The Derivation System SD

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교재 3장—진리표—과 4장—진리나무—은 논리 관계의 의미론적 접근 방식을 논의합니다. 한편,

- 교재 5장–*SD* 도출 체계–은 논리 관계의 구조론적 접근 방식을 논의합니다.
  - *SD*는 11가지 도출 법칙으로 구성되며, Fitch diagram을 통해 제시합니다.

Fitch Diagram: An Example

1	$\quad \mid \forall x(Ax \supset Rx)$	
2	$\quad \mid Aa \supset Ra$	$\forall$E, 1
3	$\quad \mid Aa$	
4	$\quad \mid Ra$	$\Rightarrow$E, 2, 3
5	$\quad \mid \sim Ra$	
6	$\sim Aa$	$\neg$I, 3–5
7	$\quad \mid \sim La$	
8	$\quad \mid \sim La \supset Aa$	
9	$\quad \mid Aa$	$\Rightarrow$E, 7, 8
10	$\quad \mid \sim Aa$	R, 6
11	La	$\neg$E, 7–10

11 Rules of SD by Names

▷ *Reiteration*

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- ▷ *Reiteration*
- ▷ 5 Introduction Rules:
  $\&$I, $\vee$I, $\sim$I, $\supset$I, $\equiv$I

11 Rules of SD by Names

- *Reiteration*

- 5 Introduction Rules:
  &I, $\vee$I, $\sim$I, $\supset$I, $\equiv$I

- 5 Eliminations Rules:
  &E; $\vee$E; $\sim$E $\supset$E; $\equiv$E

11 Rules of *SD* by Requirements

- ▷ The Non-Subderivation Rules[1] of *SD*:
  R, &E, &I, $\vee$I, $\supset$E, $\equiv$E

[1] Require No Subderivation

11 Rules of SD by Requirements

- The Non-Subderivation Rules[1] of *SD*:
  R, &E, &I, $\vee$I, $\supset$E, $\equiv$E

- The Subderivation Rules[2] of *SD*:
  $\supset$I, $\sim$I, $\sim$E, $\vee$E, $\equiv$I

[1] Require No Subderivation
[2] Require Subderivation

Reiteration (R)

1	**P**
2	**P**

1	**P**	
2	**P**	

1	$\mathcal{A}$	
2	$\mathcal{A}$	

Conjunction Elimination (&E)

$$
\begin{array}{r|l}
1 & \mathbf{P \& Q} \\
2 & \mathbf{P}
\end{array}
$$

Or,

$$
\begin{array}{r|l}
1 & \mathbf{P \& Q} \\
2 & \mathbf{Q}
\end{array}
$$

$$\begin{array}{r|l} 1 & \mathbf{P\&Q} \\ 2 & \mathbf{P} \end{array}$$

Or,

$$\begin{array}{r|l} 1 & \mathbf{P\&Q} \\ 2 & \mathbf{Q} \end{array}$$

$$\begin{array}{r|l} 1 & A\&B \\ \hline 2 & A \end{array}$$

Or,

$$\begin{array}{r|l} 1 & A\&B \\ \hline 2 & B \end{array}$$

Conjunction Introduction (&I)

1	**P**
2	**Q**
3	**P&Q**

1	**P**
2	**Q**
3	**P&Q**

1	A
2	B
3	$A\&B$

Disjunction Introduction ($\vee$I)

1	$\mathbf{P}$
2	$\mathbf{P} \vee \mathbf{Q}$

Or,

1	$\mathbf{P}$
2	$\mathbf{Q} \vee \mathbf{P}$

$$\begin{array}{l|l} 1 & \mathbf{P} \\ 2 & \mathbf{P} \vee \mathbf{Q} \end{array}$$

$$\begin{array}{l|l} 1 & \mathcal{A} \\ \cline{2-2} 2 & \mathcal{A} \vee \mathcal{B} \end{array}$$

Or,

$$\begin{array}{l|l} 1 & \mathbf{P} \\ 2 & \mathbf{Q} \vee \mathbf{P} \end{array}$$

Or,

$$\begin{array}{l|l} 1 & \mathcal{A} \\ \cline{2-2} 2 & \mathcal{B} \vee \mathcal{A} \end{array}$$

Conditional Elimination ($\supset$E)

1	$\mathbf{P} \supset \mathbf{Q}$
2	$\mathbf{P}$
3	$\mathbf{Q}$

1	$\mathbf{P} \supset \mathbf{Q}$
2	$\mathbf{P}$
3	$\mathbf{Q}$

$$\begin{array}{r|l} 1 & A \supset B \\ 2 & \underline{A} \\ 3 & B \end{array}$$

Biconditional Elimination ($\equiv$E)

