

Pairs Trading with Nonlinear and Non-Gaussian State Space Models*

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Abstract

This paper studies pairs trading using a nonlinear and non-Gaussian state space model framework. We model the spread between the prices of two assets as an unobservable state variable, and assume that it follows a mean reverting process. This new model has two distinctive features: (1) The innovations to the spread is non-Gaussianity and heteroskedastic. (2) The mean reversion of the spread is nonlinear. We show how to use the filtered spread as the trading indicator to carry out statistical arbitrage. We also propose a new trading strategy and present a Monte Carlo based approach to select the optimal trading rule. As the first empirical application, we apply the new model and the new trading strategy to two examples: PEP vs KO and EWT vs EWH. The results show that the new approach can achieve 21.86% annualized return for the PEP/KO pair and 31.84% annualized return for the EWT/EWH pair. As the second empirical application, we consider all the possible pairs among the largest and the smallest five US banks listed on the NYSE. For these pairs, we compare the performance of the proposed approach with that of the existing popular approaches, both in-sample and out-of-sample. Interestingly, we find that our approach can significantly improve the return and the Sharpe ratio in almost all the cases considered.

Keywords: pairs trading, nonlinear and non-Gaussian state space models, Quasi Monte Carlo Kalman filter.

JEL codes: C32, C41, G11, G17.

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1 Introduction

In early 1980s, a group of physicists, mathematicians and computer scientists, leaded by quantitative analyst Nunzio Tartaglia, tried to use a sophisticated statistical approach to find the opportunities of arbitrage trading (Gatev et al. 2006). Tartaglia’s strategy, later coined pairs trading, is to find a pair of two stocks whose prices have moved similarly historically, and make profit by applying the simple contrarian principles. Since then, pairs trading has become a popular short-term arbitrage strategy used by hedge funds and is often considered as the “ancestor” of statistical arbitrage.

Pairs trading works by constructing a self financing portfolio with a long position in one security and a short position in the other. Given that the two securities have moved together historically, when a temporary anomaly happens, one security would be overvalued than the other relative to the long-term equilibrium. Then, an investor may be able to make money by selling the overvalued security, buying the undervalued security, and clearing the exposure when the two securities settle back to their long-term equilibrium. Because the effect from movement of the market is hedged by this self financing portfolio, pairs trading is market-neutral.

The methods for pairs trading can be broadly divided into nonparametric and parametric methods. In particular, Gatev et al. (2006) propose a nonparametric distance based approach in determining the securities for constructing the pairs. They choose a pair by finding the securities that minimized the sum of squared deviations between the two normalized prices. They argue this approach “best approximates the description of how traders themselves choose pairs”. They find that average annualized excess returns reach 11% for the top pairs portfolios using CRSP daily data from 1962 to 2002. Other Nonparametric methods on pairs trading can also be found in Bogomolov (2013) among others. Overall, the nonparametric distance based approach provides a simple and general method of selecting “good” pairs; however, as pointed out by Krauss (2016) and others, this selection metric is prone to pick up pairs with small variance of the spread, and therefore limits the profitability of pairs trading.

In contrast, the parametric approach tries to capture the mean-reverting characteristic of the spread using a parametric model. For example, Elliott et al. (2005) propose a mean-reverting Gaussian Markov chain model for the spread which is observed in Gaussian noise. See Vidyamurthy (2004), Cummins and Bucca (2012), Tourin and Yan (2013), Moura et al. (2016), Stbinger and Endres (2018), Clegg and Krauss (2018), Elliott and Bradrania (2018), Bai and Wu (2018) for other parametric methods on pairs trading. Overall, the parametric approach provides tractable methods for the analysis of pairs trading; however, most of the existing parametric models are too simple to be capable of capturing the dynamics of asset price, which substantially limits the returns from pairs trading.

Compared with the existing methods on pairs trading, the proposed approach has the following

features: (1) It is based on a nonlinear and non-Gaussian state space model. This modelling can capture several stylized features of financial asset prices, including heavy-tailedness, heteroskedasticity, volatility clustering and nonlinear dependence. (2) The trading strategy is different from the existing ones. It utilizes the features of the model such as heteroskedasticity and volatility clustering, and it can potentially achieve significantly higher returns and Sharpe ratios. (3) The optimal trading rules is also different from the existing ones. Although this rule has no analytic solution, we show that it can be computed effectively using simulations. Finally, the optimal trading rule can adapt to various objectives, such as a high cumulative return, Sharpe ratio, or Calmar ratio.

We apply our approach to two pairs: PEP vs KO and EWT vs EWH. We find that our approach achieves an annualized return of 0.2186 and Sharpe ratio of 2.9518 on the PEP/KO pair and an annualized return of 0.3184 and Sharpe ratio of 3.8892 on the EWT/EWH pair. In comparison, a conventional approach applied to the same pairs can only achieve an annualized return of 0.1311 and Sharpe ratio of 1.1003 for the PEP/KO pair and an annualized return of 0.1480 and Sharpe ratio of 1.1277 for the EWT/EWH pair. Next, we test our approach using all the possible pairs among the largest 5 banks and the smallest 5 banks listed in NYSE. We find significant improvements over the conventional approach for almost all the pairs. We also find that the pairs between small banks produce higher return than the pairs between large banks. This is likely because the spread between small banks are more volatile, providing more opportunities for active trading.

The main contributions of this paper can be summarized as follows. On the theory side, we propose a complete set of tools for pairs trading that include a model for the dynamics of the spread, a new trading strategy and a Monte Carlo method for determining the optimal trading rule. On the empirical side, we apply our approach to various pairs in practice. The results show that the new approach can achieve significant improvements on the performance of pairs trading.

The remainder of this paper is organized as follows. In Section 2, we propose a new model for pairs trading. In Section 3, we propose a new trading strategy based on the mean-reverting property of spread, and compare it with conventional trading strategies using simulations. In Section 4, we implement the proposed approach to actual data, and in Section 5 we conclude the paper.

2 A New Model for Pairs Trading

We propose the following nonlinear and non-Gaussian state space model for pairs trading:

$$P_{A,t} = \phi + \gamma P_{B,t} + x_t + \varepsilon_t \quad (1)$$

$$x_{t+1} = f(x_t; \theta) + g(x_t; \theta) * \eta_t \quad (2)$$

where P_A is the price of security A , P_B is the price of security B , γ is the hedge ratio between two securities, and x is the true spread between P_A and P_B . We assume x follow a mean-reverting process as in (2), $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ and $\eta_t \sim p(\eta_t; \theta)$ which could be non-Gaussian. Popular choices for f , g and p could be the followings. Our framework applies to all of them.

- Linear mean-reverting (Ornstein–Uhlenbeck process): $f(x_t; \theta) = \theta_1 + \theta_2 x_t$
- Nonlinear mean-reverting model: $f(x_t; \theta) = \theta_1 + \theta_2 x_t + \theta_3 x_t^2$
- Ait-Sahalia's nonlinear mean-reverting model (Ait-Sahalia, 1996): $f(x_t; \theta) = \theta_1 + \theta_2 x_t^{-1} + \theta_3 x_t + \theta_4 x_t^2$
- Homoskedasticity model: $g(x_t; \theta) = 1$
- ARCH(m) model: $g(x_t; \theta) = \sqrt{\theta_0 + \sum_{i=1}^m \theta_i x_{t-i}^2}$
- APARCH(m, δ) model: $g(x_t; \theta) = (\theta_0 + \sum_{i=1}^m \theta_i |x_{t-i}|^\delta)^{\frac{1}{\delta}}$
- Gaussian distributed noise: $p(\eta; \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(\mu-\eta)^2}{2\sigma^2}\right)$
- Student's t distributed noise: $p(\eta; \nu) = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\nu\pi}\Gamma(\frac{\nu}{2})} \left(1 + \frac{\eta^2}{\nu}\right)^{-\frac{\nu+1}{2}}$
- Generalized error distributed noise: $p(\eta; \alpha, \beta, \mu) = \frac{\beta}{2\alpha\Gamma(\frac{1}{\beta})} \exp\left(-(|\eta - \mu|/\alpha)^\beta\right)$

In model (1)-(2), we consider x as the unobservable true spread between security A and B , which follows a mean-reverting process. P_A is the observation and P_B is the control variable. Since ϕ and θ_1 in the f function can not be identified simultaneously, we let $\phi = 0$ and denote $\psi = (\gamma, \theta, \sigma_\varepsilon)$ as the parameter of the model (1)-(2). ψ is going to determined based on data set $\{P_{A,t}, P_{B,t}\}_{t=0}^T$.

Our new model has three advantages compared with existing models for pairs trading, such as Elliott et al. (2005) and Moura et al. (2016). First, since η can be non-Gaussian, x can follow a non-Gaussian process. By allowing for this non-Gaussianity in η , the model can capture the distributional deviation from Gaussianity and reproduce heavy-tailed returns.

Second, the model captures heteroskedasticity in financial data. A well-known feature of financial time-series is volatility clustering: “large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes” (Mandelbrot, 1963). This feature was documented later in Ding, Granger and Engle (1993), and Ding and Granger (1996) among others. In model (2), the volatility persistence is represented by ARCH-style modeling. Details about the application of ARCH model in finance can be found in Bollerslev, Chou and Kroner (1992).

Third, in order to characterize the nonlinear dependence in financial data, we allow f to be nonlinear. Scheinkman and LeBaron (1989) find evidence that indicates the presence of nonlinear dependence in weekly returns on the CRSP value-weighted index. Ait-Sahalia (1996) finds nonlinearity in the drift function of interest rate and concludes that “the principal source of rejection of existing (linear drift) models is the strong nonlinearity of the drift”. We keep the functional form of f flexible and, as a result, we can capture the nonlinear dependence in financial data.

3 A New Approach to Pairs Trading

In this section, we discuss the trading strategies and trading rules for pairs trading. In this paper, a trading strategy is the method of buying and selling of assets in markets based on the estimation of the unobservable spread. A trading rule is the predefined values to generate the trading signal for a specific trading strategy with an investing objective. To implement a strategy and rule on pairs trading, we need the following quantities: (i) parameter estimates for the model (1)-(2), (ii) an estimate of the spread, and (iii) choice of a specific strategy and the optimal trading rule, and we discuss these aspects in this section. More specifically, in Section 3.1, we present an algorithm on the filtering of the unobservable spread and parameter estimation. In Section 3.2, We will discuss two benchmark trading strategies. In Section 3.3, we will present and compare three popular trading rules associated with the benchmark trading strategies. In Section 3.4, we propose a new trading strategy. In this new trading strategy, we change the way we open or close a trade, and we will discuss the benefit of this new strategy compared with the benchmark strategies. Since the existing trading rule cannot be simply applied to the model (1)-(2), we propose a new approach to calculate the optimal trading rule based on the simulation of the spread. The detail of this simulation based method is in Section 3.5. In Section 3.6, we summarize the procedure of pairs trading. This procedure can be applied to pairs trading with all of the trading strategies and trading rules discussed in this paper.

3.1 Algorithm for Filtering and Parameter Estimation

For a specification of model (1)-(2), we run the following algorithm of Quasi Monte Carlo Kalman filter for nonlinear and non-Gaussian state space models to estimate the unobservable spread and unknown parameters in the model, based on the observations $\{P_{A,t}, P_{B,t}\}_{t=0}^T$. Suppose the initial spread x_0 follows $N(\mu, \Sigma)$ for any reasonable choices of μ and Σ .

- Step 1: For non-Gaussian density $p(\eta_t)$, we use Gaussian mixture density to approximate its pdf and denote the approximation as $\tilde{p}(\eta_t) = \sum_{i=1}^m \alpha_i \phi(\eta_t - a_i, P_i)$, $\sum_{i=1}^m \alpha_i = 1$ where ϕ is

the Gaussian pdf defined by

$$\phi(v, \Sigma) = \frac{1}{(2\pi)^{1/2}|\Sigma|^{1/2}} \exp\left(-\frac{1}{2}v^T \Sigma^{-1} v\right).$$

To get this approximation, we determine the values of $\{\alpha_i, a_i, P_i\}_{i=1}^m$ by minimizing the relative entropy between the true density $p(\eta_t)$ and its approximation $\tilde{p}(\eta_t)$. The relative entropy is defined by

$$\mathcal{H}(p|\tilde{p}) = \int \left(\log \frac{p(\eta)}{\tilde{p}(\eta)} \right) \times p(\eta) d\eta.$$

If η_t is Gaussian, then this step can be dropped.

- Step 2: Generate a Box-Muller transformed Halton sequence $\{x_t^{(g)}\}_{g=1}^G$ with sequence size G from $\phi(x_t - b_{ts}, P_{ts})$. Compute and store

$$Q_{t+1i} = \frac{1}{G} \sum_{g=1}^G \left(f\left(x_t^{(g)}\right) - c_{t+1i} \right)^2 + \left(g\left(x_t^{(g)}\right) \right)^2 * P_k,$$

and

$$c_{t+1i} = \frac{1}{G} \sum_{g=1}^G f\left(x_t^{(g)}\right) + g\left(x_t^{(g)}\right) * a_k.$$

When $t = 0$, $\{x_0^{(g)}\}_{g=1}^G$ is sampled from $N(\mu, \Sigma)$.

- Step 3: Repeat Step 2 for $s = 1, 2, \dots, J_{t+1}$, $J_{t+1} = m^t$, and $k = 1, \dots, m$, and store c_{t+1i} and Q_{t+1i} for $i = 1, 2, \dots, I_{t+1}$, $I_{t+1} = J_{t+1} * m = m^{t+1}$.
- Step 4: Based on the results from Step 3, generate a Box-Muller transformed Halton sequences $\{x_{t+1i}^{(g)}\}_{g=1}^G$ from $\phi(x_{t+1} - c_{t+1i}, Q_{t+1i})$ for $i = 1, 2, \dots, I_{t+1}$, $I_{t+1} = m^{t+1}$. Then generate $P_{A,t+1i}^{(g)} = x_{t+1i}^{(g)} + \gamma * P_{B,t+1}$. Compute and store the followings

$$\bar{P}_{A,t+1i} = \frac{1}{G} \sum_{g=1}^G P_{A,t+1i}^{(g)},$$

$$V_{t+1i} = \frac{1}{G} \sum_{g=1}^G \left(P_{A,t+1i}^{(g)} - \bar{P}_{A,t+1i} \right)^2 + \sigma_\varepsilon^2,$$

$$S_{t+1i} = \frac{1}{G} \sum_{g=1}^G \left(x_{t+1i}^{(g)} - c_{t+1i} \right) \left(P_{A,t+1i}^{(g)} - \bar{P}_{A,t+1i} \right).$$

- Step 5: Compute $K_{t+1i} = S_{t+1i} V_{t+1i}^{-1}$, $P_{t+1i} = Q_{t+1i} - K_{t+1i}^2 V_{t+1i}$, and $b_{t+1i} = c_{t+1i} + K_{t+1i} (P_{A,t+1} - \bar{P}_{A,t+1})$.

- Step 6: Repeat Step 4-5 for $i = 1, 2, \dots, I_{t+1}$, $I_{t+1} = m^{t+1}$. Compute and store \bar{x}_{t+1} and \bar{P}_{t+1} where $\bar{x}_{t+1} = \sum_{i=1}^{I_{t+1}} \beta_{t+1i} b_{t+1i}$, and

$$\bar{P}_{t+1} = \sum_{i=1}^{I_{t+1}} \beta_{t+1i} (P_{t+1i} + b_{t+1i}^2) - \left(\sum_{i=1}^{I_{t+1}} \beta_{t+1i} b_{t+1i} \right)^2,$$

$$\beta_{t+1i} = \frac{\phi(P_{A,t+1} - c_{t+1i} - \gamma * P_{B,t+1}, V_{t+1i})}{\sum_{i=1}^{I_{t+1}} \phi(P_{A,t+1} - c_{t+1i} - \gamma * P_{B,t+1}, V_{t+1i})}.$$

- Step 7: Repeat Step 2-6 for $t = 0, 1, 2, \dots, T$.

