Introduction to lithospheric geodynamic modelling

Rheology of the lithosphere

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Goals of this lecture

- Present the equations used to model **nonlinear viscous flow** and **frictional plasticity**
- Discuss the **strength of the lithosphere** and how it is modelled

Terminology, clarified

- There are five words used to describe rock deformation that are frequently misused, confused and poorly understood:
  - **Brittle**
  - **Ductile**
  - **Elastic**
  - **Plastic**
  - **Viscous**

Terminology, clarified

- There are five words used to describe rock deformation that are frequently misused, confused and poorly understood:
  - **Brittle**
  - **Ductile**

  Deformation ‘types’

  - **Elastic**
  - **Plastic**
  - **Viscous**

  Deformation mechanisms

Terminology, clarified

- '**Type**' of deformation
  - **Brittle**: Fracture of rock with possible slip along the fracture surface (fault); Relatively low T and P, large forces or rapid imposed deformation
  - **Ductile**: Flow or coherent change in the rock in the solid crystalline state; Relatively high T and P, small forces, slow imposed deformation

Reminder: (Linear) Viscous deformation

- In simple shear,

  $\tau_s = \eta \dot{\varepsilon}_s$ $\quad$ η Dynamic viscosity

  Shear stress proportional to **shear strain rate**

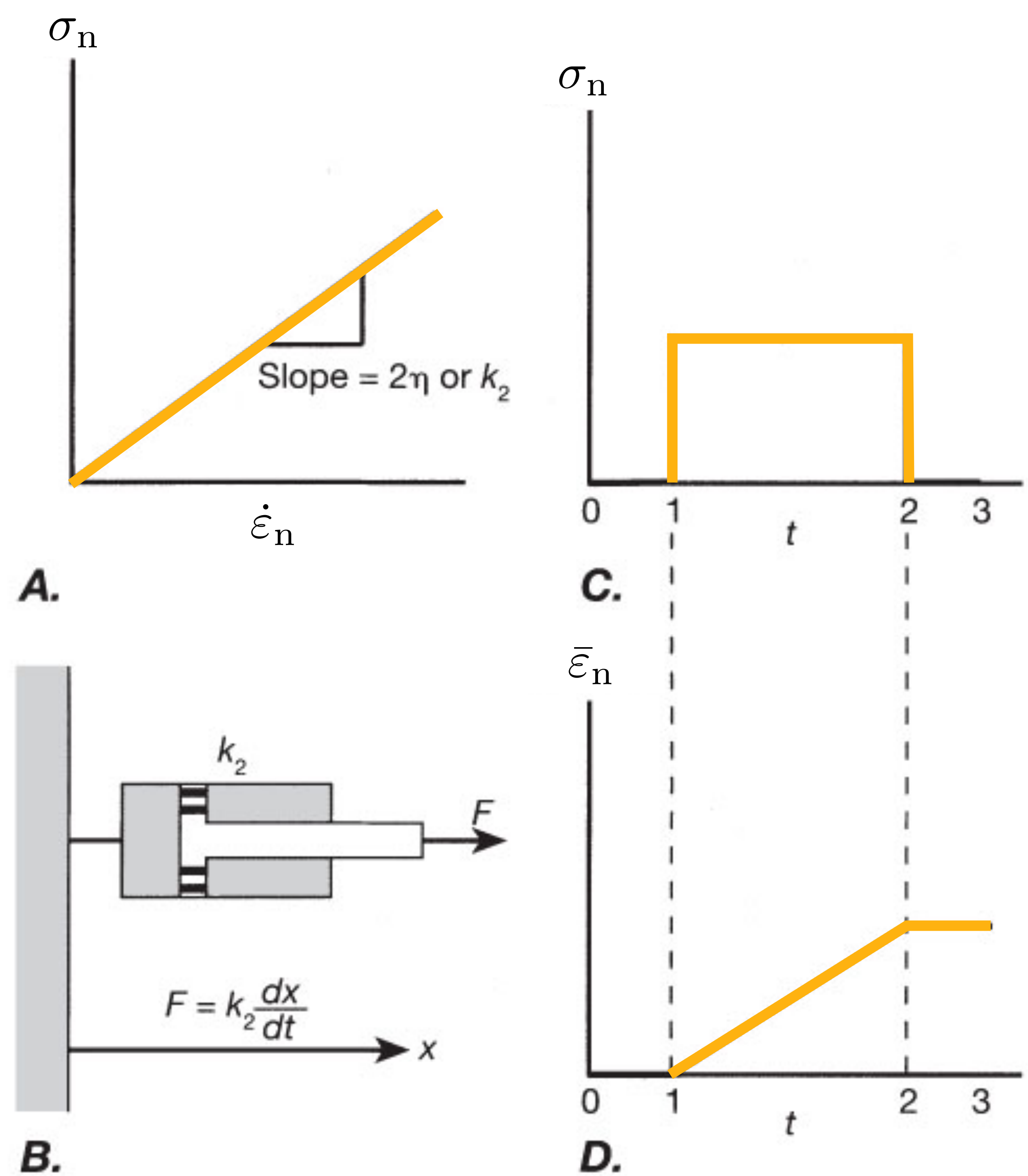
- In general,

  $\sigma' = 2\eta\dot{\varepsilon}$

  deviatoric stress is proportional to **strain rate**

- For linear viscous (Newtonian) materials, η **is constant**

- Nonrecoverable



Twiss and Moores, 2007

Viscous flow - Nonlinear viscous deformation

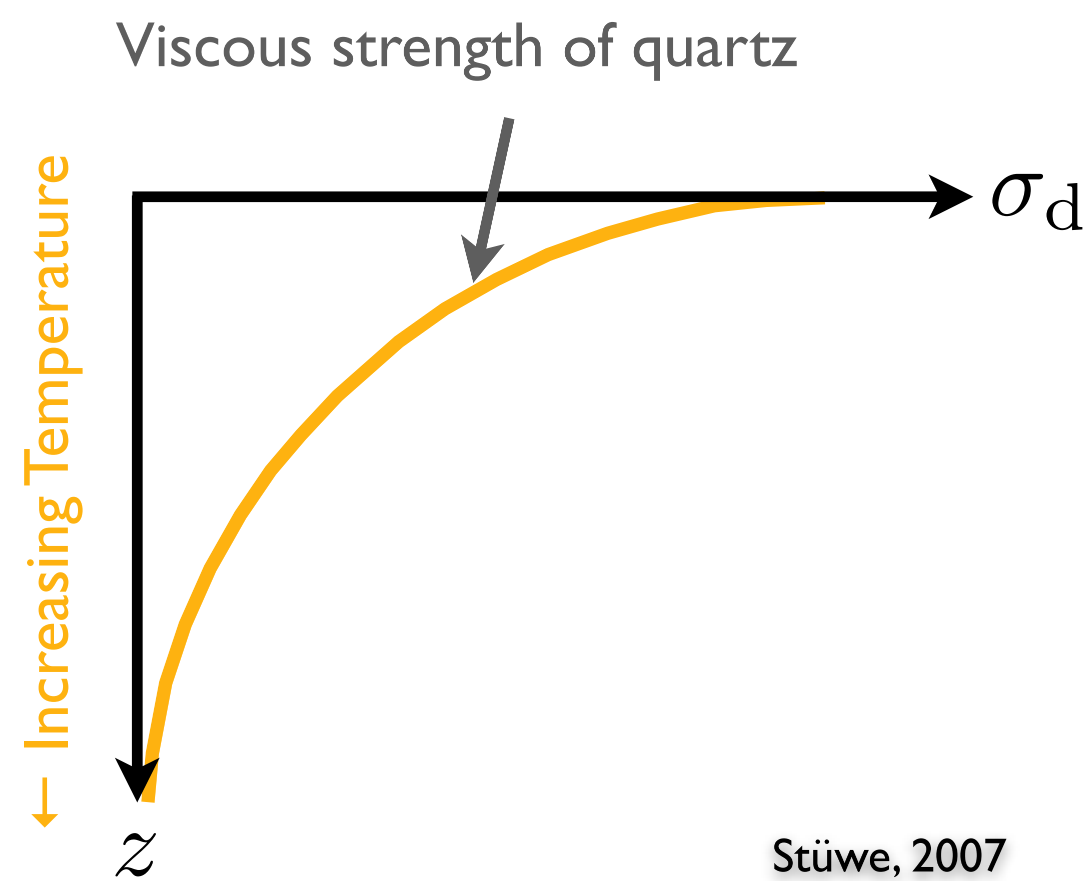
- Most rocks do not behave as Newtonian viscous materials
- Why not?

Viscous flow - Nonlinear viscous deformation

- Most rocks <u>do not</u> behave as Newtonian viscous materials
- Why not?
- Two main reasons:
    - Temperature dependence

      $$\eta = A_0 \exp\left(Q/RT_\mathrm{K}\right)$$

      A_0 is the pre-exponent constant, Q is the activation energy, R is the universal gas constant and T_K is temperature in Kelvins



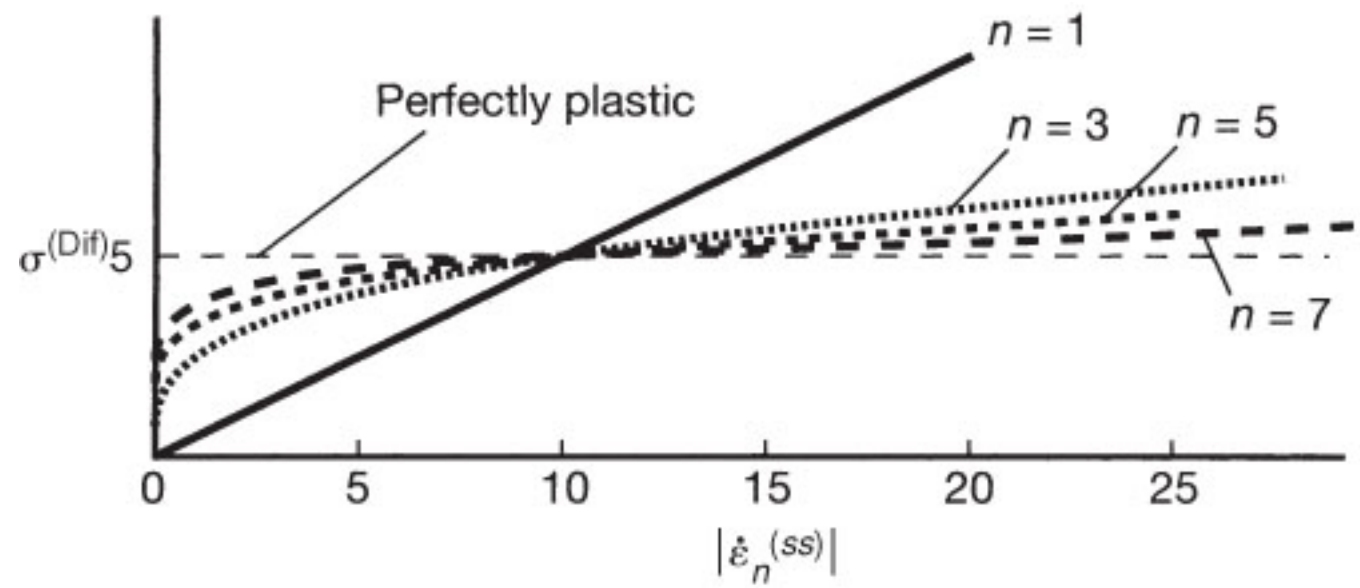
Stüwe, 2007

Viscous flow - Nonlinear viscous deformation

- Most rocks do not behave as Newtonian viscous materials
- Why not?
- Two main reasons:
  - Nonlinearity

    $\tau_s^n = A_{\mathrm{eff}} \dot{\varepsilon}_s$

    n is the power law exponent and
    A_{eff} is a material constant in $\mathrm{Pa}^n\,\mathrm{s}$
  - Many rocks deform 8 times as fast when stress is doubled

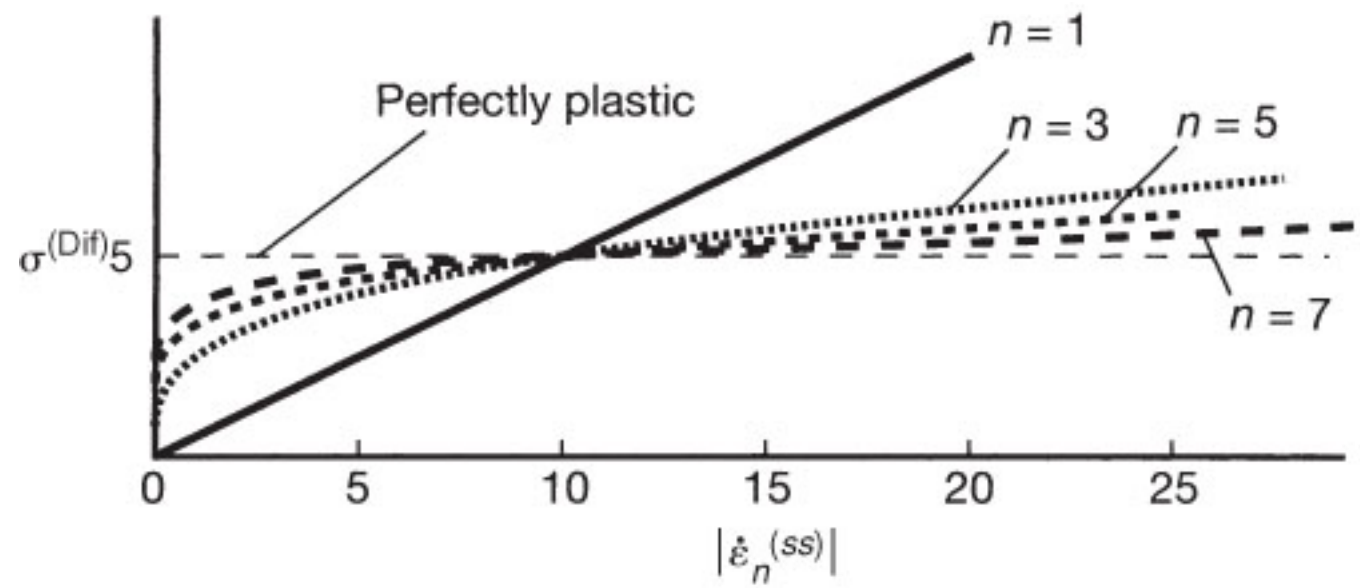


Twiss and Moores, 2007

Viscous flow - Nonlinear viscous deformation

- The units of A_{eff} are strange, so it is often easier to consider the **effective viscosity**

$$\eta_{\mathrm{eff}} = \frac{\tau_s}{\dot{\varepsilon}_s} = A_{\mathrm{eff}}^{1/n} \times \dot{\varepsilon}_s^{(1/n)-1}$$



