Introduction to lithospheric geodynamic modelling

Physics of heat transfer

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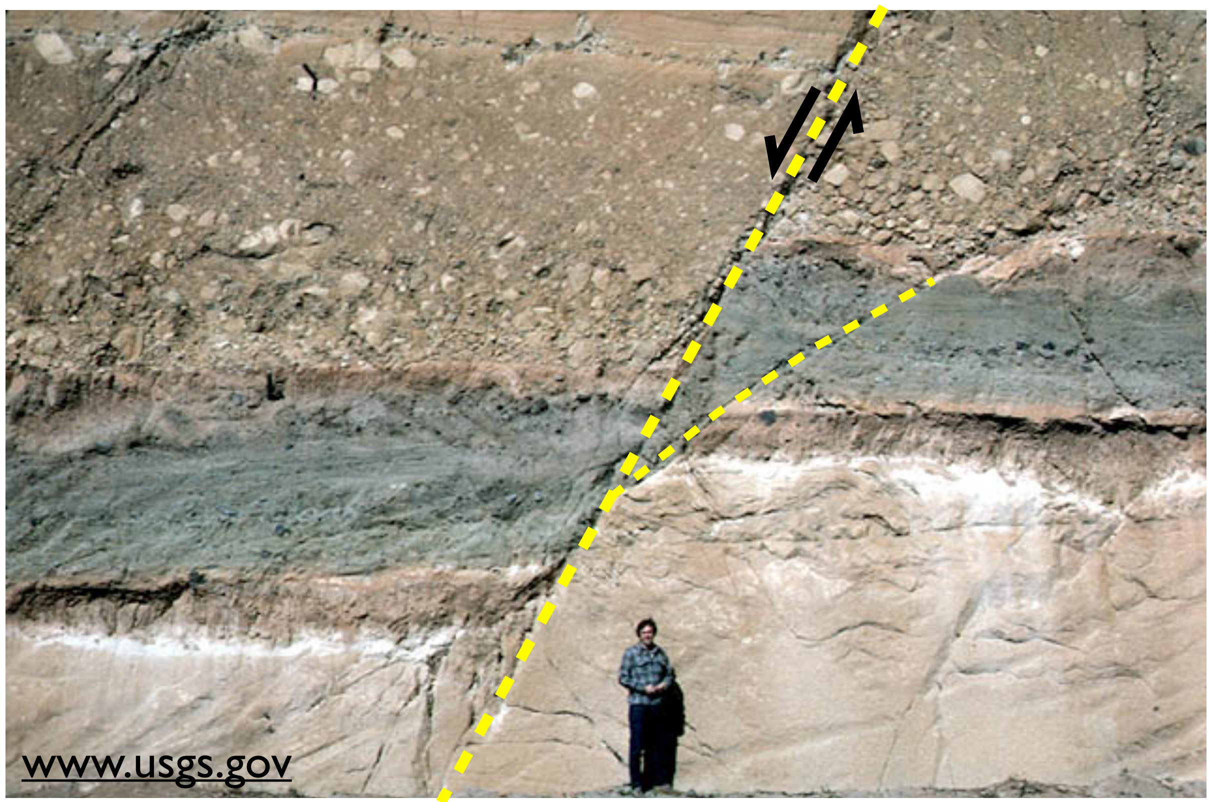
Goals of this lecture

- Make a case for why understanding **heat transfer** is important
- Present the main **heat transfer mechanisms** in the lithosphere

Why start with heat transfer?

The dynamics of the Earth are dominantly controlled by gravitational and thermal processes. In fact, you could argue that thermal processes have the largest impact. Why?

Why start with heat transfer?



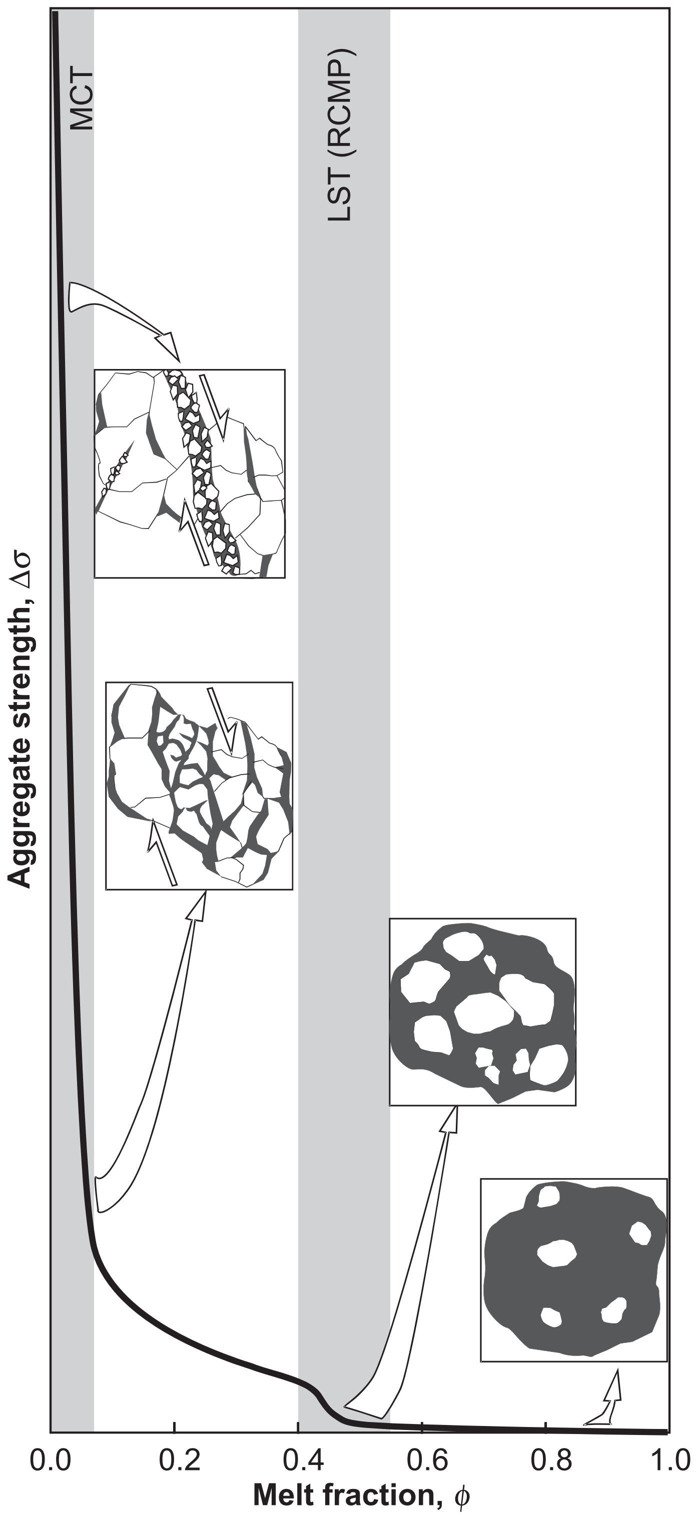
Faulting

Folding

Many rock properties are largely a function of temperature.

Faulting and brittle deformation may occur near the surface, but folding and ductile flow are dominant at depth.

The effects of partial melting on rock strength



Rosenberg et al., 2007

- Partial melting dramatically decreases rock strength
- Only about 7% melt is required for to decrease rock strength by 80-90%

Heat transfer processes in the lithosphere

- **Conduction**
- **Production**
- **Advection**

Heat transfer processes in the lithosphere

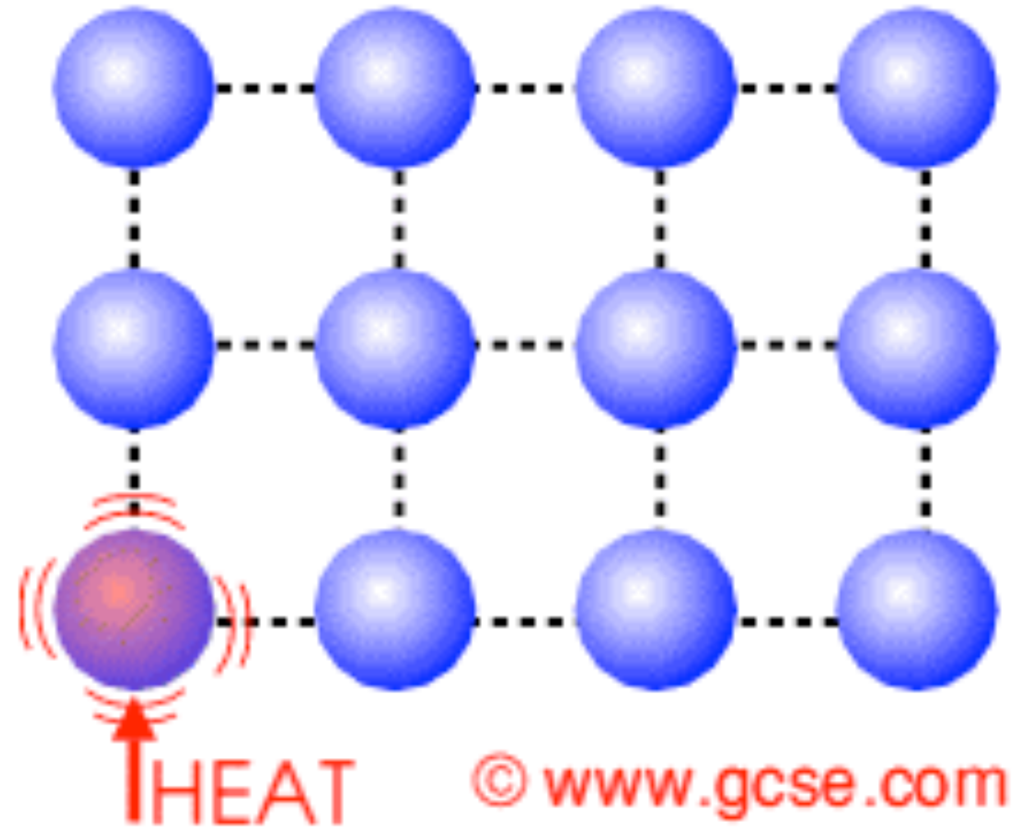
- **Conduction**: The diffusive transfer of heat by kinetic atomic or molecular interactions within the material. Also known as thermal diffusion.

- **Production**

- **Advection**

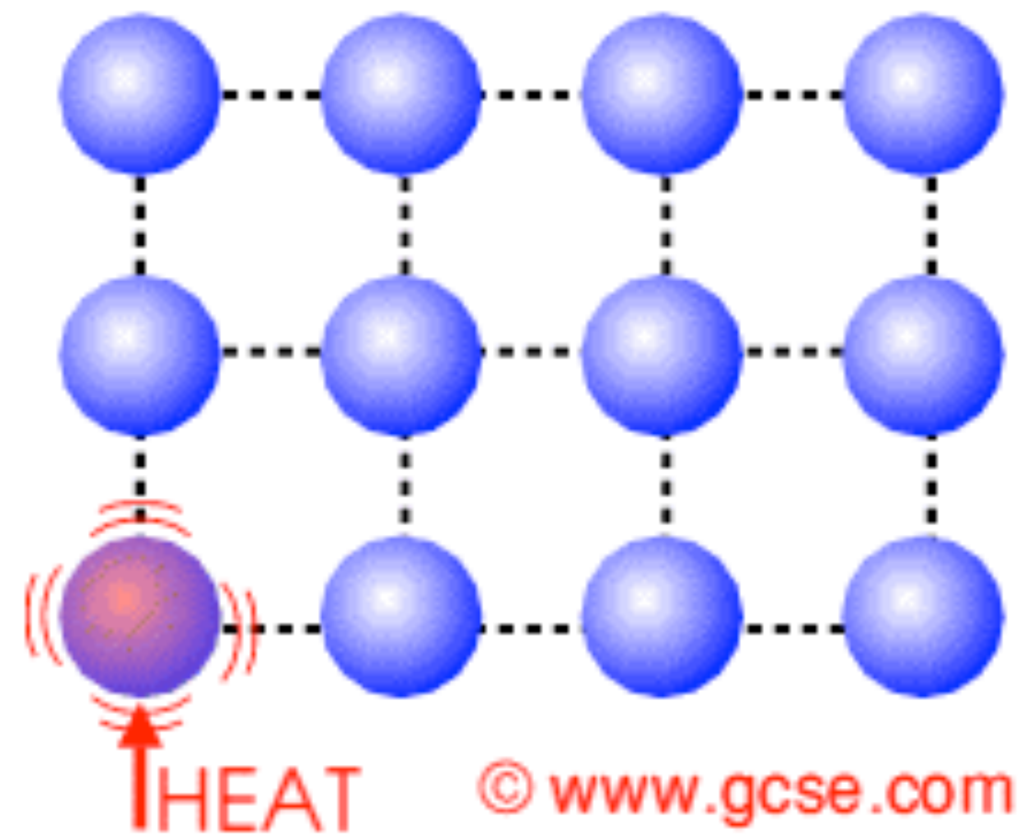
Basic ideas of heat conduction

- The conduction of heat in solids is a **diffusion** process, and well described by **Fourier's laws**, the basic mathematical relationships describing **diffusion**



