TIME SERIES

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PART 1 : Nelson-Siegel Model

Explanation and plotting

The Nelson-Siegel model is a parametric model for fitting and forecasting the yield curve of interest rates. It provides a representation of the term structure of interest rates using an exponential decay function.

$$y(\tau) = \beta_0 + \beta_1 \frac{1 - e^{-\tau/\lambda}}{\tau/\lambda} + \beta_2 \left(\frac{1 - e^{-\tau/\lambda}}{\tau/\lambda} - e^{-\tau/\lambda} \right)$$

Where:

- $y(\tau)$ represents the yield at maturity τ
- β_0, the intercept, represents the long-term level of the yield curve, it captures the general level of interest rates across all maturities
- β_1, the slope coefficient, determines the slope of the yield curve, it mainly affects the difference between short-term and long-term interest rates
- β_2 controls the curvature of the yield curve, it is most influential in the medium-term maturities
- λ determines the speed at which the impact of β_1 and β_2 fades as maturity τ increases

We have already implemented a function (NS) based on the Nelson-Siegel model. Let's begin by calling it.

```r
source("C:\\Users\\Sebastian\\OneDrive - Université Paris-Dauphine\\DAUPHINE\\01_M1\\S2\\Info\\NS_fct.R
```

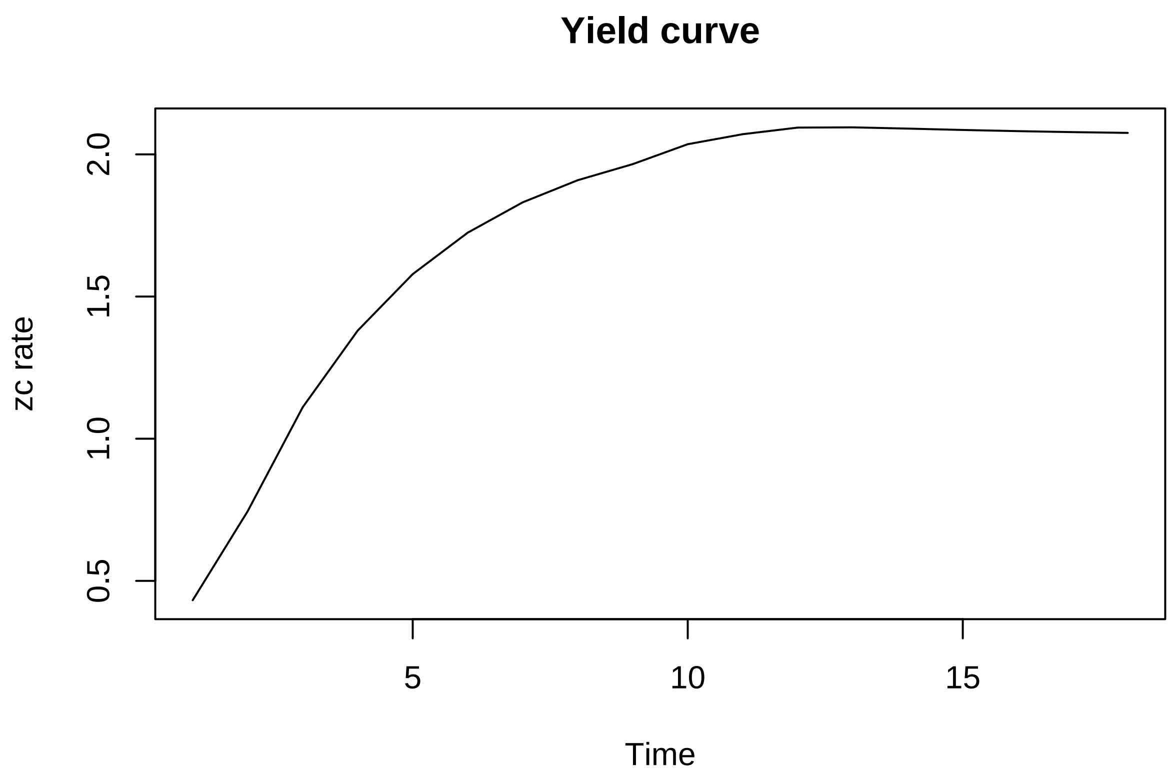
To store our time-to-maturity values, we define a vector t, representing τ The numbers 1, 3, 6, etc., correspond to maturities in months, which we convert into years by dividing by 12.

```r
t<-( c(1,3,6,9,12,15,18,21,24,30,36,48,60,72,84,96,108,120)/12)
```