Twiss and Moores, 2007

Viscous flow - Dorn's law

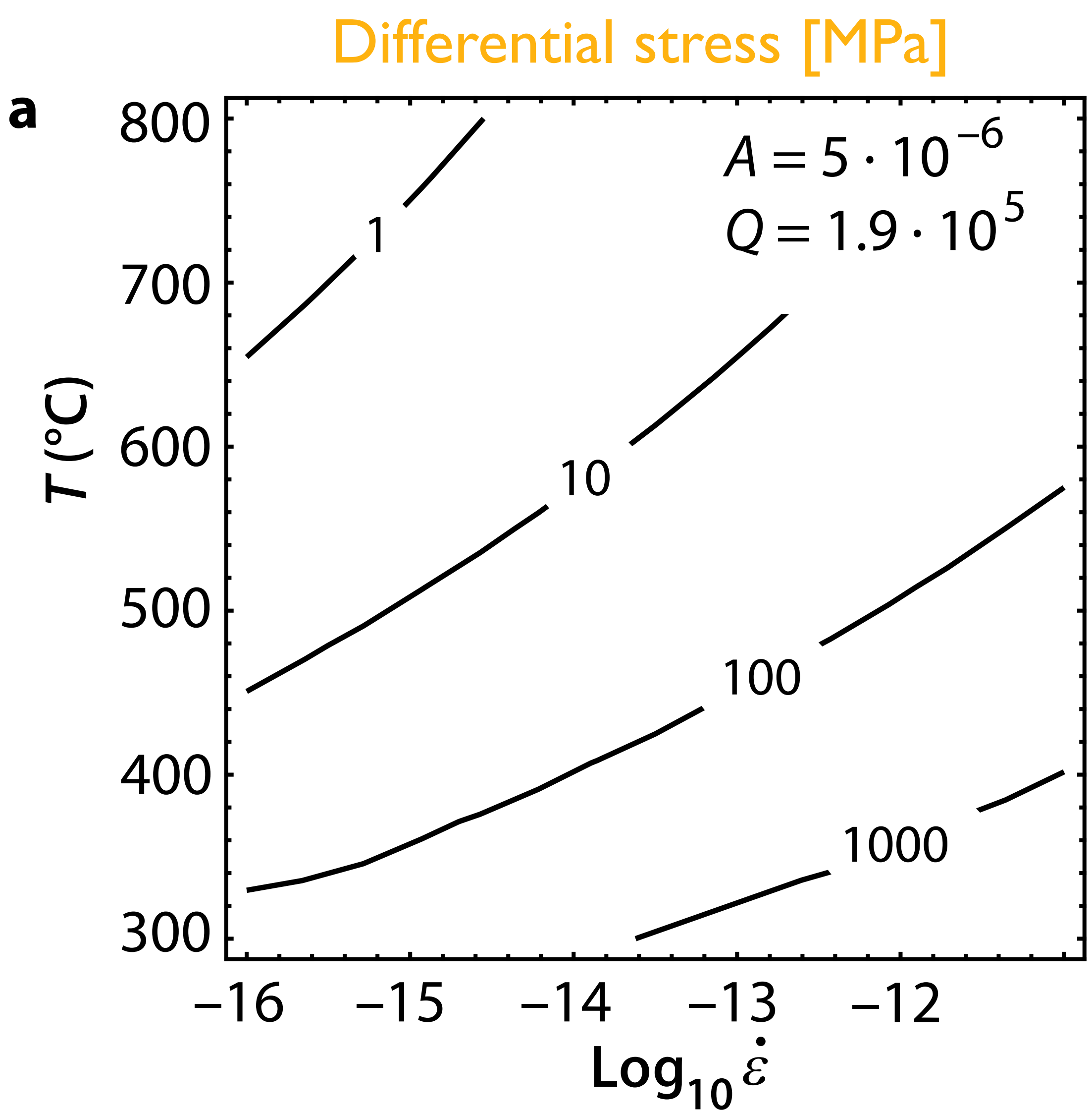


Fig. 5.12, Stüwe, 2007

- To investigate the behavior of **viscous** flow in the lithosphere, the power law viscosity equation is usually coupled with the Arrhenius relationship and given in terms of differential stress and uniaxial shortening rate

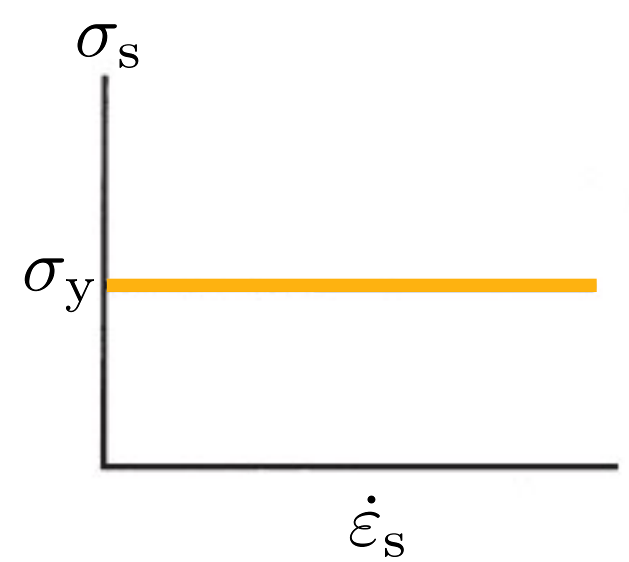
$$\sigma_{\mathrm{d}} = (\sigma_1 - \sigma_3) = \left(\frac{\dot{\varepsilon}_l}{A}\right)^{(1/n)} \exp\left(\frac{Q}{nRT}\right)$$

where A, Q, and n can be constrained experimentally

Plasticity - Perfectly plastic behavior

Twiss and Moores, 2007

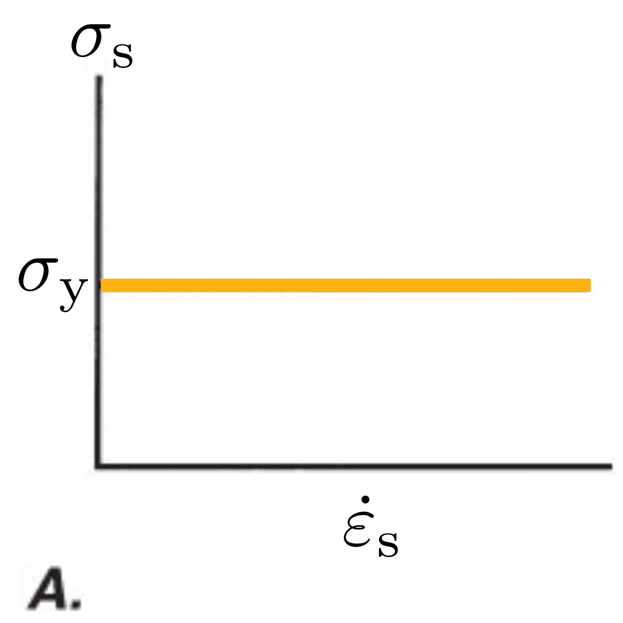
- Constant stress required for deformation



A.

Plasticity - Perfectly plastic behavior

Twiss and Moores, 2007



- **Constant stress** required for deformation
  - No deformation prior to exceeding yield stress
  - Infinite deformation if applied stress equals (or exceeds) yield stress

$$\begin{cases} \sigma < \sigma_y & \text{no deformation} \\ \sigma = \sigma_y & \text{failure; infinite deformatio} \end{cases}$$

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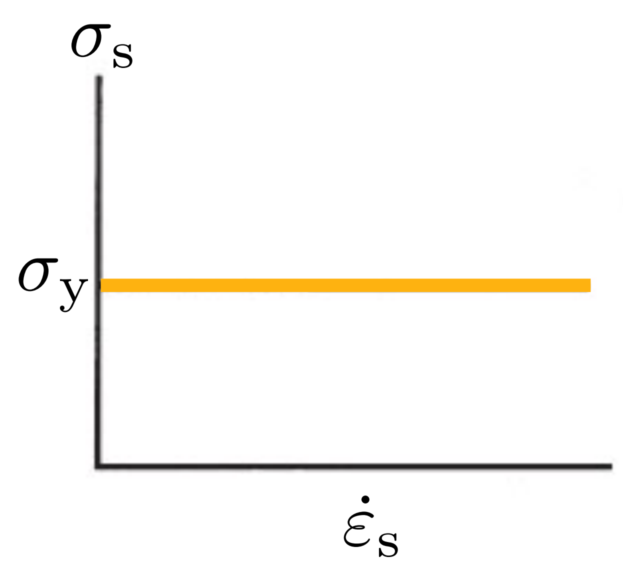
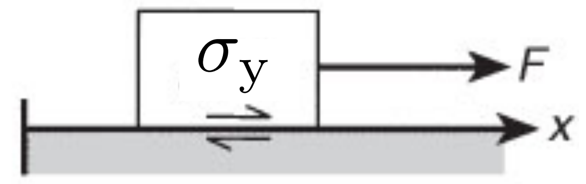


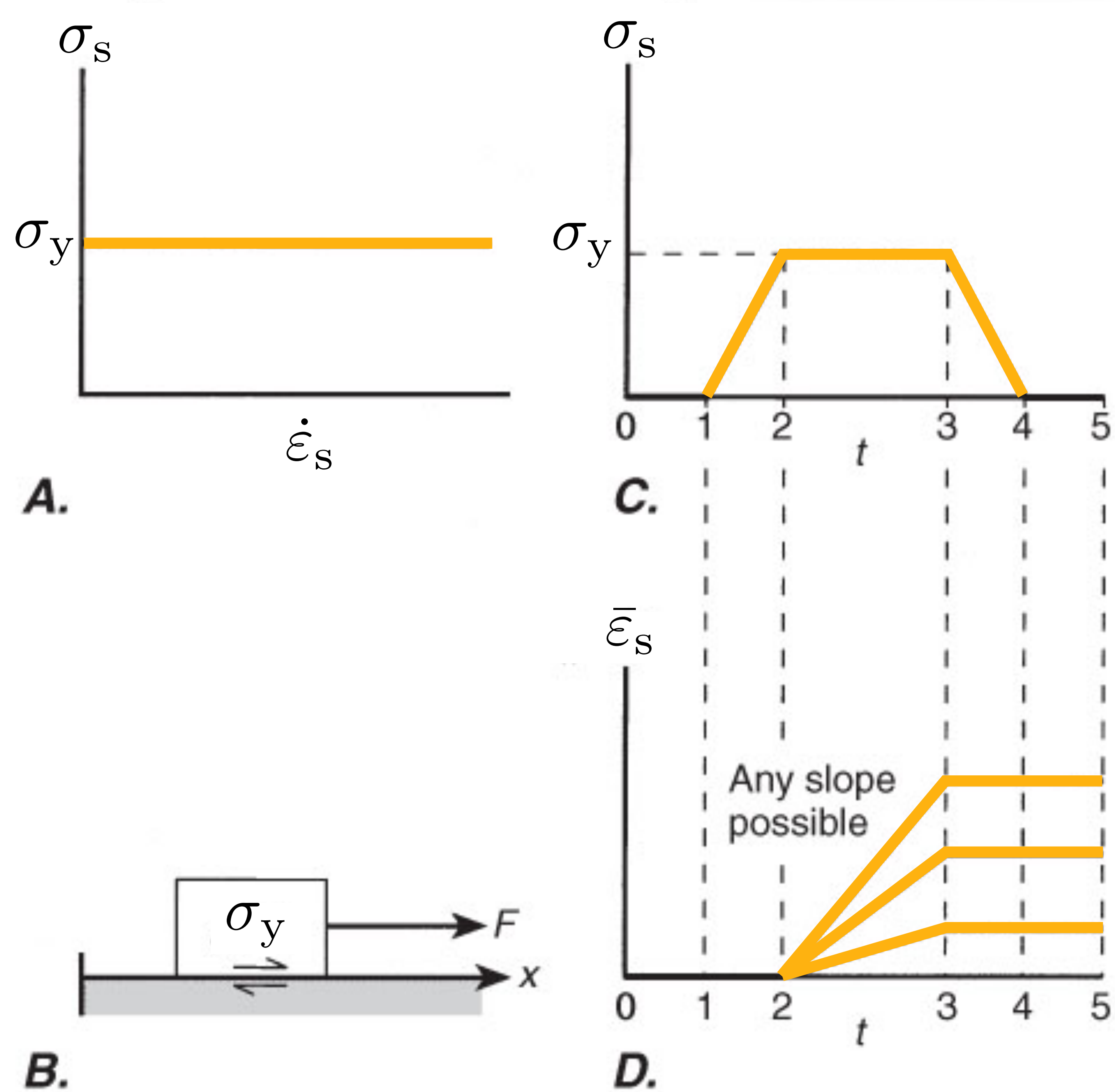
A.



B.