1	$\mathbf{P} \equiv \mathbf{Q}$
2	$\mathbf{P}$
3	$\mathbf{Q}$

Or,

1	$\mathbf{P} \equiv \mathbf{Q}$
2	$\mathbf{Q}$
3	$\mathbf{P}$

$$\begin{array}{r|l} 1 & \mathbf{P} \equiv \mathbf{Q} \\ 2 & \mathbf{P} \\ 3 & \mathbf{Q} \end{array}$$

$$\begin{array}{r|l} 1 & A \equiv B \\ 2 & A \\ \hline 3 & B \end{array}$$

Or,

$$\begin{array}{r|l} 1 & \mathbf{P} \equiv \mathbf{Q} \\ 2 & \mathbf{Q} \\ 3 & \mathbf{P} \end{array}$$

Or,

$$\begin{array}{r|l} 1 & A \equiv B \\ 2 & B \\ \hline 3 & A \end{array}$$

Conditional Introduction ($\supset$I)

1	$\quad\quad \mid \mathbf{P}$
	$\quad\quad \mid\underline{\quad\quad}$
2	$\quad\quad \mid \mathbf{Q}$
3	$\mathbf{P} \supset \mathbf{Q}$

1	$\quad\mid$ $\mathbf{P}$
2	$\quad\mid$ $\mathbf{Q}$
3	$\mathbf{P} \supset \mathbf{Q}$

1	$\quad\mid$ A
2	$\quad\mid$ B
3	$A \supset B$

Negation Introduction ($\sim$I)

1		$\mathbf{P}$
2		$\mathbf{Q}$
3		$\sim \mathbf{Q}$
4	$\sim \mathbf{P}$	

$$\begin{array}{r|l|l}
1 & & \mathbf{P} \\ \cline{3-3}
2 & & \mathbf{Q} \\
3 & & \sim \mathbf{Q} \\
4 & \multicolumn{2}{l}{\sim \mathbf{P}}
\end{array}
\qquad\qquad\qquad
\begin{array}{r|l|l}
1 & & A \\ \cline{3-3}
2 & & B \\
3 & & \sim B \\
4 & \multicolumn{2}{l}{\sim A}
\end{array}$$

Negation Elimination ($\sim$E)

$$
\begin{array}{l|l|l}
1 & & \sim \mathbf{P} \\ \cline{3-3}
2 & & \mathbf{Q} \\
3 & & \sim \mathbf{Q} \\
4 & \mathbf{P} &
\end{array}
$$

1		$\sim \mathbf{P}$
2		$\mathbf{Q}$
3		$\sim \mathbf{Q}$
4	$\mathbf{P}$	

1		$\sim A$
2		B
3		$\sim B$
4	A	

Disjunction Elimination ($\vee$E)

$$
\begin{array}{l|l}
1 & \mathbf{P} \vee \mathbf{Q} \\
2 & \quad \begin{array}{|l} \mathbf{P} \\ \hline \end{array} \\
3 & \quad \begin{array}{|l} \mathbf{R} \end{array} \\
4 & \quad \begin{array}{|l} \mathbf{Q} \\ \hline \end{array} \\
5 & \quad \begin{array}{|l} \mathbf{R} \end{array} \\
6 & \mathbf{R}
\end{array}
$$

1	$\mathbf{P} \vee \mathbf{Q}$	
2		$\mathbf{P}$
3		$\mathbf{R}$
4		$\mathbf{Q}$
5		$\mathbf{R}$
6	$\mathbf{R}$	

1	$A \vee B$	
2		A
3		C
4		B
5		C
6	C	

Biconditional Introduction ($\equiv$I)

$$
\begin{array}{r|l}
1 & \quad \begin{array}{|l} \mathbf{P} \\ \hline \end{array} \\
2 & \quad \begin{array}{|l} \mathbf{Q} \end{array} \\
3 & \quad \begin{array}{|l} \mathbf{Q} \\ \hline \end{array} \\
4 & \quad \begin{array}{|l} \mathbf{P} \end{array} \\
5 & \mathbf{P} \equiv \mathbf{Q}
\end{array}
$$

$$
\begin{array}{r|l}
1 & \quad \begin{array}{|l} \mathbf{P} \\ \hline \mathbf{Q} \end{array} \\
3 & \quad \begin{array}{|l} \mathbf{Q} \\ \hline \mathbf{P} \end{array} \\
5 & \mathbf{P} \equiv \mathbf{Q}
\end{array}
\qquad\qquad
\begin{array}{r|l}
1 & \quad \begin{array}{|l} A \\ \hline B \end{array} \\
3 & \quad \begin{array}{|l} B \\ \hline A \end{array} \\
5 & A \equiv B
\end{array}
$$