- Fourier's first law states that the flux of heat in a material q is <u>directly proportional to the temperature gradient</u>

Basic ideas of heat conduction



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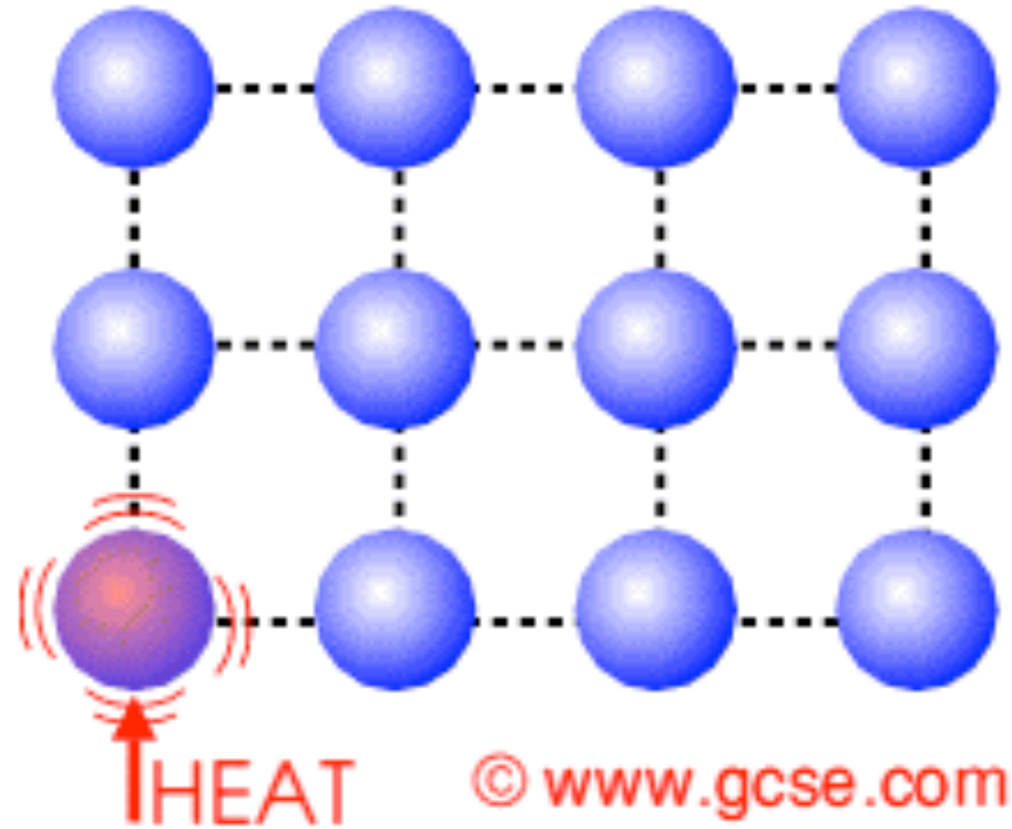
Basic ideas of heat conduction



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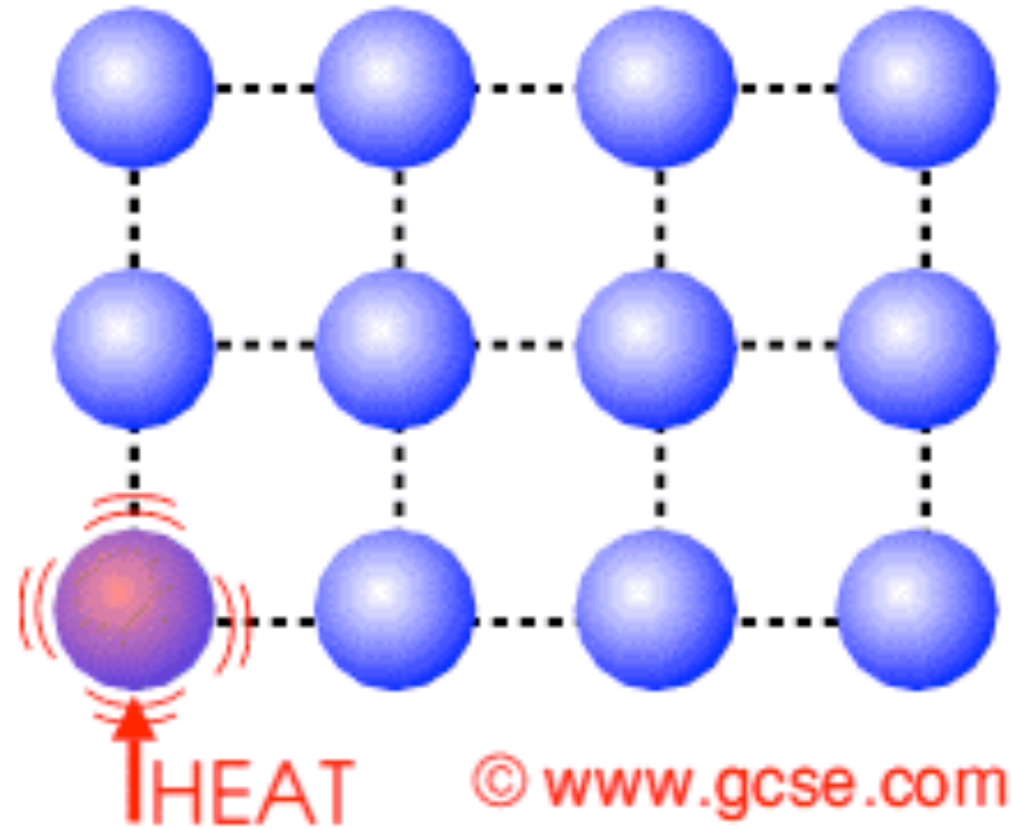
- What would this relationship look as an equation?

Basic ideas of heat conduction



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- What would this relationship look as an equation?

Fourier's first law

- In 1D, the mathematical translation of "Heat flux q is *directly* proportional to the thermal gradient in a material" is

$$q = -k \frac{dT}{dz}$$

- Here, T represents temperature and z represents spatial position, depth in the Earth for our example
- Thus, dT/dz is the change in temperature with depth, or the **thermal gradient**
- The proportionality constant k is known as the **thermal conductivity**

Fourier's first law

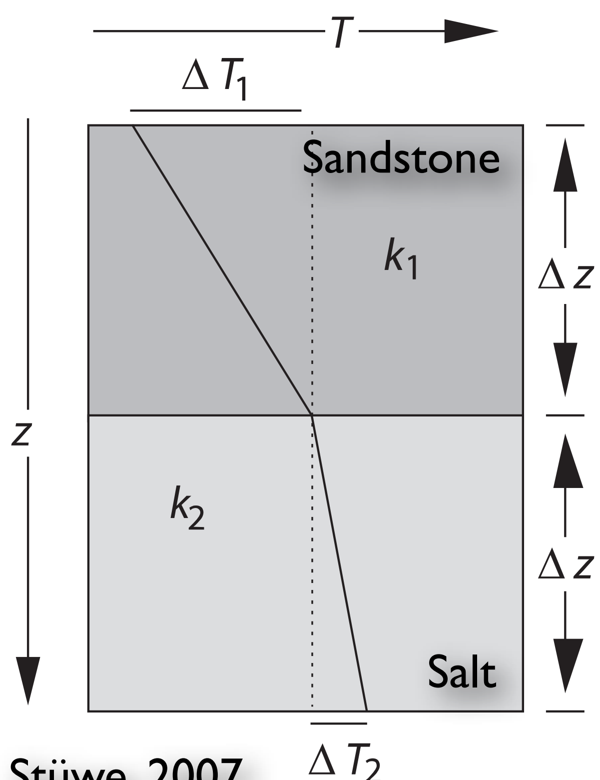
- In 1D, the mathematical translation of "Heat flux q is *directly* proportional to the thermal gradient in a material" is

$$q = -k \frac{dT}{dz}$$

- **Why is there a negative sign?**

What is thermal conductivity?

rock type	k
sandstone	1.5-4.2
gneiss	2.1-4.2
amphibolite	2.5-3.8
granite	2.4-3.8
ice	2.2
water	0.58
salt	5.4-7.2
iron	73



Stüwe, 2007

- The mathematical translation of "Heat flux q is *directly* proportional to the thermal gradient in a material" is

$$q = -k \frac{dT}{dz}$$

- Thermal conductivity is a proportionality factor
- As you can easily see, rocks with a "high" thermal conductivity will produce a large heat flow, whereas rocks with a "low" thermal conductivity will have very low heat flow
- Thermal conductivity of most crustal rocks is 2-3 W m^{-1} K^{-1}

Heat transfer processes in the lithosphere

- **Conduction**: The diffusive transfer of heat by kinetic atomic or molecular interactions within the material. Also known as thermal diffusion.

- **Production**: Not really a heat transfer process, but rather a source of heat. Sources in the lithosphere include radioactive decay, friction in deforming rock or chemical reactions such as phase transitions.

- **Advection**

What is heat production?

- Heat production in the Earth is precisely what it is called, heat generated by various processes, including
    - Radioactive isotope decay - **Radiogenic heat production**
    - Fault slip or shearing - **Mechanical heat production**
        - Also called shear heating or viscous dissipation
    - Metamorphic reactions - **Chemical heat production**

Radiogenic heat production

- **Radiogenic heat production**, A or H, results from the decay of radioactive elements in the Earth, mainly ^{238}U, ^{235}U, ^{232}Th and ^{40}K. A is generally used for volumetric heat production and H for heat production by mass.

Rock Type	Concentration U (ppm)	Th (ppm)	K (%)
Reference undepleted (fertile) mantle	0.031	0.124	0.031
"Depleted" peridotites	0.001	0.004	0.003
Tholeiitic basalt	0.07	0.19	0.088
Granite	4.7	20	4.2
Shale	3.7	12	2.7
Average continental crust	1.42	5.6	1.43
Chondritic meteorites	0.008	0.029	0.056

Turcotte and Schubert, 2014

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- These elements occur in the mantle, but are concentrated in the crust, where radiogenic heating can be significant
  - The surface heat flow in continental regions is ~65 mW m^{-2} and ~37 mW m^{-2} is from radiogenic heat production (57%)

		Concentration	
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Radiogenic heat production

Rock type	Heat production		
	By mass, H [W kg^{-1}]	By volume, A [W m^{-3}]	By volume, A [μW m^{-3}]
Reference undepleated (fertile) mantle	7.39E-12	2.44E-08	0.024
"Depleated" peridotites	3.08E-13	1.02E-09	0.001
Tholeiitic basalt	1.49E-11	4.41E-08	0.044
Granite	1.14E-09	3.01E-06	**3.008**
Shale	7.74E-10	1.86E-06	**1.857**
Average continental crust	3.37E-10	9.26E-07	**0.927**
Chondritic meteorites	3.50E-12	1.15E-08	0.012

Calculated from Turcotte and Schubert, 2014

- Typical heat production values for upper crustal rocks are 2-3 μW m^{-3}, but the crustal average is <1 μW m^{-3}