Plasticity - Perfectly plastic behavior

Twiss and Moores, 2007



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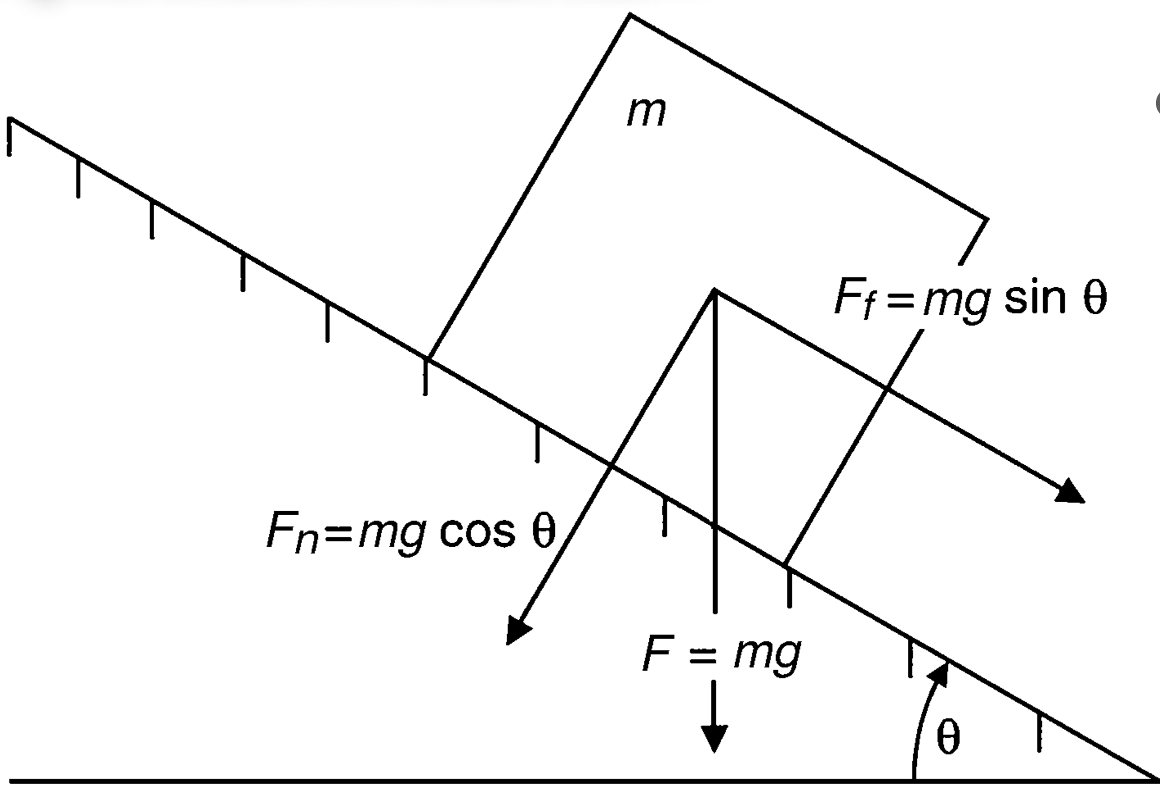
    - After exceeding the frictional resistance, slip will occur on the fault or shear zone
    - Known as **frictional plasticity**

Plasticity - Friction in rocks

Fig. 8.5, Turcotte and Schubert, 2014



Normal stress $\sigma_{\mathrm{n}} = \dfrac{mg\cos\theta}{A}$

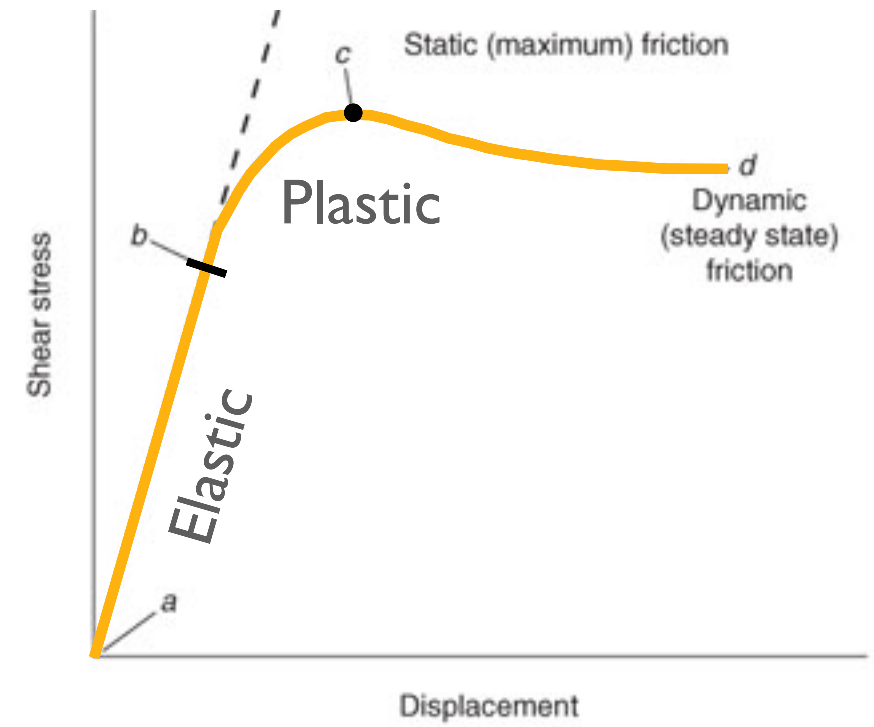
Shear stress $\sigma_{\mathrm{s}} = \dfrac{mg\sin\theta}{A}$

- Fault slip accounts for a large portion of deformation of the upper crust
- What must be overcome for slip to occur? **Friction**
  - After exceeding the frictional resistance, slip will occur on the fault or shear zone
  - Known as **frictional plasticity**
  - The basic relationship for static friction is $\tau_{fs} = f_s \sigma_n$ (Amonton's law)

    where f_s is the **coefficient of static friction**, and τ_{fs} is the **static frictional stress** required for slip

Plasticity - Friction in rocks

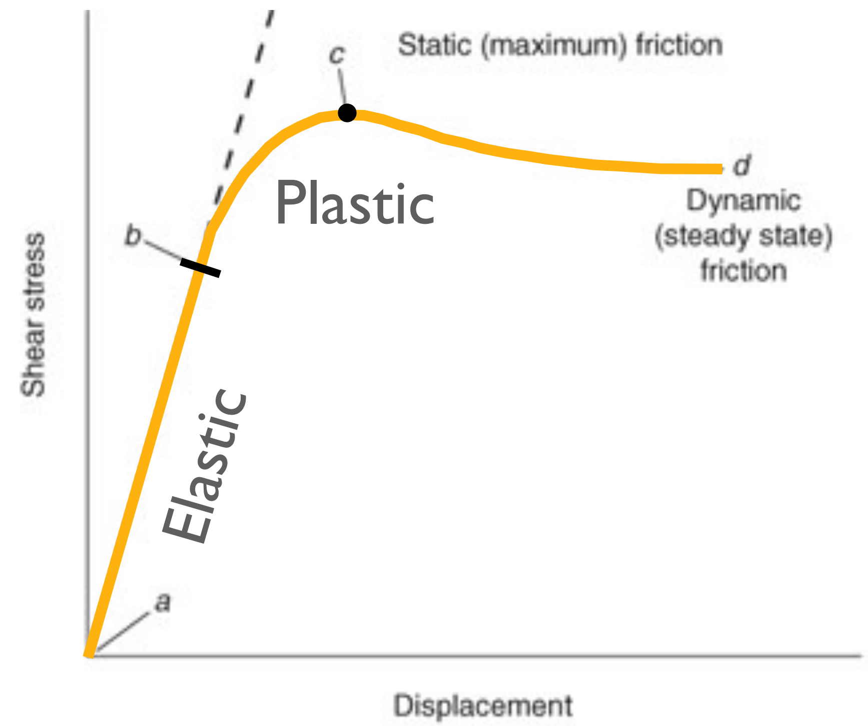
- Upper crustal rocks generally behave as elastic-perfect plastic



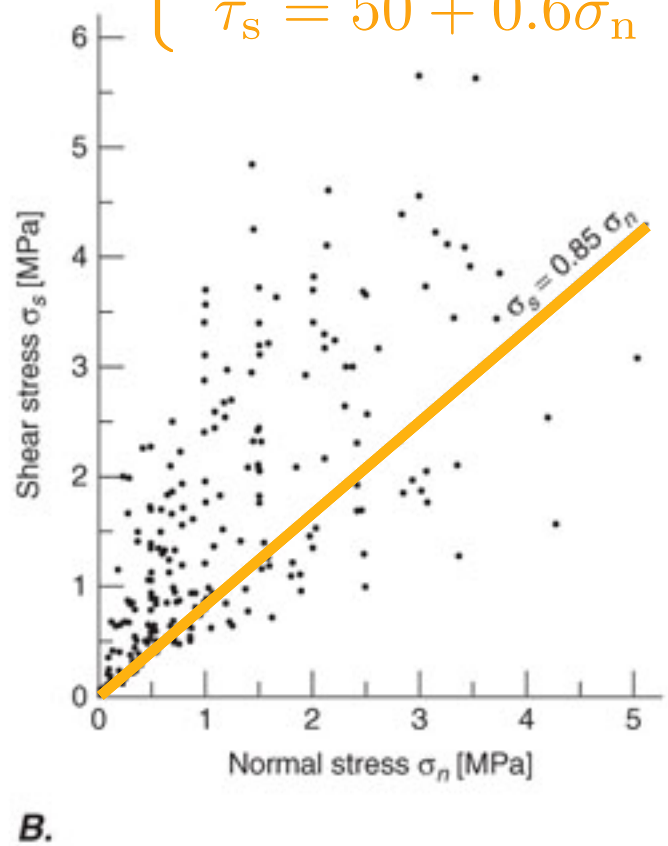
Twiss and Moores, 2007

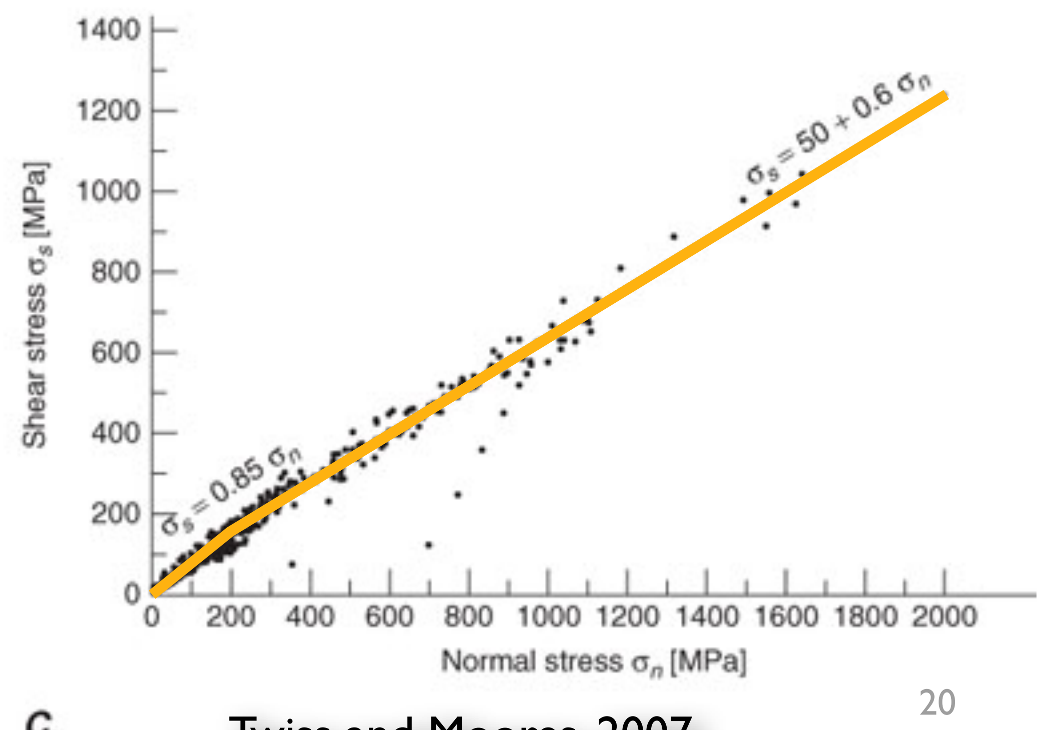
Plasticity - Friction in rocks

- Upper crustal rocks generally behave as elastic-perfect plastic
- For frictional slip, rock property measurements suggest the shear stress required for fault slip increases with normal force in two ‘domains’
- These are known as Byerlee’s laws



$$\begin{cases} \tau_\mathrm{s} = 0.85\sigma_\mathrm{n} \text{ [MPa]} & \text{for } 5 \text{ MPa} < \sigma_\mathrm{n} \leq 200 \text{ MPa} \\ \tau_\mathrm{s} = 50 + 0.6\sigma_\mathrm{n} \text{ [MPa]} & \text{for } \sigma_\mathrm{n} \geq 200 \text{ MPa} \end{cases}$$





Twiss and Moores, 2007

Plasticity - Mohr-Coulomb criterion

- Amonton's law as we saw it does not account for rock cohesion

$$\tau_{fs} = f_s \sigma_n$$

- Including cohesion c we can modify Amonton's law to

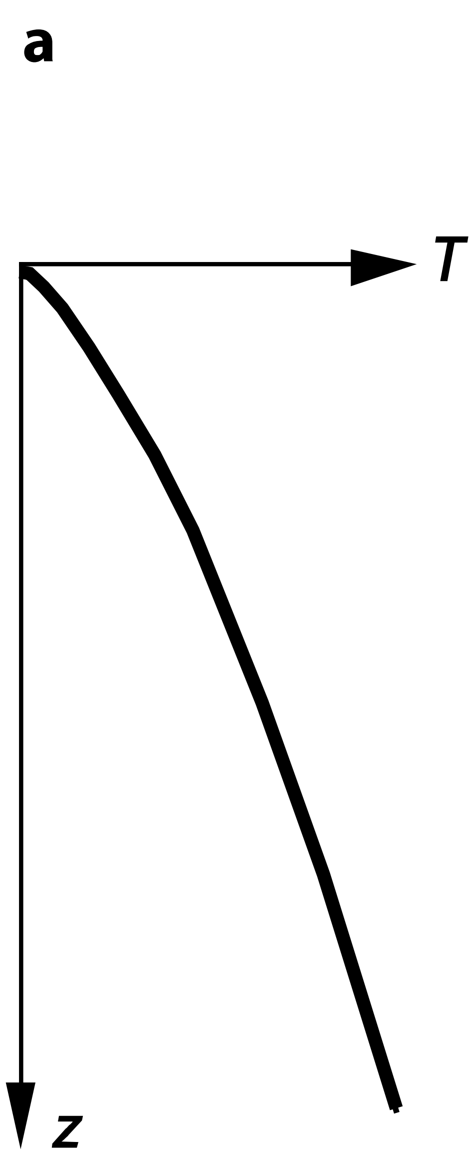
$$\tau_{fs} = c + f_s \sigma_n$$

- This is known as the **Coulomb criterion**

Strength of the lithosphere

- We've now discussed **viscous** and **plastic** deformation
- How do we put these together to get a sense of the rheology of the lithosphere?
- What behaviors do we expect in which regions and why?