$\{\bar{x}_t\}_{t=1}^T$ from Step 6 is our estimation of the spread. To estimate the unknown parameter in the model, we first write the log-likelihood function as

$$L_T^G(\psi) \equiv \sum_{t=0}^T \log f^G(\psi; P_{A,t}, P_{B,t}) =$$

$$= \sum_{t=1}^T \log \left[\sum_i^{I_{t+1}} \frac{1}{\sqrt{2\pi |V_{t+1i}|}} \exp \left(-\frac{(P_{A,t+1} - \bar{P}_{A,t+1i})^2}{2 * V_{t+1i}} \right) \right]$$

and MLE of the unknown parameter would be determined to maximize the above likelihood, that is,

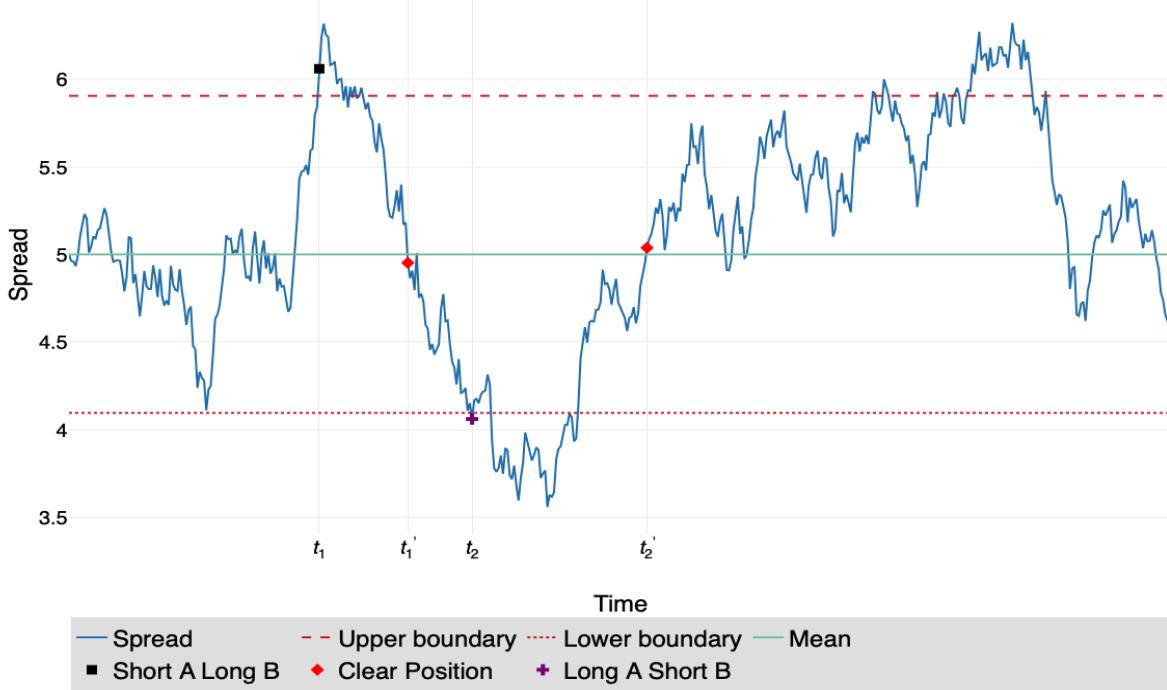
$$\hat{\psi}_{MLE} = \underset{\psi \in \Phi}{\operatorname{argmax}} L_T^G(\psi).$$

3.2 Benchmark Trading Strategies

As we discussed in Section 1, the basic idea for pairs trading is to open a trade (short one asset and long the other one) when the spread deviates from the equilibrium and close the trading when the spread settle back to the equilibrium. The trading strategies for pairs trading are constructed based on this idea. We use Figure 1 and Figure 2 to illustrate two benchmark trading strategies (hereafter Strategy A and Strategy B). In Figure 1 and Figure 2, the same estimated spread is plotted as solid lines, and a preset upper-boundary U and a preset lower-boundary L are plotted as dashed lines. We will discuss how to choose the optimal U and L in Section 3.2. The upper-boundary and lower-boundary act as thresholds to determine whether the spread deviates from the long-term equilibrium enough, and we use these two criteria to open a trade. Also, a preset value C acts as a threshold to determine whether the spread settles back to the long-term equilibrium, and we use this criterion to close a trade. In this paper, we take C as the mean of the spread, and plot it as solid green line in both Figure 1 and Figure 2.

In Strategy A (illustrated in Figure 1), a trade is opened at t_1 when the spread is higher than or equal to U . In this case, we sell 1 share of stock A and buy γ share of stock B. At t'_1 when the

Figure 1: Trading Strategy A



spread is less than or equal to the mean (i.e., C), we close the trade and clear the position. The return from this trade is thus $U - C$. At t_2 when the spread is less than or equal to L , , we open a trade by buying 1 share of stock A and sell γ share of stock B. We close this trade and clear the position at t'_2 when the spread is higher than or equal to the mean. The return from this trade is $C - L$.

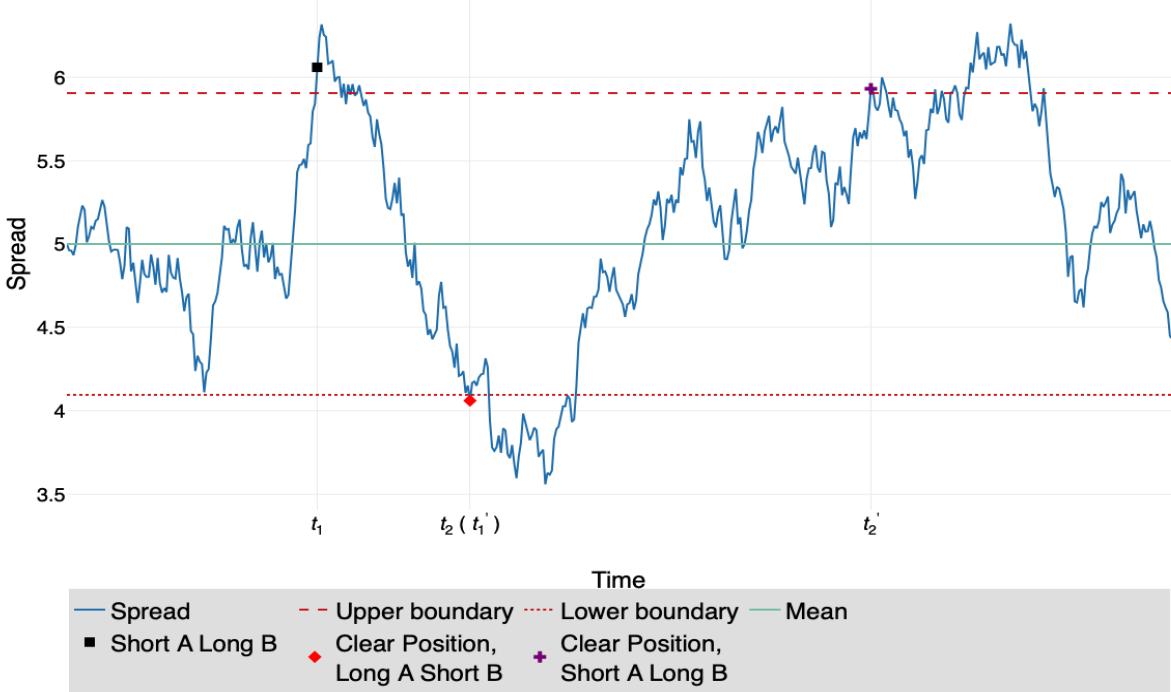
In Strategy B (illustrated in Figure 2), we open a trade when the spread cross the upper-boundary from below (e.g., at t_1) or cross the lower-boundary from above (e.g., at t_2). Unlike the Strategy A, We will hold the portfolio until we need to switch the position. Thus in Strategy B, we clear the exposure at the same time when we open a new trade (i.e., t_2 and t'_1 coincide).

3.3 Conventional Trading Rules

In the implementation of pairs trading, trading rule for a specific trading strategy is the computation of optimal thresholds U and L based on that strategy to fulfill an investing objective¹. There are three popular approaches for computing the optimal thresholds U and L when the model (2) is linear, homoscedastic and Gaussian (i.e., f is linear, g is a constant and η is a Gaussian noise).

¹Investing objective could be various, such as maximizing the expected cumulative return or maximizing the Sharpe ratio.

Figure 2: Trading Strategy B



The optimal trading rule for a general specification of model (2) will be given in Section 3.4.

- Rule I: Ad hoc boundaries

Rule I takes U to be one ($1-\sigma$ rule) or two ($2-\sigma$ rule) standard deviations above the mean, L to be one or two standard deviations below the mean and C to be the mean of the spread. This rule is simple and popular in practice. In particular, the $2-\sigma$ rule was first applied by Gatev et al. (2006) and later checked by Moura et al. (2016), Zeng and Lee (2014) and Cummins and Bucca (2012). The $1-\sigma$ rule was discussed in Zeng and Lee (2014) and the performance of $1-\sigma$ rule and $2-\sigma$ rule was compared in the same paper.

- Rule II : Boundaries based on the first-passage-time

This rule was first adopted by Elliott et al. (2005) and later by Moura et al. (2016). Suppose Z_t follows a standardized Ornstein–Uhlenbeck process:

$$dZ_t = -Z_t dt + \sqrt{2} dW_t$$

Let T_{0,Z_0} be the first passage time of Z_t :

$$T_{0,Z_0} = \inf\{t \geq 0, Z(t) = 0 | Z(0) = Z_0\}.$$

T_{0,Z_0} has a pdf known explicitly:

$$f_{0,Z_0}(t) = \sqrt{\frac{2}{\pi}} \frac{|Z_0| e^{-t}}{(1 - e^{-2t})^{3/2}} \exp\left(-\frac{Z_0^2 e^{-2t}}{2(1 - e^{-2t})}\right)$$

$f_{0,Z_0}(t)$ can be maximized at t^* given by:

$$t^* = \frac{1}{2} \ln \left[1 + \frac{1}{2} \left(\sqrt{(Z_0^2 - 3)^2 + 4Z_0^2} + Z_0^2 - 3 \right) \right]$$

Here t^* is the most possible time, given the value of current spread, that the spread will settle back to the mean. In model (2), if the spread x follows (discrete time) Ornstein–Uhlenbeck process, then we can first standardize x , and then above formula for t^* can be used to construct the optimal C . Similar idea can be applied to compute the optimal upper-boundary U and lower-boundary L .

- Rule III: Boundaries based on the renewal theorem

This rule was first proposed by Bertram (2010), and then extended by Zeng and Lee (2014). In this rule, each trading cycle is separated into two parts, where τ_1 can be used to denote the time from taking (long or short) position to clearing the position, and τ_2 can be used to denote the time from clearing position to opening next trading. That is,

$$\tau_1 = \inf \{t; \hat{x}_t = C | \hat{x}_0 = U\}$$

$$\tau_2 = \inf \{t; \hat{x}_t = U | \hat{x}_0 = C\}$$

Suppose T is the total trading duration we have for a pair, and N_T is the number of transactions we can have in the period $[0, T]$. Then, by the renewal theorem, the return per unit time is given by:

$$(U - C) \lim_{T \rightarrow \infty} \frac{E(N_T)}{T} = \frac{U - C}{E(\tau_1 + \tau_2)}.$$

where $E(\tau_1)$ and $E(\tau_2)$ can be computed based on the density of first passage time, mentioned in Rule II.

The problem of this rule is, as Zeng and Lee (2014) have pointed out, that when there is no transaction cost, this strategy implies U (and L) will be arbitrarily close to C . This implies that the trader values the trading frequency more than the profit per trade. Consequently, this could increase the risk of the portfolio significantly.

3.4 The New Trading Strategy

We summarize the new trading strategy (hereafter Strategy C) in Figure 3. The basic idea of Strategy C is similar to both Strategy A and Strategy B: open a trade when the spread is far away from the equilibrium and close the trade when the spread settle back to the equilibrium. Unlike

the Strategy A and B, in Strategy C, we open a trade when the spread cross the upper-boundary from above (or cross the lower-boundary from below), and we clear the position when the spread cross the mean, or cross the boundaries (U and L) after a trade has been opened (i.e., the spread cross the upper-boundary from below or the lower-boundary from above). For example, in Figure 3a for a homoscedastic model, at t_1, t_2, t_3 and t_4 we open a trade; and at t'_1, t'_2, t'_3 , and t'_4 we clear the exposure. In Figure 3b for a heteroscedastic model, we open a trade at t_1 and t_2 ; and we close the trade at t'_1 , and t'_2 .

We now discuss the properties of this trading strategy when the model (2) is homoscedastic (i.e., the g function is constant) and when it is heteroscedastic (i.e., g is a general function). In the first situation, the main benefit of Strategy C is that we can avoid holding the portfolio when the spread is larger than the upper boundary (or smaller than the lower boundary). This would significantly decrease the risk and drawdown of the portfolio. The main drawback of Strategy C is that the return can be lower because we open the trade when the spread is closer to the mean of the spread than in Strategy A. Therefore, there is a tradeoff between the risk and the return. In the situation when the model (2) is heteroscedastic, this strategy can not only reduce the risk, it can also improve the return. This is because the opening of a trade now depends on the level of the volatility and, as a result, the boundaries are no longer constant over time. The logic of this new strategy is illustrated in Figure 3a and 3b, for homoscedastic and heteroscedastic cases, respectively.

3.5 Simulation Based Method for Optimal Trading Rule

For a general specification of model (1)-(2), the conventional trading rules in Section 3.2 are difficult to be applied. For example, the $1-\sigma$ rule or $2-\sigma$ rule cannot be applied when the model (2) is heteroscedastic; for a complicated specification of model (2), it's impossible to derive the density of the first passage time explicitly, thus Rule II and Rule III are unavailable in this case.

To compute the optimal trading rule under model (2) for all of the trading strategies, we propose to select the optimal boundaries (U and L , we set C as the mean of spread by default) based on the Monte Carlo simulation of the spread (equation (2) given the estimation of the unknown parameters). Different criterion or investing objectives, such as expected return, Sharpe ratio or Calmar ratio² could be used to determine the optimal boundaries for a given trading strategy.

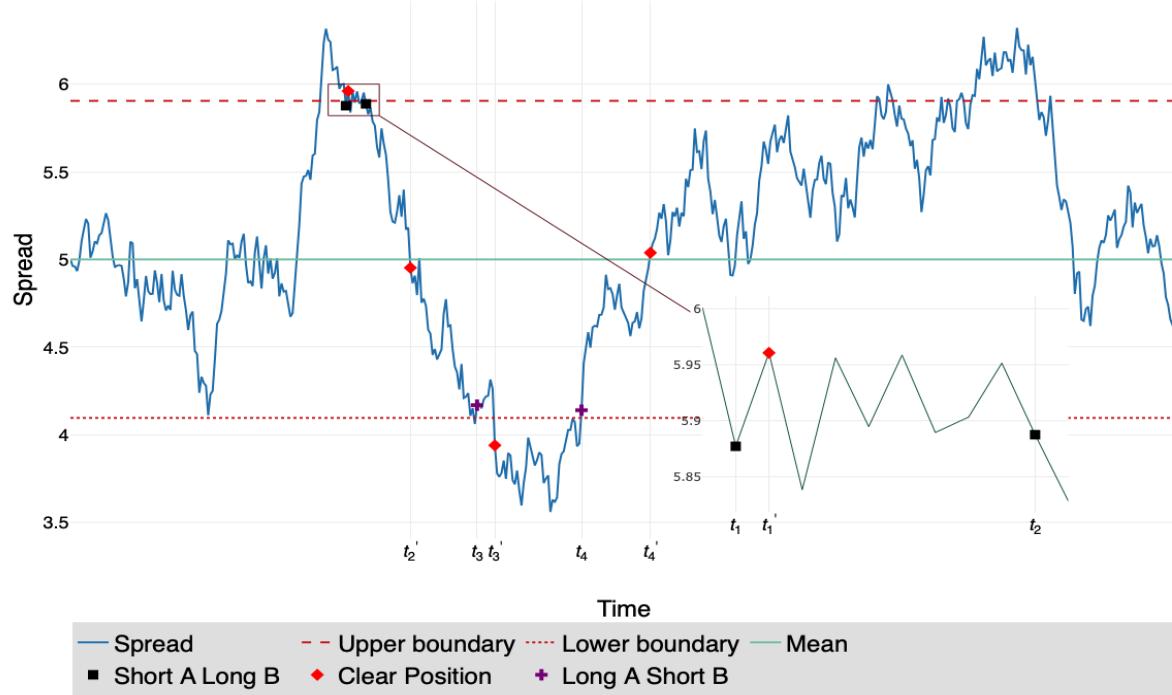
Now we use the following four specifications of model (2) to describe the detail about the computation of the new trading rules.