1D steady-state heat conduction + production

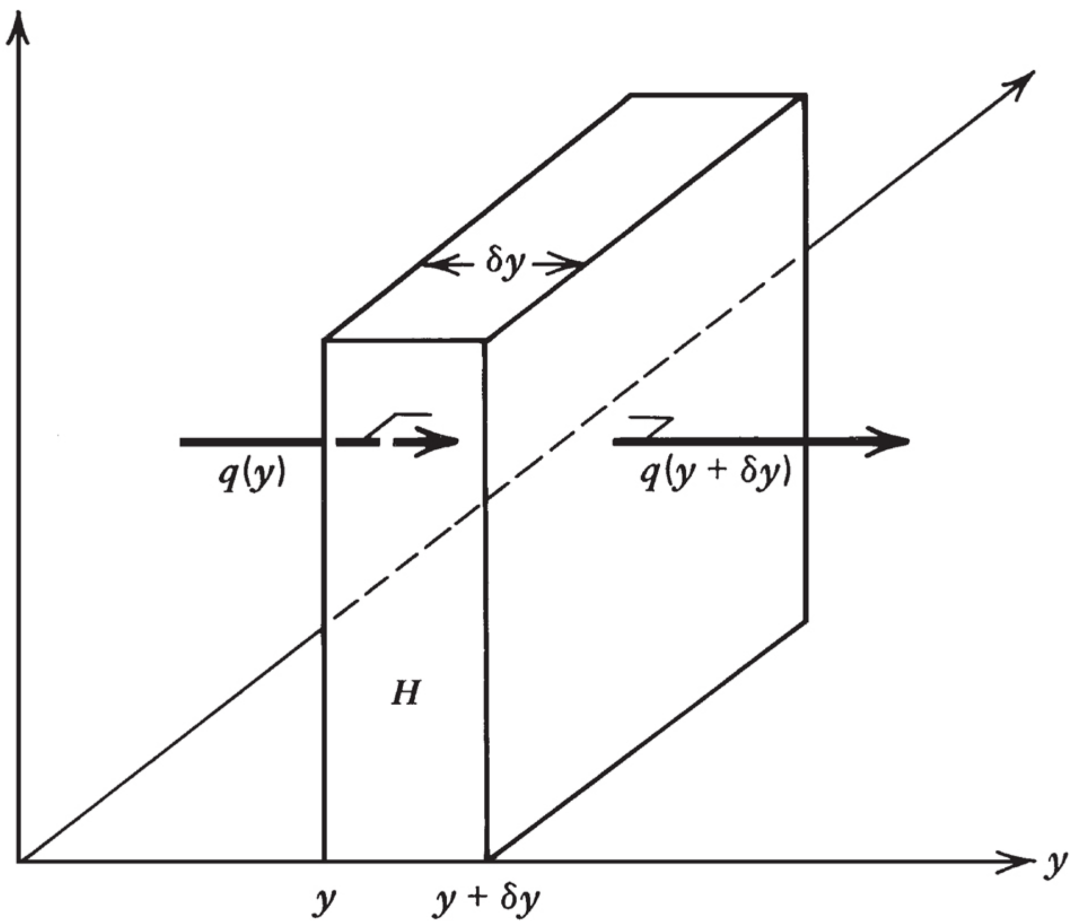


Fig. 4.5, Turcotte and Schubert, 2014

- Consider the heat flux across a slab of thickness δz
- The net heat flux is simply the heat flux out minus the heat flux in, or

$$q(z+\delta z) - q(z)$$

- Using Taylor series expansion and Fourier's first law, it can be shown that

$$q(z+\delta z) - q(z) = \delta z \frac{dq}{dz} = \delta z \frac{d}{dz}\left[-k\left(\frac{dT}{dz}\right)\right]$$
$$= \delta z\left[-k\left(\frac{d^2T}{dz^2}\right)\right]$$

for constant thermal conductivity k

1D steady-state heat conduction + production

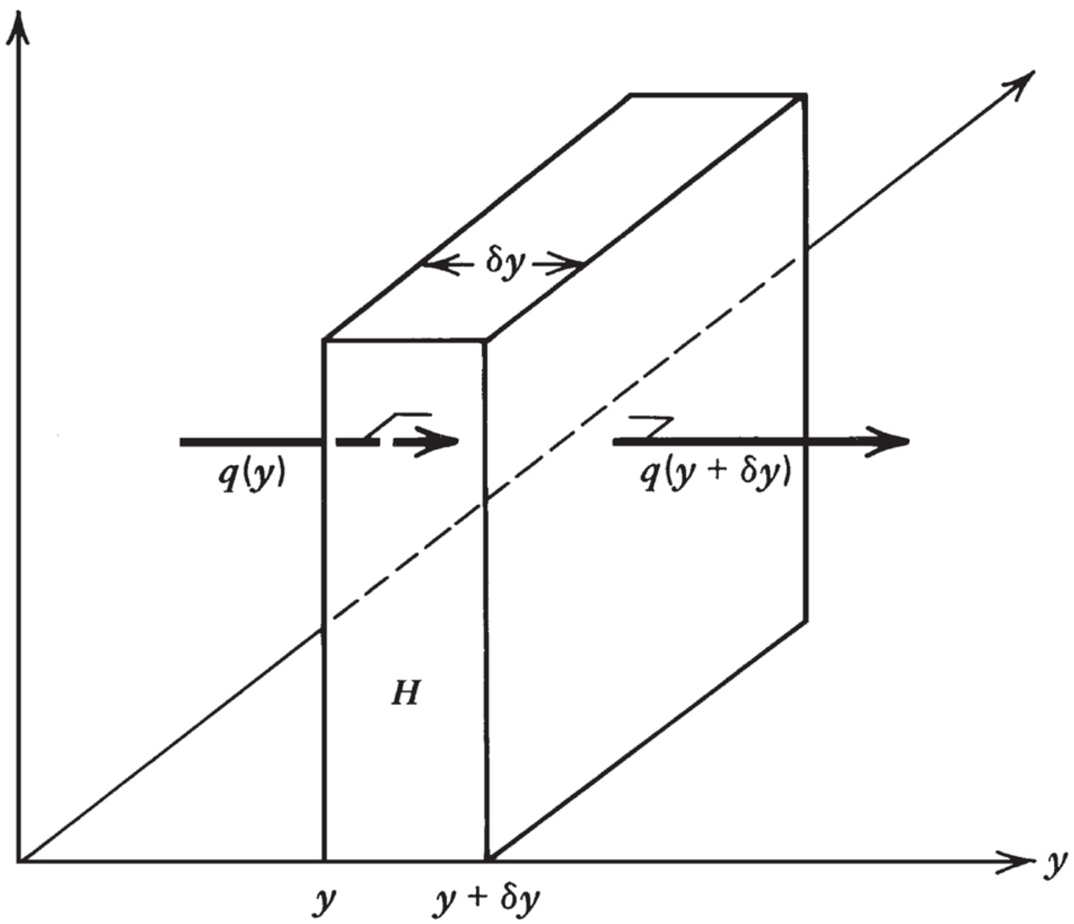


Fig. 4.5, Turcotte and Schubert, 2014

- If we assume the only source of changing the heat flux in the slab is radiogenic heat, we can say

$$\rho H \delta z = \delta z \left[-k \left(\frac{d^2T}{dz^2} \right) \right]$$

or

$$k \frac{d^2T}{dz^2} + \rho H = 0$$

where ρ is rock density

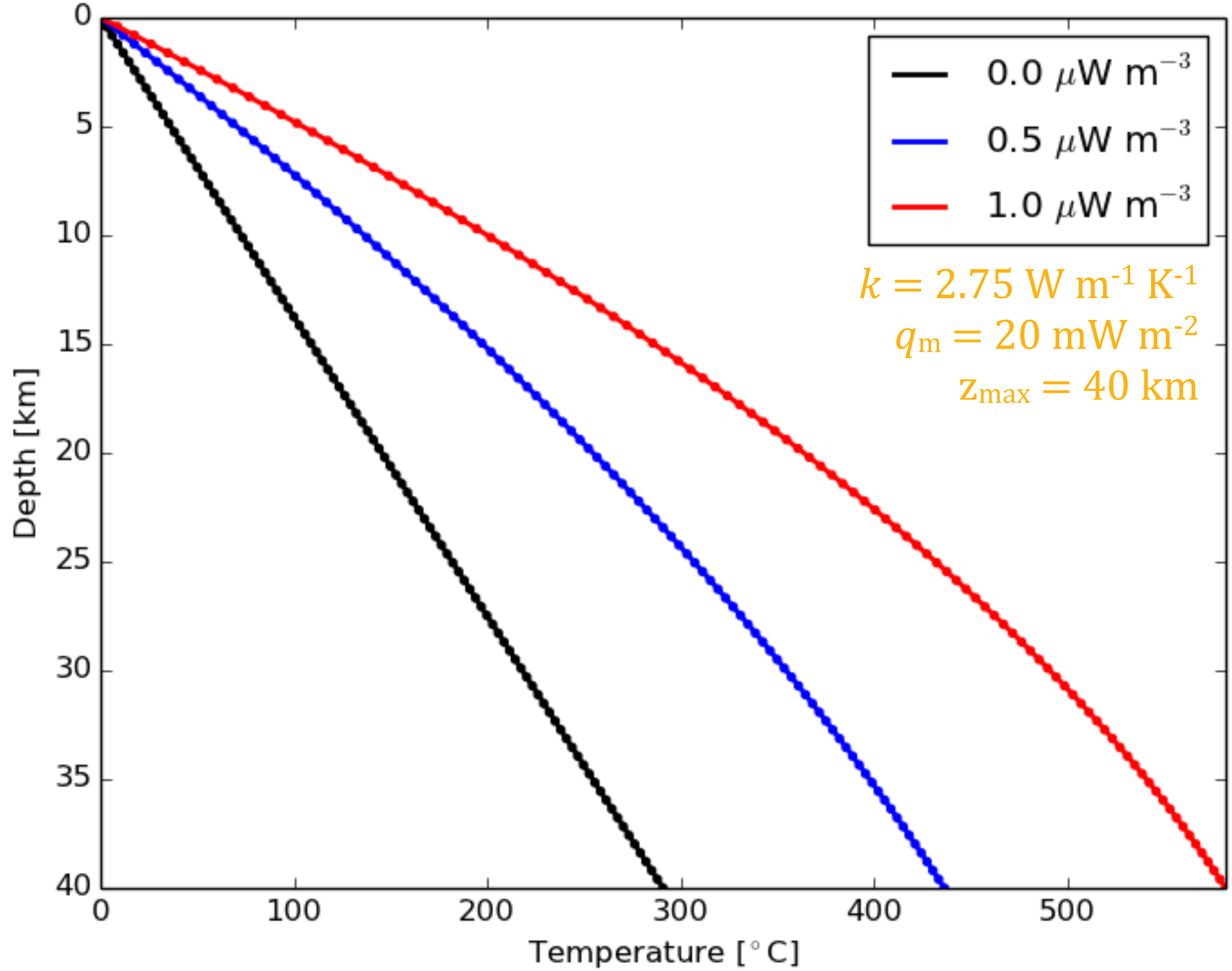
- This is the **steady-state** heat conduction equation including heat production in 1D
  - Steady state is the same as time independent, meaning the solution does not change with time

With a solution to this equation, we can plot a **geotherm**, or temperature with depth

Geotherms in the continental crust

With a 40-km-thick crust, you can see that heat production can increase the Moho temperature considerably

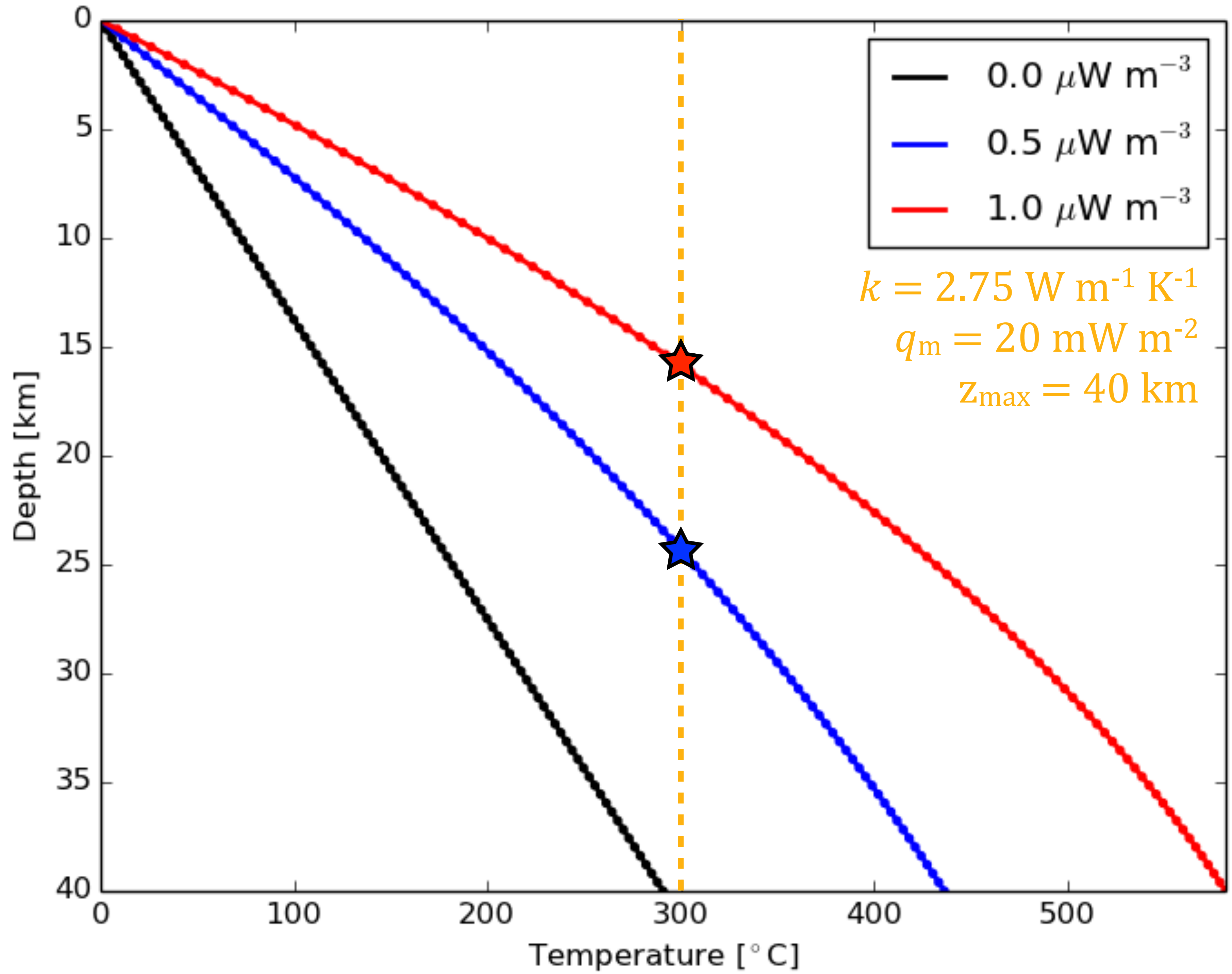
Note, that this model has a constant heat flow basal boundary condition