We execute our function, ensuring that the chosen parameters produce a normal yield curve. To achieve this, I set β_1 (short-term effect) smaller than β_2 (medium-term hump effect), which in turn is smaller than β_0 (long-term level).

```r
NS_1 <- NS(2.05,-1.80,2.1,0.85,t)
```

```r
layout(matrix(1:1,1,1))
plot.ts(NS_1,ylab="zc rate", main="Yield curve")
```



Extension on β_1 and β_2 parameters

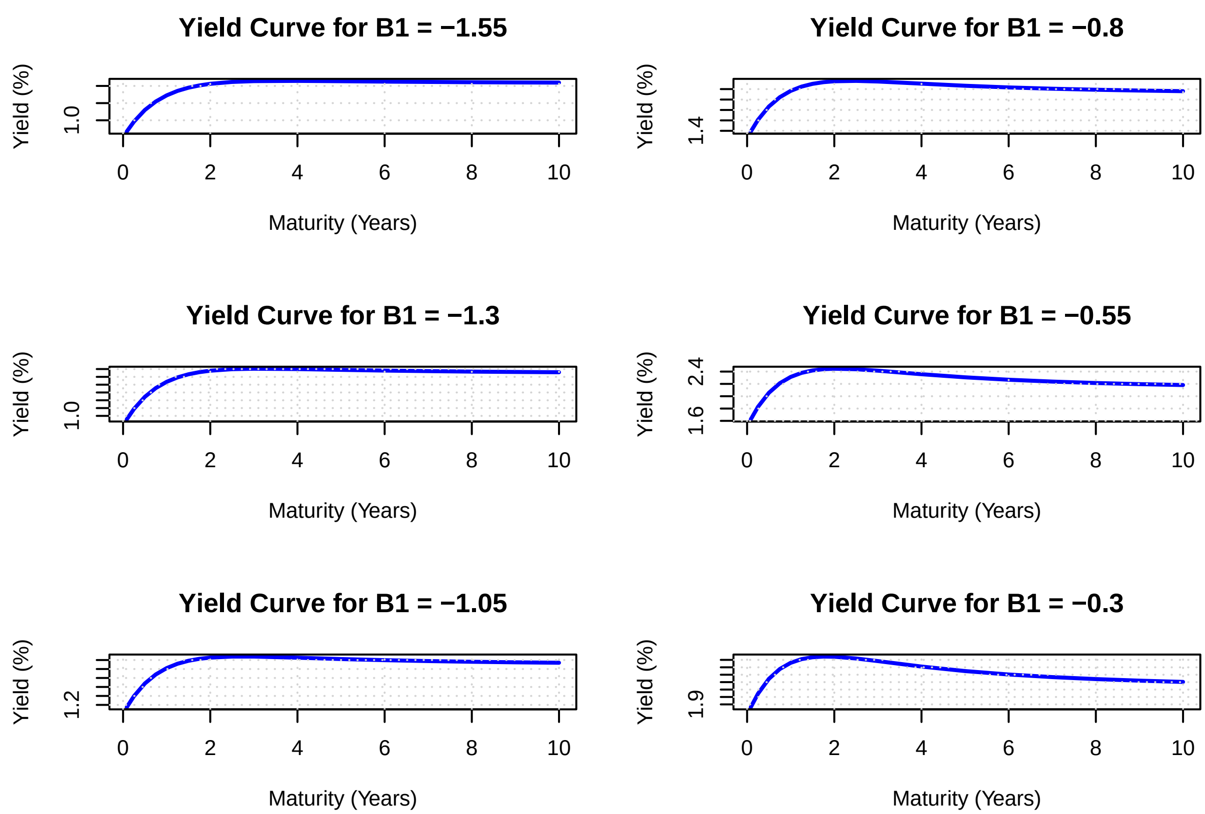
As previously stated:

- β_1 determines the **slope** of the yield curve, primarily affecting short-term rates.
- β_2 controls the **curvature**, influencing mid-term rates by introducing a hump effect.