²Let $CR_{a,t}$ be the cumulative return of portfolio a at time t , and we define the maximum drawdown of the cumulative return across time 0 to T as $MD_{a,T}$:

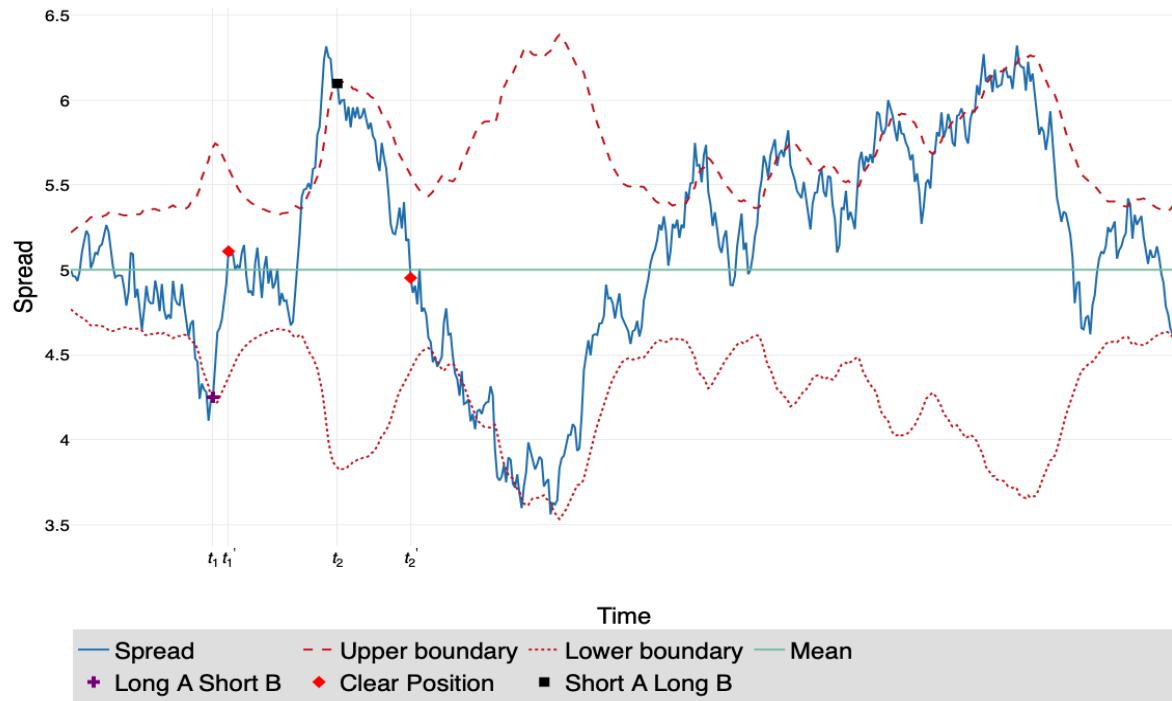
$$MD_{a,T} = \sup_{t \in [0,T]} \left[\sup_{\tau \in [0,t]} CR_{a,\tau} - CR_{a,t} \right].$$

Figure 3: Trading Strategy C

(a) Trading Strategy C in Homoscedastic Model



(b) Trading Strategy C in Heteroscedastic Model



- Model 1: $x_{t+1} = 0.9590 * x_t + 0.0049 * \eta_t$, $\eta_t \sim N(0, 1)$
- Model 2: $x_{t+1} = 0.9 * x_t + 0.5590 * x_t^2 + 0.0049 * \eta_t$, $\eta_t \sim N(0, 1)$
- Model 3: $x_{t+1} = 0.9590 * x_t + \sqrt{(0.00089 + 0.08 * x_t^2)} * \eta_t$, $\eta_t \sim N(0, 1)$
- Model 4: $x_{t+1} = 0.9590 * x_t + \frac{0.0049}{\sqrt{3}} * \eta_t$, $\eta_t \sim t_3$

Model 1 is a linear, homoscedastic, and Gaussian model. This is the most popular model used for pairs trading. See Elliott et al. (2005) and Moura et al. (2016) for examples of this model. Model 2 is a nonlinear model, Model 3 is a heteroscedastic model, and Model 4 is a non-Gaussian model. The last three models are different extensions of Model 1 and have never been discussed in the literature on pairs trading. These four models can be considered as the benchmark models for pairs trading. Further extensions are available based on the combination of these four models, and our simulation based method for optimal trading rule can also be applied to them.

For every specification of Model 1-4, we will calculate the optimal trading rules through the N simulations of the spread for Strategy A, B and C respectively, and compare the resulting performances of the three strategies based on the expected return, Sharpe ratio. More specifically, across all of the examples, we represent the optimal trading rule (upper-boundary U and lower-boundary L) as the ratio to one standard deviation of the spread, and we consider the upper-boundary U between $[0.1, 2.5]$ and lower-boundary L between $[-2.5, -0.1]$ for a grid size of 0.1. For every specification of Model 1-4 and every realization of the process of the spread $\{x_t^{(m,n)}\}_{t=0}^T$, where $m = 1, 2, 3, 4; n = 1, \dots, N$, we choose U_i from $[0.1, 2.5]$ and L_j from $[-2.5, -0.1]$, where $i, j = 1, \dots, 25$, and compute the resulting cumulative return and Sharpe ratio for difference strategies. More specifically, We denote the cumulative return and Sharpe ratio as $CR_{i,j}^{m,k,n}$ and $SR_{i,j}^{m,k,n}$ respectively, where m is for different models, k is for difference strategies and n is for different realization of the spread in simulation. For Model m and strategy k , the resulting expected cumulative return $CR_{i,j}^{m,k}$ and Sharpe ratio $SR_{i,j}^{m,k}$ are computed as

$$CR_{i,j}^{m,k} = \frac{1}{N} \sum_{n=1}^N CR_{i,j}^{m,k,n}$$

$$SR_{i,j}^{m,k} = \frac{1}{N} \sum_{n=1}^N SR_{i,j}^{m,k,n}.$$

Then the Calmar ratio can be defined in a similar way as the Sharpe ratio:

$$Calmar_a \equiv \frac{E(R_a)}{MD_{a,T}}$$

where $E(R_a)$ is the expected return of portfolio a .

Then the optimal trading rule $(U_{m,k}^*, L_{m,k}^*)$ is selected to maximize $CR_{i,j}^{m,k}$ or $SR_{i,j}^{m,k}$, that is,

$$[U_{m,k}^*, L_{m,k}^*] = \arg \max_{U_i, L_j} z_{i,j}^{m,k}$$

where $z = CR$ or SR . Across all of the examples, we set the total trading period to be 1000 trading days (or approximately four years), and we set the simulation size to be $N = 10000$. For simplicity, we assume the transaction cost is 20 bp (0.2%)³, and annualized risk free rate is set to be 0.

In Table 1, we report the optimal trading rule for every combination of the 4 models and 3 strategies, and the resulting expected cumulative return and Sharpe ratio⁴. As we can find from this table, Strategy C outperforms other two strategies when the model is heteroscedastic in both the cumulative return and the Sharpe ratio; also, for other homoscedastic models (Model 1, 2 and 4), the Sharpe ratio of Strategy C is competitive, although the cumulative return is not. This supports our discussion of this new strategy in Section 3.3.

We leave the detailed results of simulation method in appendix. More precisely, the expected cumulative returns and Sharpe ratio as functions of various choices of U and L are given in Figure A1-A4 for every possible combination of the three strategies and four models. The return is displayed in number, not in percentage through all figures.

3.6 Summary

We are now in a position to summarize the procedure for pairs trading based on model (1)-(2) and conclude this section.

- Step 1: Choose a specific model for (1)-(2). Given this model and observations $\{P_{A,t}, P_{B,t}\}_{t=0}^T$, we run Quasi Monte Carlo Kalman filter and get the filtered estimation of the spread $\{\bar{x}_t\}_{t=0}^T$ and the estimation of the unknown parameter $\hat{\psi}$ in the model. The detail of running QMCKF has been discussed in Section 3.1.
- Step 2: Choose a trading strategy, and determine the optimal trading rule (the optimal U and L) for a specific criterion based on Monte Carlo simulation based on the data until time T . The detail of this step can be found in Section 3.2-3.5.
- Step 3: For $t > T$, we run QMCKF and estimate \bar{x}_t with $\psi = \hat{\psi}$, the estimate of the parameter we get in Step 1. We use this $\{\bar{x}_t\}_{t>T}$ and follow the preset trading strategy and optimal trading rule from Step 2 to generate the trading signal for trading.

³This transaction cost is on one asset of the pair. Since a complete trading includes transactions on two assets, the total transaction cost of one complete trading is 40 bp.

⁴If the spread and the strategy is symmetric around the mean, then the optimal upper boundary and lower boundary should also be symmetric around the mean, i.e., $U^* = -L^*$. However, due to the approximation error in gridding, the absolute values of U^* and L^* may not be exactly the same in Table 1.

Table 1: Optimal selection of trading rule for cumulative return and Sharpe ratio

Model	Strategy	U^*	L^*	CR	U^*	L^*	SR
Model 1	A	0.7	-0.7	0.2508	1.1	-1	0.0573
	B	0.5	-0.5	0.2745	0.5	-0.5	0.0522
	C	1	-1	0.1934	0.9	-0.9	0.0679
Model 2	A	0.8	-0.8	0.2749	1.2	-1.3	0.1302
	B	0.6	-0.6	0.3016	0.6	-0.6	0.1198
	C	1.2	-1.3	0.1640	1.2	-1.3	0.1162
Model 3	A	0.3	-0.2	3.9413	0.4	-0.4	0.0751
	B	0.1	-0.1	4.0139	0.1	-0.1	0.0743
	C	0.8	-0.8	6.6763	0.1	-0.1	0.2499
Model 4	A	0.6	-0.6	0.3792	1	-1	0.0881
	B	0.4	-0.5	0.4071	0.5	0.5	0.0782
	C	1	-1	0.2243	1	-1	0.0829

Note: The third and forth columns are the optimal upper-boundary and lower-boundary based on maximizing the cumulative return, and the fifth column is the resulting cumulative return. The sixth and seventh columns are the optimal upper-boundary and lower-boundary based on maximizing the Sharpe ratio, and the eighth column is the resulting Sharpe ratio. The cumulative return is displayed in number, not in percentage.

4 Applications

In this section, we test the performance of Pairs Trading through nonlinear and non-Gaussian state space modeling for different trading strategies. Across all of the applications in this section, we assume the transaction cost is 20 bp and the annualized risk free rate is 2%, and we test the performance of Strategy A, B and C for two specifications of model (2):

- Model I: $x_{t+1} = \theta_0 + \theta_1 x_t + \theta_2 * \eta_t$, $\eta_t \sim N(0, 1)$
- Model II: $x_{t+1} = \theta_0 + \theta_1 x_t + \sqrt{\theta_2 + \theta_3 x_t^2} * \eta_t$, $\eta_t \sim N(0, 1)$

4.1 Pepsi vs Coca

In this example, we examine the performance of Pairs Trading for PEP (Pepsi) and KO (Coca). The data is the daily observation of adjusted closing prices of PEP and KO from 01/03/2012-06/28/2019.

Table 2 reports the parameter estimation of both Model I and Model II for this pair. The trading signal for Model I is given in Figure A5 and that for Model II is given in Figure A6, and the annualized performance (annualized return, annualized Std Dev, annualized Sharpe ratio and Calmar ratio, and annualized Pain index) is given in Table 3. The plot of the cumulative return and drawdown of every strategy through the whole trading period for both models are given in Figure A7 and A8. It's easy to find that in Model II, the annualized return of Strategy C is almost 50% higher than those of Strategy A and B, while Strategy C keeps the risk (measured by Annualized Std Dev) almost half of Strategy A or B. By comparing the Sharpe ratio, Calmar ratio and Pain index, we can find this improvement is significant. While the difference of performances of Strategy A and Strategy B across the two models is limited. This implies the effect of heteroskedasticity modelling to the performances of Strategy A and B is not significant. This is because in Strategy A and B, the hedging portfolio will be held until the spread is around the mean, so the frequency of changing positions is low in Strategy A or B than that in Strategy C. This can be easily confirmed by counting the trading numbers based on Figure A5 and Figure A6.

Table 2: Parameter estimation of Model I and Model II on PEP vs KO

	Model I	Model II
γ	1.98	2.03
σ_{ε}^2	0.012	0.011
θ_0	-0.0001	-0.001
θ_1	0.9572	0.9330
θ_2	0.029	0.0003
θ_3	-	0.1283

4.2 EWT vs EWH

In this example, we examine the performance of Pairs Trading for EWT and EWH. The data is the daily observation of adjusted closing prices of EWT and EWH from 01/01/2012-05/01/2019. EWT is the iShares MSCI Taiwan ETF managed by BlackRock, which seeks to track the investment results of an index composed of Taiwanese equities, and EWH is that for Hong Kong equities. Following the example of PEP vs KO, we will test the performance of Strategy A, B and C for Model I and Model II. We report the parameter estimation in Table 4 and the trading signal in Figure A9 and Figure A10. By comparing the annualized performance in Table 5, we can find the heteroskedasticity modeling can improve the performance of Strategy C significantly, while has no effect on Strategy A or B. Also, the riskiness of Strategy B (small Sharpe ratio and Calmar

Table 3: Annualized Performance of Pairs Trading on PEP vs KO

	Return	Std Dev	Sharpe	Calmar	Pain index
Strategy A, Model I	0.1311	0.0988	1.1003	1.3742	0.0195
Strategy B, Model I	0.1385	0.1153	1.0052	1.2204	0.0334
Strategy C, Model I	0.0618	0.0534	0.7649	0.8243	0.0087
Strategy A, Model II	0.1340	0.1038	1.0751	1.4040	0.0200
Strategy B, Model II	0.1407	0.1139	1.0366	1.2398	0.0258
Strategy C, Model II	0.2186	0.0659	2.9518	8.2384	0.0030

Note: The data is from 01/03/2012-06/28/2019. The return is displayed in number, instead of in percentage.

ratio and high annualized standard variance) is confirmed again in this example. We also plot the cumulative return and drawdown of every strategy through the whole trading period for both models in Figure A11 and A12.

Table 4: Parameter estimation of Model I and Model II on EWT vs EWH

	Model I	Model II
γ	1.40	1.42
σ_ε^2	0.0007	0.0006
θ_0	-0.0004	-0.0015
θ_1	0.9898	0.9589
θ_2	0.0337	0.0016
θ_3	-	0.1136

4.3 Pairs Trading on US Banks Listed on NYSE

We use this example to illustrate the improvement of our new modelling and strategy by implementing pairs trading on US banks listed on NYSE during 01/01/2013-01/10/2019. To avoid data snooping and make our results more concrete, we use a simple way to choose assets and construct pairs. More precisely, based on the market capacity, we select the 5 largest banks to construct the group of large banks and the 5 smallest banks to construct the group of small banks. The large bank group includes: JPM, BAC, WFC, C and USB⁵, and the small bank group includes: CPF, BANC,

⁵JPM is for J P Morgan Chase & Co; BAC is for Bank of America Corporation; WFC is for Wells Fargo & Company; C is for Citigroup Inc.; USB is for U.S. Bancorp.

Table 5: Annualized Performance of Pairs Trading on EWT vs EWH

	Return	Std Dev	Sharpe	Calmar	Pain index
Strategy A, Model I	0.1480	0.1111	1.1277	1.3042	0.0156
Strategy B, Model I	0.1109	0.1362	0.6531	0.7836	0.0328
Strategy C, Model I	0.1294	0.0740	1.4458	3.0926	0.0080
Strategy A, Model II	0.1402	0.1223	0.9622	1.2354	0.0196
Strategy B, Model II	0.1093	0.1349	0.6473	0.7717	0.0306
Strategy C, Model II	0.3184	0.0752	3.8892	10.3005	0.0032

Note: The data is from 01/03/2012-06/28/2019. The return is displayed in number, instead of in percentage.

CUBI, NBHC, FCF⁶. We compare the performance between Model I combined with Strategy A and Model II combined with Strategy C. Model I combined with Strategy A is a popular approach in the existing literature on pairs trading, and it can be a good benchmark for comparison.

