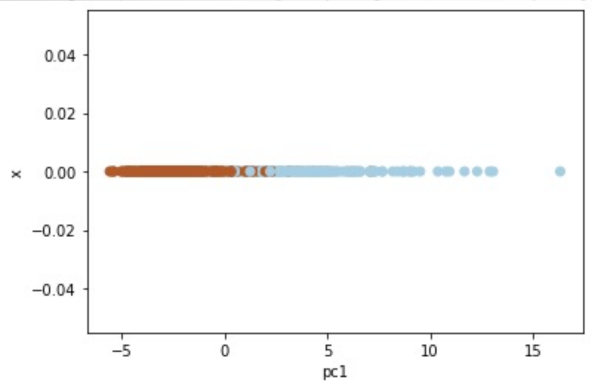
- Eigenvector (vector propio)
- Eigenvalue (valor propio)

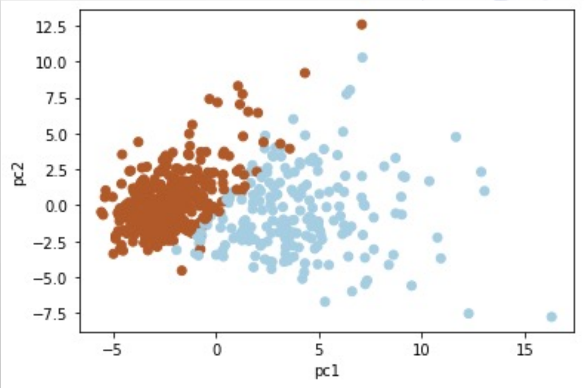


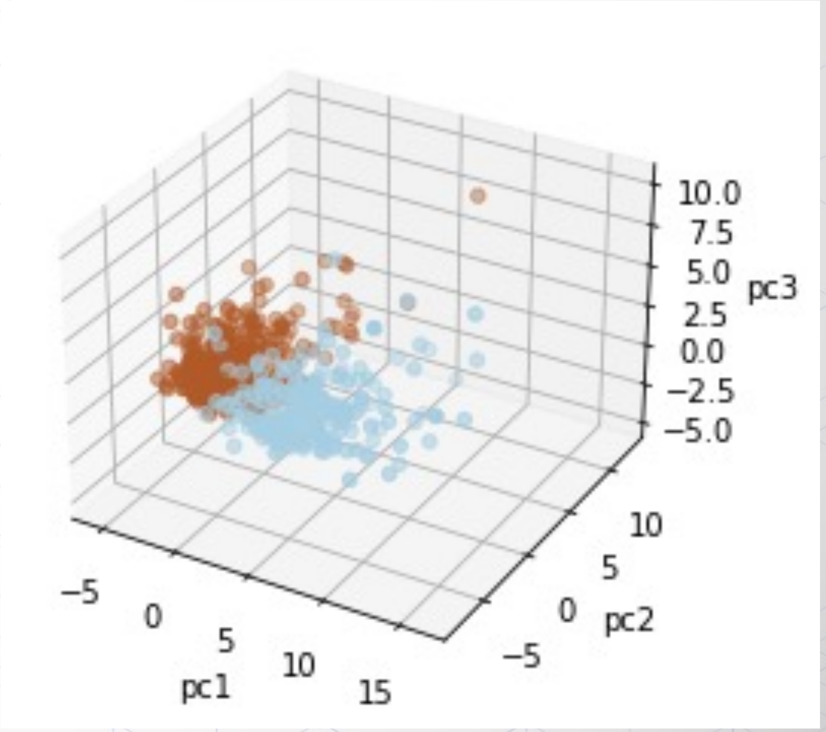
Código

Ejemplo

Breast cancer: Dos clases, 30 dim

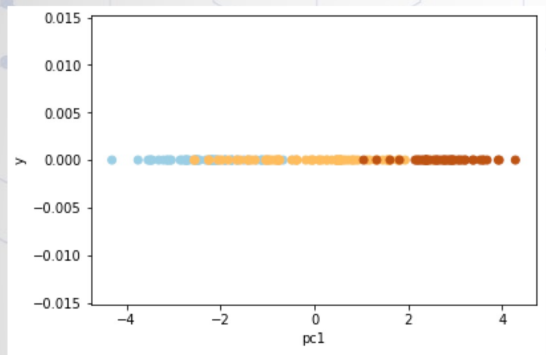