Brace-Goetze lithosphere a.k.a. the "jelly sandwich"

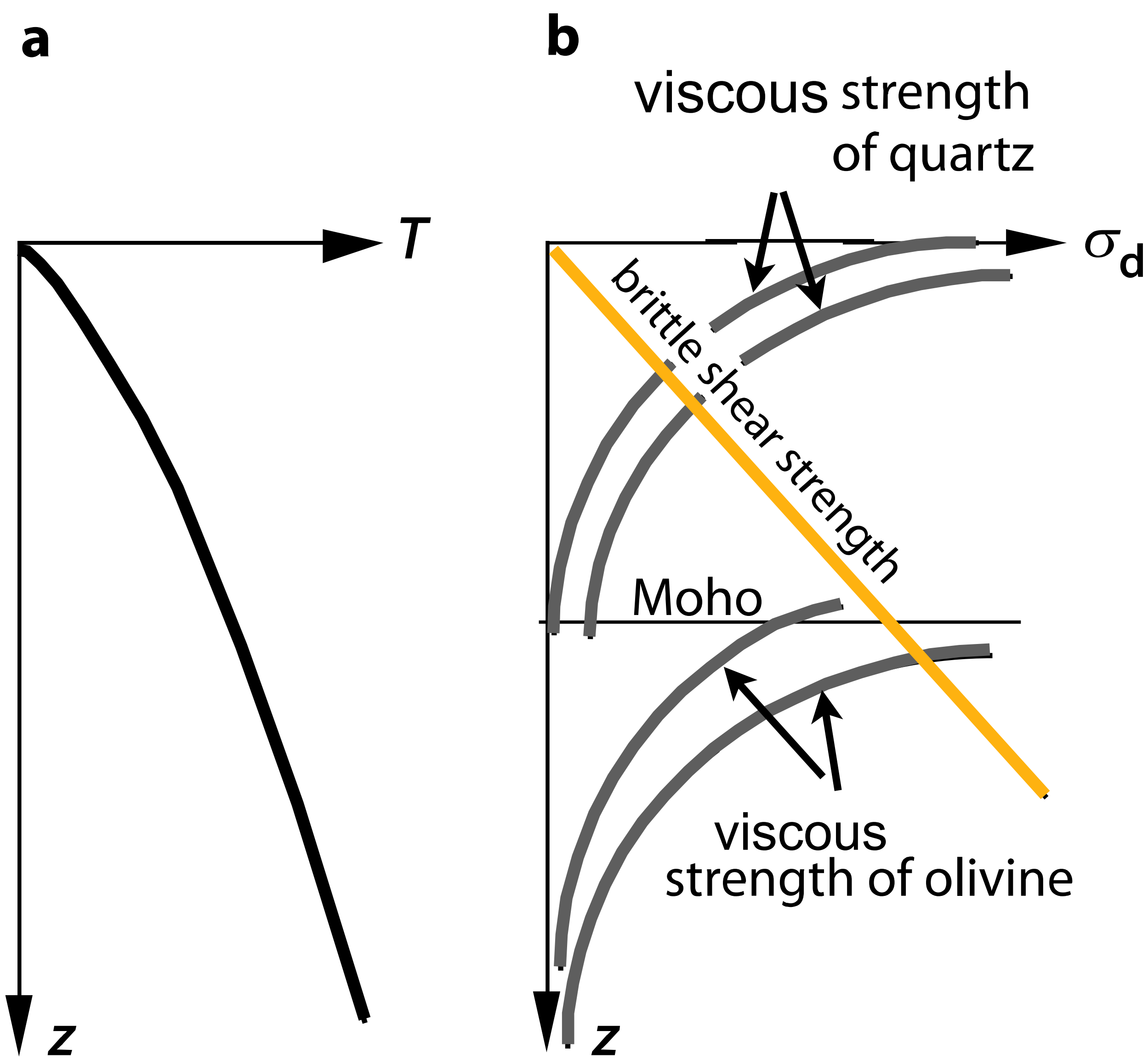


What to we 'know'?

Temperature in the Earth increases with depth

Figure 5.13 - Stüwe, 2007

Brace-Goetze lithosphere a.k.a. the “jelly sandwich”



Rocks with a **frictional plastic** rheology will show a <u>near linear increase in strength with depth</u>

Why?

Figure 5.13 - Stüwe, 2007

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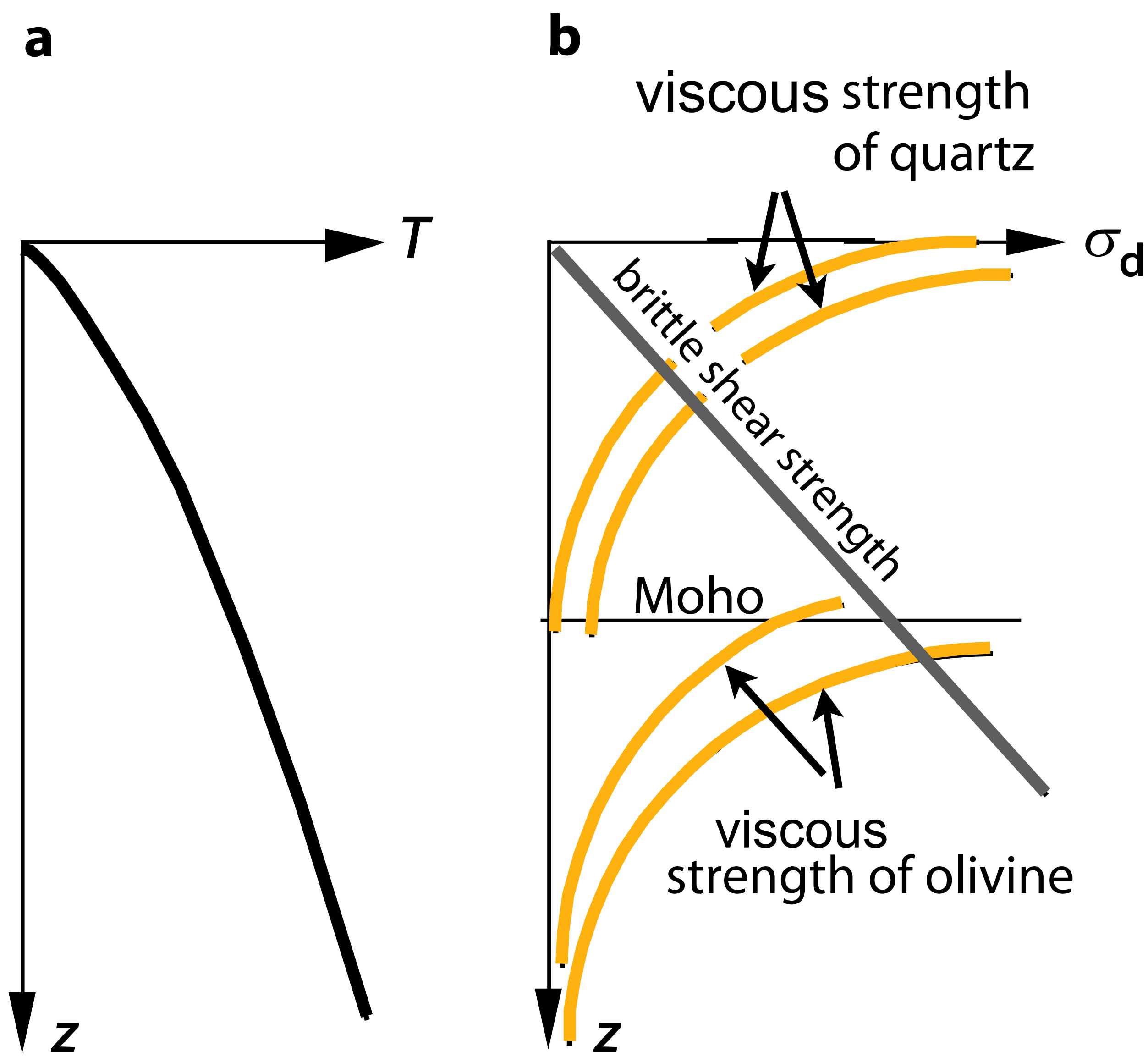
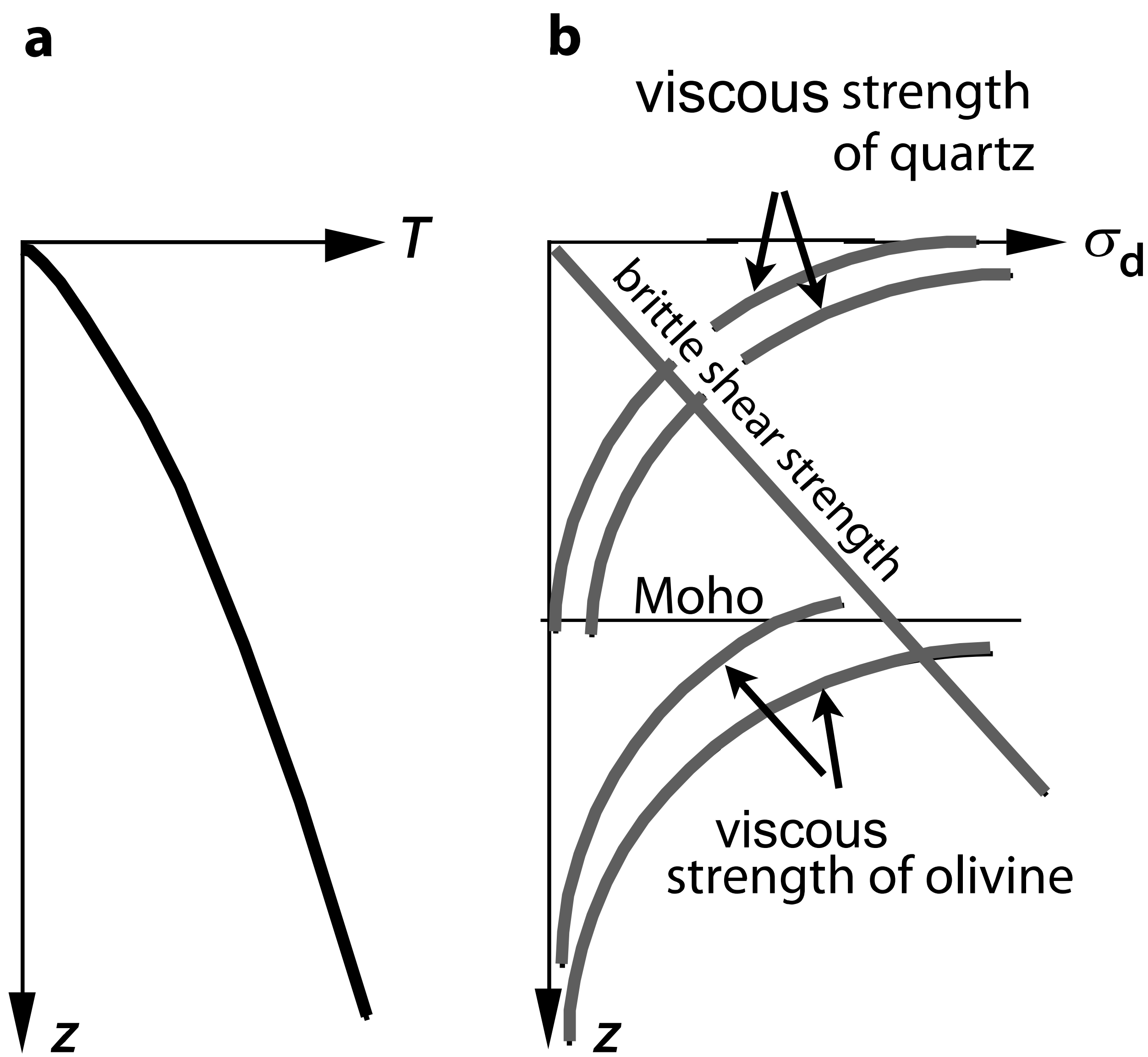


Viscosity will <u>decrease with increasing temperature</u>

If viscous, rocks will <u>get weaker with depth</u>

Figure 5.13 - Stüwe, 2007

Brace-Goetze lithosphere a.k.a. the “jelly sandwich”



Rocks will fail by the <u>weakest available mechanism</u>

Figure 5.13 - Stüwe, 2007

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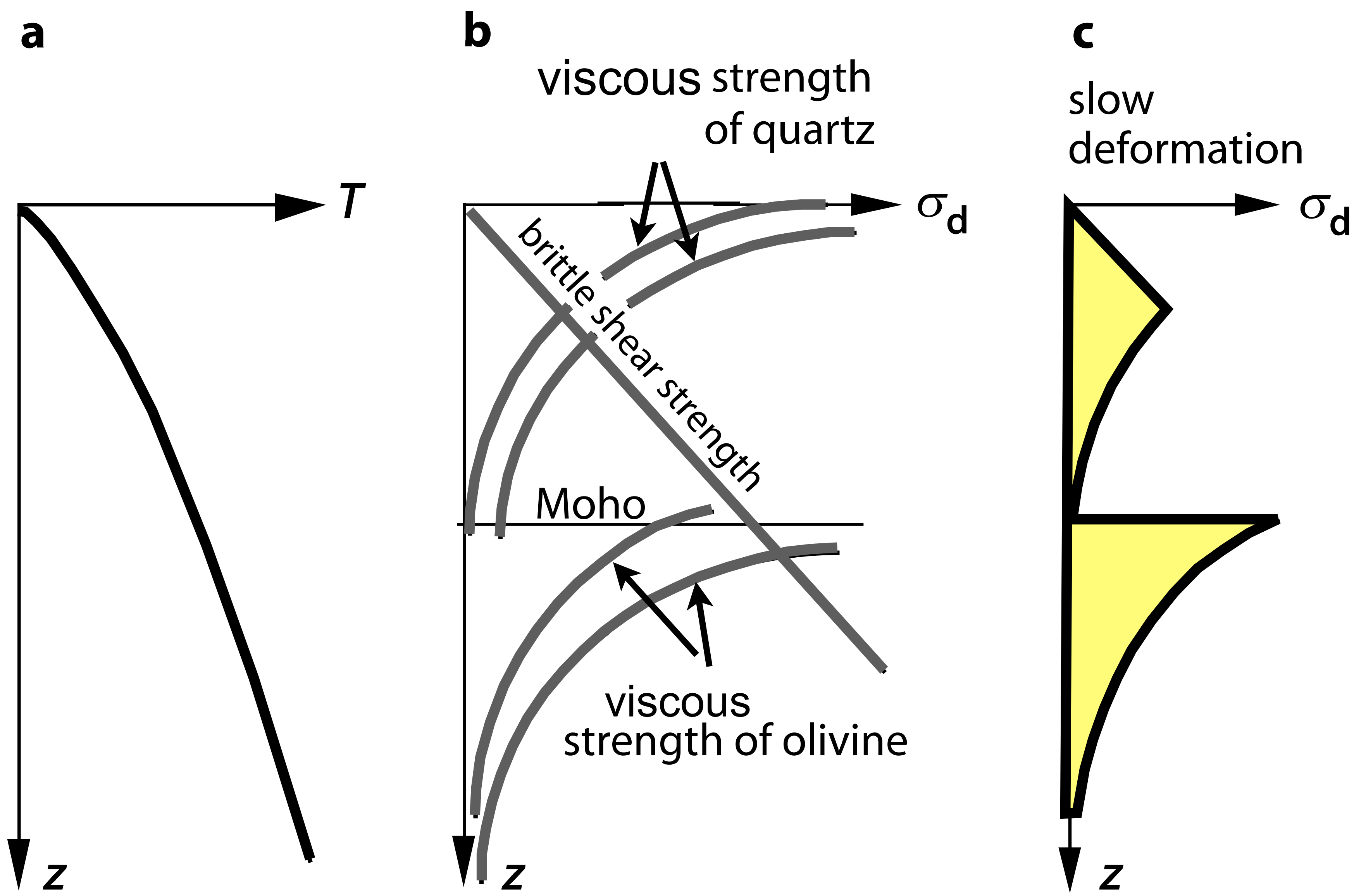