In Table A1, we report the performance of these two approaches on 10 pairs among the large banks. The performance on 10 pairs among the small banks is given in Table A2. It's easy to find that Model II combined with Strategy C outperforms Model I combined with Strategy A through almost all of the pairs, either in the sense of annualized return or annualized Sharpe ratio. And the improvement of Model II combined with Strategy C in Sharpe ratio is much more significant than that in return. For example, when trading is implemented on pairs among large banks, the improvement on return is 41.29%, and the improvement on Sharpe ratio is 89.23%; and if trading is implemented on pairs among small banks, the improvement on return is 74.41%, and the improvement on Sharpe ratio is 151.8%.

Also, by comparing the results in Table A1 and A2, we can find that the performance of pairs among small banks would be better than that among large banks, either Model I combined with Strategy A or Model II combined with Strategy C is applied for trading. For example, if we exercise Model I combined with Strategy A, the mean of returns of all pairs among large banks would be 0.0703, that among small banks can be improved to 0.1524; and if Model II combined with Strategy C is exercised, we could get an improvement of 0.1664 (from 0.0994 to 0.2658) by switching from trading on large banks to trading on small banks. This is because the movement of prices of small banks is more volatile than that of large banks, and thus the volatility of the spread between small banks is bigger than that between large banks.

In Table A3, we report the performance of the two approaches of pairs trading on all possible

⁶CPF is for CPB Inc.; BANC is for Banc of California, Inc.; CUBI is for Customers Bancorp, Inc.; NBHC is for National Bank Holdings Corporation; FCF is for First Commonwealth Financial Corporation.

pairs between large banks and small banks, that is, we pair one large bank with one small bank. For some pairs, such as JPM/CUBI and BAC/CUBI, the resulting spread is far from mean-reverting, thus the performance of pairs trading is poor for these pairs. Similar to our findings from Table A1 and A2, in this exercise, we can also find that the improvement of Model II combined with Strategy C with respect to Model I combined with Strategy A on Sharpe ratio would be more significant than return (208.4% on Sharpe ratio, and 103.6% on return).

The results of Table A1-A3 are also plotted in Figure A13 and A14 to give a more straightforward comparison of the performances.

To further investigate the performance of pairs trading, we check the out-of-sample performance of the two approaches on the 10 bank stocks. More precisely, we separate 01/10/2012-01/12/2019 into two periods: 01/10/2012-01/01/2018 as in-sample period and 01/01/2018-01/12/2019 as out-of-sample period. We use the in-sample data to train the model, estimate the parameter of the model, and determine the optimal trading rules. In out-of-sample period, we use the parameters and optimal trading rules based on in-sample data to generate the trading signal. The results are given in Table A4-A9. We can confirm our earlier findings through these tables also: (1) Model II combined with Strategy C outperforms Model I combined with Strategy A in both return and Sharpe ratio, and the improvement is more significant in Sharpe ratio. (2) The performance of pairs trading on small banks would be better than large banks. Also, by comparing the performance through in-sample period to out-of-sample period, we can find that pairing large bank with small bank would be more robust than pairing large banks only or small banks only.

5 Conclusion

Pairs trading is a statistical arbitrage involves the long/short position of overpriced and underpriced assets. Our result in this paper shows that digging into the modeling and trading strategy can improve the performance of pairs trading significantly and implies the great potential of pairs trading on financial market. This can help the empirical research on the general profitability of pairs trading and discussion on the tests of market efficiency, and we leave this for future research.

References

- Agns Tourin and Raphael Yan, 2013, *Dynamic pairs trading using the stochastic control approach*, Journal of Economic Dynamics and Control, 37 (2013) 1972-1981.
- Ait-Sahalia, Y., 1996, *Testing Continuous-Time Models of the Spot Interest Rate*, Review of Financial Studies, 9, 385-426
- Avellaneda, M., and J.-H. Lee. 2010. *Statistical arbitrage in the US equities market*. Quantitative Finance 10:761–782.
- Benoit B. Mandelbrot, 1971, *When Can Price be Arbitraged Efficiently? A Limit to the Validity of the Random Walk and Martingale Models*, The Review of Economics and Statistics, Vol. 53, No. 3 (Aug., 1971), pp. 225-236
- Bogomolov, T. 2013. *Pairs trading based on statistical variability of the spread process*, Quantitative Finance 13:1411–1430.
- Carlos Eduardo de Moura, Adrian Pizzinga and Jorge Zubelli (2016), *A pairs trading strategy based on linear state space models and the Kalman filter*, Quantitative Finance
- Clegg, Matthew and Krauss, Christopher. 2018, *Pairs trading with partial cointegration*, Quantitative Finance 18 (1), 121–138.
- Cummins, Mark and Bucca, Andrea, 2012, *Quantitative spread trading on crude oil and refined products markets*, Quantitative Finance. Dec2012, Vol. 12 Issue 12, p1857-1875.
- David A. Hsieh, 1989, *Testing for Nonlinear Dependence in Daily Foreign Exchange Rates*, The Journal of Business, Vol. 62, No. 3 (Jul., 1989), pp. 339-368
- Ding, Z., Granger, C.W.J. *Modeling volatility persistence of speculative returns: A new approach*, Journal of Econometrics, 1996, vol. 73, issue 1, 185-215
- E. F. Fama and James D. MacBeth. *Risk, return, and equilibrium*, The Journal of Political Economy 79.1 (1971), pp. 30–55
- Elliott, R. J., J. Van Der Hoek, and W. P. Malcolm. 2005. *Pairs trading*. Quantitative Finance 5:271–276
- Elliott, R. J. ; Bradrania, R, 2018, *Estimating a regime switching pairs trading model*, Quantitative Finance, 2018, Vol.18(5), pp.877-883

Eugene F. Fama, 1970, *Efficient Capital Markets: A Review of Theory and Empirical Work*, The Journal of Finance, Vol. 25, No. 2, Papers and Proceedings of the TwentyEighth Annual Meeting of the American Finance Association New York, N.Y. December, 28-30, 1969 (May, 1970), pp. 383-417

Gatev, E.G., Goetzmann, W.N. and Rouwenhorst, K.G. (2006). *Pairs Trading: Performance of a Relative Value Arbitrage Rule*. The Review of Financial Studies, 19, 797-827.

Jose A. Scheinkman and Blake LeBaron, 1989, *Nonlinear dynamics and stock returns*, *The Journal of Business*, Vol. 62, No. 3 (Jul., 1989), pp. 311-337

Kiyoshi Suzuki, 2018, *Optimal pair-trading strategy over long/short/square positions—empirical study*, Quantitative Finance, Volume 18, 2018 - Issue 1

Kon S, 1984, *Models of stock returns: a comparison*, J. Finance XXXIX 147–65

Mandelbrot B, 1963, *The variation of certain speculative prices*, J. Business XXXVI 392–417

Ole E. Barndorff-Nielsen, and Neil Shephard, 2001, *Non-Gaussian Ornstein-Uhlenbeck-based models and some of their uses in financial economics*, J. R. Statist. Soc. B (2001) 63, Part 2, pp. 167-241

Rad, H., R. K. Y. Low, and R. Faff. 2016. *The profitability of pairs trading strategies: distance, cointegration and copula methods*. Quantitative Finance 16:1541–1558.

Rama Cont, 2001, *Empirical properties of asset returns: stylized facts and statistical issues*, Quantitative Finance VOL 1 (2001) 223–236

Sergio M. Focardi, Frank J. Fabozzic, Ivan K. Mitov, 2016, *A new approach to statistical arbitrage: Strategies based on dynamic factor models of prices and their performance*. Journal of Banking & Finance, 65 (2016) 134-155

Stbinger, Johannes and Endres, Sylvia, 2018, *Pairs trading with a mean-reverting jump-diffusion model on high-frequency data*, Quantitative Finance Volume 18, 2018 - Issue 10

Tim Bollerslev, Ray Y. Chou and Kenneth F. Kroner, 1992, *ARCH modeling in finance: A review of the theory and empirical evidence*, Journal of Econometrics 52 (1992) 5-59.

Vidyamurthy, G., 2004. *Pairs trading: Quantitative methods and analysis*. J. Wiley, Hoboken, N.J.

Yang Bai and Lan Wu, 2018, *Analytic value function for optimal regime-switching pairs trading rules*, Quantitative Finance, 2018, Vol.18(4)

Yang, S. Y., Qiao, Q., Beling, P. A., Scherer, W. T., Kirilenko, A. A., 2015. *Gaussian process-based algorithmic trading strategy identification*. Quantitative Finance 0 (0), 1–21.

Yaoting Lei and Jing Xu, 2015, *Costly arbitrage through pairs trading*, Journal of Economic Dynamics & Control, 56 (2015) 1-19

Zeng, Z., Lee, C. G., 2014. *Pairs trading: optimal thresholds and profitability*. Quantitative Finance 14 (11), 1881–1893.

Zhuanxin Ding, Clive W.J. Granger, Robert F. Engle (1993) *A long memory property of stock market returns and a new model*,Journal of Empirical Finance, Volume 1, Issue 1, 1993, Pages 83-106

Table A1: Performance of Pairs Trading on Intergroup Pairs of Big Banks

Pair	Stock #1	Stock #2	Model I + Strategy A		Model II + Strategy C		Improvement (in %)	
			Return	Sharpe	Return	Sharpe	Return	Sharpe
1	JPM	BAC	0.1185	1.0030	0.0961	1.1126	-18.90	10.93
2	JPM	WFC	0.0229	0.2268	0.0581	0.7434	153.7	227.8
3	JPM	C	0.0567	0.5359	0.1049	1.3486	85.01	151.7
4	JPM	USB	0.0412	0.3971	0.0663	0.7832	60.92	97.23
5	BAC	WFC	0.0451	0.3455	0.0695	0.6046	54.10	74.99
6	BAC	C	0.0874	0.8158	0.1369	1.7516	56.64	114.7
7	BAC	USB	0.0554	0.3786	0.0923	1.0077	66.61	166.2
8	WFC	C	0.1031	0.8041	0.1014	0.9731	-1.649	21.02
9	WFC	USB	0.0591	0.5631	0.0674	0.8934	14.04	58.66
10	C	USB	0.1140	0.9040	0.2009	2.0862	76.23	130.8
Mean		0.0703	0.5974	0.0994	1.1304	41.29	89.23	
Min		0.0229	0.2268	0.0581	0.6046	153.7	166.6	
Max		0.1185	1.0030	0.2009	2.0862	69.54	108.0	
Median		0.0579	0.5495	0.0942	0.9904	62.69	80.24	

Note: Return is the annualized return, displayed in number, not in percentage. Sharpe is the annualized Sharpe ratio. Improvement is defined as $\frac{(\text{Model II+Strategy C}) - (\text{Model I+Strategy A})}{|\text{Model I+Strategy A}|}$ for return and Sharpe ratio respectively, measured in percentage.

Table A2: Performance of Pairs Trading on Intergroup Pairs of Small Banks

Pair	Stock #1	Stock #2	Model I + Strategy A			Model II + Strategy C			Improvement (in %)
			Return	Sharpe	Return	Sharpe	Return	Sharpe	
1	CPF	BANC	0.1832	0.6745	0.2158	1.3428	17.79	99.08	
2	CPF	CUBI	0.1092	0.4736	0.2374	1.3563	117.4	186.4	
3	CPF	NBHC	0.1436	0.7694	0.1912	1.2573	33.15	63.41	
4	CPF	FCF	0.1162	0.7127	0.2175	1.7210	87.18	141.5	
5	BANC	CUBI	0.1583	0.5199	0.4820	1.9742	204.5	279.7	
6	BANC	NBHC	0.2105	0.8353	0.1807	1.1435	-14.16	36.90	
7	BANC	FCF	0.1669	0.5830	0.3094	2.1898	85.38	275.6	
8	CUBI	NBHC	0.1575	0.6049	0.2392	1.4485	51.87	139.5	
9	CUBI	FCF	0.1362	0.5593	0.2718	1.5292	99.56	173.4	
10	NBHC	FCF	0.1425	0.8161	0.3132	2.5273	119.8	209.7	
Mean		0.1524	0.6549	0.2658	1.6490	74.41	151.8		
Min		0.1092	0.4736	0.1807	1.1435	65.48	141.4		
Max		0.2105	0.8353	0.4820	2.5273	129.0	202.6		
Median		0.1506	0.6397	0.2383	1.4889	58.29	132.7		

Note: Return is the annualized return, displayed in number, not in percentage. Sharpe is the annualized Sharpe ratio.
Improvement is defined as that in Table A1

Table A3: Performance of Pairs Trading on Intragroup Pairs.

Pair	Stock #1	Stock #2	Model I + Strategy A		Model II + Strategy C		Improvement (in %)	
			Return	Sharpe	Return	Sharpe	Return	Sharpe
1	JPM	CPF	0.0670	0.3965	0.1833	1.4799	173.6	273.2
2	JPM	BANC	0.0587	0.2396	0.0935	0.8334	59.28	247.8
3	JPM	CUBI	-0.0604	-0.2669	0.0423	0.3536	170.0	232.5
4	JPM	NBHC	0.1860	0.9750	0.2683	2.1385	44.25	119.3
5	JPM	FCF	0.1151	0.7230	0.2594	2.3479	125.4	224.7
6	BAC	CPF	0.0778	0.3770	0.2486	1.5596	219.5	313.7
7	BAC	BANC	0.0565	0.2124	0.1383	0.7916	144.8	272.7
8	BAC	CUBI	-0.0959	-0.3612	0.0473	0.5852	149.4	262.0
9	BAC	NBHC	0.1942	0.9496	0.3420	2.4948	76.11	162.7
10	BAC	FCF	0.1729	0.9061	0.2541	2.1954	46.96	142.3
11	WFC	CPF	0.0420	0.2149	0.1138	1.2746	171.0	493.1
12	WFC	BANC	0.1671	0.6058	0.2071	1.0214	23.94	68.60
13	WFC	CUBI	0.0606	0.2572	0.2053	1.3002	238.8	405.5
14	WFC	NBHC	0.1410	0.7844	0.1237	0.9464	-12.27	20.65
15	WFC	FCF	0.1058	0.5948	0.1366	1.3104	29.11	120.3
16	C	CPF	0.1421	0.7000	0.2214	2.1513	55.81	207.3
17	C	BANC	0.0244	0.0961	0.1999	1.1101	719.3	1055
18	C	CUBI	-0.0031	-0.0138	0.0617	0.4357	2090	3257
19	C	NBHC	0.2164	1.0536	0.2927	2.3896	35.26	126.8
20	C	FCF	0.1520	0.7687	0.2246	1.8611	47.76	142.1
21	USB	CPF	0.0782	0.4494	0.2408	2.0902	207.9	365.1
22	USB	BANC	0.1435	0.5450	0.2361	1.7444	64.53	220.1
23	USB	CUBI	-0.0678	-0.2938	0.0700	0.3497	203.2	219.0
24	USB	NBHC	0.1911	1.2574	0.2384	2.1422	24.74	70.37
25	USB	FCF	0.0789	0.5077	0.1206	1.1142	52.85	119.5
Mean			0.0898	0.4671	0.1828	1.4409	103.6	208.4
Min			-0.0959	-0.3612	0.0423	0.3497	144.1	196.8
Max			0.2164	1.2574	0.3420	2.4948	58.04	98.41
Median			0.0789	0.5077	0.2053	1.3104	160.2	158.1

Note: Return is the annualized return, displayed in number, not in percentage. Sharpe is the annualized Sharpe ratio. Improvement is defined as that in Table A1

Table A4: In Sample Performance of Pairs Trading on Intergroup Pairs of Big Banks

Pair	Stock #1	Stock #2	Model I + Strategy A			Model II + Strategy C			Improvement (in %)
			Return	Sharpe	Return	Sharpe	Return	Sharpe	
1	JPM	BAC	0.1145	0.8864	0.1501	1.8003	31.09	103.1	
2	JPM	WFC	0.0160	0.1461	0.0795	0.9451	396.9	546.9	
3	JPM	C	0.0664	0.5686	0.1013	1.5193	52.56	167.2	
4	JPM	USB	0.0186	0.2172	0.0629	1.4293	238.2	558.1	
5	BAC	WFC	0.0027	0.0179	0.0568	0.4748	2004	2553	
6	BAC	C	0.0920	0.7252	0.1193	1.5417	29.67	112.6	
7	BAC	USB	0.0603	0.3936	0.1535	1.5144	154.6	284.8	
8	WFC	C	0.0827	0.5918	0.1219	1.2283	47.40	107.6	
9	WFC	USB	0.0600	0.6432	0.0739	0.9603	23.17	49.30	
10	C	USB	0.1146	0.8553	0.1695	1.7648	47.91	106.3	
Mean			0.0628	0.5045	0.1089	1.3178	73.42	161.2	
Min			0.0027	0.0179	0.0568	0.4748	2004	2553	
Max			0.1146	0.8864	0.1695	1.8003	47.91	103.1	
Median			0.0634	0.5802	0.1103	1.4719	74.11	153.7	

Note: The data is from 01/10/2012 to 01/01/2018. Return is the annualized return, displayed in number, not in percentage. Sharpe is the annualized Sharpe ratio. Improvement is defined as that in Table A1.

