# CM1020: Discrete Mathematics
# Peer-Graded Assignment 2.109

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## Question 1

We can show a function $A \to B$ to be injective by showing that if $f(x_1) = f(x_2)$ for arbtirary $x_1, x_2 \in A$, then $x_1 = x_2$. We can disprove it by showing that more than one function value is possible for a given element.

We can show a function $A \to B$ to be surjective by showing that for an arbitrary element $y \in B$ there is an element $x \in A$ such that $f(x) = y$. We can disprove it by finding an element that does not have a function value.

**Function 1** To show that the function

$$f_1 : \mathbb{R} \to \mathbb{R} \text{ where } f(x) = x^2 + 1$$

is injective, we see if $f(x) = f(y)$ for arbitrary $x_1, x_2 \in A$, then $x = y$.

$$\begin{aligned} {x_1}^2 + 1 &= {x_2}^2 + 1 && (1) \\ {x_1}^2 &= {x_2}^2 && (2) \\ \pm x_1 &= \pm x_2 && (3) \end{aligned}$$

Therefore $x_1 \neq x_2$ and the function is **not injective**. For example: $x_1 = -2, x_2 = 2$ and $f(x_1) = f(x_2) = 4$. The function is **not surjective** as there exist elements in the co-domain that do not have a function value $f(x)$, for example $f(x) = -2$.

**Function 2** The function

$$f_2 : \mathbb{R} \to [-1; \infty) \text{ where } f(x) = x^2 + 1$$

is **not injective** as per the reasoning for function $f_1$. As the co-domain is restricted to positive real numbers, The function is **surjective** as there exist pre-images $x$ for any $f(x)$.

**Function 3** The function

$$f_2 : \mathbb{R} \to \mathbb{R} \text{ where } f(x) = x^3$$

can be shown to be **injective** as it is a **strictly increasing** and **real-valued** function. Therefore for any given $x_1, x_2$ we know that $f(x_1) \neq f(x_2)$. The plot for this function is shown in Figure 1. The function is **surjective** as $f(x)$ is defined for any $x$.

**Function 4** The function

$$f_2 : \mathbb{R} \to [-1; \infty) \text{ where } f(x) = 2x + 3$$

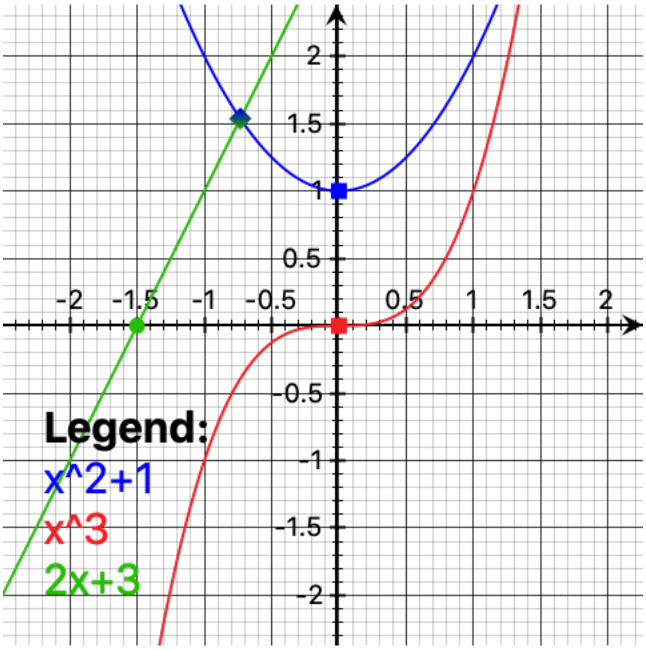
is **injective** because it is a **strictly increasing** and **real-valued** function. For each $x$ there is a unique $f(x)$ in the co-domain. The function is **surjective** as $f(x)$ is defined for any $x$.

**Function 5** The function

$$f_2 : \mathbb{Z} \to \mathbb{Z} \text{ where } f(x) = 2x + 3$$

is **injective** because it is a **strictly increasing** and **integer-valued** function. For each $x$ there is a unique $f(x)$ in the co-domain. The function is **not surjective** as $f(x)$ is not defined for any $x$, for example $f(1)$ is not defined in the co-domain $\mathbb{Z}$.

**Figure 1.** *Plots for Question 1*




## Question 2

1. The function $f_2 : \mathbb{R} \to (-1; \infty)$ where $f(x) = e^x + 1$ is a bijection if it is both injective and surjective. It is injective because it is **strictly increasing** and each $x \in \mathbb{R}$ has a unique $f(x) \in \mathbb{R}$.

2. The inverse of the function is given as follows:

$$f^{-}1 = e^y + 1 = x \tag{4}$$
$$e^y = x - 1 \tag{5}$$
$$\ln(e^y) = \ln(x-1) \tag{6}$$
$$y = \ln(x-1) \tag{7}$$

3. The plots are given in Figure 2.

4. The graphs are both strictly increasing. The graph of $f$ is exponential growth, while the graph of $f^{-}1$ is logarithmic growth. The graphs are symmetric.

**Figure 2.** *Plots for Question 2*