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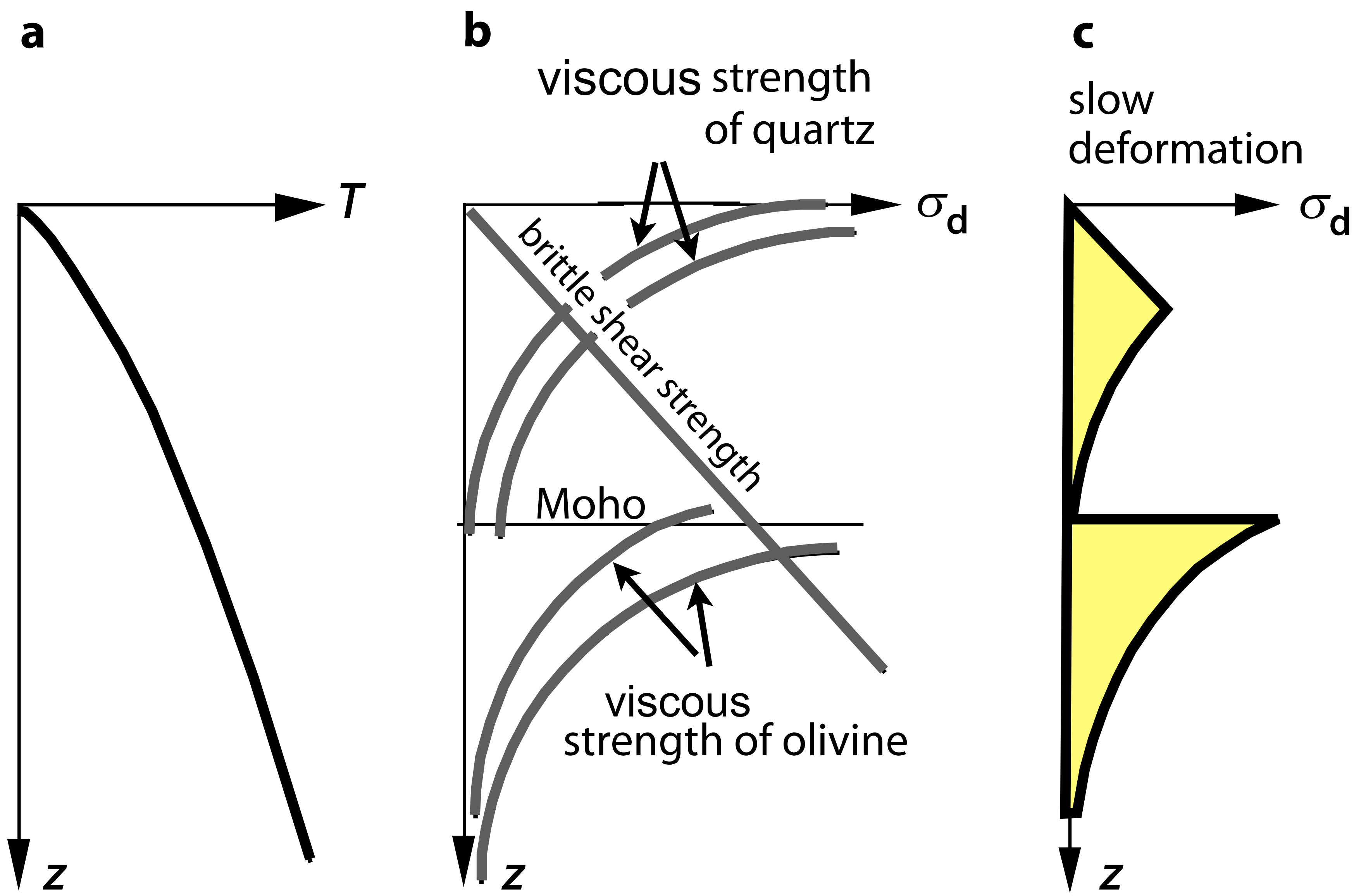

Figure 5.13 - Stüwe, 2007

What will happen if the deformation rate is increased?

Brace-Goetze lithosphere a.k.a. the “jelly sandwich”

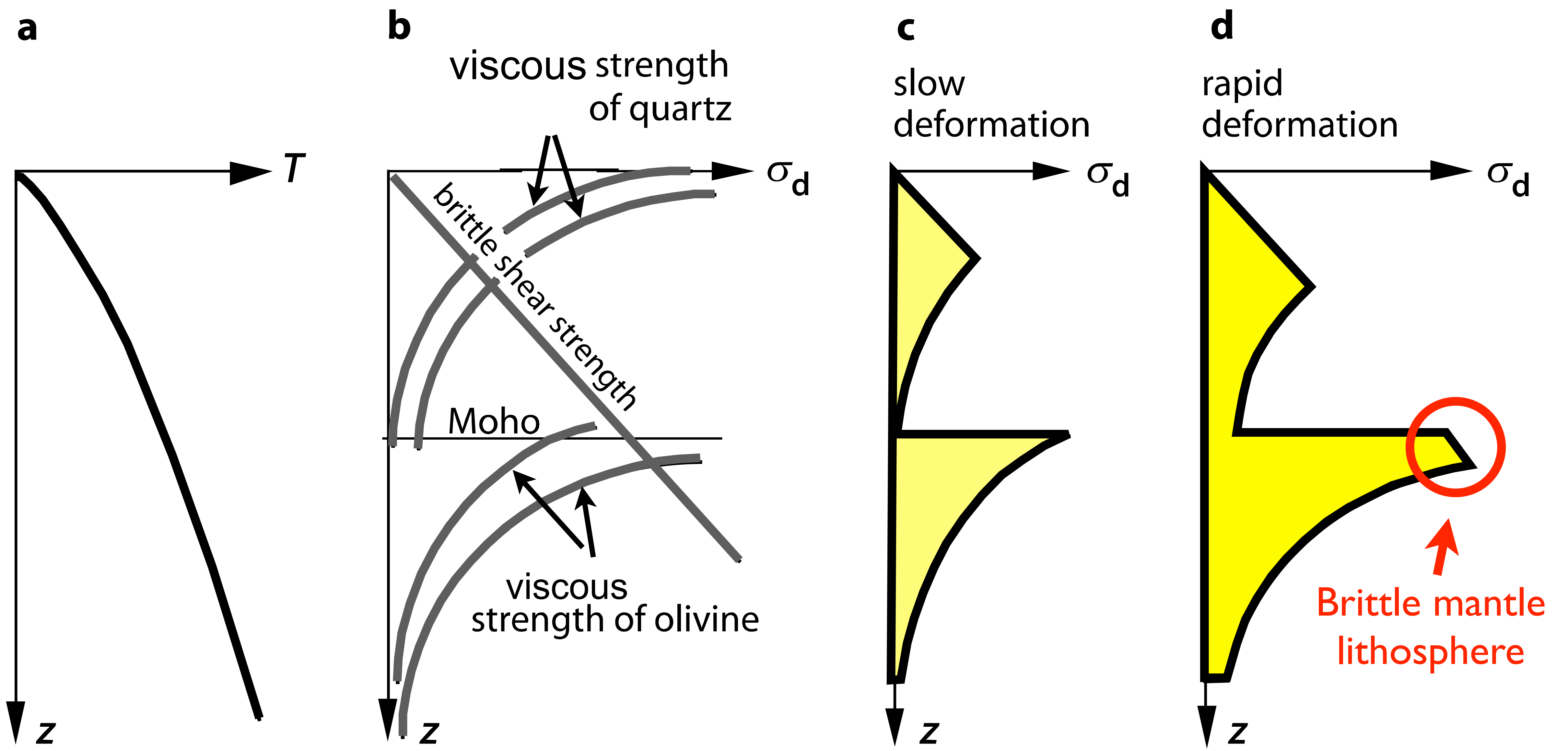

Figure 5.13 - Stüwe, 2007

Strength envelopes challenge #1

- Start by navigating to the directory `NGWM2016-modelling-course/Lessons/06-Rheology-of-the-lithosphere/scripts`
- Right-click on the Python script called `strength-envelope-uniform-crust.py` and choose "Edit with IDLE"

Strength envelopes challenge #1

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- Right-click on the Python script called `strength-envelope-uniform-crust.py` and choose "Edit with IDLE"
- The script cannot currently be run because it is missing
  - 2 equations for rock rheology
  - 2 conditions for conditional (`if`/`else`) statements
  - 2 value assignments in the `if`/`else` conditional statements
- Your task is to input the missing code to get the script working and save a copy of the plot it produces

