CITS5501 Software Testing and Quality Assurance
Formal methods

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Specification languages

Sources

- Pressman, R., *Software Engineering: A Practitioner's Approach*, McGraw-Hill, 2005
- Huth and Ryan, *Logic in Computer Science*
- Pierce et al, *Software Foundations vol 1*
- Alloy tutorial at http://alloytools.org/tutorials/online/

Some problems in specifying systems

System specifications can suffer from a few potential problems.

- *Contradictions*. In a very large set of specifications, it can be difficult to tell whether there are requirements that contradict each other.
    - Can arise where e.g. specifications are obtained from multiple users/stakeholders
    - Example: one requirement says "all temperatures" in a chemical reactor must be monitored, another (obtained from another member of staff) says only temperatures in a specific range.

Some problems in specifying systems

- *Ambiguities*. i.e., statements which can be interpreted in multiple different ways.
  "The operator identity consists of the operator name and password; the password consists of six digits. It should be displayed on the security screen and deposited in the login file when an operator logs into the system."

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  "The operator identity consists of the operator name and password; the password consists of six digits. It should be displayed on the security screen and deposited in the login file when an operator logs into the system."
- . . . Does "it" refer to the identity, or the password?
- "Should" can be ambiguous – does "The system should do X" mean the system *must* do X, or that it is optional but desirable that the system do X?
- Many terms have both technical and non-technical meanings (possibly multiple of each): for instance, "reliable", "robust", "composable", "category", "failure", "orthogonal", "back end", "kernel", "platform", "entropy" . . .

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- "is tall" is vague: some people are definitely tall, and some are definitely short, but it can be difficult to tell when exactly someone meets the criterion of being tall.
- Likewise "fast", "performant", "efficiently", "scalable", "flexible", "is user-friendly", "should be secure", "straightforward to understand" are all vague.

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- e.g. An obviously incomplete requirement: “A user may specify normal or emergency mode when requesting a system shutdown. In normal mode, pending operations shall be logged and the system shut down within 2 minutes.”
- . . . So what happens in emergency mode?
- But other cases of incompleteness may be harder to spot.

Some problems in specifying systems

- In addition to these, there are many other ways requirements can be written poorly –
  e.g. Overly long and complex sentences, mixed levels of abstraction (mixing high-level, abstract statements with very low-level ones $\rightarrow$ difficult to distinguish high-level architecture from low-level details), undefined jargon terms, specifying implementation rather than requirements (how vs what), over-specifying, don't satisfy business needs, etc.

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- Formal specifications can potentially help avoid ambiguity, vagueness, contradiction and some gaps in completeness.
- Other problems, not so much. Just as it's possible to write programs badly in any language, it's also possible to write formal specifications badly.
  There is still a need for *review* of specifications, as with any artifact.

Some problems in specifying systems

Some example requirements[1]:

- "The system shall be highly reliable."
- "Credit card details must be encrypted."
- "The system should be easy to use by medical staff and should be organized in such a way that user errors are minimized."
- "The system shall have a response time of under 0.1 seconds."
- "If the LAUNCH-MISSILE signal is set to TRUE and the ABORT-MISSILE signal is set to TRUE then do not launch the missile, unless the ABORT-MISSILE signal is set to FALSE and the ABORT-MISSILE-OVERRIDE is also set to FALSE, in which case the missile is not to be launched."

[1] Sources: Pressman; LaPlante, *Requirements Engineering*; bitter experience.

Formal specifications

- Formal specifications can help with ameliorating these problems.
- Sometimes, just the process of attempting to formalize a requirement can reveal problems with it.
- Using a formal model can help reveal *ambiguity* and *vagueness* and allow them to be eliminated
- It may also be possible (depending on the mathematical model used) to detect inconsistencies
- Detecting whether a specification is *complete* is more difficult.
  - Some gaps may be able to be detected
  - But there are nearly always some details that are left undefined, or scenarios that may not have been considered.

Formal specifications

- Formal specifications have a meaning defined in terms of mathematics.
- Similar to programming languages, there are different sorts of formal specification languages and tools, with different sorts of scope.

Types of specification languages/notations

Some are small and specific in scope. For instance:

- *State charts*, which we have seen, define states and transitions between them.
- *Backus-Naur Form*, which we have also seen, defines languages – sets of strings.
- *Regular expressions* also define sets of strings (most programming languages have an implementation of them)
- π-calculus is used to represent concurrent systems

These examples highlight another reason to use formal specifications: sometimes they're the most concise way of saying something. Writing a grammar or set of state transitions in natural language (rather than BNF or as a state chart) would be a gruelling and horrible exercise.

Types of specification languages/notations

- Some languages/notations more general, and are intended to be able to describe a wide range of systems. They may be based on numerous different mathematical formalisms – predicate logic + set theory, category theory, etc.

Formal specification languages

Some examples of general-purpose specification languages:

- **Z notation**
  - based on set theory and predicate logic
  - developed in the 1970s.
  - Now has an ISO standard, and variations (e.g. object-oriented versions)
- **TLA+**:
  - Stands for "Temporal Logic of Actions"
  - A general-purpose specification language
  - Especially well-suited for writing specifications of concurrent and distributed systems
  - For finite state systems, can check (up to some number of steps) that particular properties hold (e.g. safety, no deadlock)

Formal specification languages

- We'll be using the **Alloy** specification language
- Alloy is both a language for describing structures, and a tool (written in Java) for exploring and checking those structures.
- Influenced by Z notation, and modelling languages such as UML (the Unified Modelling Language).
- Website: http://alloy.mit.edu/ (The Alloy Analyzer tool can be downloaded from here.)
- Online demo: the website at http://alloy4fun.inesctec.pt allows Alloy to be run via a web browser.