Table A5: Out of Sample Performance of Pairs Trading on Intergroup Pairs of Big Banks

Pair	Stock #1	Stock #2	Model I + Strategy A			Model II + Strategy C			Improvement (in %)	
			Return	Sharpe	Return	Sharpe	Return	Sharpe	Return	Sharpe
1	JPM	BAC	-0.0503	-0.4730	-0.0500	-0.4760	0.5964	-0.6342		
2	JPM	WFC	-0.0809	-0.5693	-0.0361	-0.3281	55.38	42.37		
3	JPM	C	-0.0841	-0.6845	0.0299	0.3228	135.6	147.2		
4	JPM	USB	0.0867	0.9267	0.1297	1.6816	49.60	81.46		
5	BAC	WFC	0.0364	0.4593	0.0464	0.4636	27.47	0.9362		
6	BAC	C	-0.0512	-0.3766	0.0149	0.2612	129.1	169.4		
7	BAC	USB	-0.0037	-0.0252	0.0587	0.5169	1686	2151		
8	WFC	C	-0.0586	-0.3472	0.0698	0.7619	219.1	319.5		
9	WFC	USB	-0.1029	-0.6961	0.0269	0.3591	126.4	151.6		
10	C	USB	-0.0486	-0.2948	0.0942	0.7796	293.8	364.5		
	Mean		-0.0357	-0.2081	0.0384	0.4343	207.6	308.7		
	Min		-0.1029	-0.6961	0.0500	-0.4760	51.41	31.62		
	Max		0.0867	0.9267	0.1297	1.6816	49.60	81.46		
	Median		-0.0508	-0.3619	0.0382	0.4114	175.2	213.7		

Note: The data is from 01/01/2018 to 01/12/2019. Return is the annualized return, displayed in number, not in percentage. Sharpe is the annualized Sharpe ratio. Improvement is defined as that in Table A1.

Table A6: In Sample Performance of Pairs Trading on Intergroup Pairs of Small Banks

Pair	Stock #1	Stock #2	Model I + Strategy A			Model II + Strategy C			Improvement (in %)	
			Return	Sharpe	Return	Sharpe	Return	Sharpe	Return	Sharpe
1	CPF	BANC	0.2713	0.9758	0.3513	2.0574	29.56	110.8		
2	CPF	CUBI	0.1226	0.4404	0.4457	1.9114	263.5	334.0		
3	CPF	NBHC	0.1905	0.9823	0.2559	1.7188	34.33	74.98		
4	CPF	FCF	0.1855	1.2385	0.2453	2.5505	32.24	105.9		
5	BANC	CUBI	0.2500	0.6928	0.4076	1.9505	63.04	181.5		
6	BANC	NBHC	0.2406	0.8926	0.1699	1.4127	-29.38	58.27		
7	BANC	FCF	0.2056	0.7819	0.3308	1.8279	60.89	133.8		
8	CUBI	NBHC	0.1130	0.3808	0.2164	1.8059	91.50	374.2		
9	CUBI	FCF	0.1125	0.4133	0.1886	1.1579	67.64	180.2		
10	NBHC	FCF	0.1026	0.5723	0.2523	1.8035	145.9	215.1		
	Mean		0.1794	0.7371	0.2864	1.8197	59.63	146.9		
	Min		0.1026	0.3808	0.1699	1.1579	65.59	204.1		
	Max		0.2713	1.2385	0.4457	2.5505	64.28	105.9		
	Median		0.1880	0.7374	0.2541	1.8169	35.16	146.4		

Note: The data is from 01/10/2012 to 01/01/2018. Return is the annualized return, displayed in number, not in percentage. Sharpe is the annualized Sharpe ratio. Improvement is defined as that in Table A1.

Table A7: Out of Sample Performance of Pairs Trading on Intergroup Pairs of Small Banks

Pair	Stock #1	Stock #2	Model I + Strategy A			Model II + Strategy C			Improvement (in %)
			Return	Sharpe	Return	Sharpe	Return	Sharpe	
1	CPF	BANC	0.1856	0.7541	0.1649	0.8297	-11.15	10.03	
2	CPF	CUBI	-0.0924	-0.3528	0.2424	1.8467	362.3	623.4	
3	CPF	NBHC	-0.0769	-0.3944	0.1621	1.0216	310.8	359.0	
4	CPF	FCF	-0.0373	-0.1906	0.2094	1.4249	661.4	847.6	
5	BANC	CUBI	0.1266	0.7454	0.4109	2.5902	224.6	247.5	
6	BANC	NBHC	-0.1577	-0.6720	-0.0797	-0.3926	49.46	41.58	
7	BANC	FCF	0.0107	0.0821	0.1601	1.3930	1396	1596	
8	CUBI	NBHC	-0.1475	-0.5514	0	-	100	100	
9	CUBI	FCF	-0.1137	-0.4079	0	-	100	100	
10	NBHC	FCF	-0.0578	-0.3088	0.1520	1.0421	363.0	437.4	
	Mean		-0.0360	-0.1296	0.1422	0.9756	494.6	852.6	
	Min		-0.1577	-0.6720	-0.0797	-0.3926	49.46	41.58	
	Max		0.1856	0.7541	0.4109	2.5902	121.4	243.5	
	Median		-0.0674	-0.3308	0.1611	1.0319	339.2	411.9	

Note: The data is from 01/01/2018 to 01/12/2019. Return is the annualized return, displayed in number, not in percentage. Sharpe is the annualized Sharpe ratio. Improvement is defined as that in Table A1. The returns for CUBI/NBHC and CUBI/FCF are 0 because no trading is opened for these two pairs during the out-of-sample period, and the Sharpe ratios are undefined.

Table A8: In Sample Performance of Pairs Trading on Intragroup Pairs

Pair	Stock #1	Stock #2	Model I + Strategy A		Model II + Strategy C		Improvement (in %)	
			Return	Sharpe	Return	Sharpe	Return	Sharpe
1	JPM	CPF	0.1668	0.9415	0.2866	3.0567	71.82	224.7
2	JPM	BANC	0.2067	0.7134	0.2581	1.5501	24.87	117.3
3	JPM	CUBI	0.0649	0.9832	0.2576	1.6633	296.9	69.17
4	JPM	NBHC	0.1505	0.8387	0.2735	2.2745	81.73	171.2
5	JPM	FCF	0.2083	1.3273	0.3281	2.9235	57.51	120.3
6	BAC	CPF	0.1572	0.7484	0.2099	1.7310	33.52	131.3
7	BAC	BANC	0.2361	0.7452	0.1708	1.0044	-27.66	34.78
8	BAC	CUBI	0.0789	0.2755	0.1669	1.4519	111.5	427.0
9	BAC	NBHC	0.2608	1.2323	0.3354	2.5663	28.60	108.3
10	BAC	FCF	0.1918	1.0401	0.2653	2.3337	38.32	124.4
11	WFC	CPF	0.0376	0.1924	0.0988	0.6388	162.8	232.0
12	WFC	BANC	0.2371	0.8323	0.2165	1.0599	-8.690	27.53
13	WFC	CUBI	0.0729	0.2682	0.2307	1.9597	216.5	630.7
14	WFC	NBHC	0.0974	0.5548	0.0917	0.6167	-5.850	11.16
15	WFC	FCF	0.0656	0.3971	0.1413	1.1406	115.4	187.2
16	C	CPF	0.0571	0.2873	0.1766	1.4015	206.3	387.8
17	C	BANC	0.2454	0.8899	0.2154	1.9512	-12.22	119.3
18	C	CUBI	0.0715	0.2696	0.1589	1.0954	122.2	306.3
19	C	NBHC	0.1279	0.6511	0.2125	1.5321	66.15	135.3
20	C	FCF	0.1160	0.6154	0.1790	1.3736	54.31	123.2
21	USB	CPF	0.0654	0.4915	0.2126	1.9990	225.1	306.7
22	USB	BANC	0.2164	0.7529	0.3389	1.9118	56.61	153.9
23	USB	CUBI	0.0565	0.2443	0.2826	1.9450	400.2	696.2
24	USB	NBHC	0.1340	0.9289	0.1947	1.5321	45.30	64.94
25	USB	FCF	0.0922	0.6221	0.2167	2.1579	135.0	246.9
Mean			0.1366	0.6737	0.2208	1.7148	61.61	154.5
Min			0.0376	0.1924	0.0917	0.6167	143.9	220.5
Max			0.2608	1.3273	0.3389	3.0567	29.95	130.3
Median			0.1279	0.7134	0.2154	1.6633	68.41	133.2

Note: The data is from 01/10/2012 to 01/01/2018. Return is the annualized return, displayed in number, not in percentage. Sharpe is the annualized Sharpe ratio. Improvement is defined as that in Table A1.

Table A9: Out of Sample Performance of Pairs Trading on Intragroup Pairs

Pair	Stock #1	Stock #2	Model I + Strategy A		Model II + Strategy C		Improvement (in %)	
			Return	Sharpe	Return	Sharpe	Return	Sharpe
1	JPM	CPF	0.1514	0.8997	0.2731	2.3058	80.38	156.3
2	JPM	BANC	0.2190	0.9752	0.2023	1.1630	-7.626	19.26
3	JPM	CUBI	0.0965	1.1227	0.1610	1.0135	66.84	-9.727
4	JPM	NBHC	0.0303	0.1492	0.1799	1.8165	493.7	1117
5	JPM	FCF	0.0878	0.4209	0.1682	1.0338	91.57	145.6
6	BAC	CPF	0.0379	0.1702	0.1592	1.3579	320.1	697.8
7	BAC	BANC	0.1763	0.6913	0.1693	0.8830	-3.971	27.73
8	BAC	CUBI	0.0926	0.3435	0.1014	0.4298	9.503	25.12
9	BAC	NBHC	-0.0212	-0.0999	0.0144	0.7148	167.9	815.5
10	BAC	FCF	0.0196	0.0899	0.1117	0.8152	469.9	8.6.8
11	WFC	CPF	-0.0625	-0.2981	-0.0061	0.6388	90.24	314.3
12	WFC	BANC	0.0583	0.2249	0.1282	0.6058	119.9	169.4
13	WFC	CUBI	-0.0181	-0.0652	0.2826	1.5870	1661	2534
14	WFC	NBHC	-0.1181	-0.5631	0.0447	0.2594	137.8	146.1
15	WFC	FCF	-0.0821	-0.3725	0.1225	0.8413	249.2	325.9
16	C	CPF	-0.0072	-0.0314	0.1433	1.1894	2090	3888
17	C	BANC	0.1238	0.4691	0.0839	0.6480	-32.23	38.13
18	C	CUBI	0.0459	0.1692	0.2568	1.2778	459.5	655.2
19	C	NBHC	-0.0648	-0.2911	0.2108	2.1138	425.3	826.1
20	C	FCF	-0.0265	-0.1143	0.2174	1.2651	920.4	1207
21	USB	CPF	0.2108	2.2429	0.2652	2.4946	25.81	11.22
22	USB	BANC	0.1951	0.8939	0.1909	1.3332	-2.153	49.14
23	USB	CUBI	0.1516	0.7685	0.2356	1.5712	55.41	104.5
24	USB	NBHC	-0.0242	-0.1258	0.1514	0.9637	725.6	866.1
25	USB	FCF	0.0037	0.0192	0.1979	1.2151	5249	6229
Mean			0.0510	0.3076	0.1626	1.1815	218.6	284.2
Min			-0.1181	-0.5631	-0.0061	0.2594	94.84	146.4
Max			0.2190	2.2429	0.2826	2.4946	29.04	11.22
Median			0.0379	0.1692	0.1682	1.1630	343.8	587.4

Note: The data is from 01/01/2018 to 01/12/2019. Return is the annualized return, displayed in number, not in percentage. Sharpe is the annualized Sharpe ratio. Improvement is defined as that in Table A1.

