MATH FINANCE LAB 1:

In this lab you will price calls and puts, using a binomial tree. The strike price (of both call and put) $K = 22$, initial stock price is $S_0 = 20$, volatility is $\sigma = 20\%$, the interest rate is $r = 12\%$, and expiration is $n = 5$ years. The time-step of the lattice is $dt = 1$ year.

The tree is recombining, that is, $d = 1/u$.

Each node on the tree is labelled by (t, i) where t refers to time ($t = 0, ..., n$) and i refers to how many time nature "choose to go up" (by time t). So for instance, the node $(3, 2)$ is the node for time $t = 3$ where the stock price went up twice ($i = 2$), and went down once ($t - i = 1$). Convince yourself that the value of S at node (t, i) is:

$$S(t, i) = S0u^i d^{t-i}$$

The value of the European call at node (t, i) is the conditional expectation of the European call value at next time:

$$C^{EUR}(t, i) = e^{-r}\tilde{E}_t[C^{EUR}(t + 1)]$$

The continuation value of the US call at node (t, i) is:

$$C^{cont}(t, i) = e^{-r}\tilde{E}_t[C^{US}(t + 1)]$$

In an American call, one receives the maximum of the continuation value and the exercise value, where the exercise value is $Callex(t, i) = \max(S(t, i) - K, 0)$:

$$C^{US}(t, i) = \max(C^{cont}(t, i), Callex(t, i))$$