Geotherms in the continental crust

Why might this matter?
The approximate depth at which crustal rocks transition from brittle deformation to ductile deformation corresponds roughly to the 300°C isotherm in the crust

Based on the models here, that depth could vary from ~15-40 km (!), which has serious implications for how the crust deforms



Distribution of heat producing elements

Rock type	Heat production By mass, H [W kg^{-1}]	By volume, A [W m^{-3}]	By volume, A [μW m^{-3}]
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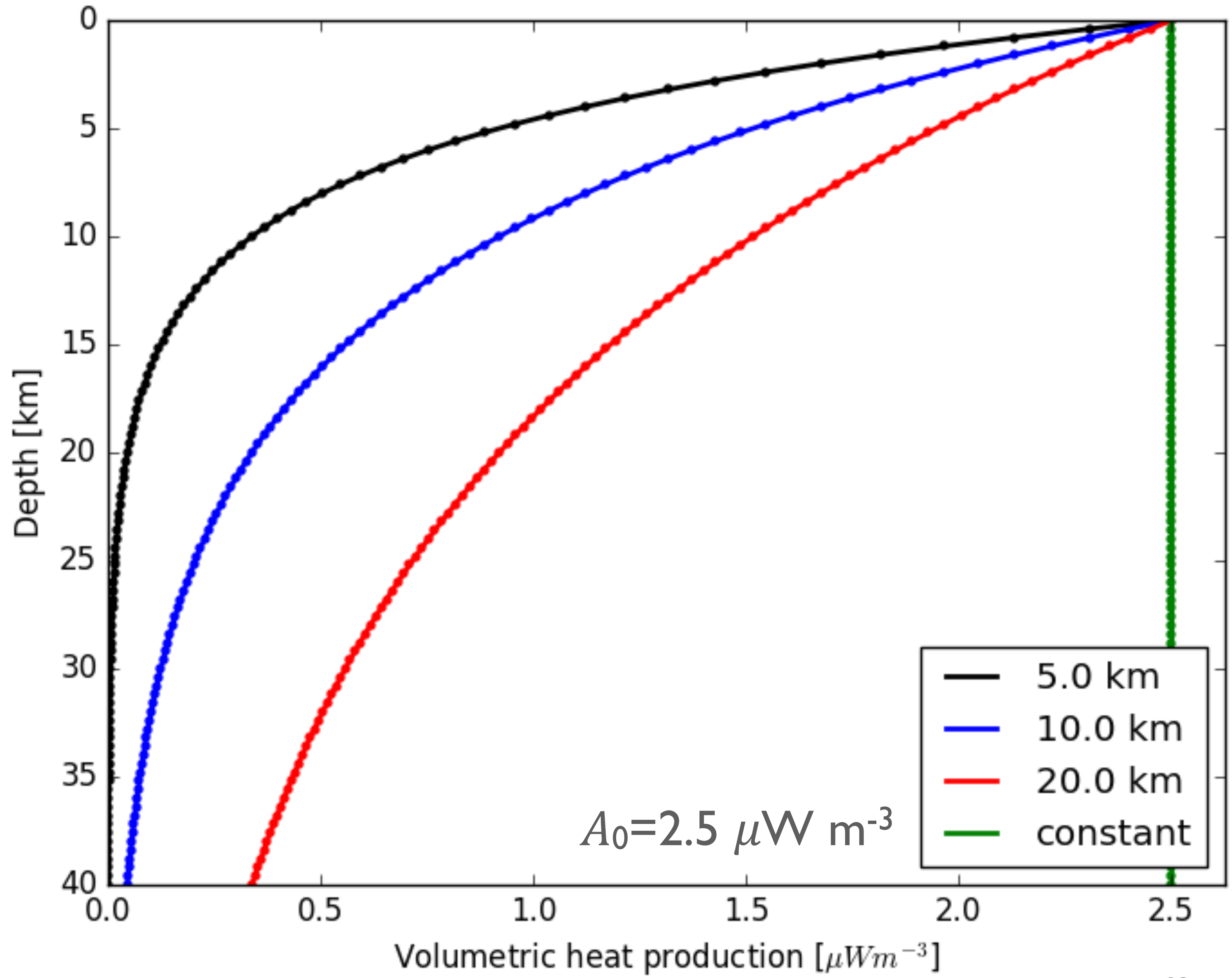
- As we've seen, however, the concentration of heat-producing elements is not constant in the crust, but decreases with depth

Exponential decay of heat production

Heat production is commonly modelled as decreasing exponentially with depth, with the rate of exponential set by the **e-folding depth**, the depth at which the concentration decreases by 1/e from the concentration at the Earth's surface

Note: e = 2.71828182846



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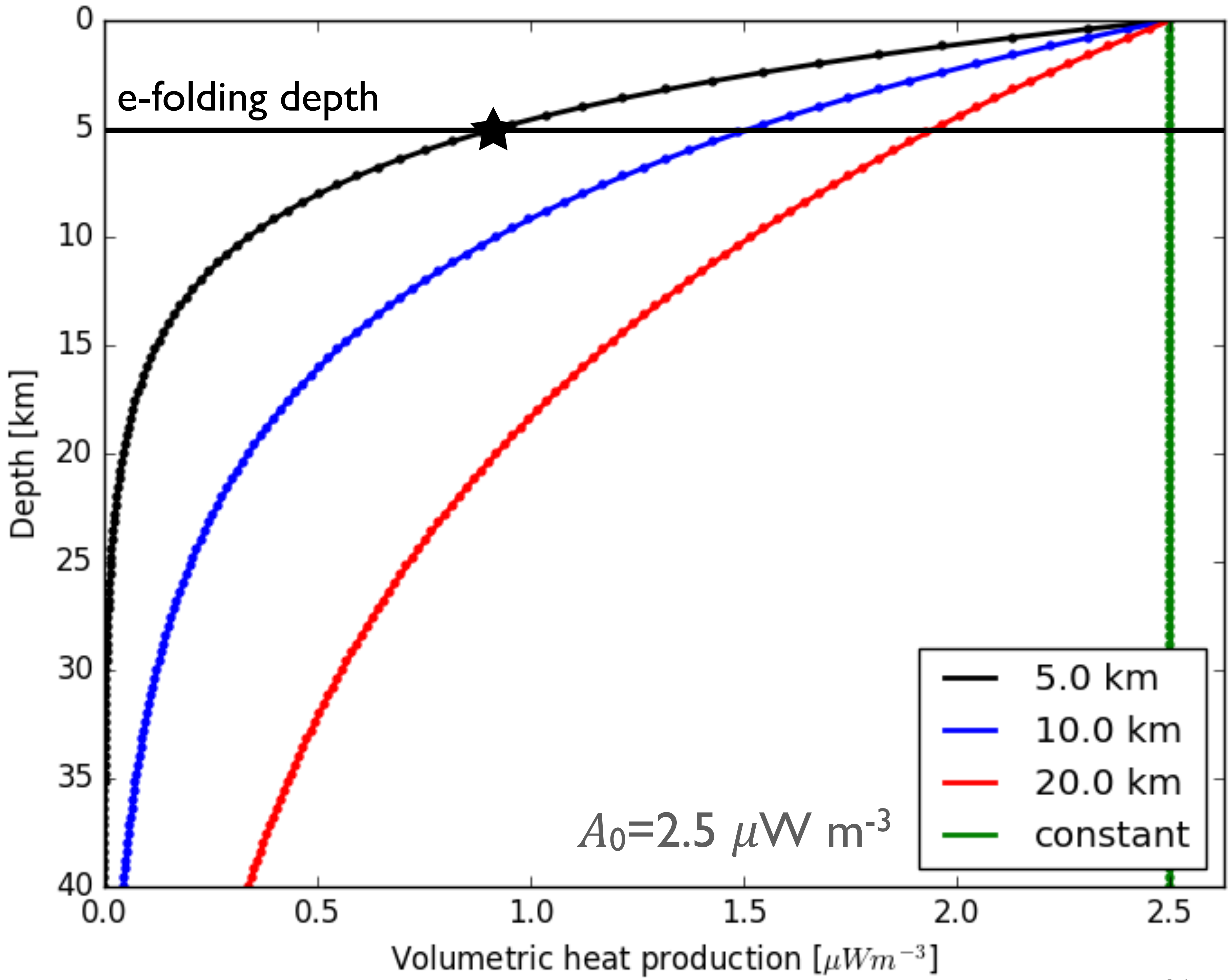

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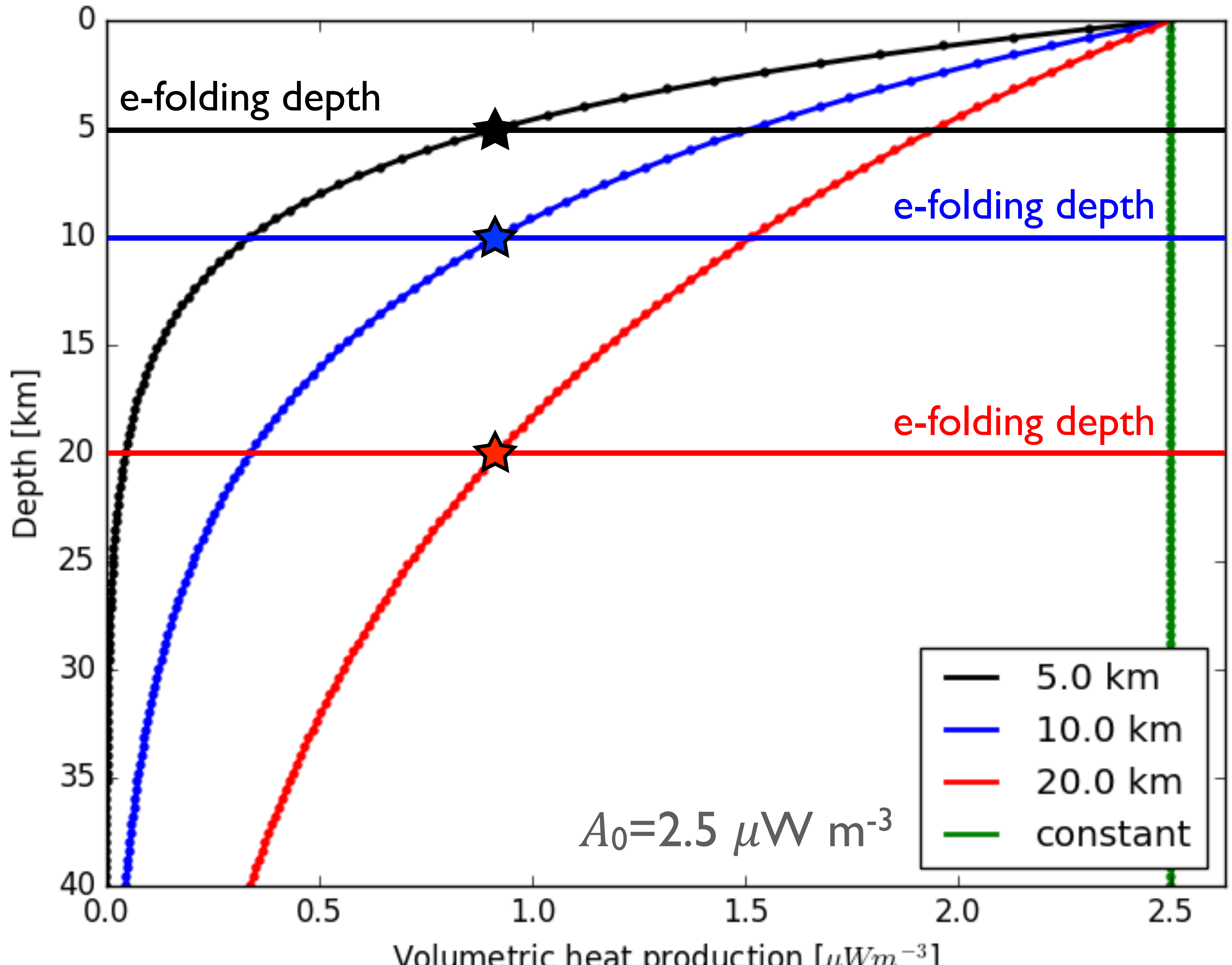