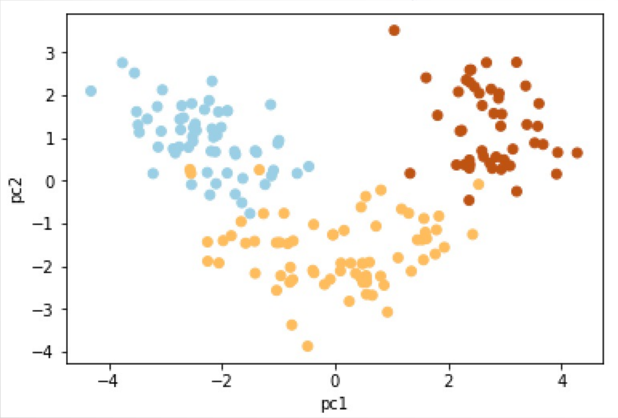


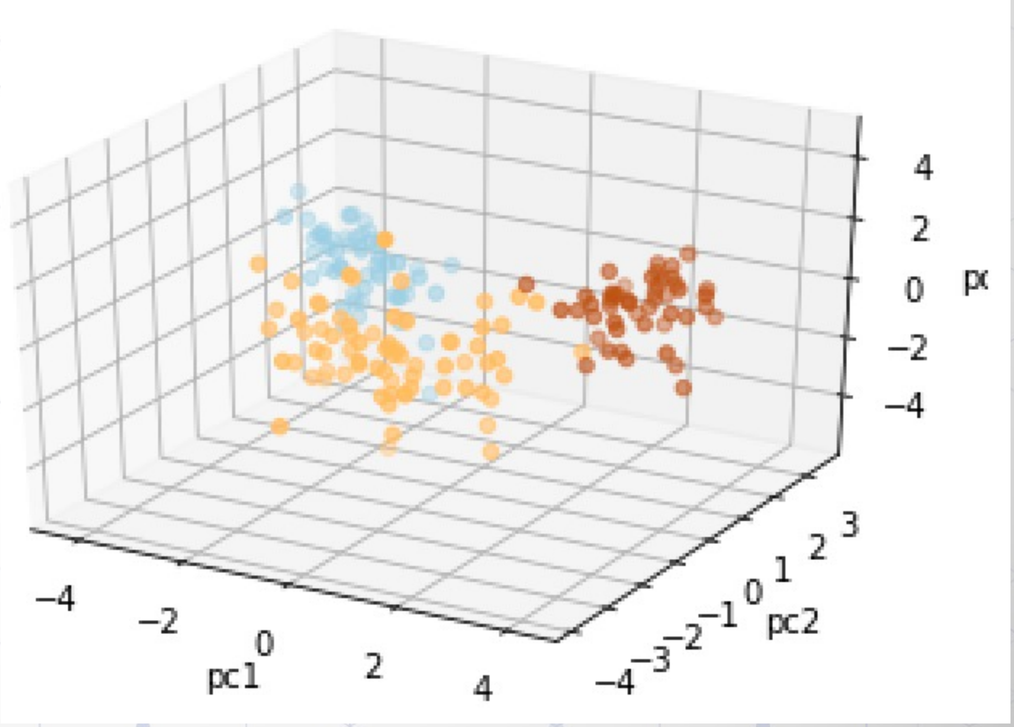


Ejemplo 2

Wine: Tres clases, 13 dim

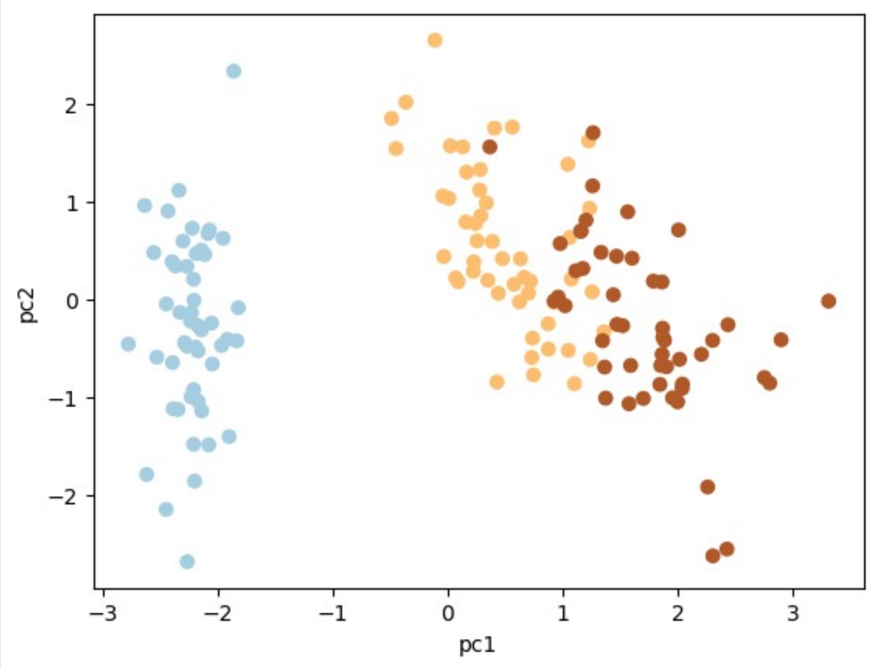


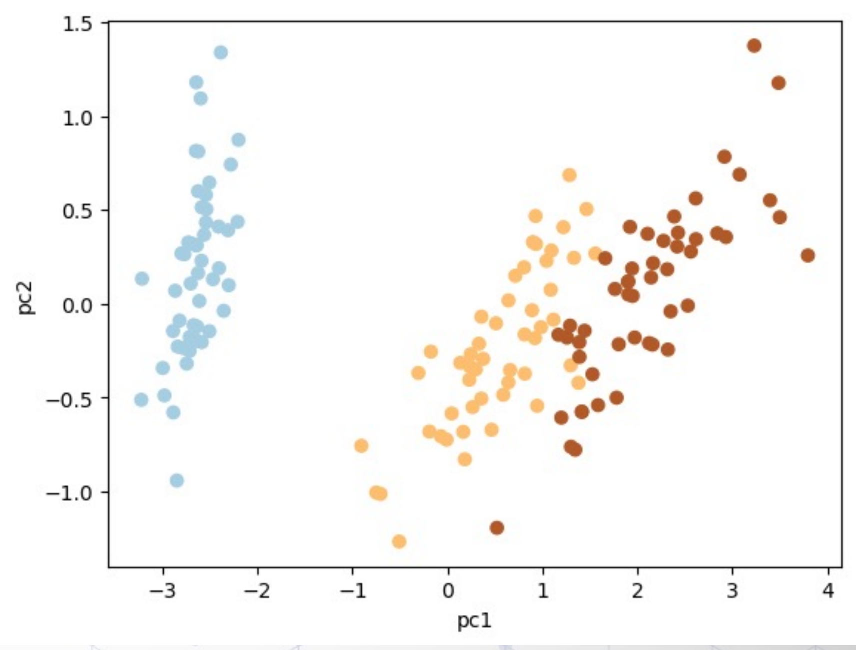




Ejemplo 3

Iris: Tres clases, dim 4






```python
# PCA

# Calcular matriz de covarianza
matriz_cov = np.cov(xNew.T)

# Calcular vectores y valores propios
eig_val, eig_vecs = np.linalg.eig(matriz_cov)
```

```python
from sklearn.decomposition import PCA

pca = PCA(n_components=2)
X_pca = pca.fit(X).transform(X)
```