We will now analyze how the yield curve reacts to changes in these parameters.

Effect of β_1 (Slope Parameter) We create a loop to observe how modifying β_1 impacts the yield curve.

```
layout(matrix(1:6, ncol = 2))
B0 <- 2.05
B2 <- 2.1
L  <- 0.85
B1 <- -1.8
for (i in 1:6) {
  B1 <- B1 + 0.25
  yield_curve <- NS(B0, B1, B2, L, t)
  plot(t, yield_curve, type = "l", col = "blue", lwd = 2,
       xlab = "Maturity (Years)", ylab = "Yield (%)",
       main = paste("Yield Curve for B1 =", round(B1, 2)))
  grid()
}
```

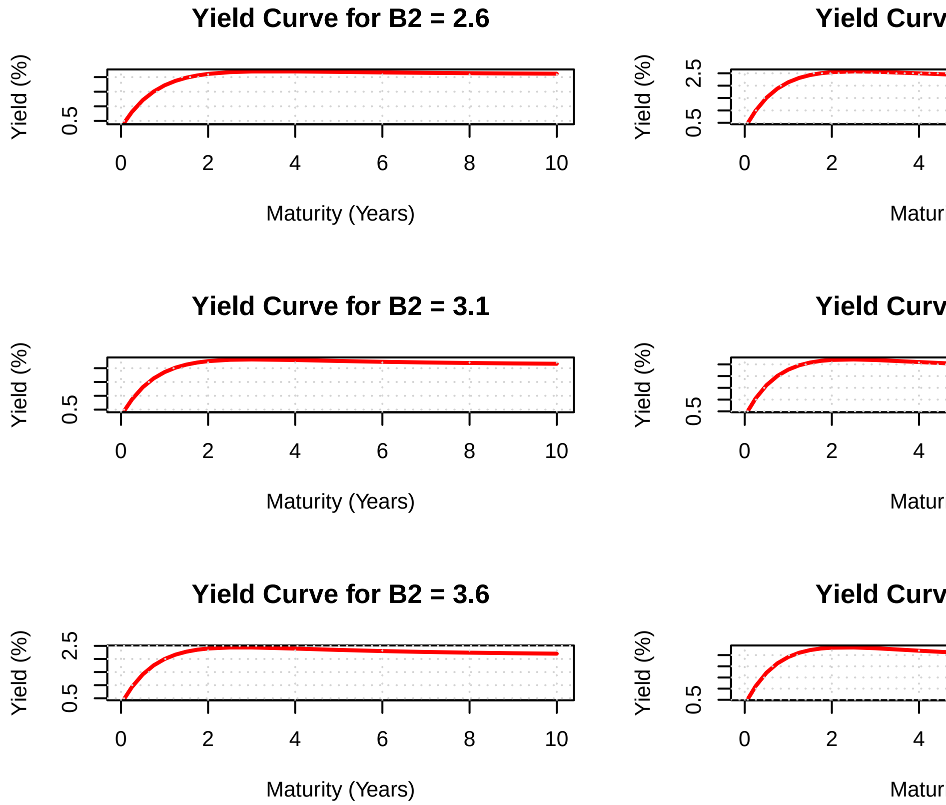


Observations on β_1:

As we increase β_1:

- The slope of the yield curve increases, meaning short-term rates become lower relative to long-term rates.
- A higher β_1 leads to a steeper yield curve, consistent with a normal yield curve.

```r
layout(matrix(1:6, ncol = 2))
B0 <- 2.05
B1 <- -1.8
L  <- 0.85
B2 <- 2.1
for (i in 1:6) {
  B2 <- B2 + 0.5  # Increment B2
  yield_curve <- NS(B0, B1, B2, L, t)  # Compute yield curve
  plot(t, yield_curve, type = "l", col = "red", lwd = 2,
       xlab = "Maturity (Years)", ylab = "Yield (%)",
       main = paste("Yield Curve for B2 =", round(B2, 2)))
  grid()
}
```



Effect of β_2 (Curavature Parameter)

Observations on β_2:

As we increase β_2:

- The hump in the mid-maturity range(2-5 years) becomes more pronounced
- A higher β_2 introduces more curvature, making medium-term rates deviate more from short and long-term rates.