Figure A1: Performance of Strategy A, B and C, based on Model 1

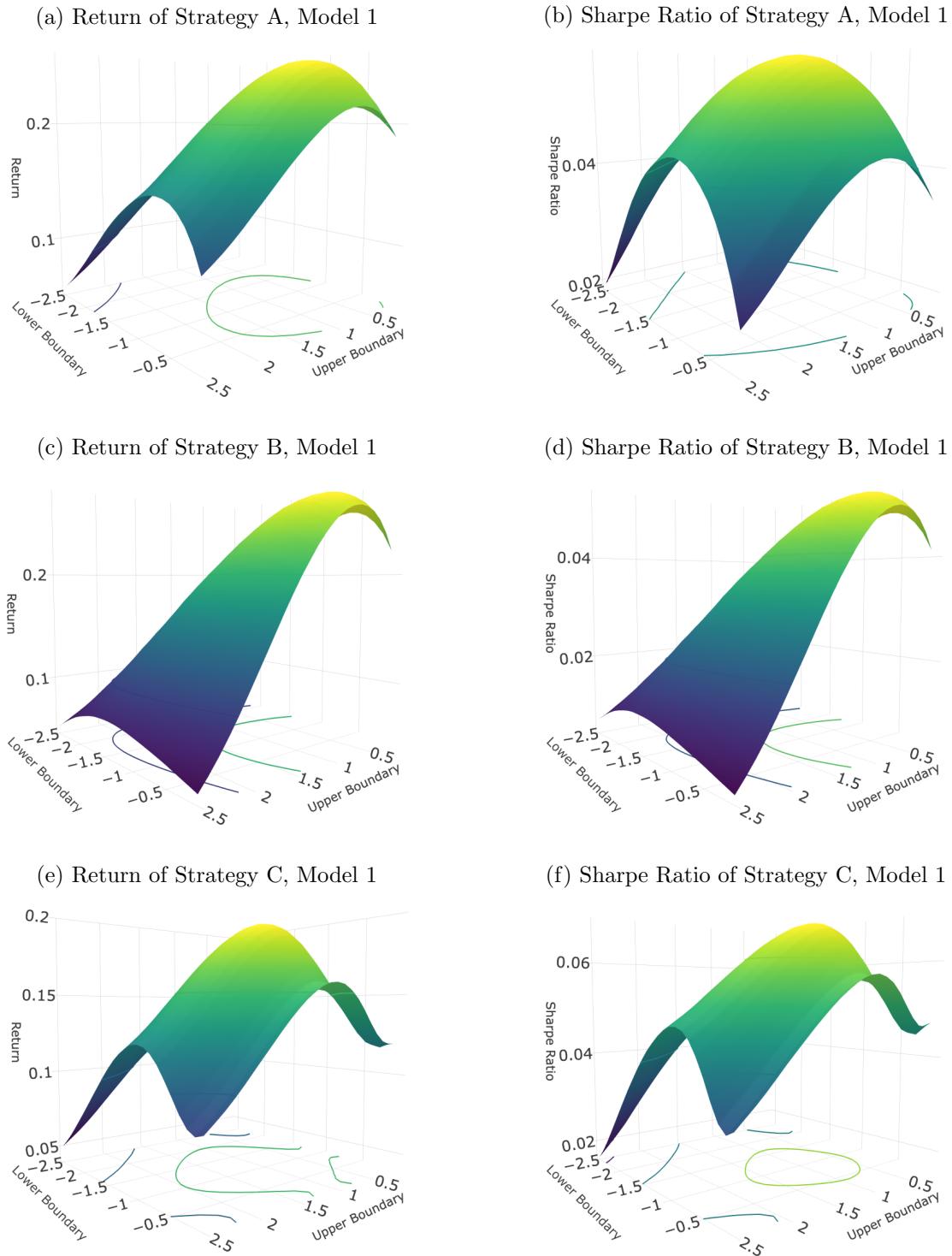


Figure A2: Performance of Strategy A, B and C, based on Model 2

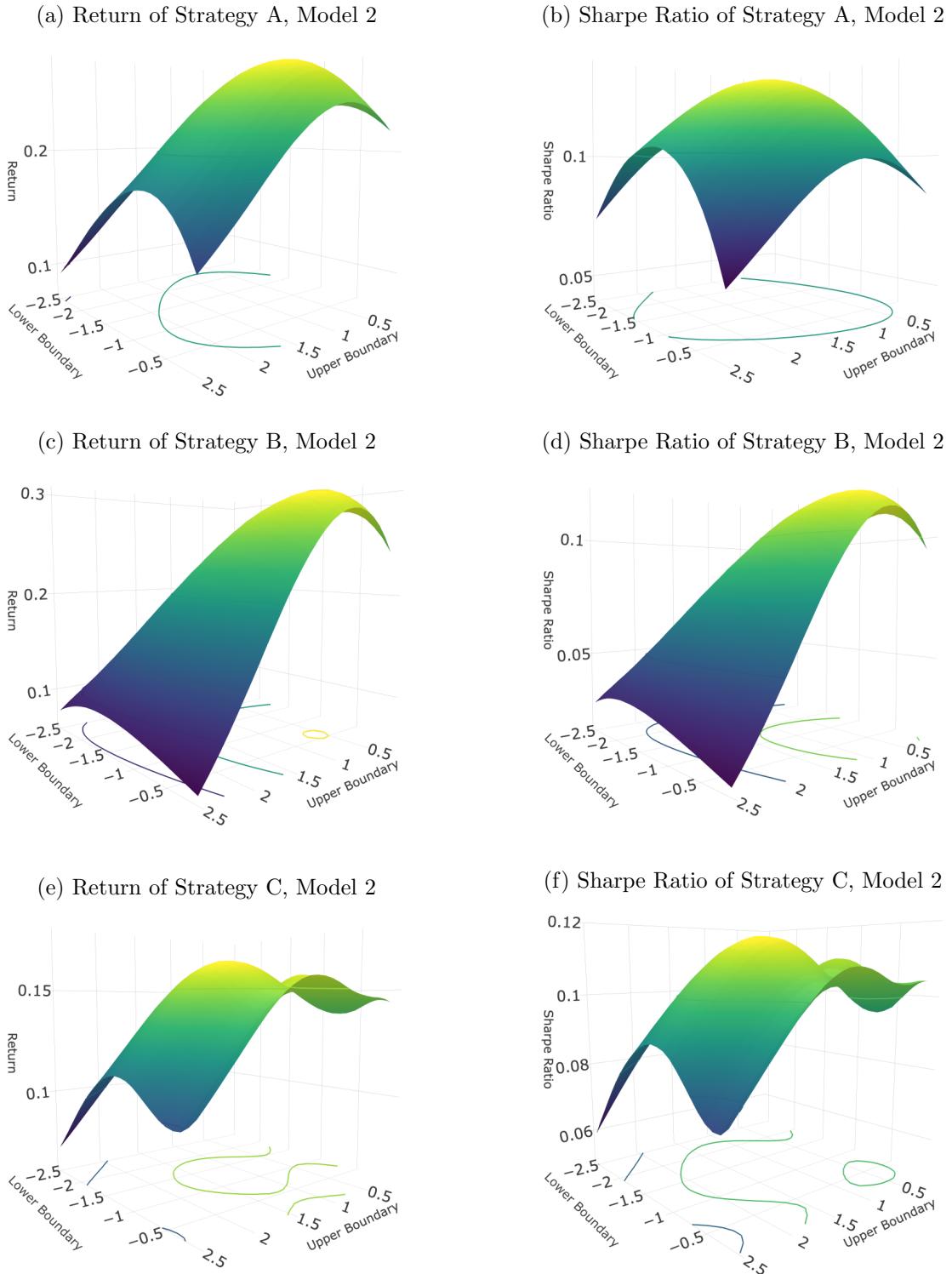


Figure A3: Performance of Strategy A, B and C, based on Model 3

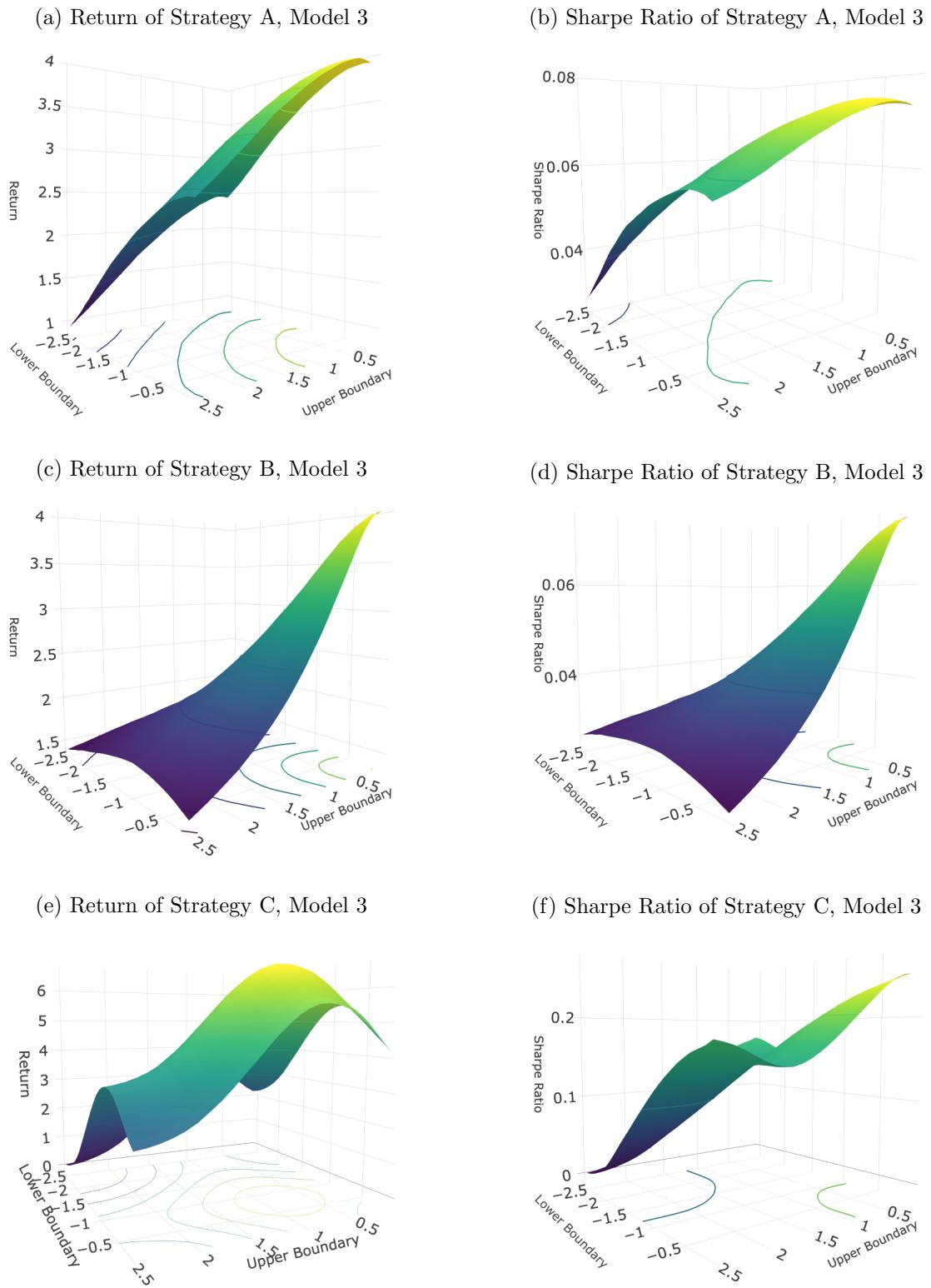


Figure A4: Performance of Strategy A, B and C, based on Model 4

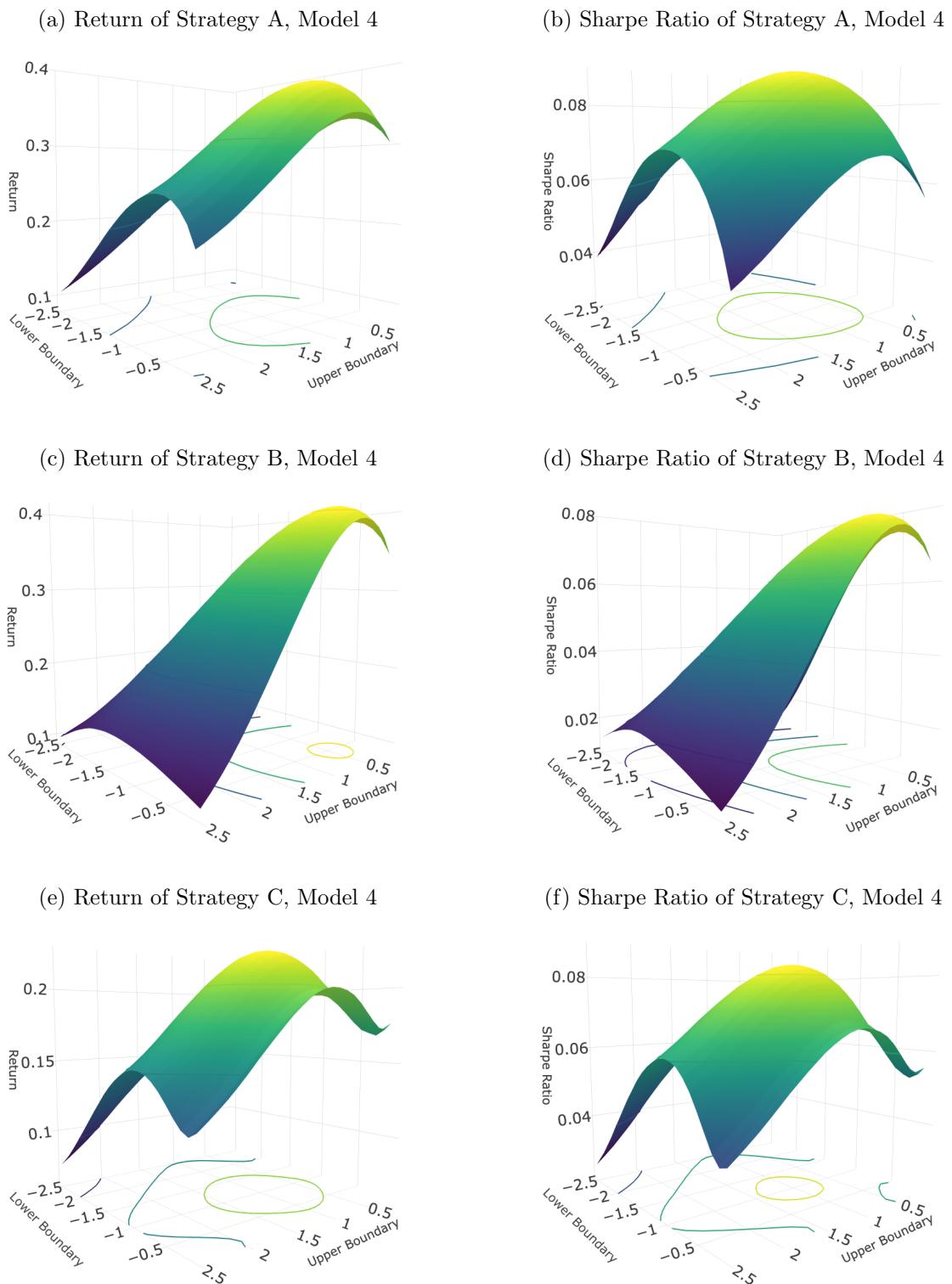
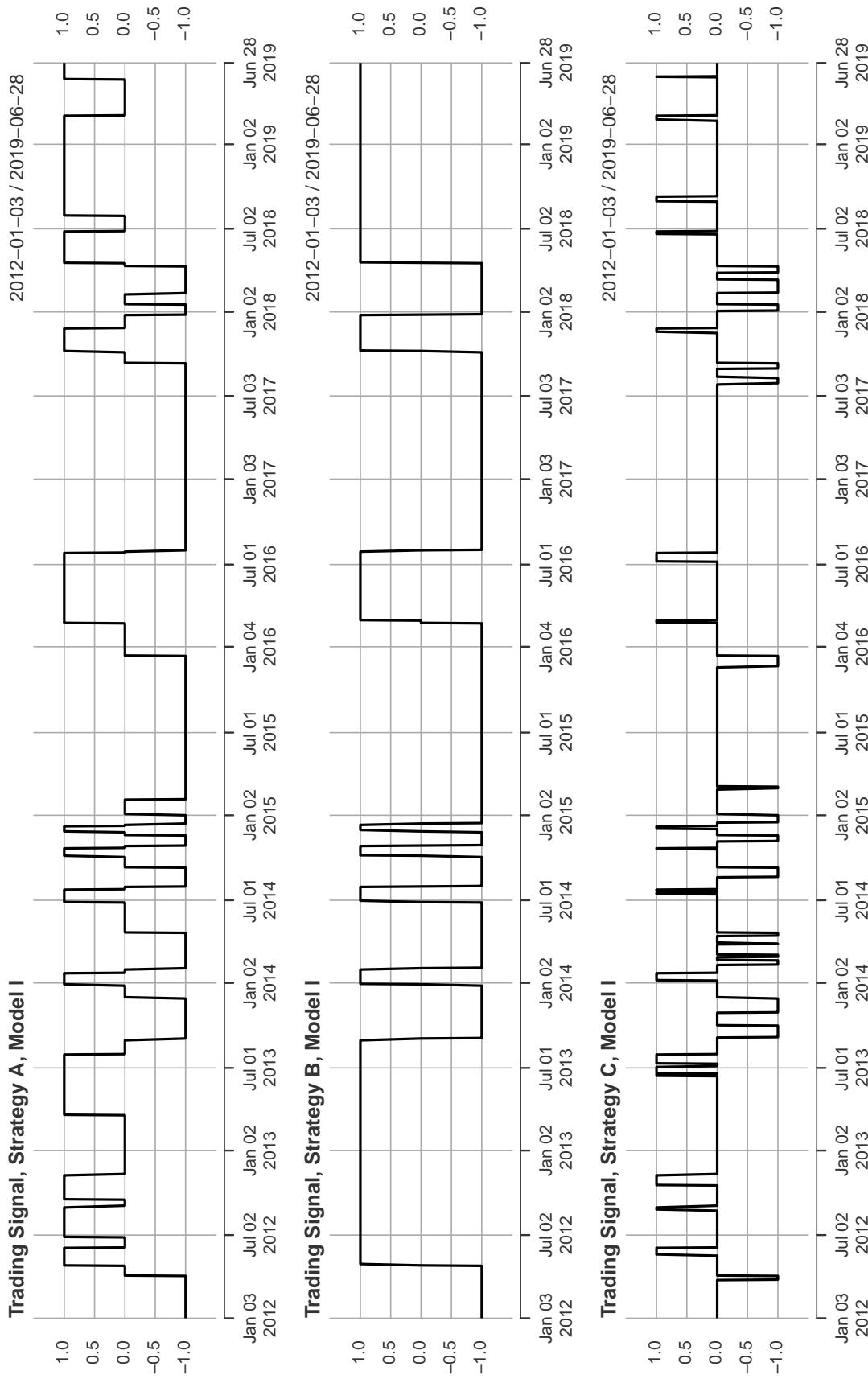
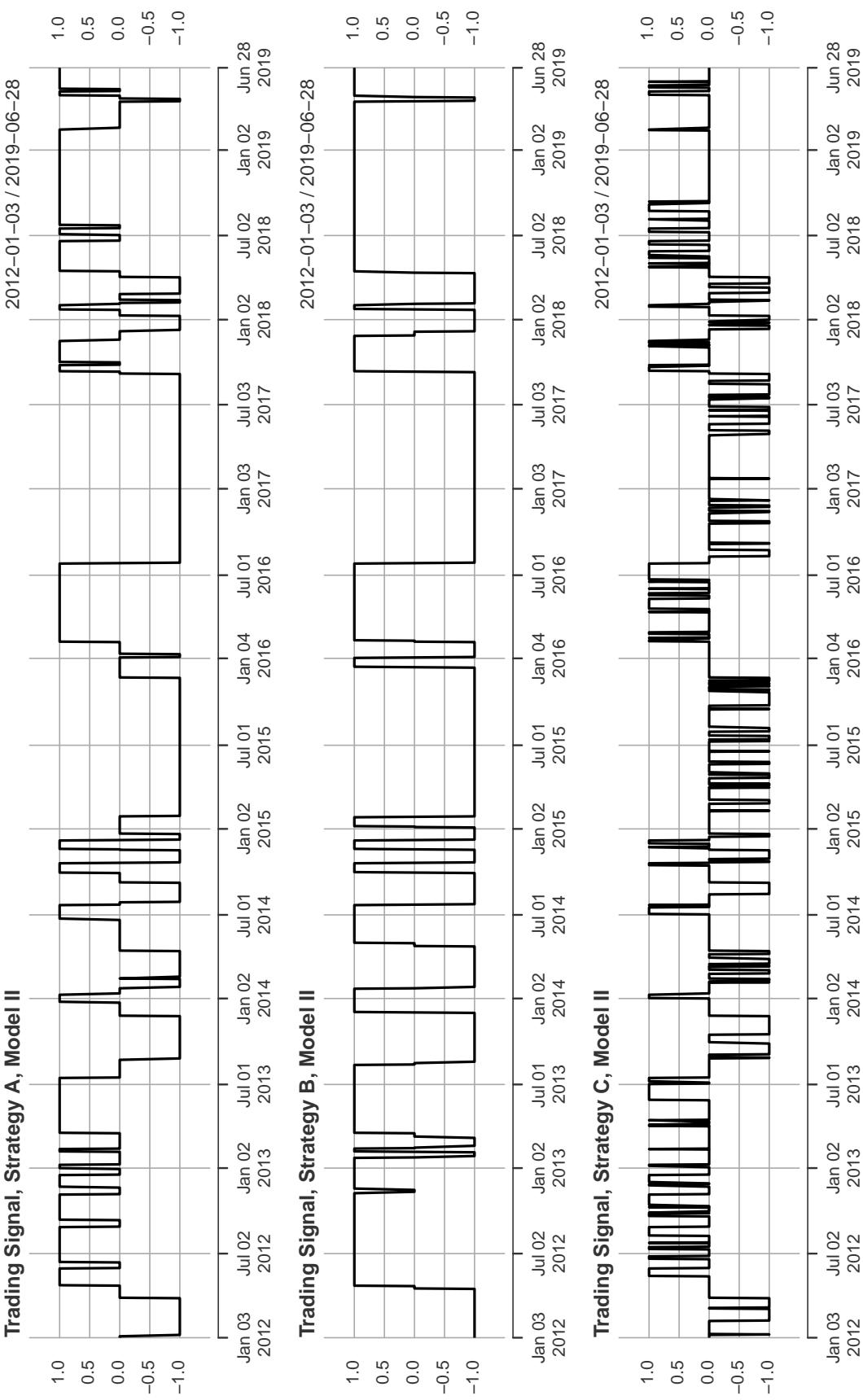


