MATH FINANCE LAB 3

You will use the explicit finite difference method to price a European call with maturity $T = 0.1$ and strike $K = 21$. The stock price is $S(0) = 20$, the interest rate is $r = 0.12$. Use $m = 10$ time steps. Denote by $f(i,j)$ the value of the call when time $t = i * \Delta t$ and the stock price is $S = j * dS$. Take $\Delta S = 2$. Recall the Black-Scholes equation:

$$rf(t,S) = \frac{\partial f(t,S)}{\partial t} + rS\frac{\partial f(t,S)}{\partial S} + \frac{1}{2}\sigma(S)^2 S^2 \frac{\partial^2 f(t,S)}{\partial S^2} \tag{1}$$

In this lab, you will take $\sigma(S) = 0.2 * (1 + \exp(-S))$.

a) Without looking at the book, find the coefficients a_j^*, b_j^* and c_j^* of the explicit difference recursion.

$$f(i,j) = a^*(j) * f(i+1,j-1) + b^*(j) * f(i+1,j) + c^*(j) * f(i+1,j+1)$$

by doing the following substitutions in (1)

$$\begin{aligned}
S &\rightarrow j * \Delta S \\
f(t,S) &\rightarrow f(i,j) \\
\frac{\partial f(t,S)}{\partial t} &\rightarrow \frac{f(i+1,j) - f(i,j)}{\Delta t} \\
\frac{\partial f(t,S)}{\partial S} &\rightarrow \frac{f(i+1,j+1) - f(i+1,j-1)}{2\Delta S} \\
\frac{\partial^2 f(t,S)}{\partial S^2} &\rightarrow \frac{f(i+1,j+1) - 2f(i+1,j) + f(i+1,j-1)}{(\Delta S)^2}
\end{aligned}$$

b)Run the code.

c) Compare the value of the option at time $t = 0$, i.e., $f(0,10)$, with the value that you obtain with Monte Carlo simulation using the Euler scheme, i.e.:

$$\frac{\exp(-rT)}{\Omega} \sum_{\omega=1}^{\Omega} \max(S(T,\omega) - K, 0)$$

where

$$S(t+\Delta t,\omega) - S(t,\omega) = rS(t,\omega)\Delta t + \sigma(S(t,\omega))S(t,\omega)z(t,\omega)\sqrt{\Delta t}$$

where $\{z(t)\}$ is a collection of standard normal random variables.