PART 2 : Nelson-Siegel-Svensson Model

Explanation and Plotting

The **Nelson-Siegel-Svensson model** is an extension of the Nelson-Siegel model, adding an extra term to better fit the yield curve, particularly allowing for a second hump.

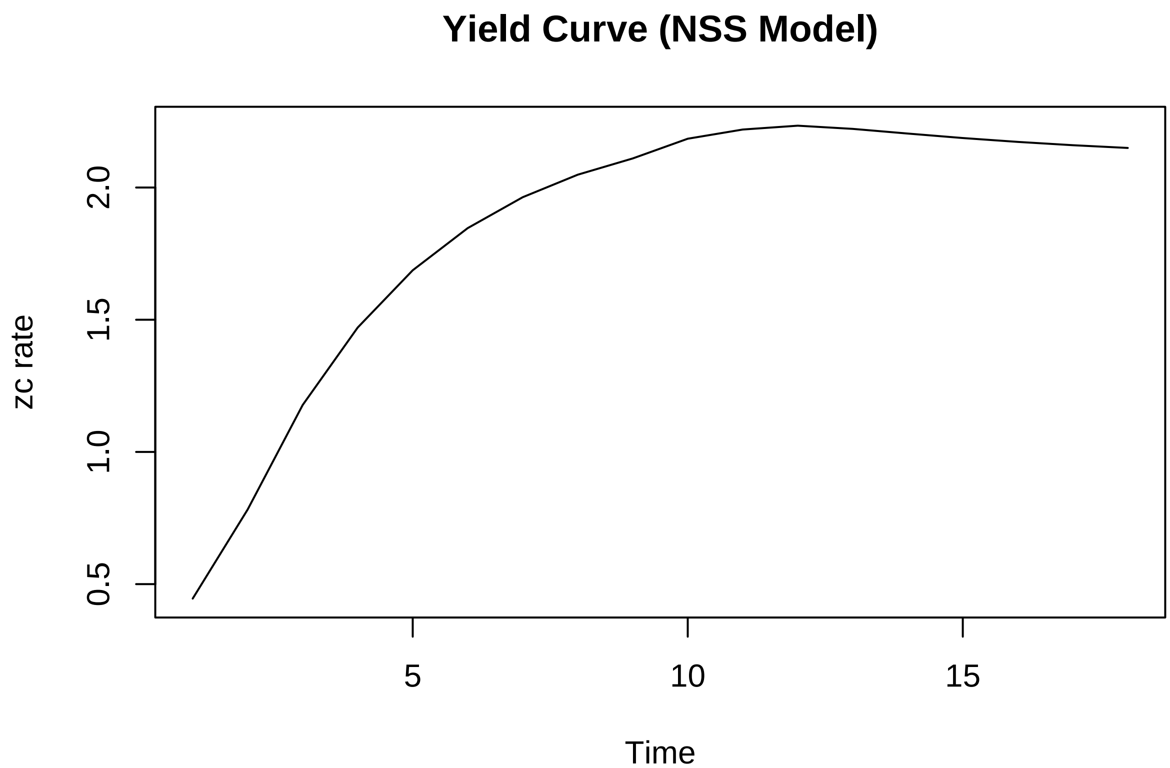
$$y(\tau) = \beta_0 + \beta_1 \frac{1 - e^{-\tau/\lambda_1}}{\tau/\lambda_1} + \beta_2 \left(\frac{1 - e^{-\tau/\lambda_1}}{\tau/\lambda_1} - e^{-\tau/\lambda_1} \right) + \beta_3 \left(\frac{1 - e^{-\tau/\lambda_2}}{\tau/\lambda_2} - e^{-\tau/\lambda_2} \right)$$

Where:

- $y(\tau)$ represents the yield at maturity τ.
- β_0, the intercept, represents the **long-term level** of the yield curve.
- β_1, the slope coefficient, determines the **steepness** of the curve.
- β_2, the curvature parameter, influences the **medium-term hump**.
- β_3, the second curvature parameter, adds **additional flexibility**, allowing a second hump or dip.
- λ_1 and λ_2 determine how quickly the effects of β_1, β_2, and β_3 decay as maturity increases.