Figure A5: Trading signal of Strategy A, B and C on PEP vs KO for Model I



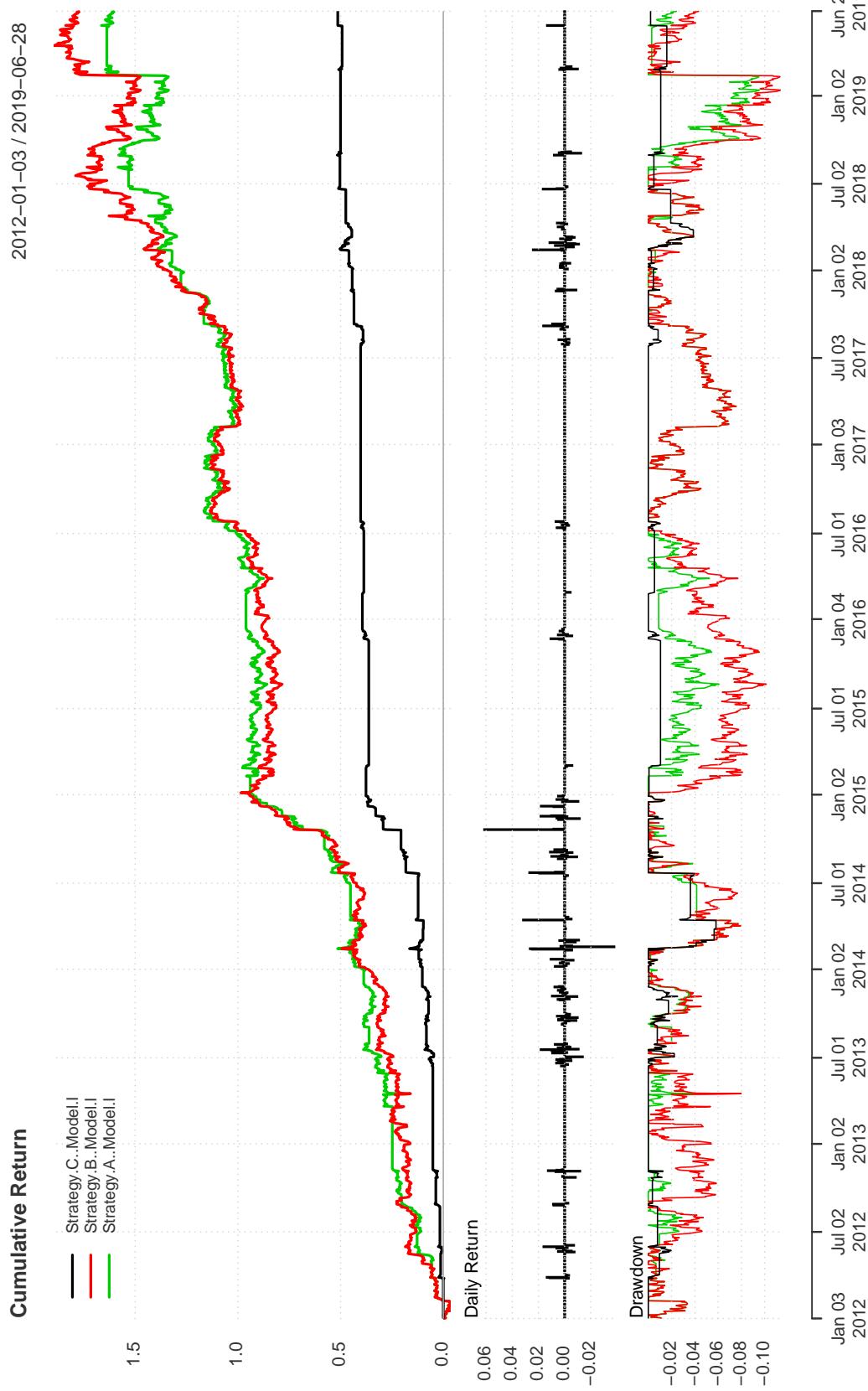
Note: When the trading signal is 1 we short PEP and long KO; when the trading signal is -1 we short KO and long PEP; when the trading signal is 0 we clear the position and hold no asset.

Figure A6: Trading signal of Strategy A, B and C on PEP vs KO for Model II



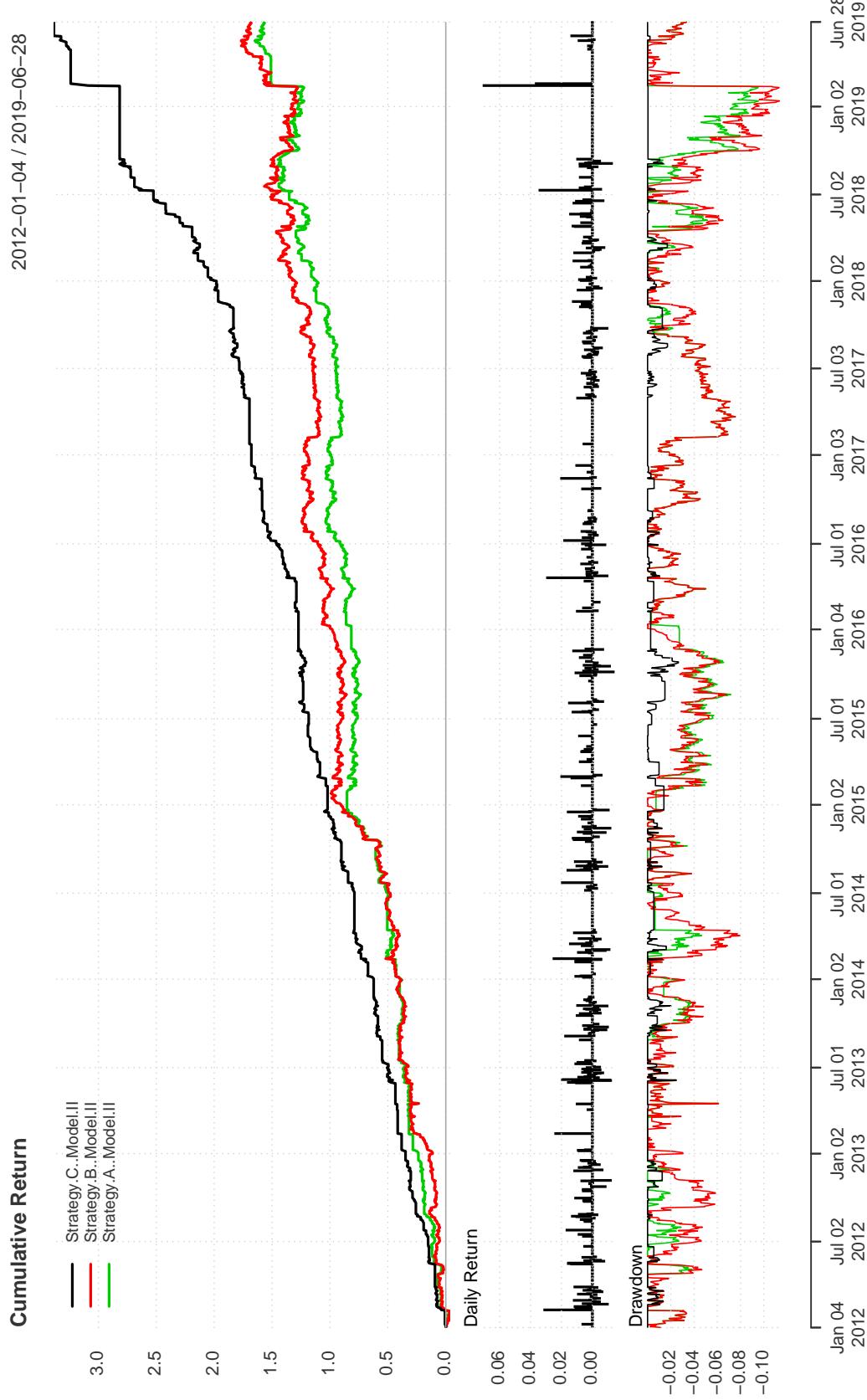
Note: When the trading signal is 1 we short PEP and long KO; when the trading signal is -1 we short KO and long PEP; when the trading signal is 0 we clear the position and hold no asset.

Figure A7: Trading Performance of Strategy A, B and C on PEP vs KO for Model I



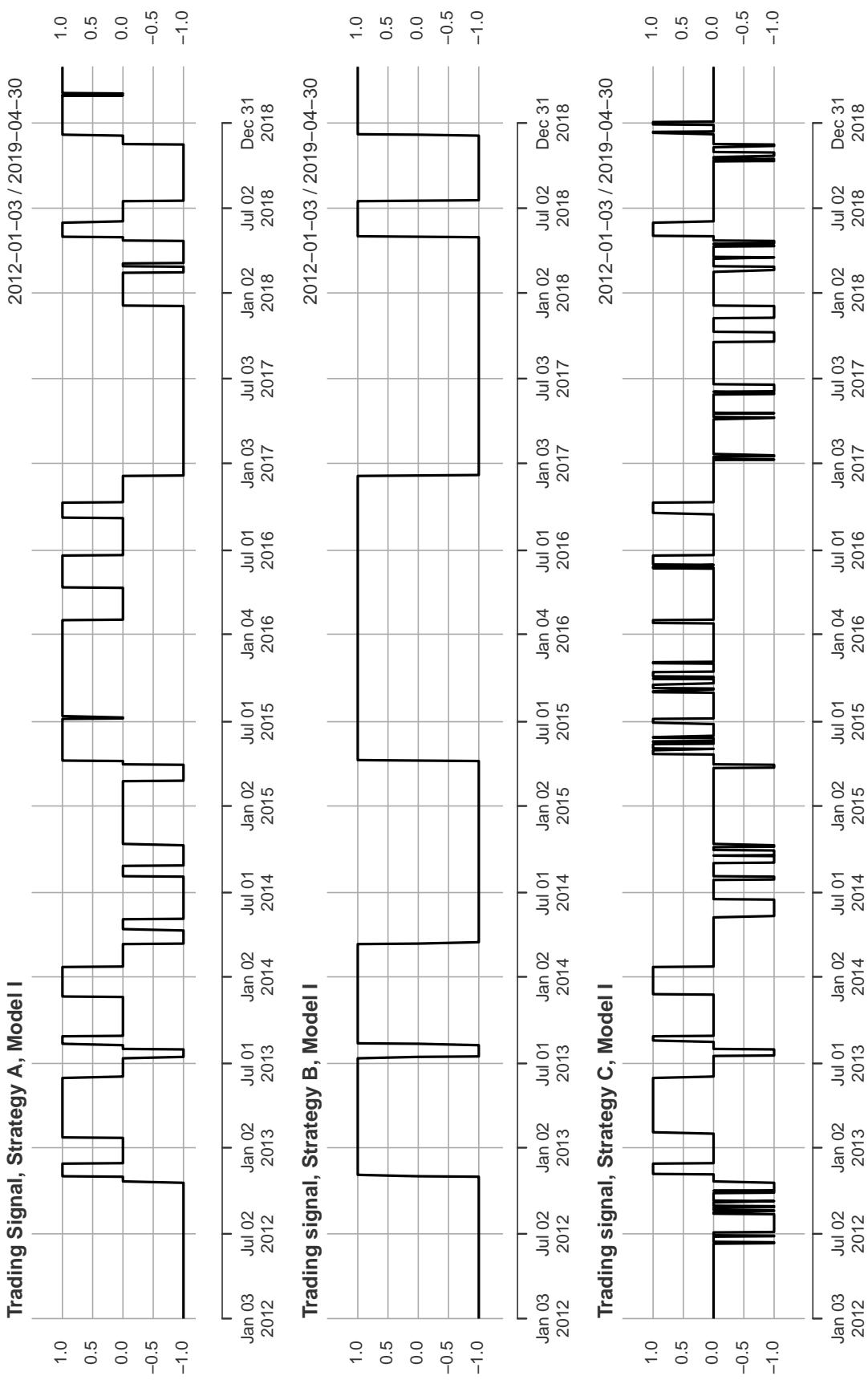
Note: Black curves are the results of Strategy C; red curves are the results of Strategy B; green curves are the results of Strategy A. The Daily Return diagram is only for Strategy C