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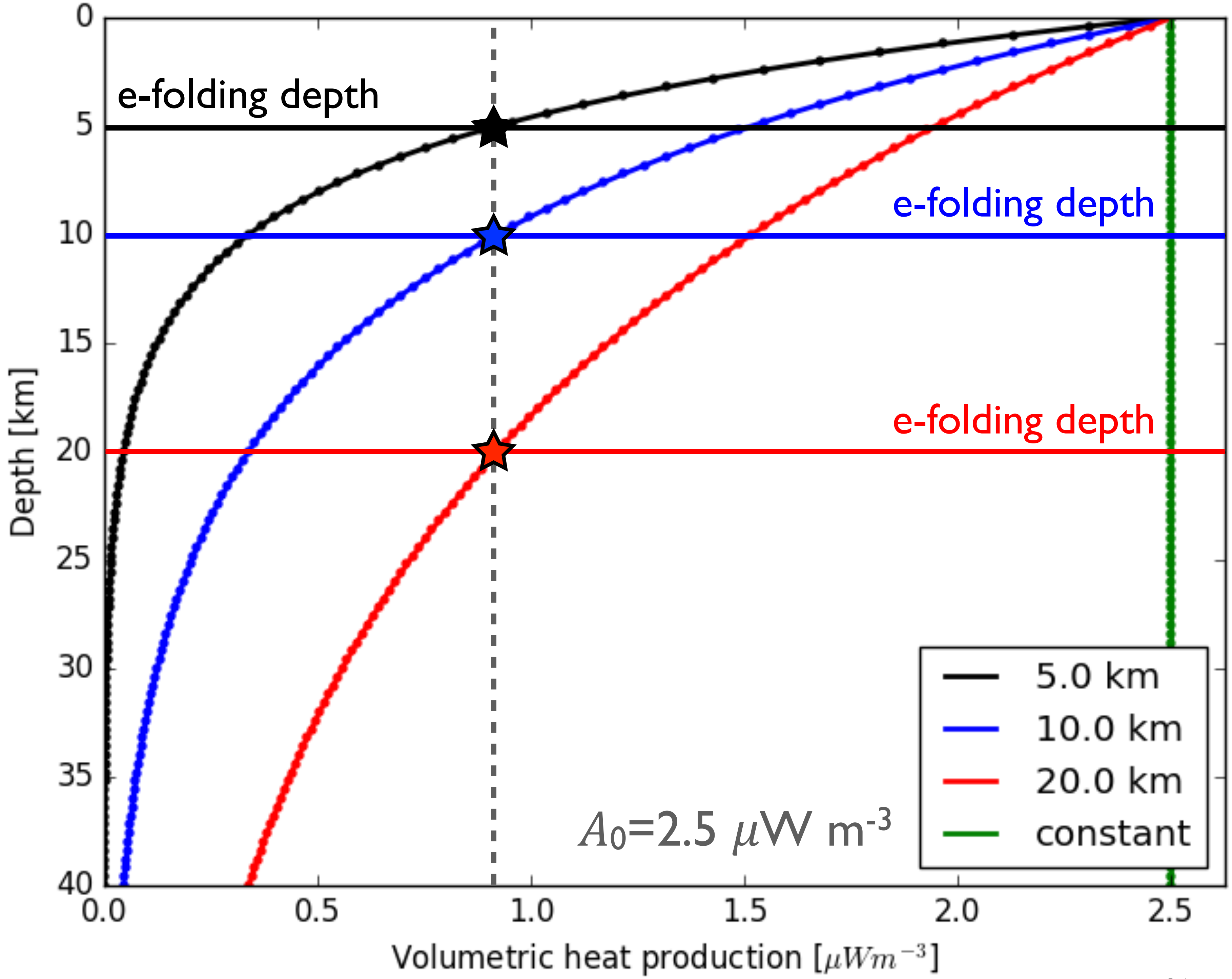
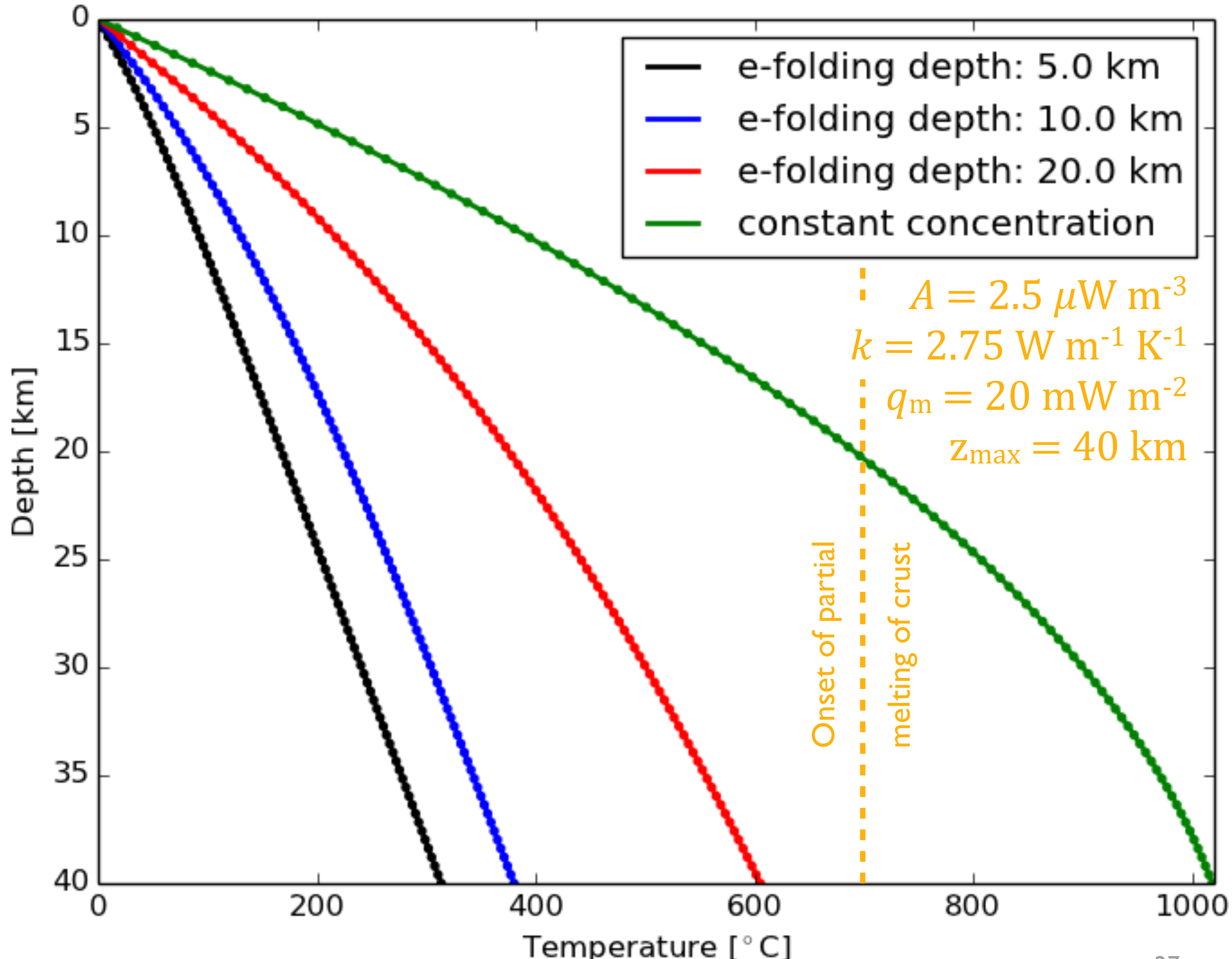
2.5 / e = 0.9197



Geotherms in the continental crust

As before, changing the concentration of heat-producing elements shows us something important about how we might expect the crust to deform

Here, <u>we might predict partial (to complete) melting of the crust and very weak rocks if we don't consider decreasing heat production</u>



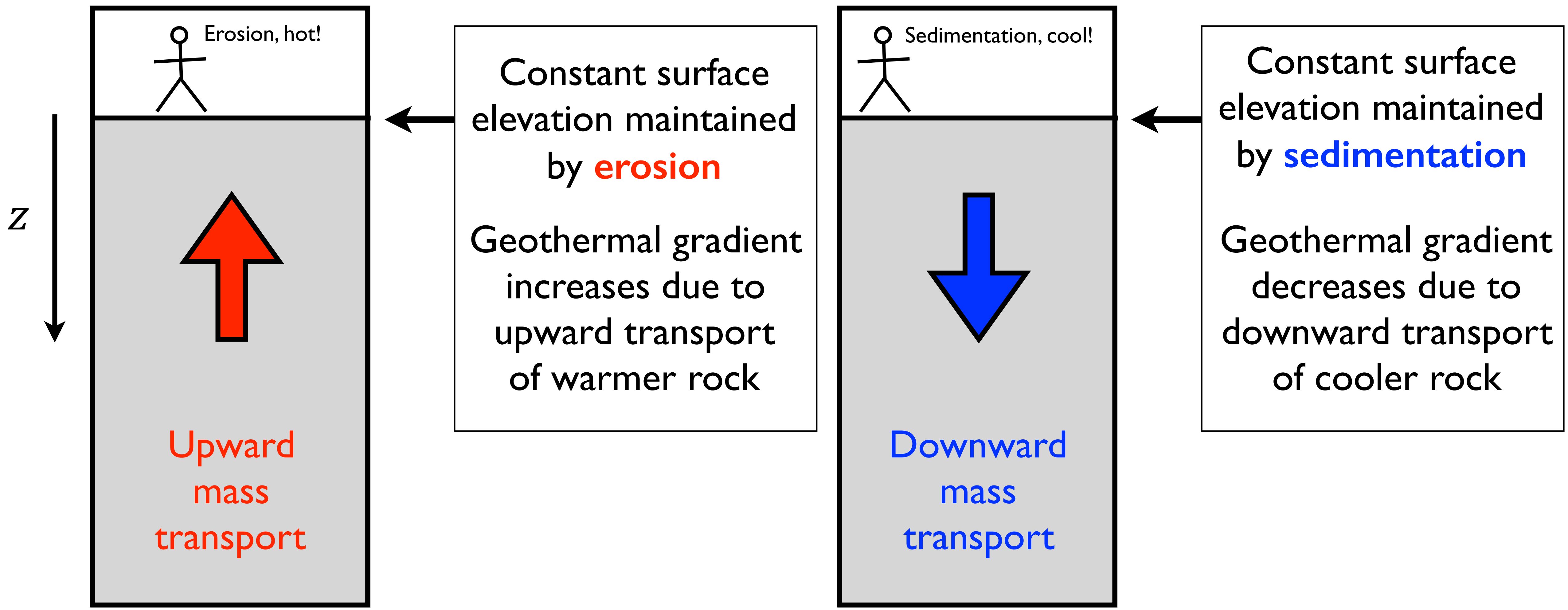
Heat transfer processes in the lithosphere

- **Conduction**: The diffusive transfer of heat by <u>kinetic atomic or molecular interactions</u> within the material. Also known as thermal diffusion.

- **Production**: Not really a heat transfer process, but rather a source of heat. Sources in the lithosphere include radioactive decay, friction in deforming rock or chemical reactions such as phase transitions.