Note that our NSS function is contained in the same file as the NS function so we dont need to call it again.

```
NSS_1 <- NSS(2.05,-1.80,2.1,0.5,0.85,1.5,t)
layout(matrix(1:1,1,1))
plot.ts(NSS_1, ylab="zc rate", main="Yield Curve (NSS Model)")
```



Extension on β_3 parameter

As previously stated:

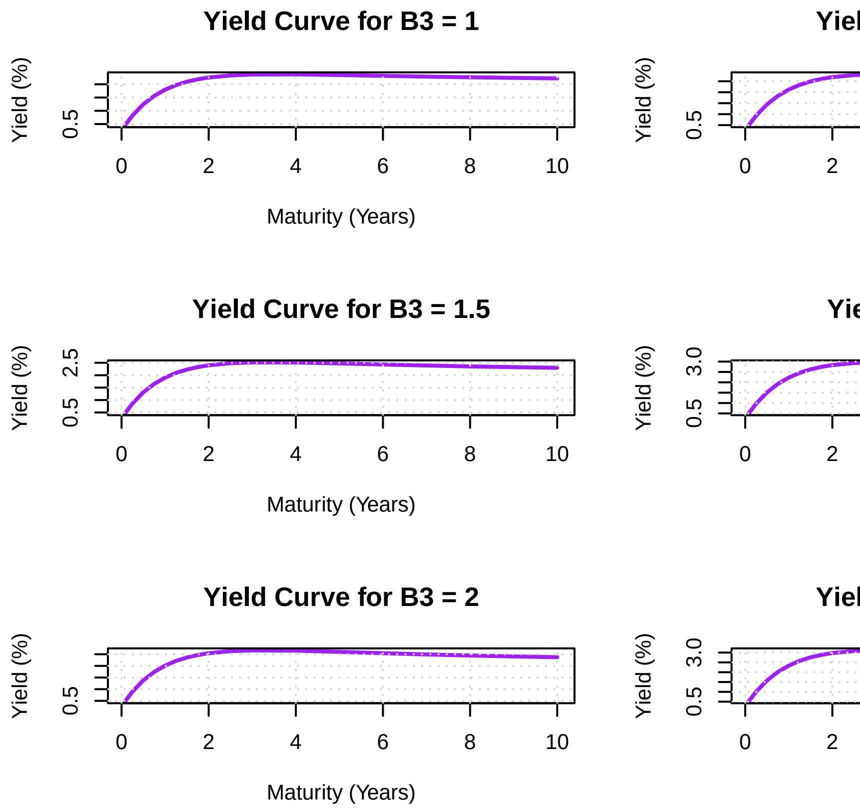
- β_1 determines the **slope** of the yield curve, primarily affecting short-term rates.
- β_2 controls the **curvature**, influencing mid-term rates by introducing a hump effect.
- β_3 allows a **second curvature**, allowing for more flexibility.

As we already analyzed β_1 and β_2 in part 1, we will only analyze β_3 now.

```
layout(matrix(1:6, ncol = 2))
B0 <- 2.05
B1 <- -1.8
B2 <- 2.1
L1  <- 0.85
L2  <- 1.5
B3 <- 0.5

for (i in 1:6) {
```

```
  B3 <- B3 + 0.5
  yield_curve <- NSS(B0, B1, B2, B3, L1, L2, t)
  plot(t, yield_curve, type = "l", col = "purple", lwd = 2,
       xlab = "Maturity (Years)", ylab = "Yield (%)",
       main = paste("Yield Curve for B3 =", round(B3, 2)))
  grid()
}
```



Effect of β_3 (Second Curvature Parameter)

Observations on β_3:

- Increasing β_3 adds a scond hump in long maturities.
- Higher β_3 increases yield curve flexibility.