Strength envelopes challenge #1

- Conditional statements allow you to run certain parts of a computer code based on meeting certain conditions
  - Basically, these are if-then, types of statements
  - For instance, the following is a working example in Python:

```
if temperature < 0.0:                  | if (boolean condition):
    print(“Better wear a hat!”)        |     (executed if condition is true)
else:                                  | else:
    print(“Hat is optional!”)          |     (executed is condition is false)
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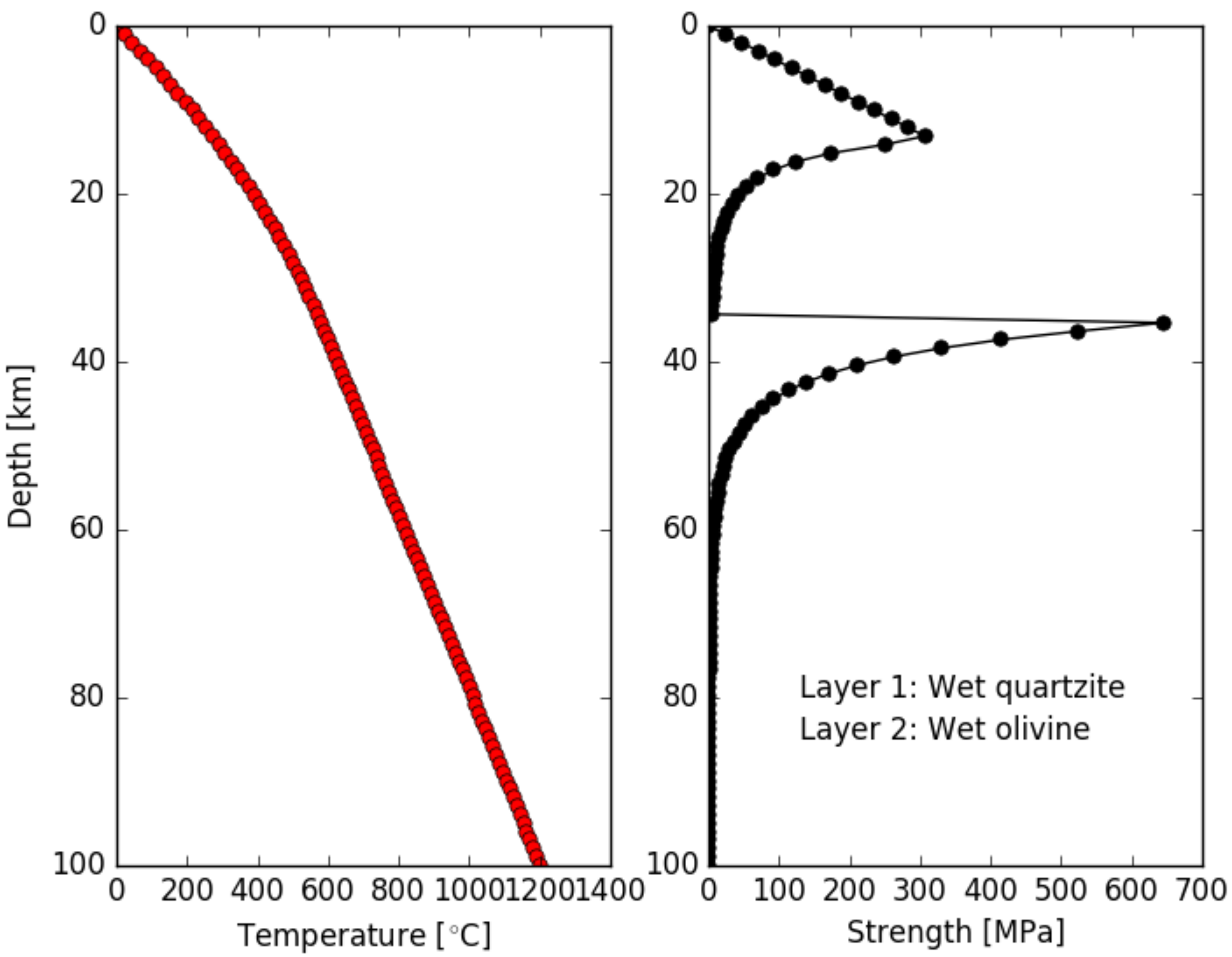
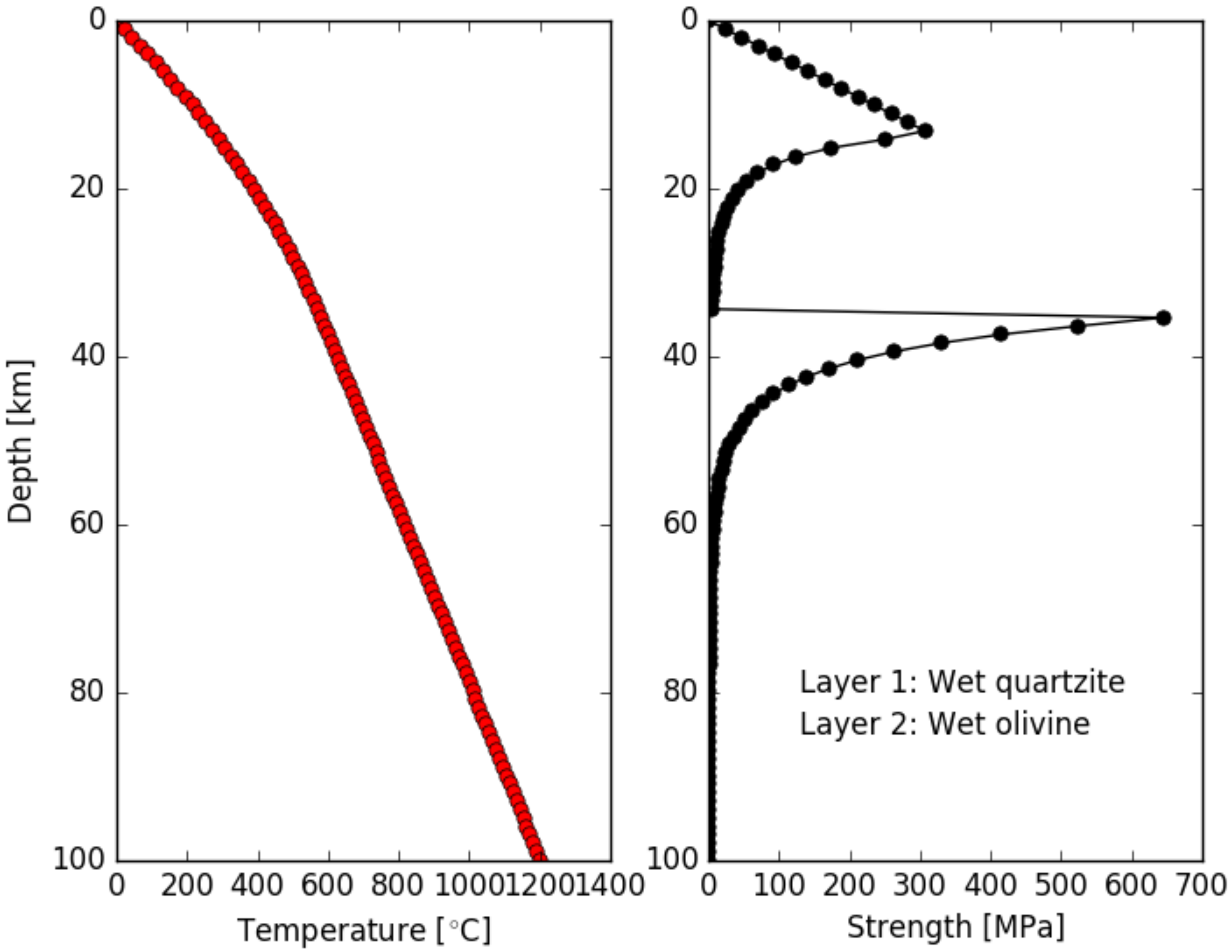
- With this in mind, fill in (a) the two missing rheology equations, (b) the two conditions for the conditional statements in the code, and (c) the two assigned values missing in the conditional statements
  - It will help to read over the entire code before making changes
  - All missing items are marked with "????"
  - Don't forget to save a copy of the plot the code produces!

Strength envelopes challenge #1



- If all has gone well, your saved plot should look like this
- Temperatures are calculated using a finite difference formulation of the heat transfer equation with spatially variable heat production
- The strength profile considers 2 material behaviors (plasticity and viscous flow) for a two-layer lithosphere (wet quartzite crust, wet olivine mantle)

Strength envelopes challenge #1



- Is the mantle "brittle" or "ductile" in this case?

Key features of the Brace-Goetze lithosphere

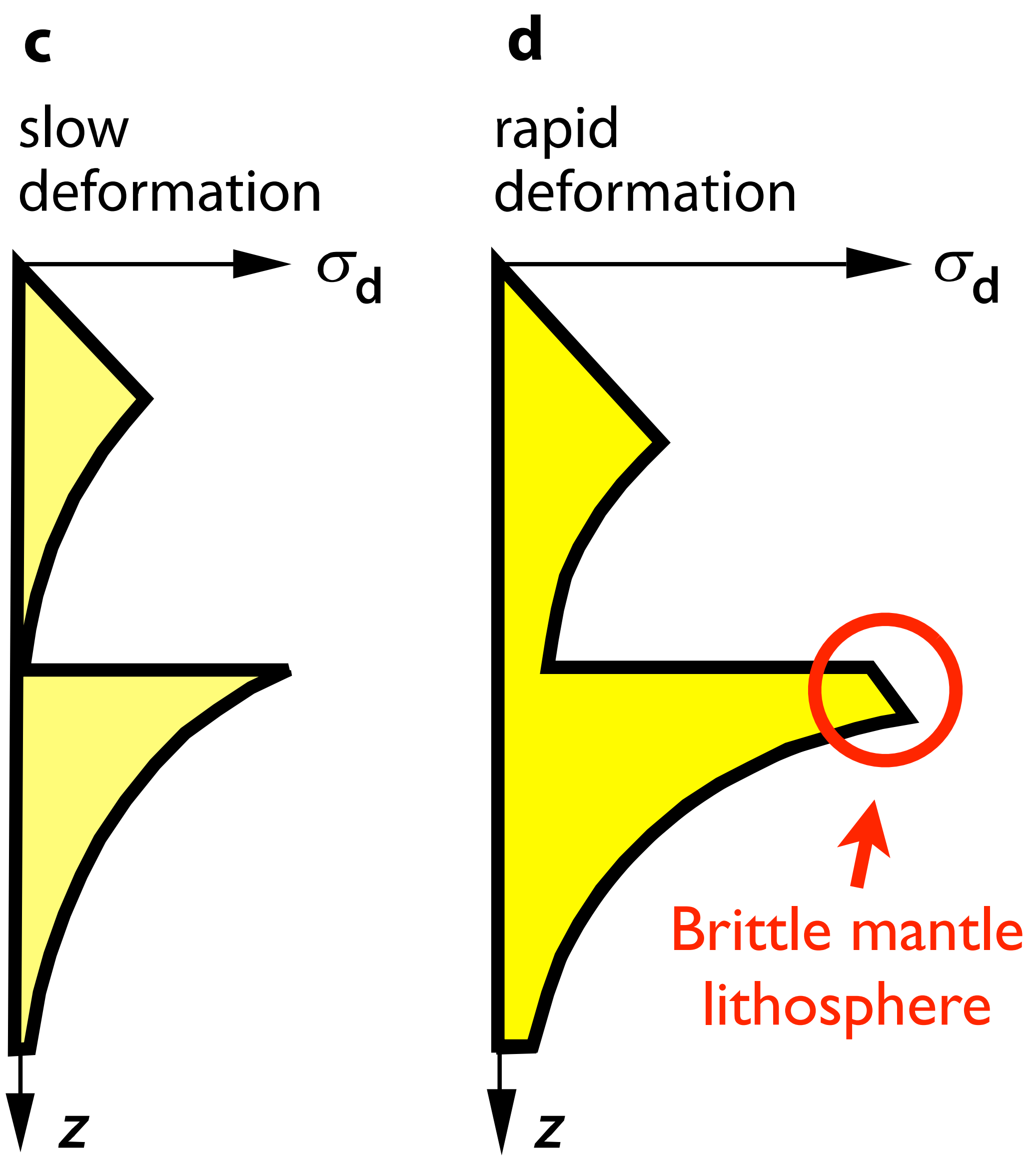


Figure 5.13 - Stüwe, 2007

- Brittle fracture in the mantle
  - In some cases, the mantle can be **brittle**
    - Rapid strain rates
    - Any other ideas?

Key features of the Brace-Goetze lithosphere

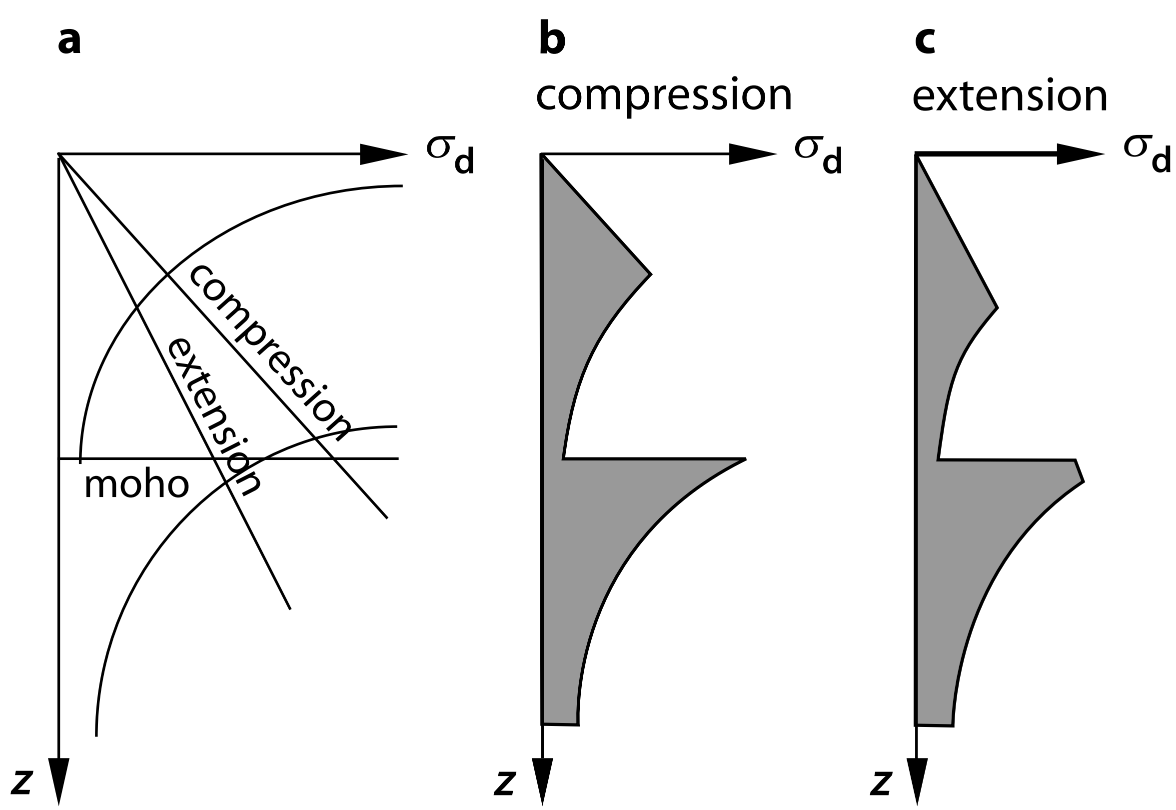


Figure 5.14 - Stüwe, 2007

- Brittle fracture in the mantle
  - In some cases, the mantle can be **brittle**
    - Rapid strain rates
    - Decrease in brittle strength (transition from compression to extension, for example)

Key features of the Brace-Goetze lithosphere

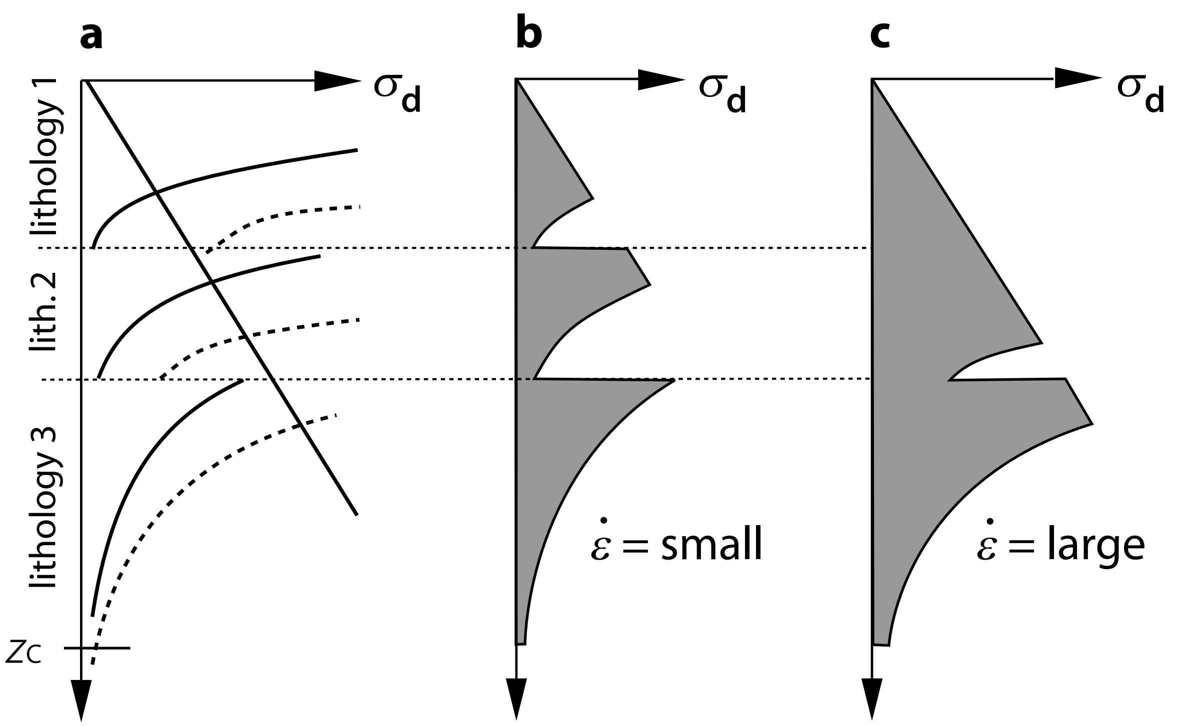
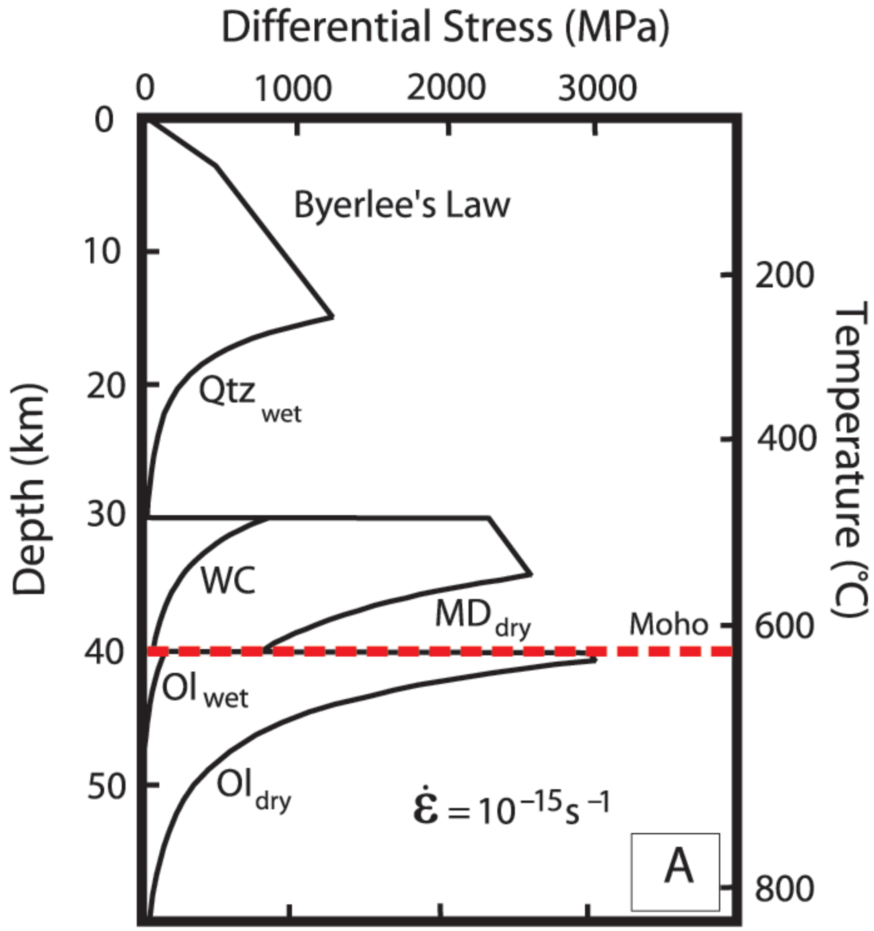


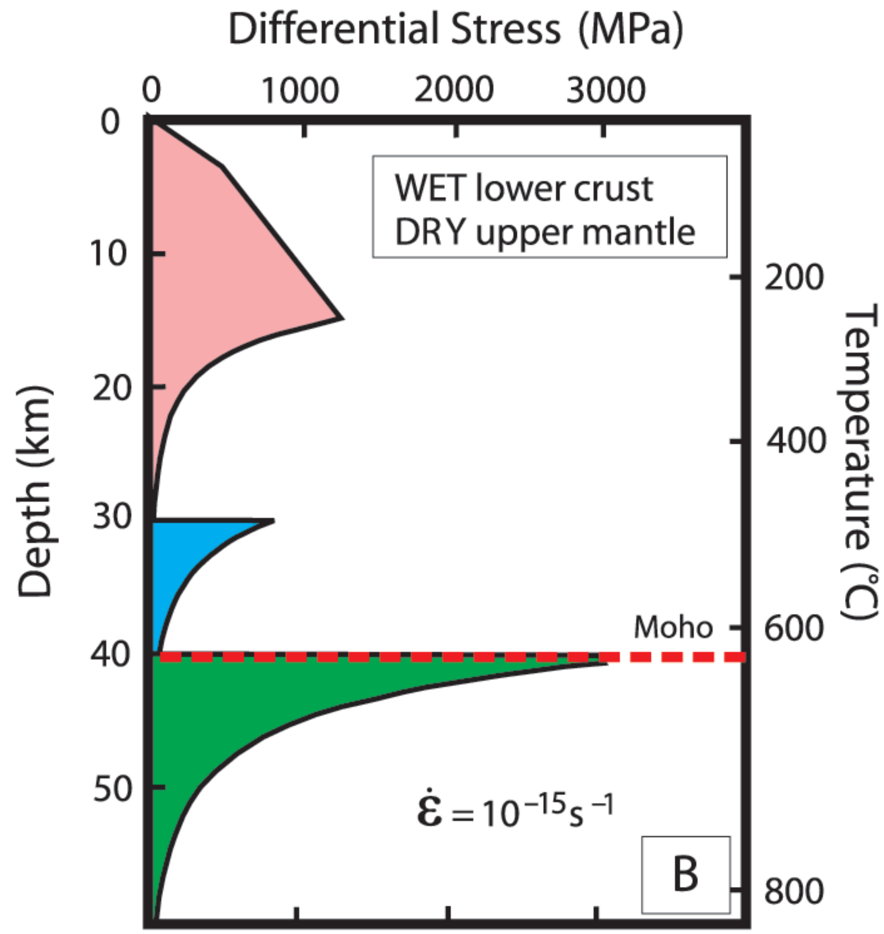
Figure 5.15 - Stüwe, 2007

- Variable rheological stratification