Figure A8: Trading Performance of Strategy A, B and C on PEP vs KO for Model II



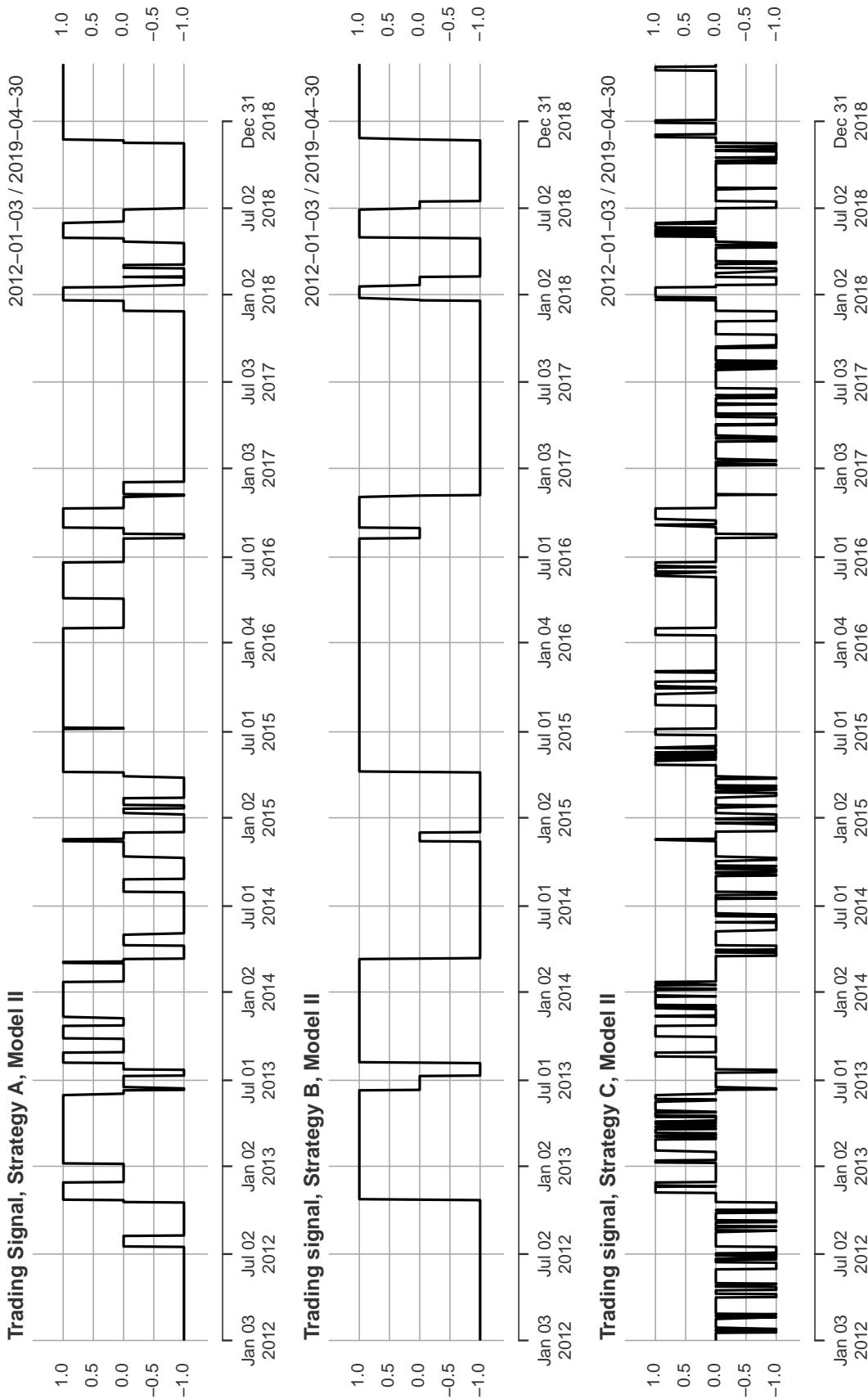
Note: Black curves are the results of Strategy C; red curves are the results of Strategy A; green curves are the results of Strategy B. The Daily Return diagram is only for Strategy C

Figure A9: Trading signal of Strategy A, B and C on EWT vs EWH for Model I



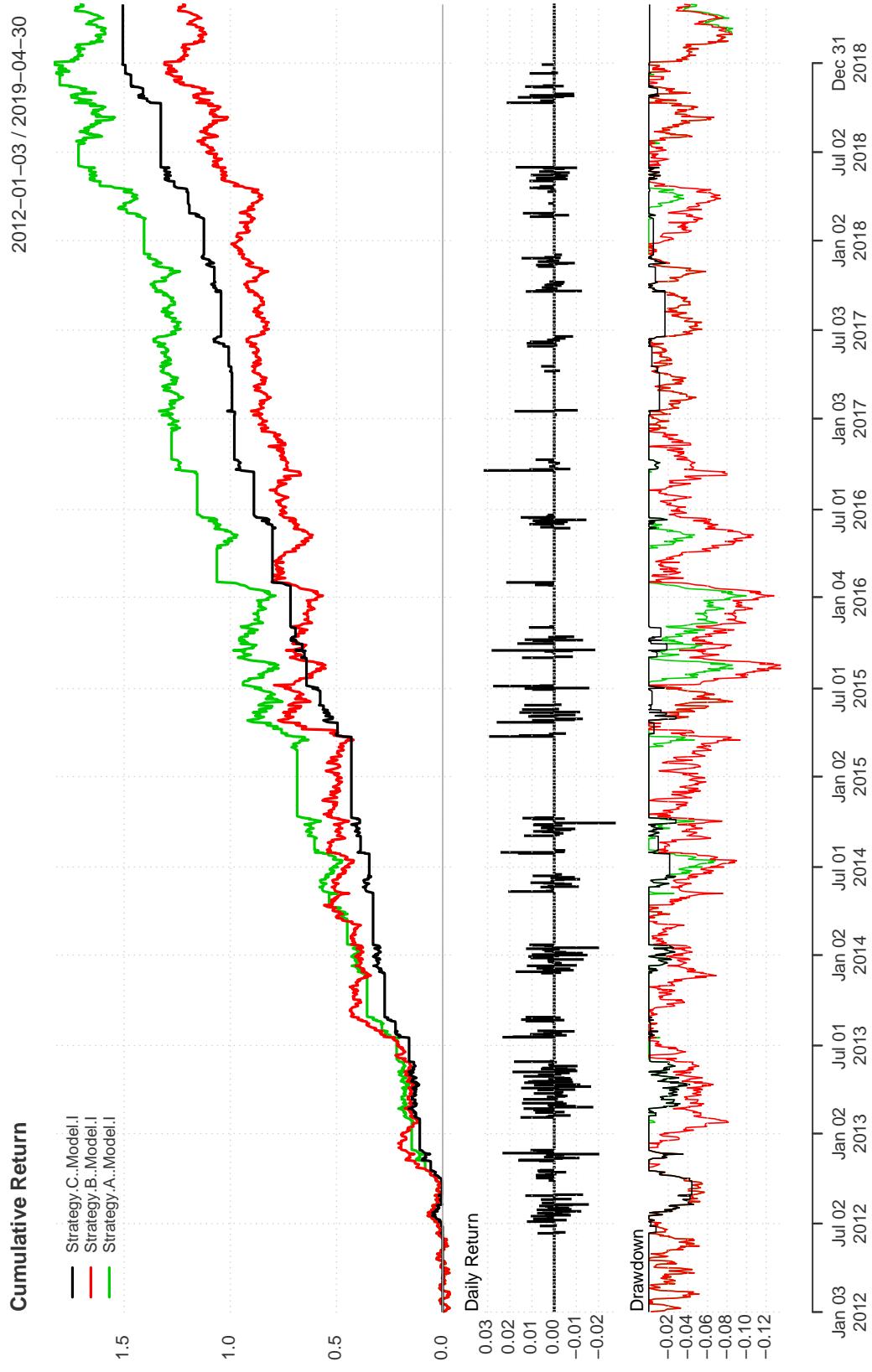
Note: When the trading signal is 1 we short EWT and long EWH; when the trading signal is -1 we short EWH and long EWT; when the trading signal is 0 we clear position and hold no asset.

Figure A10: Trading signal of Strategy A, B and C on EWT vs EWH for Model II



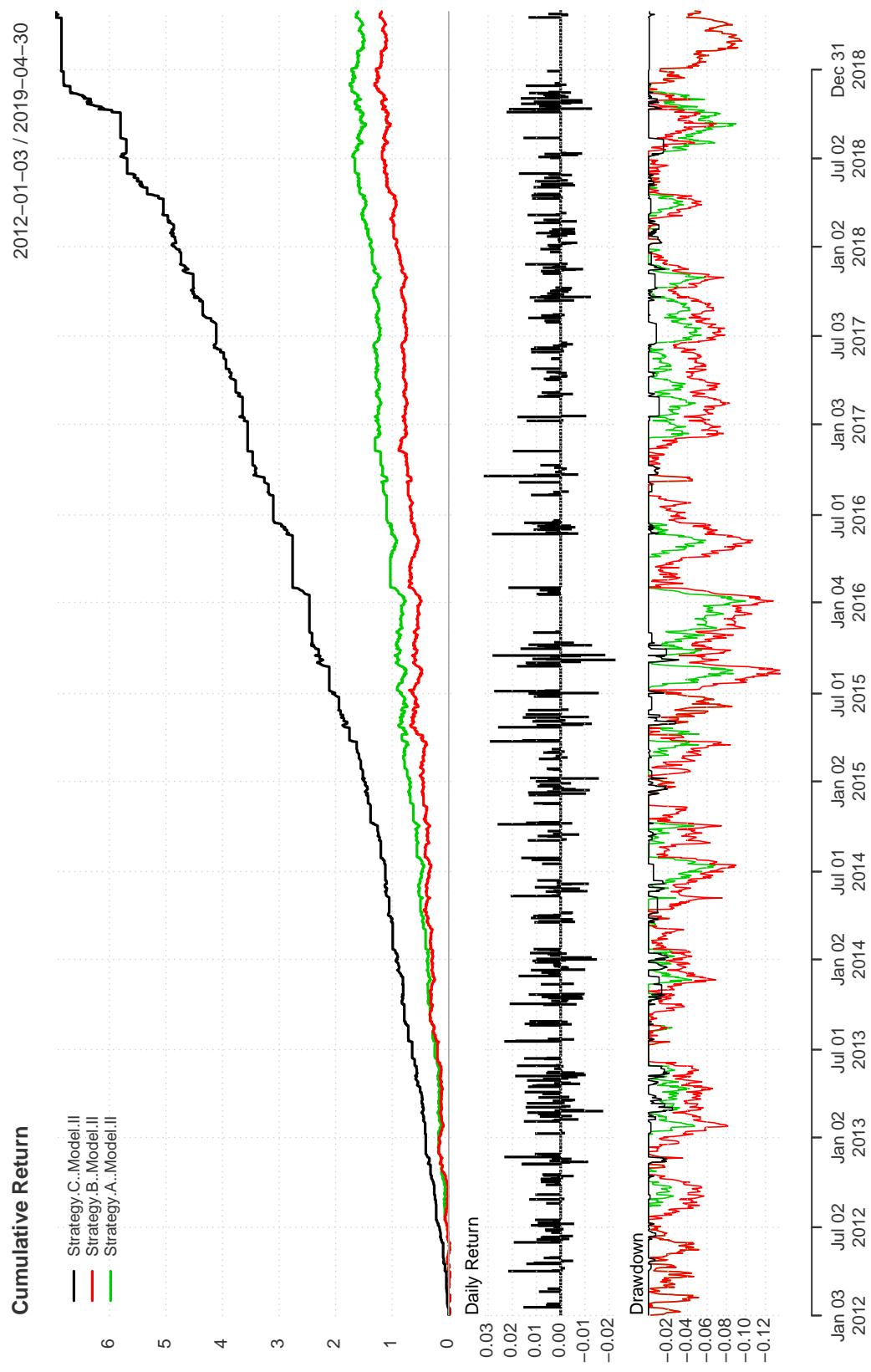
Note: When the trading signal is 1 we short EWT and long EWH; when the trading signal is -1 we short EWH and long EWT; when the trading signal is 0 we clear position and hold no asset.

Figure A11: Trading Performance of Strategy A, B and C on EWT vs EWH for Model I



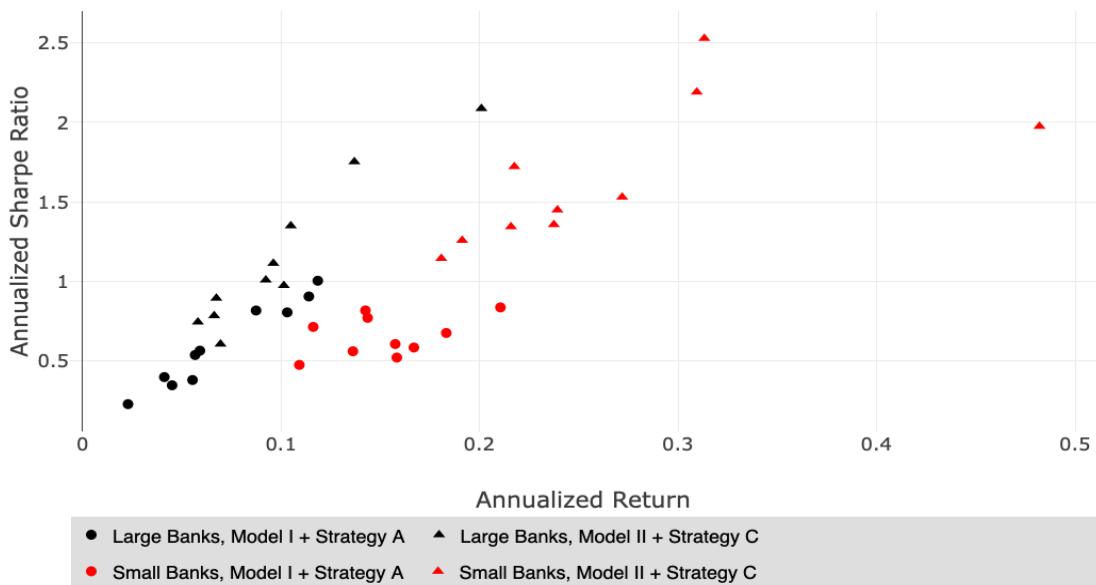
Note: Black curves are the results of Strategy C; red curves are the results of Strategy B; green curves are the results of Strategy A. The Daily Return diagram is only for Strategy C

Figure A12: Trading Performance of Strategy A, B and C on EWT vs EWH for Model II



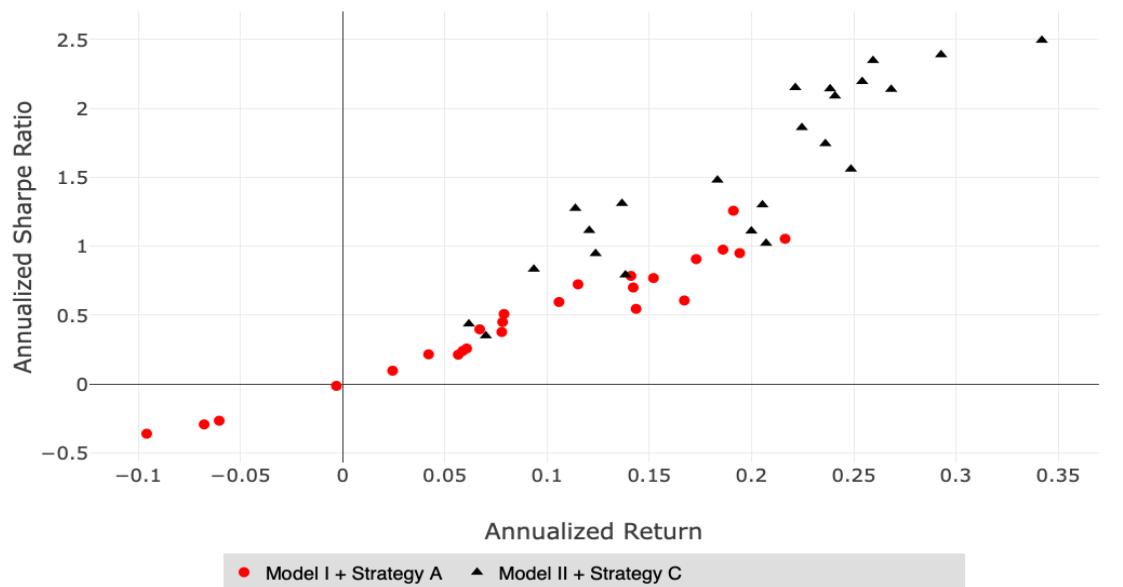
Note: Black curves are the results of Strategy C; red curves are the results of Strategy A; The Daily Return diagram is only for Strategy C

Figure A13: Annualized Return and Sharpe Ratio of Pairs Trading on Intergroup Pairs of Large Banks and Small Banks



Note: Black circles are the performances of Model I + Strategy A on pairs of large banks, red circles are the performances of Model I + Strategy A on pairs of small banks, black triangles are the performances of Model II + Strategy C on pairs of large banks, and red triangles are the performances of Model II + Strategy C on pairs of small banks.

Figure A14: Annualized Return and Sharpe Ratio of Pairs Trading on Intragroup Pairs



Note: Red circles are the performances of Model I + Strategy A on intragroup pairs: one from the group of large banks and the other one from the group of small banks; the black triangles are the performances of Model II + Strategy C