- **Advection**: The transfer of heat by <u>physical movement</u> of molecules or atoms within a material. A type of convection, mostly applied to heat transfer in solid materials.

Advection by movement of solid rock



- Advection of heat by movement of solid rock is common
  - Both erosion (upward advection of rock) and sedimentation (downward advection of rock) can significantly modify the thermal field in the crust

Mathematical description of advection

- Advection in the vertical direction at velocity v_z at steady state can be represented mathematically as

$$v_z \frac{\partial T}{\partial z} = 0$$

- If we ignore heat production, we can combine the equation above with the 1D heat conduction equation to get the **1D advection-diffusion equation**

$$\underbrace{\kappa \frac{\partial^2 T}{\partial z^2}}_{\text{Diffusion}} + \underbrace{v_z \frac{\partial T}{\partial z}}_{\text{Advection}} = 0$$

where κ is a material property called the **thermal diffusivity**, which is the thermal conductivity k divided by the material density ρ and heat capacity c_p

$$\kappa = k / \rho c_\mathrm{p}$$

Time-dependent advection and diffusion

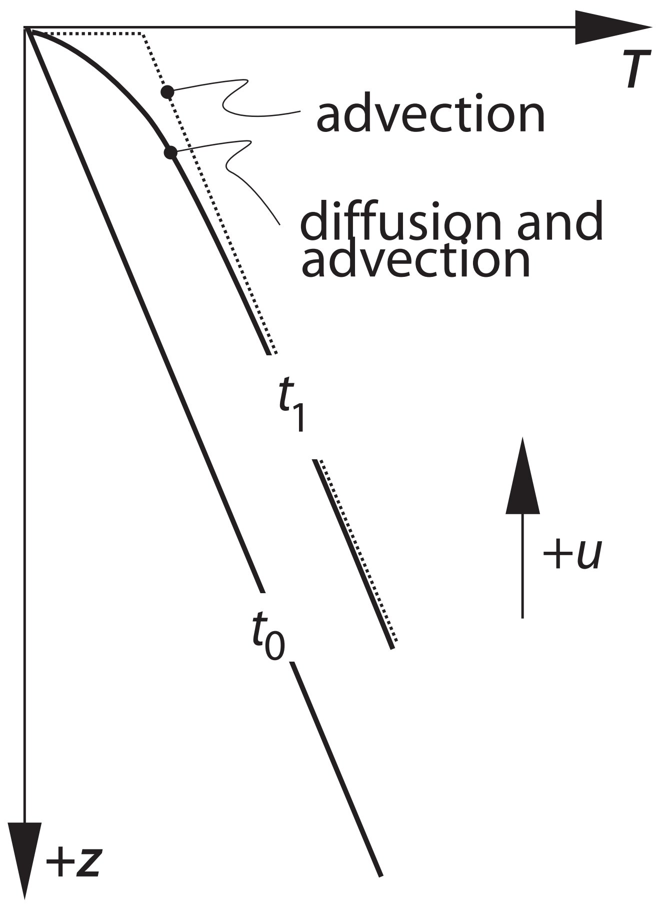
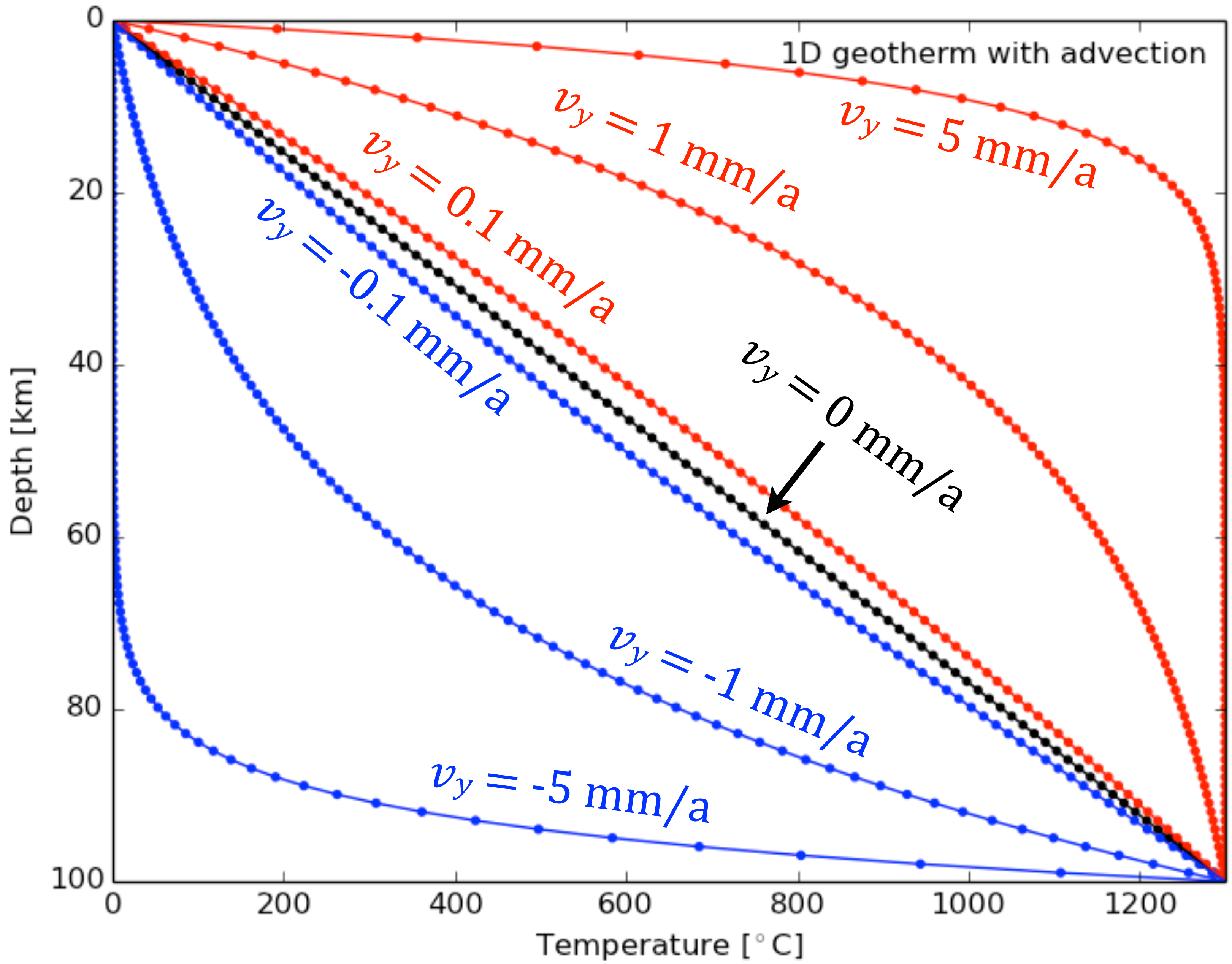


Fig. 3.13, Stüwe, 2007

When does advection matter?

- Advection is particularly relevant in two geodynamic situations
  - Advection by **movement of solid rock**
    - Example: Erosion, faulting
  - Advection by **fluid circulation**
    - Example: Groundwater flow

Advection by movement of solid rock



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Time dependent heat transfer

- Thus far, we have considered heat conduction and heat production in a **steady state**, meaning the temperature equations did not depend on time
- For example, we saw the 1D steady-state heat conduction equation with heat production
$$k\frac{d^2T}{dz^2} + \rho H = 0$$
which clearly does not depend on time (no t or derivatives with respect to t in the equation)

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- For example, we saw the 1D steady-state heat conduction equation with heat production
$$k\frac{d^2T}{dz^2} + \rho H = 0$$
which clearly does not depend on time (no t or derivatives with respect to t in the equation)
- In some geologic scenarios, the assumption of steady state is reasonable, but many others require consideration of their time evolution
  - For example, problems involving emplacement of magma require time to consider cooling of the magma body

Time dependent heat transfer

- In steady state, the 1D heat conduction equation with heat production is

$$0 = k\frac{d^2T}{dz^2} + \rho H$$

where k is thermal conductivity, ρ is density and H is heat production by mass

Time dependent heat transfer

- In steady state, the 1D heat conduction equation with heat production is

$$0 = k\frac{d^2T}{dz^2} + \rho H$$

where k is thermal conductivity, ρ is density and H is heat production by mass

- The time-dependent equivalent equation for heat conduction with production is

$$\rho c\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial z^2} + \rho H$$

where c is heat capacity, with typical values of 800-1000 J kg^{-1} K^{-1} for rock

Time dependent heat transfer

- If we ignore heat production, the equation simplifies to

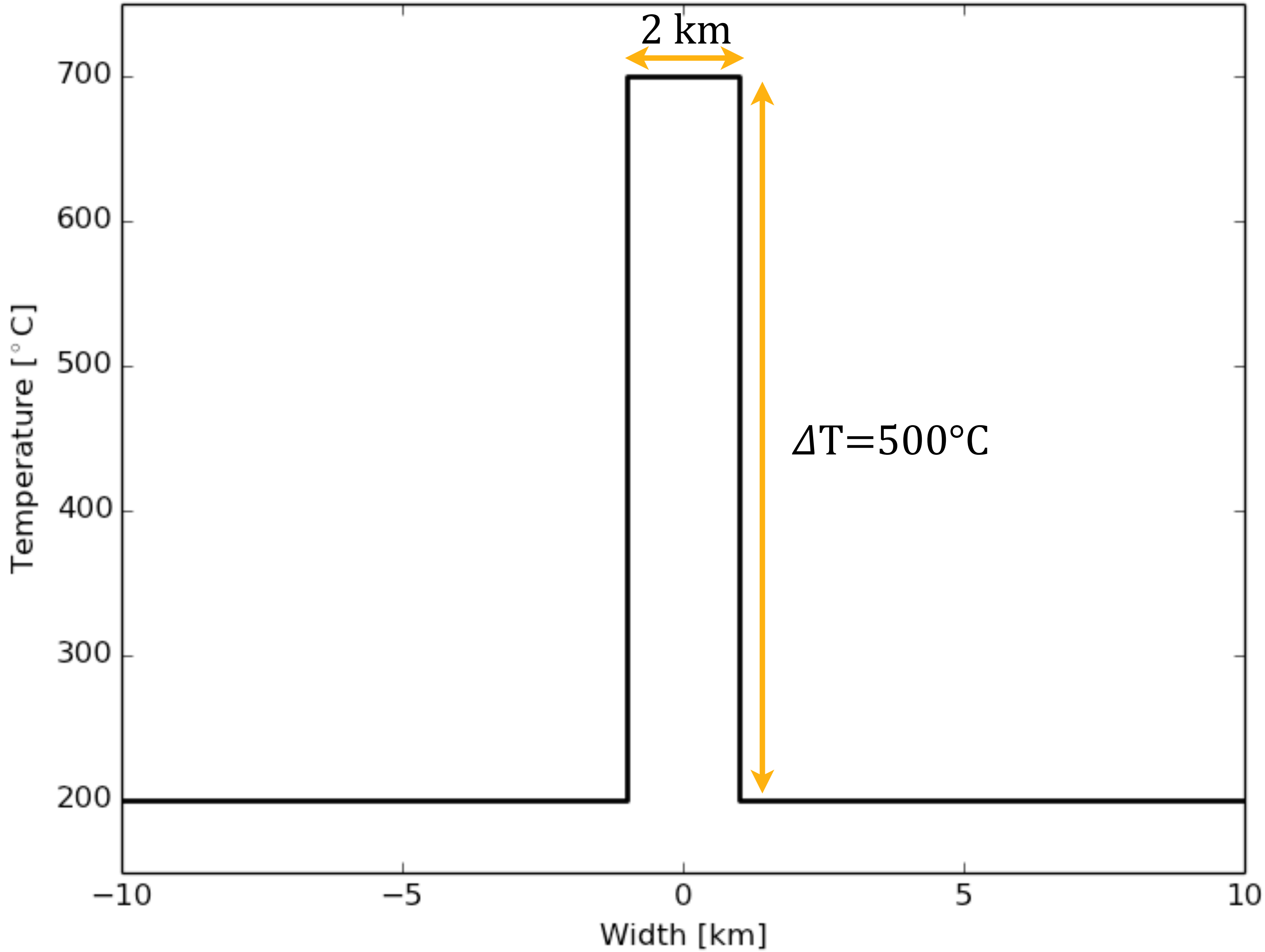
$$\rho c \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial z^2}$$

and if we divide both sides by ρc then we find a typical form of the time-dependent heat conduction equation in 1D