    - In a three-layer lithosphere, there may or may not be a mechanical separation of the top two layers
    - Depends on strain rate, temperature and the material properties

Key features of the Brace-Goetze lithosphere

- Variable geotherm
  - Viscous behavior will change depending on the thermal field
- Strength change due to metamorphism
  - Metamorphism can change rheological properties
    - Clay may metamorphose to garnet mica schist, a rheologically stronger rock



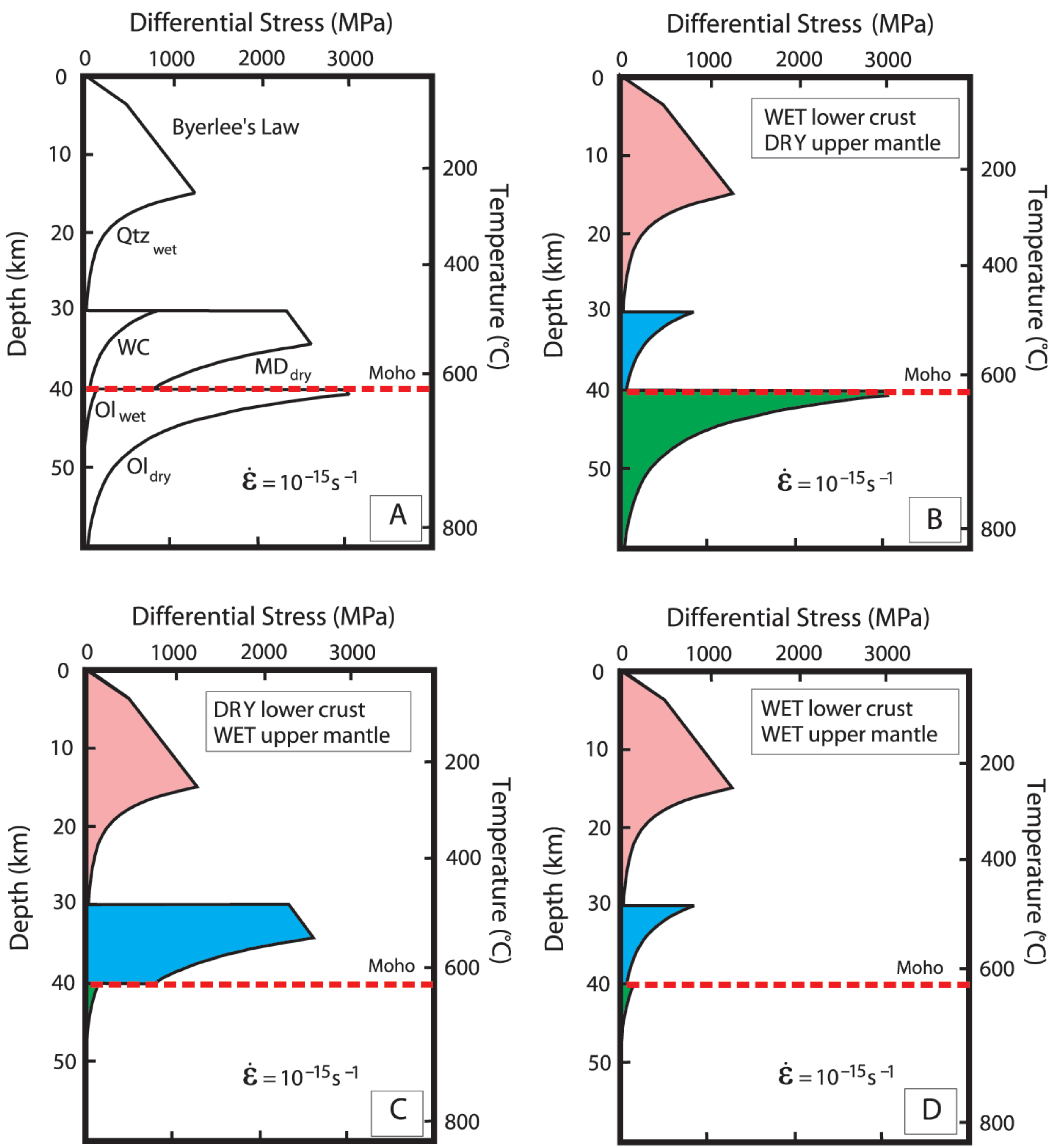


Lithospheric strength envelopes

- There are two popular types of lithospheric strength envelope models thought to represent the Earth
- Jelly sandwich
  - A - Brace-Goetze
  - B - Wet LC

Jackson, 2002

Lithospheric strength envelopes



- There are two popular types of lithospheric strength envelope models thought to represent the Earth
- **Jelly sandwich**
  - A - Brace-Goetze
  - B - Wet LC
- **Crème brûlée**
  - C - Wet UM
  - D - Wet LC, UM

Jackson, 2002

Strength envelopes challenge #2

- Start by making a copy of your `strength-envelope-uniform-crust.py` and saving it as `strength-envelope-3layer-crust.py`
- Open the script in IDLE and make the following changes

Strength envelopes challenge #2

- Start by making a copy of your `strength-envelope-uniform-crust.py` and saving it as `strength-envelope-3layer-crust.py`
- Open the script in IDLE and make the following changes
  1. Add definitions of two more crustal materials where materials are defined at the top of the code using suitable middle and lower crustal rock types such as diorite and diabase from Table 2 of Burov, 2015 (see `Reference materials` folder)

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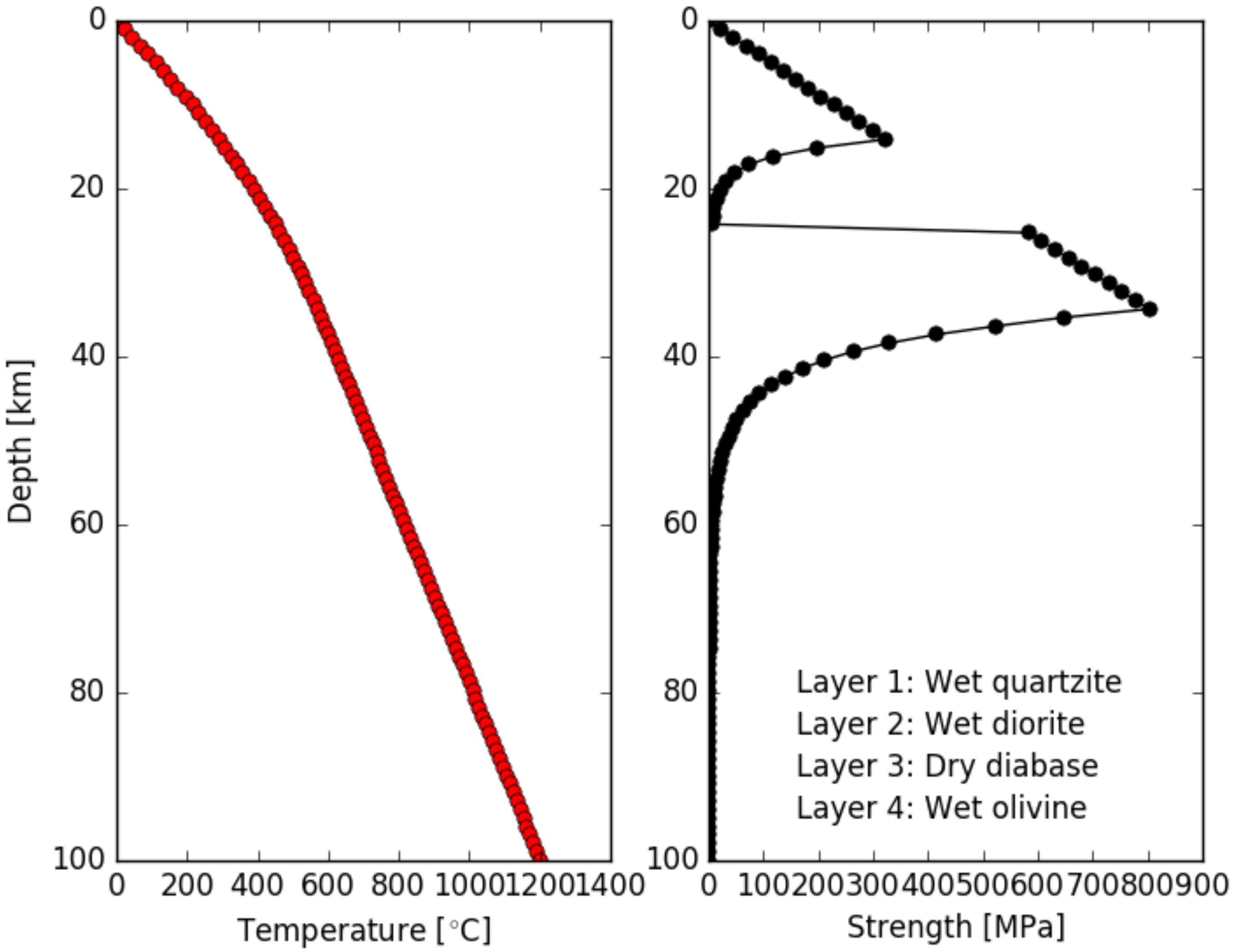
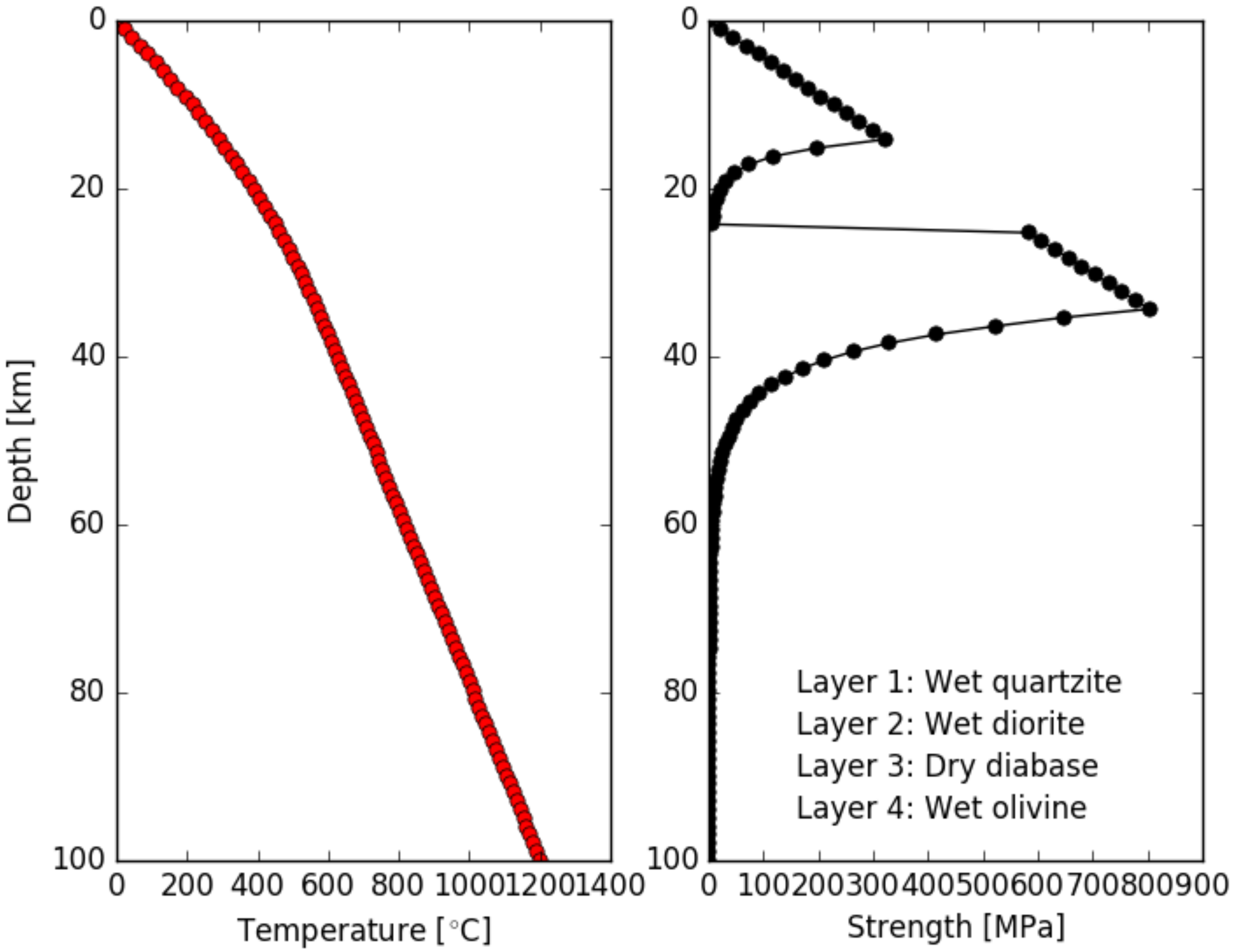
1. Add definitions of two more crustal materials where materials are defined at the top of the code using suitable middle and lower crustal rock types such as diorite and diabase from Table 2 of Burov, 2015 (see `Reference materials` folder)
   - Be sure to add lines in the material property conversions!
2. Define two additional layer depths so that you have the following:
   Upper crust: 10 km, middle crust: 25 km, lower crust: 35 km
3. Adjust the conditional statements for filling the material arrays using `elif` statements (`elif` is short for *`else if (condition)`*)
4. Add two extra text labels on the plot for the new materials