Ejercicio

- PCA para clasificación
- Conjunto de datos con más de 10 variables

PCA

- Simple
- No paramétrico
- Extrae información relevante a partir de un conjunto de datos
- Provee forma de reducir un conjunto de datos (redundante/complicado) a otro con dimensión menor
- Revela estructuras simplificadas (algunas veces ocultas)
- Permite remover ruido, información no relevante
- “también es considerado como un método de aprendizaje no supervisado”

Selección de atributos

- A favor
  - Interpretabilidad de las dimensiones
  - Podría reducir la complejidad de procesamiento
- En contra
  - Perdida potencial de información

Engineering X

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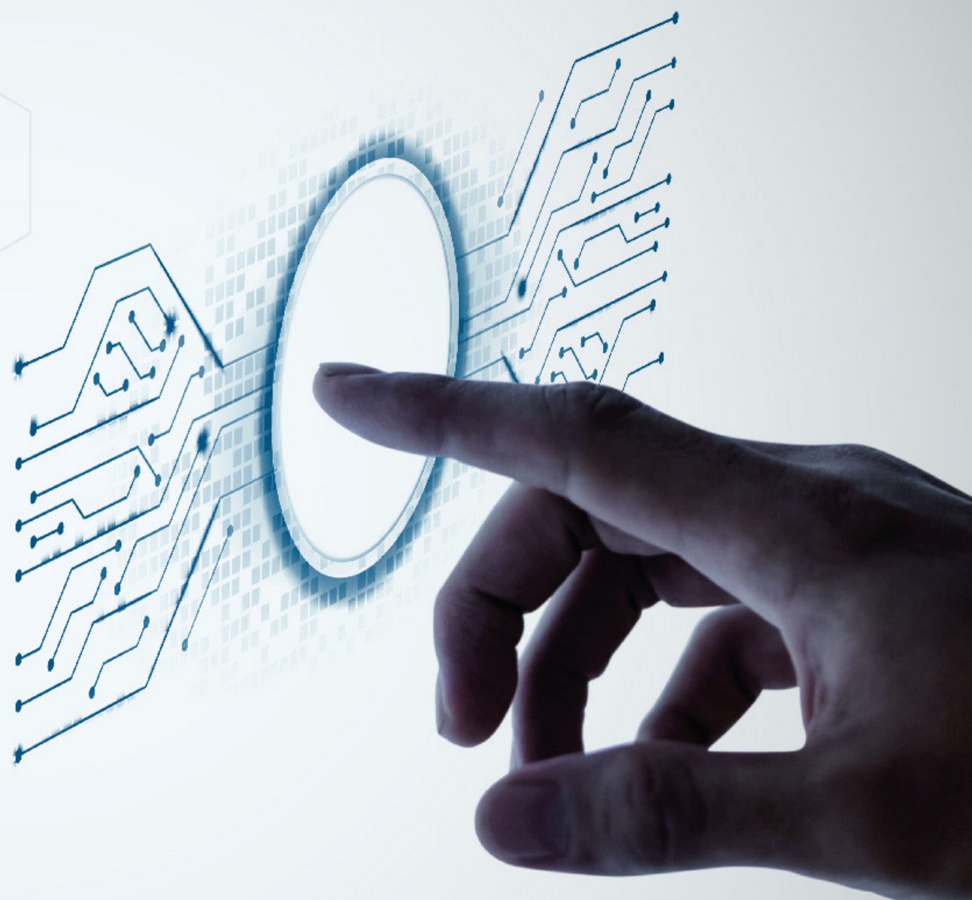
GRACIAS



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