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial z^2}$$

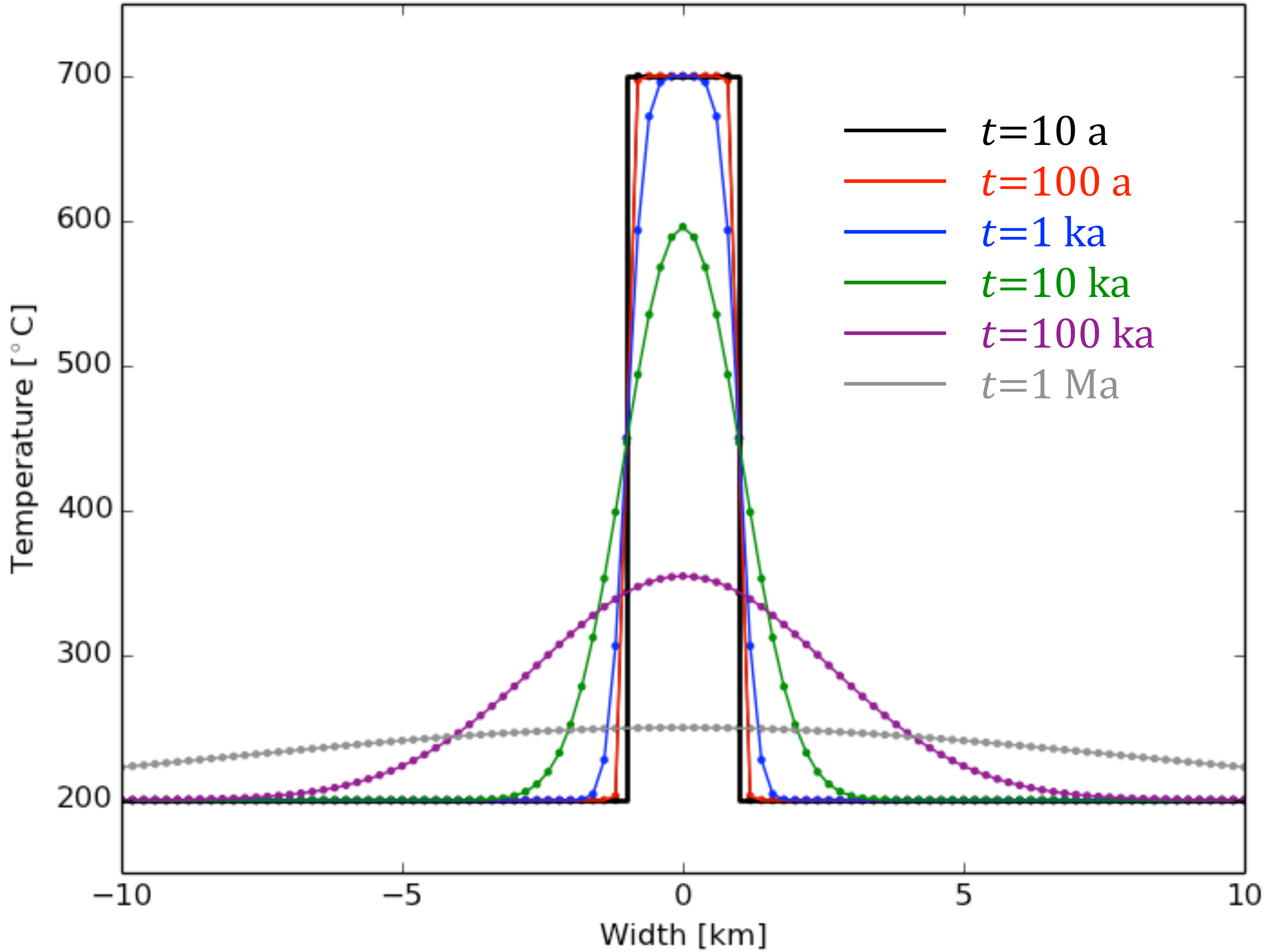
where κ is the thermal diffusivity with a typical value of 10^{-6} m^2 s^{-1}

Thermal evolution of a 1D intrusion



- The intrusion is one-dimensional with a defined thickness and infinite length and depth

Thermal evolution of a 1D intrusion

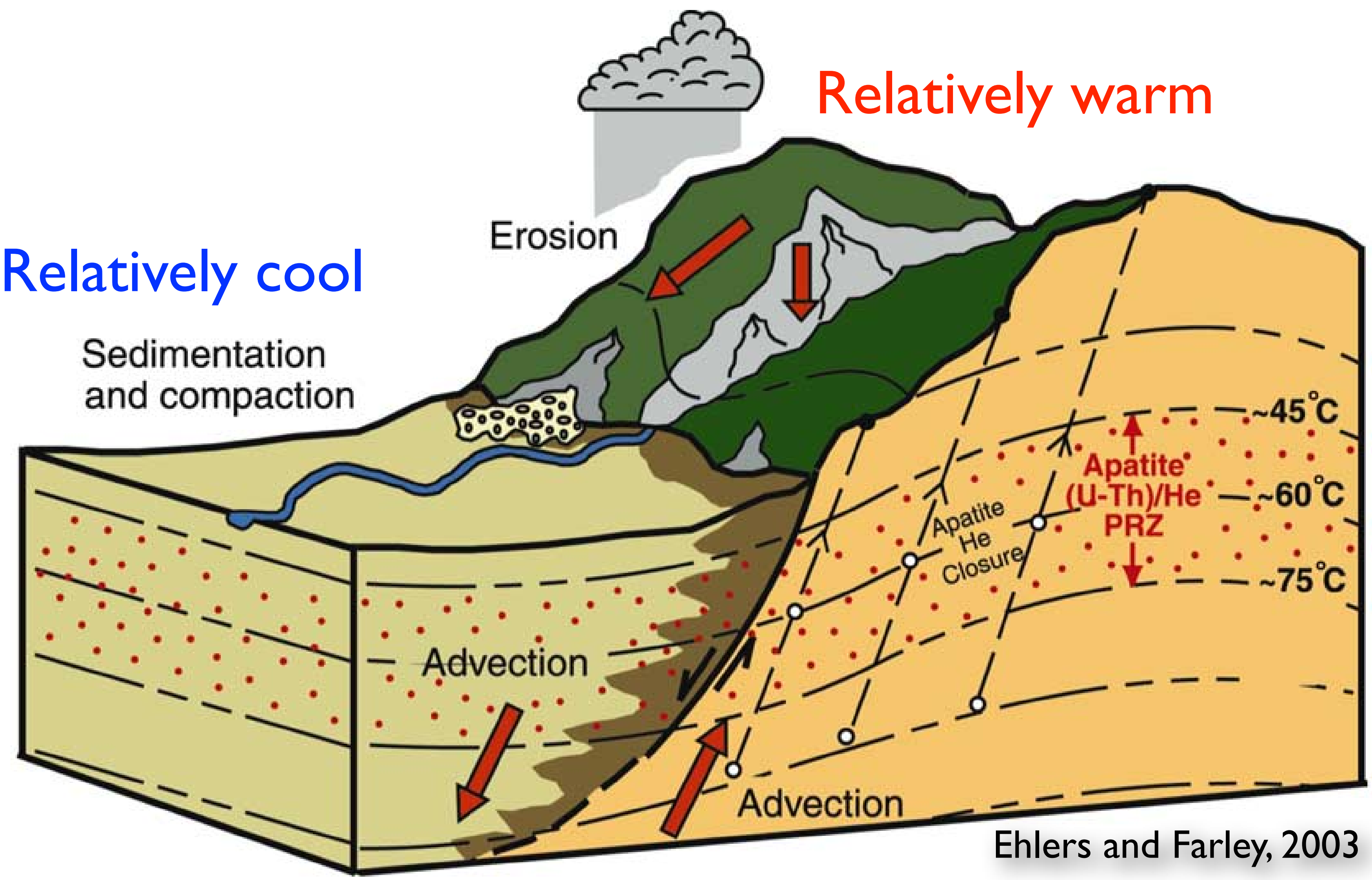


- The intrusion slowly cools over time, in this case ignoring the heat released during solidification of the body

Thermal structure of active tectonic areas

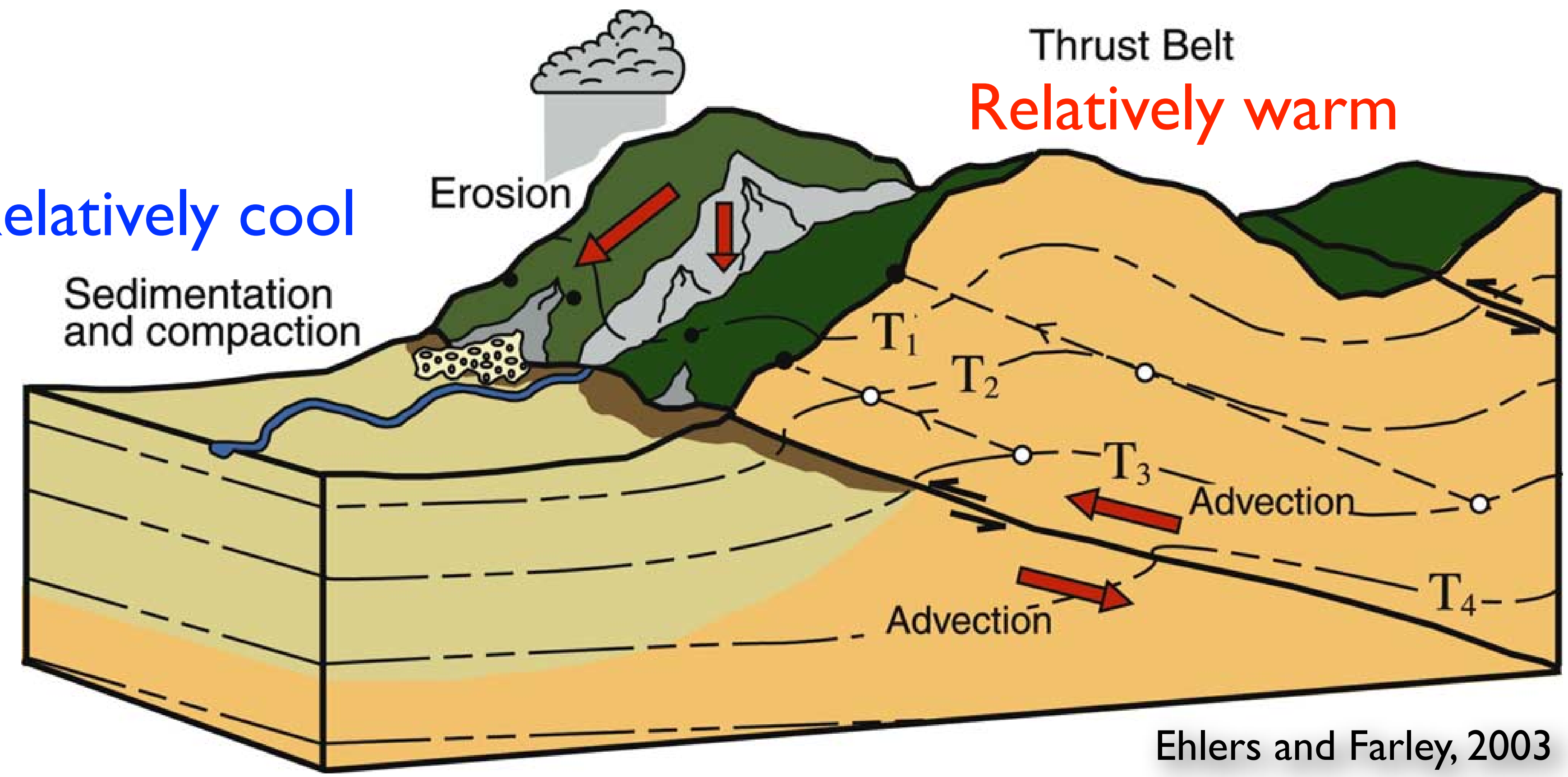
- Active tectonic areas
  - Mountainous active orogens and foreland basins in extensional and contractional settings
  - Subduction zones

Thermal structure in extensional settings



- In active extensional orogens, erosion of the uplifting footwall increases the thermal gradient, whereas sedimentation in the neighboring basin decreases the geothermal gradient
  - This combined effect, from slip on the bounding fault can significantly bend shallow crustal isotherms