5. Save a copy of the plot once the code works, then feel free to explore different values for heat production, layer depths, strain rate, etc.

Strength envelopes challenge #2

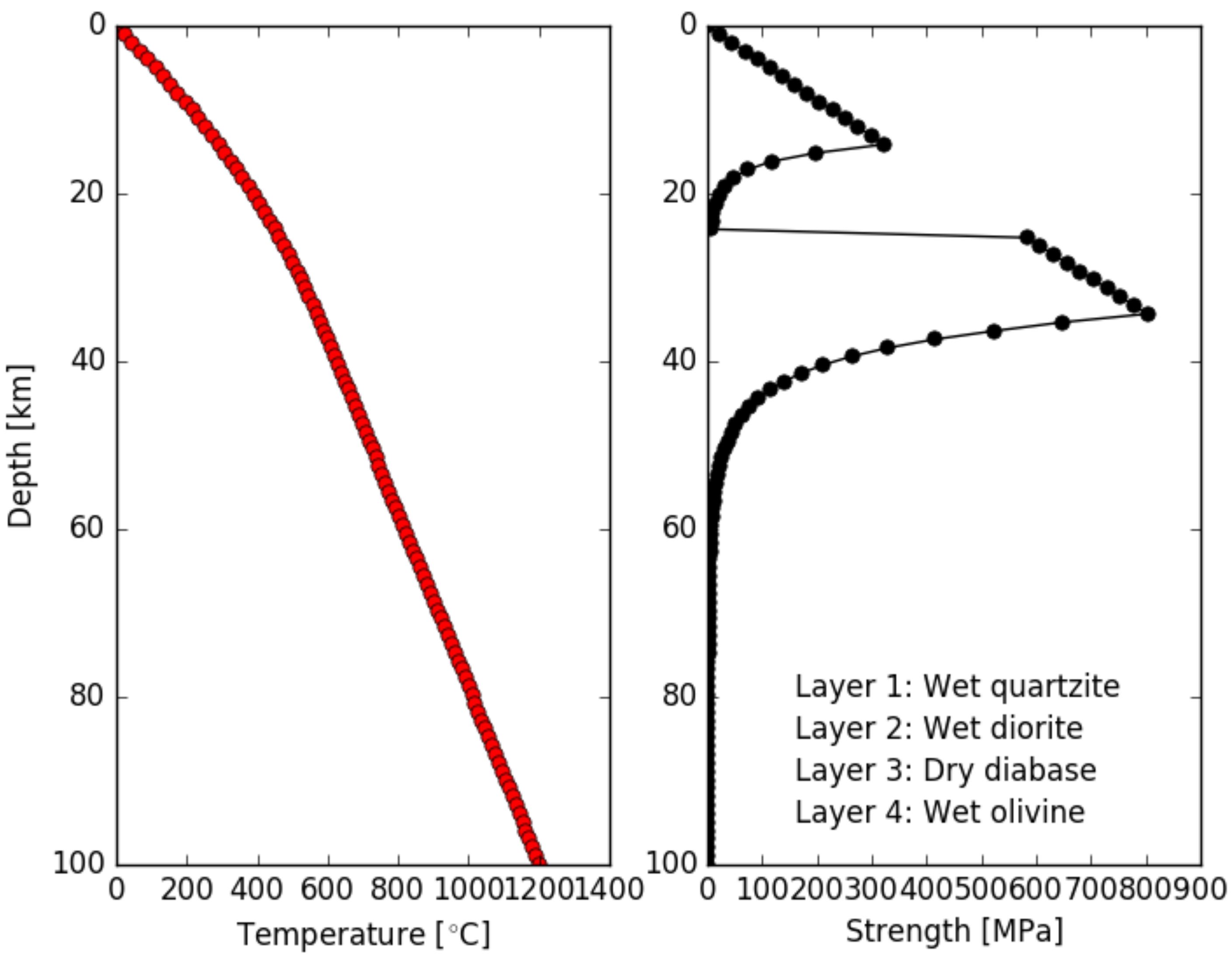


Strength envelopes challenge #2



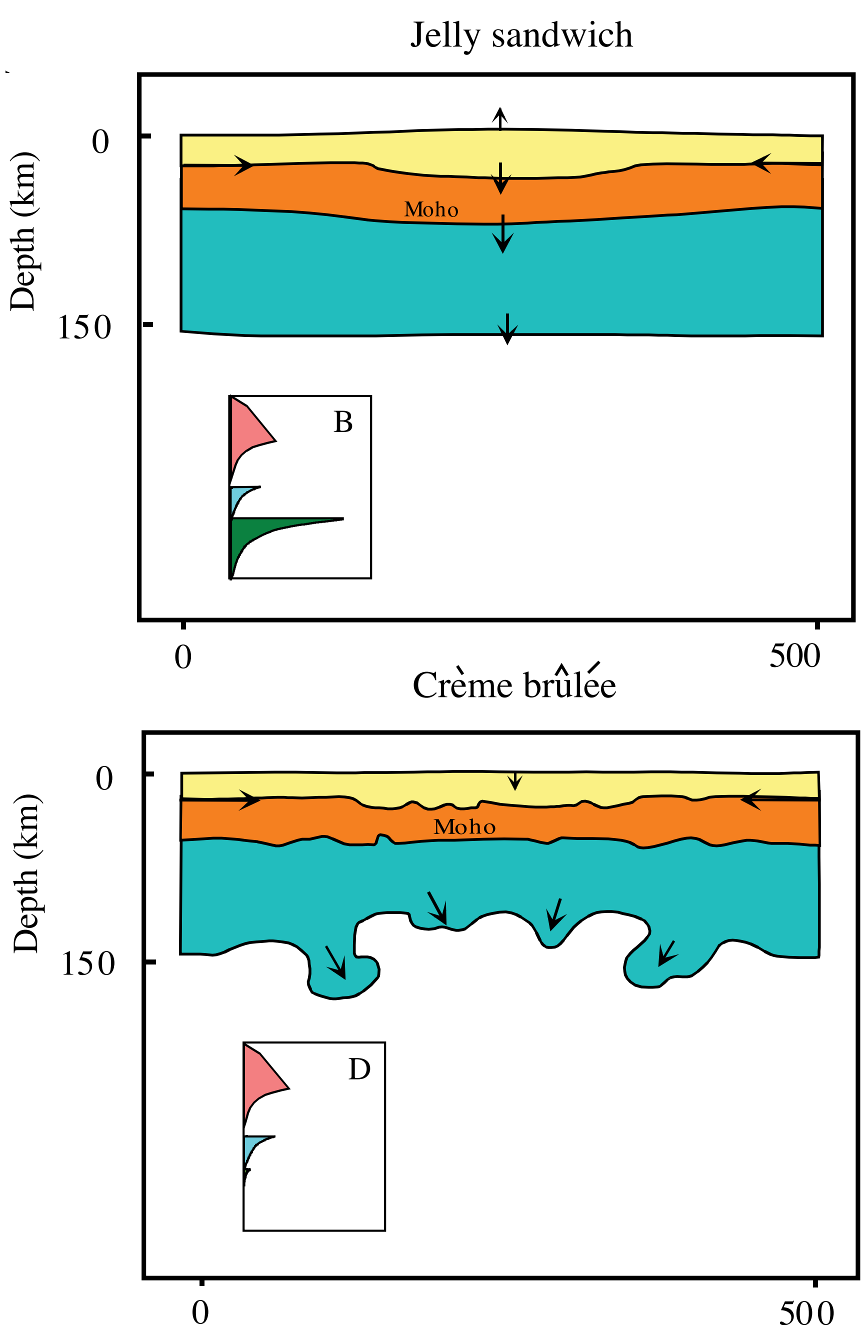
- Here, we can clearly see at least 2 crustal layers, though we've defined three
  - The upper crust is entirely brittle in this case and mechanically indistinguishable from the top of the middle crust

Strength envelopes challenge #2



- Here, we can clearly see at least 2 crustal layers, though we've defined three
    - The upper crust is entirely brittle in this case and mechanically indistinguishable from the top of the middle crust
- The lower crust is strong, giving us 2 brittle-ductile transitions and a weak middle crust (crème brûlée)
- Your plots may differ, depending on the chosen rock types

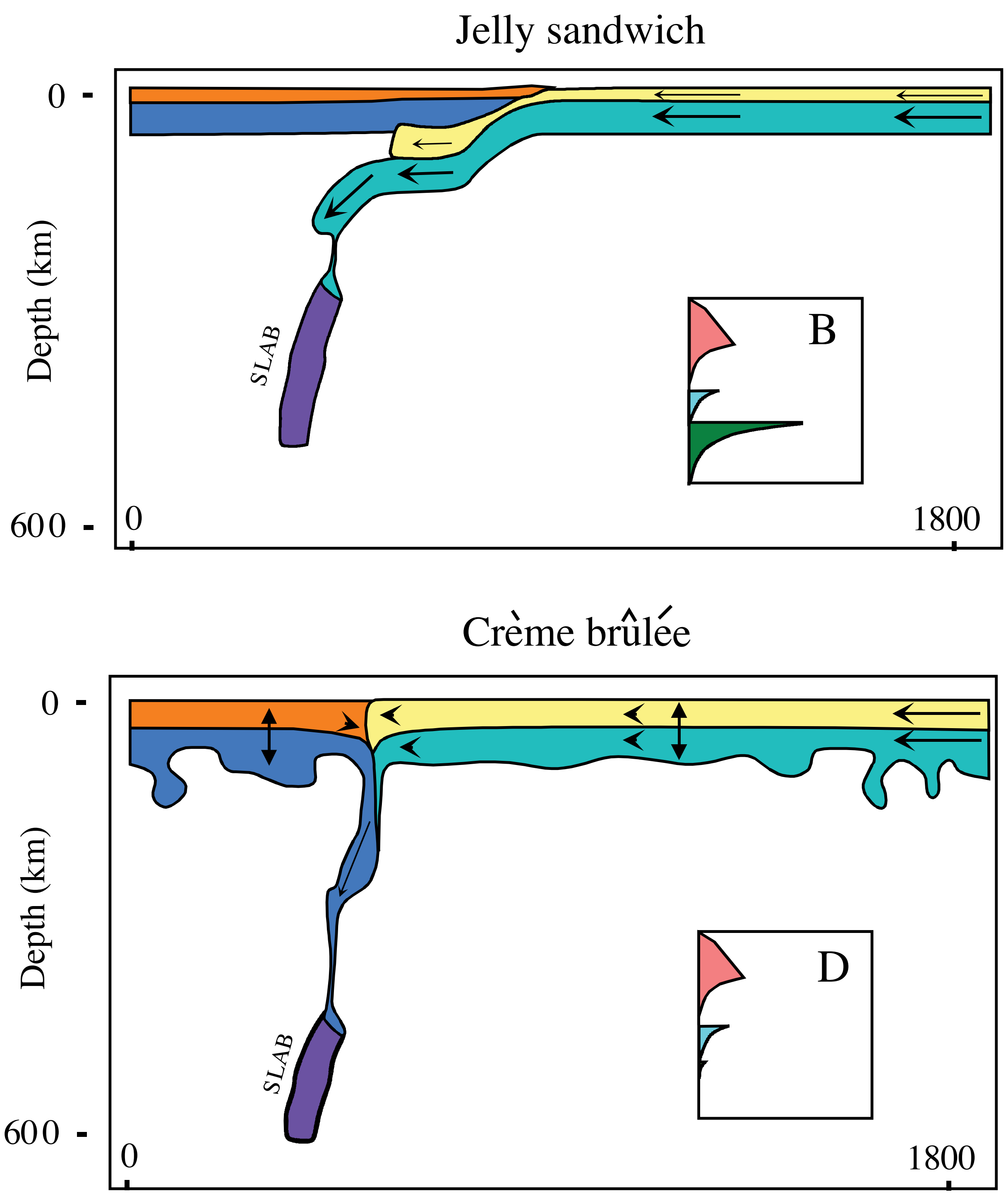
Why is crustal rheology important?



Burov and Watts, 2006

- The rheology of the lithosphere has several important implications
  - Definition of the seismogenic zone
  - Determining the overall strength of the lithosphere
  - Controlling the way in which the lithosphere deforms, its stability and ultimately its long-term evolution

Why is crustal rheology important?



Burov and Watts, 2006

- Here we see a major difference in geodynamic behavior based solely on rheology
  - The "jelly sandwich" model has stable subduction much like what we observe on Earth
  - The "crème brûlée" model is unstable with no significant subduction and lithospheric delamination

Summary

- At long timescales, deformation of the lithosphere is the result of the combination of frictional plastic and nonlinear viscous deformation
  - Frictional plastic materials have increasing strength with depth
  - Viscous materials in the Earth will generally have decreased strength with depth
- Lithospheric strength models combine these behaviors to provide insight into the overall lithosphere's strength and its variations in strength with depth
  - The rheological layering in these models has important geodynamic implications

References

Burov, E. B., & Watts, A. B. (2006). The long-term strength of continental lithosphere:" jelly sandwich" or" crème brûlée"?. GSA today, 16(1), 4.

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