

1.

Method	Vector.cs	Mecrosoft .Net framework
Count()	O(1)	O(1)
Capacity()	O(1)	O(1)
Add()	O(1) if count< capacity O(n) if count>= capacity	O(1) if count< capacity O(n) if count>= capacity
IndexOf()	O(n)	O(n)
Insert()	O(n)	O(n)
Clear()	O(n)	O(n)
Contains()	O(n)	O(n)
Remove()	O(n)	O(n)
RemoveAt()	O(n)	O(n)

2

The below function is $O(n \log(n))$ and $\Omega(n \log(n))$, therefore, it's $\theta(n \log n)$

```
private void MergeSort<K>(K[] sequence, IComparer<K> comparer) where K :
    IComparable<K>
{
    int n = sequence.Length;
    //when array length is 1, it is sorted
    if (n < 2) return;

    //chop chop
    int mid = n / 2;
    K[] s1 = sequence.Take(mid).ToArray();
    K[] s2 = sequence.Skip(mid).ToArray();

    //conquer, sort the 2 sequences recursively
    MergeSort(s1, comparer);
    MergeSort(s2, comparer);

    //Merge result to original
    Merge(s1, s2, sequence, comparer);
}
```

The name of this function is Merge Sort.

3

- a. $f = x^{1/2}$
 $g = \log(n)$

$$\lim_{n \rightarrow \infty} \frac{n^{\frac{1}{2}}}{\log n} = \infty$$

f grows faster than g
 $f \in \Omega(g)$

- b. $f = 1500$
 $g = 2$

$$\lim_{n \rightarrow \infty} \frac{1500}{2} = 750$$

f is 750 times bigger than g , they grow at the same rate ($f = 750g$)
 $f \in \theta(g)$

- c. $f = 800 \cdot 2^n$
 $g = 3^n$

$$\lim_{n \rightarrow \infty} \frac{800 \cdot 2^n}{3^n} = 0$$

f grows slower than g
 $f \in O(g)$

- d. $f = 4^{(n+13)}$
 $g = 2^{(2n+2)}$

$$\lim_{n \rightarrow \infty} \frac{4^{n+13}}{2^{2n+2}} = 16777216$$

f is 16777216 bigger than g , they grow at the same rate ($f(n) = 16777216 \cdot g(n)$)
 $f \in \theta(g)$

- e. $f = 9n \log(n)$
 $g = n \log(9n)$

$$\lim_{n \rightarrow \infty} \frac{9n \cdot \log(n)}{n \cdot \log(9n)} = 9$$

f is 9 times bigger than g , they grow at the same rate ($f(n) = 9 \cdot g(n)$)
 $f \in \theta(g)$

- f. $f = O(g)$
 $f = n!$
 $g = (n+1)!$

$$\lim_{n \rightarrow \infty} \frac{n!}{(n+1)!} = 0$$

f grows slower than g
 $f \in O(g)$