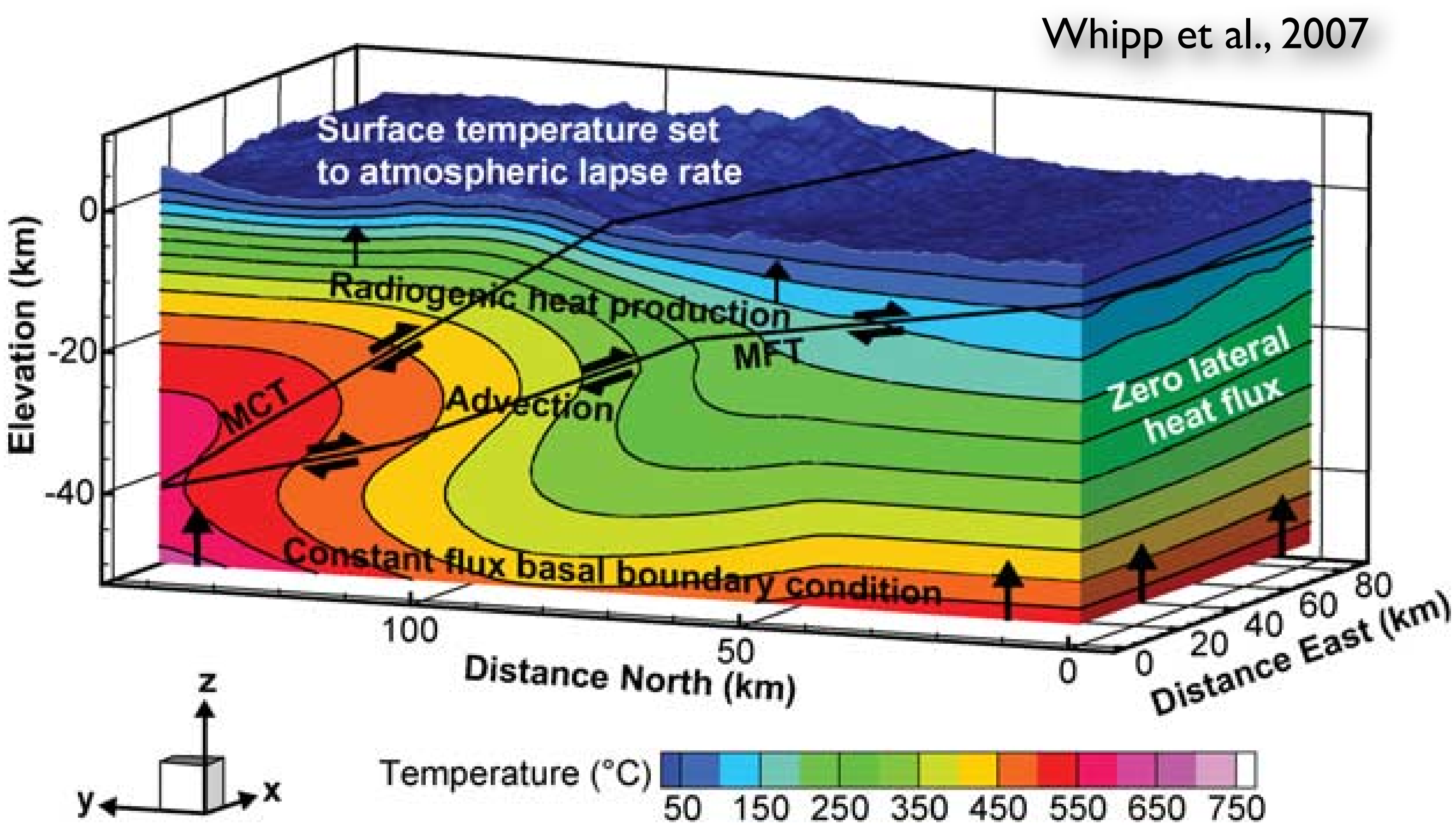
Thermal structure in contractional settings



Ehlers and Farley, 2003

- In active contractional orogens, the hanging wall of thrusts are relatively warm due to uplift and erosion, with sedimentation cooling the foreland basin region
- Isotherms in the shallow and even middle crust <u>can be heavily perturbed, even inverting the thermal gradient in places</u>

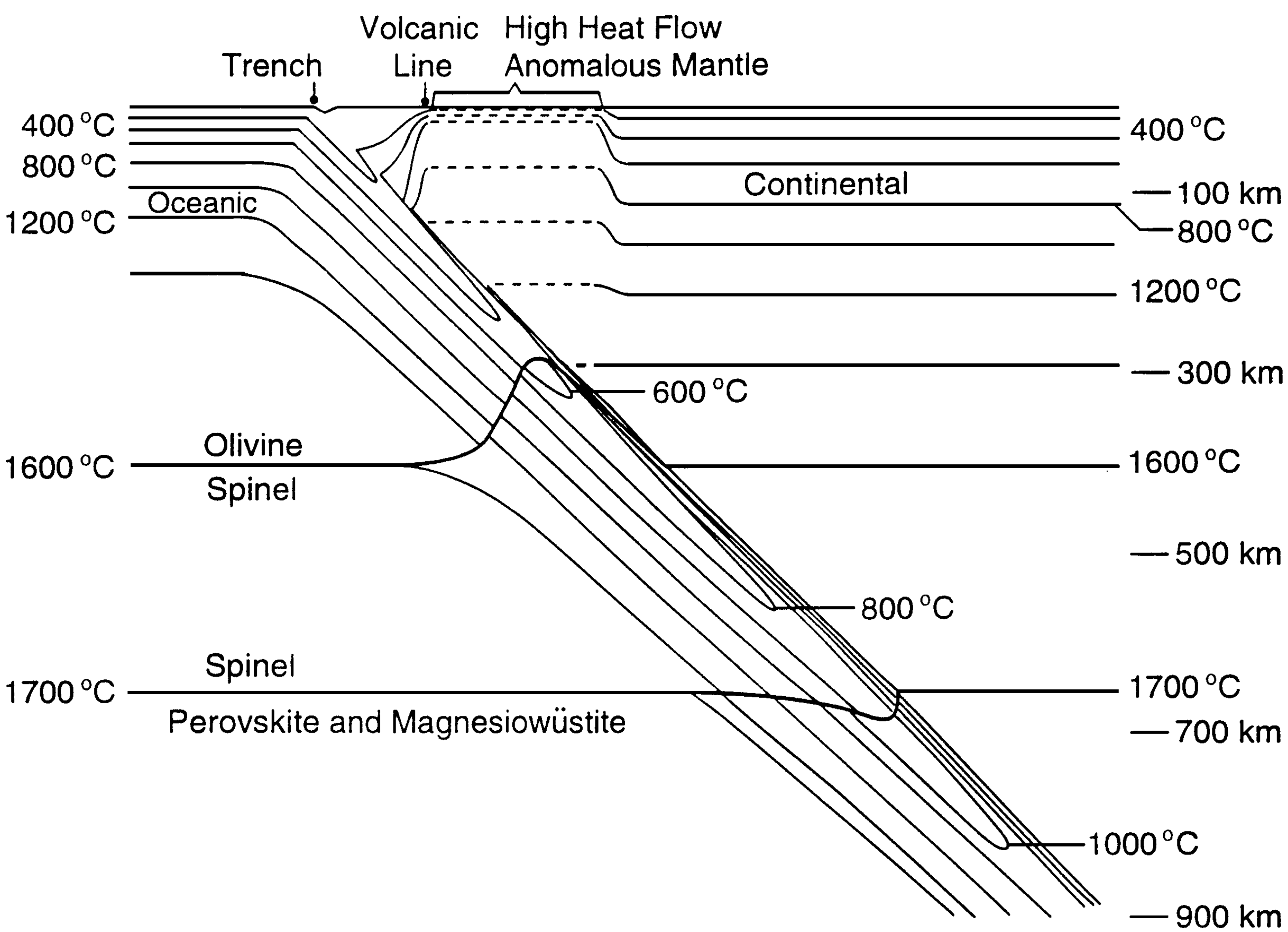
3D thermal model of the Nepalese Himalaya



- This is an example where rapid convergence (20 mm/a) is accommodated by a basal thrust fault

- Slip on this fault and advection of heat bends the isotherms and inverts the thermal gradient locally in the footwall

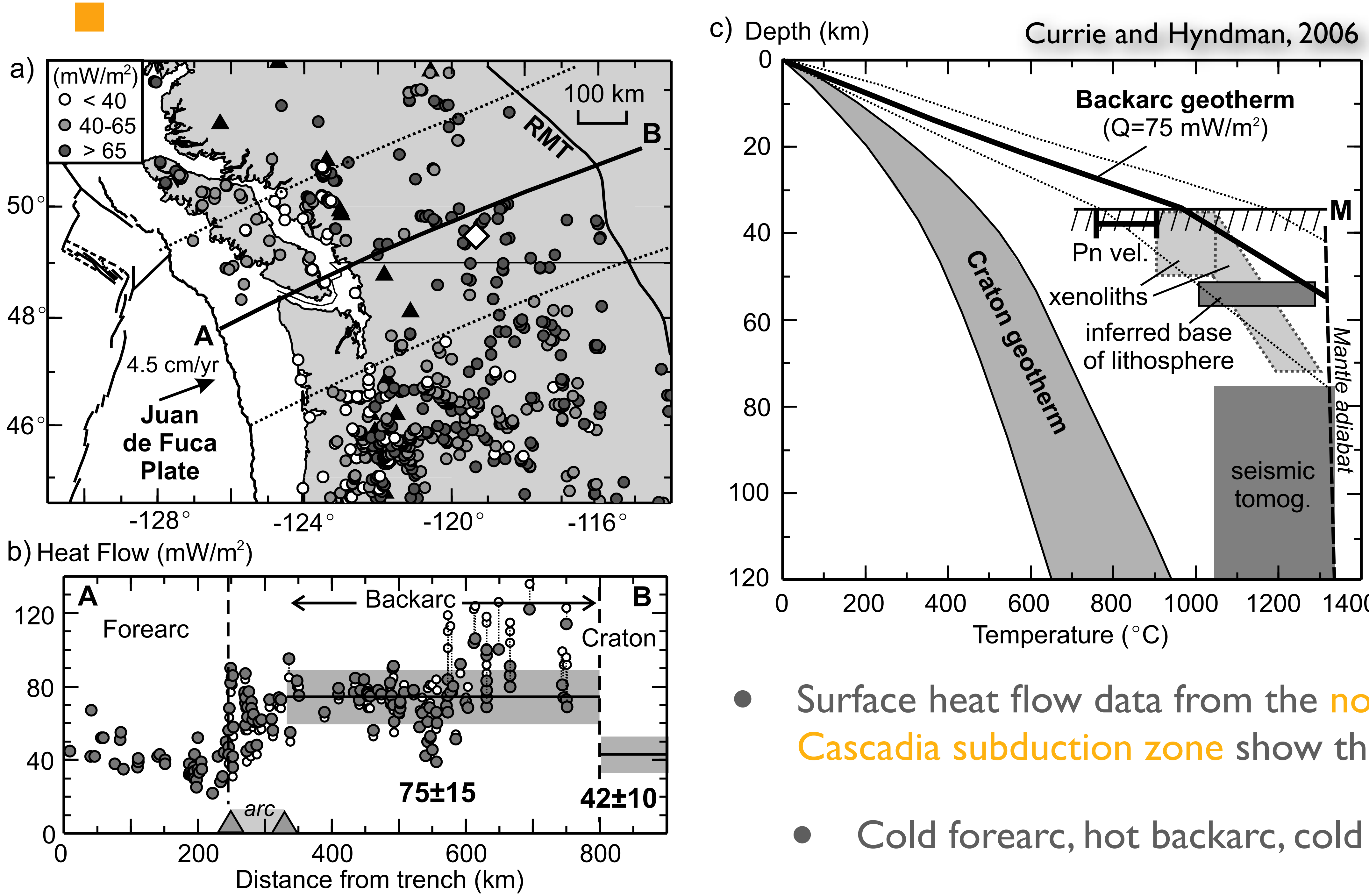
Thermal field in subduction zones



Turcotte and Schubert, 2014

- Subduction zones have a highly deformed thermal structure
- Advection resulting from the downgoing cold oceanic slab penetrates deeply into the mantle
- The forearc, on the oceanic side of the volcanic line, is generally cool
- In the backarc, corner flow in the mantle heats the overlying lithosphere
- Further inboard, the continental cratonic region shows a typical thermal structure (fairly cool)

Thermal field in subduction zones



- Surface heat flow data from the northern Cascadia subduction zone show the ‘typical’ signal
  - Cold forearc, hot backarc, cold craton
  - Steep backarc geotherm, shallow cratonic geotherm

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