[**Robust Optimization**]

$$\text{(Original)} \quad \min \quad d^T \Omega d - \lambda d^T \alpha$$

Let set $U := \{\tilde{\alpha} \mid \tilde{\alpha}_i = \hat{\alpha}_i + \bar{\alpha}_i \gamma_i, \quad -1 \le \gamma_i \le 1, \quad \sum_i |\gamma_i| = \Gamma\}$.

$$\text{(robustness)} \quad \min \quad d^T \Omega d - \lambda \min_{\tilde{\alpha} \in U} d^T \tilde{\alpha}$$

$$\Rightarrow \quad \min_{\tilde{\alpha} \in U} d^T \tilde{\alpha} = \min \sum_i d^T (\hat{\alpha}_i + \bar{\alpha}_i \gamma_i)$$

$$\begin{aligned}
\text{(Dual)} \Rightarrow \quad \max \quad & \Gamma \pi + \sum_i \theta_i \\
\text{s.t} \quad & \pi + \theta_i \le \bar{\alpha} d_i, \quad \forall i \in N \\
& \pi \ge 0, \\
& \theta_i \ge 0, \quad \forall i \in N
\end{aligned}$$

$$\begin{aligned}
\text{(ALL)} \quad \min \quad & d^T \Omega d - \lambda (\Gamma \pi + \sum_i \theta_i) \\
\text{s.t} \quad & (1) - (11) \\
& \pi + \theta_i \le \tilde{\alpha} d_i, \quad \forall i \in N \\
& \pi \ge 0 \\
& \theta_i \ge 0, \quad \forall i \in N
\end{aligned}$$

We may a simplified formulation because the worst-case only occurs when $\tilde{d}_i = \hat{d}_i - \bar{\alpha}_i, \forall i \in N (\text{i.e} \gamma_i = -1)$.

Let set $U := \{\tilde{\alpha} \mid \tilde{\alpha}_i = \hat{\alpha}_i + \bar{\alpha}_i \gamma_i, \quad 0 \le \gamma_i \le 1, \quad \sum_i \gamma_i = \Gamma\}$.

For a given α,

$$\begin{aligned}
\text{(robustness)} \quad \min_{\tilde{\alpha} \in U} d^T \tilde{\alpha} = \min & \sum_i d^T (\hat{\alpha}_i - \bar{\alpha}_i \gamma_i) \\
& \sum_i \gamma_i \le \bar{\alpha}_i d_i, \quad \forall i \in N \\
& 0 \le \gamma_i \le 1, \quad \forall i \in N
\end{aligned}$$

$$\begin{aligned}
\text{(Dual)} \Rightarrow \quad \max \quad & \Gamma \pi + \sum_i \theta_i \\
\text{s.t} \quad & \pi + \theta_i \le \bar{\alpha} d_i, \quad \forall i \in N \\
& \pi \ge 0 \\
& \theta_i \ge 0, \quad \forall i \in N
\end{aligned}$$

$$
\begin{aligned}
\text{(ALL)} \quad \min \quad & d^T \Omega d - \lambda(\Gamma \pi + \sum_i \theta_i) \\
\text{s.t} \quad & (1) - (11) \\
& \pi + \theta_i \leq \bar{\alpha} d_i, \quad \forall i \in N \\
& \pi \geq 0 \\
& \theta_i \geq 0, \quad \forall i \in N
\end{aligned}
$$