Alloy language

- We'll look at a simple model of a file system (based on the Alloy tutorial at http://alloytools.org/tutorials/online/)
- To a first approximation, Alloy looks a little like Java:

```
// A file system object in the file system
sig FSObject { parent: lone Dir }

// A directory in the file system
sig Dir extends FSObject { contents: set FSObject }

// A file in the file system
sig File extends FSObject { }
```

Alloy

In Alloy, we declare rules about a mini-universe: things that exist, and properties that should be true of them.

- "There are things called animals"

  ```
  sig Animal {}
  ```

- "A cat is a sort of animal"

  ```
  sig Cat extends Animal {}
  ```

Alloy – relations

Alloy's semantics are defined in terms of mathematical *relations*.

Example relations:

- "Is less than". e.g. "$2 < 4$", "$10 < 9$".
- "Is the blood relative of". e.g. "Alice is the blood relative of Bob".
- "Shares an office with". e.g. "Bob shares an office with Carol".

These are all *binary relations*. Statements about two entities, which can be true or false.

Alloy – relations

Relations can also be *unary* (about one entity):

- "Is even". e.g. "$even(2)$.
- "Is an employee". e.g. "Dan is an employee".

Alloy – relations

They can be ternary:

- "_ is delivered to _, by _". e.g. "The *blue book* was delivered to *Alice*, by *Bob*".

Or, in general, they can be *n*-ary – a statement about *n* things.

Alloy – relations

We can think of predicates as being functions – an *n*-ary predicate isn't true or false in itself, until we supply it with *n* arguments.

- "Is less than" isn't true or false, but "$2 < 4$" is.

Alloy – relations

Another way of viewing relations is as being a sort of table – containing all the things of which the predicate is true.

e.g. "shares an office with":

Person A	Person B
Alice	Bob
Bob	Alice
Dan	Eve
Eve	Dan

Alloy – relations

Relations can be finite, or infinite.

An infinite relation: “is less than”

Number A	Number B
1	2
1	3
2	3
...	...

Alloy – sigs

`sig Animal {}` says "There are things called animals".

It defines a unary relation, "Animal". Something thing can be-an-animal, or not.

Alloy – sigs

`sig Cat extends Animal {}` says "Cats are a sort of animal".

If something has the property "is-an-animal", *then* it might also have the property "is-a-cat".

We can read "extends" as also meaning "is a kind of", or "is a subtype of".

Alloy – subtypes

- So, **`extends`** indicates subtypes (similar to Java).
- Here, **`Dir`** and **`File`** are both subtypes of **`FSObject`**:

```
sig FSObject {}

sig Dir extends FSObject {}

sig File extends FSObject {}
```

- When we declare **`Dir`** or a **`File`** to be sub-types of **`FSObject`**, they are considered to be *mutually disjoint* sets
- The above says "There are things called FSObjects. An FSObject might be a Dir or it might be a File, but not both".

Alloy – properties

We can specify *properties* of entities:

```
// A file system object in the file system
sig FSObject { parent: lone Dir }

// A directory in the file system
sig Dir extends FSObject { contents: set FSObject }

// A file in the file system
sig File extends FSObject { }
```

Alloy – properties

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// A file system object in the file system
sig FSObject { parent: lone Dir }

// A directory in the file system
sig Dir extends FSObject { contents: set FSObject }

// A file in the file system
sig File extends FSObject { }
```

These are usually written within the sig of an entity.

They actually represent *relations* between entities.

Alloy – properties

```
// A file system object in the file system
sig FSObject { parent: lone Dir }
```

There are multiple ways of reading this:

- "There are such things as FSObjects. An FSObject has the property 'parent'. An FSObject can have zero or one parents." Or –
- "A relation 'parent' exists between FSObjects and Dirs. Whenever an FSObject appears in the relation, it can be association with at most one Dir."

These are exactly equivalent.

Alloy – properties

```
// A file system object in the file system
sig FSObject { parent: lone Dir }
```

- The "`lone`" means "zero or one". It is a *cardinality*.
- Other possible cardinalities are:
  - "some" (one or more)
  - "one" (exactly one)
  - "set" (zero or more)
- When we specify a property using a colon in this way, the default multiplicity is one.
- We can use cardinalities whenever we are specifying a set or relation: since sigs also represent sets (e.g. the set of Dirs), we can give them cardinalities, too.

Alloy – properties

```
one sig RootDir extends Dir { }
```

There exists a “RootDir”, but only one of them.

Exercise

Games:

- There are things called games.
- Games can be board games, or field games.
- There may be other sorts of games.

Alloy language – comments

```
// A file system object in the file system
sig FSObject { parent: lone Dir }

// A directory in the file system
sig Dir extends FSObject { contents: set FSObject }

// A file in the file system
sig File extends FSObject { }
```

- Comments can be written in multiple ways

Alloy language – comments

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- Comments can be written in multiple ways
  - single-line comments with “//” or “--”

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- Comments can be written in multiple ways
  - single-line comments with "//" or "--"
  - multi-line comments with "/* ... */"

Alloy – facts

- How can we express that any **`FSObject`** is either a **`Dir`** or a **`File`**? (i.e., there are no other sorts of **`FSObject`**)

Alloy – facts

- How can we express that any **FSObject** is either a **Dir** or a **File**? (i.e., there are no other sorts of **FSObject**)
- Alloy also allows us to specify *constraints*. These are introduced with the keyword **fact**.

```
sig FSObject { parent: lone Dir }
sig Dir extends FSObject { contents: set FSObject }
sig File extends FSObject { }

// All file system objects are either files or directories
fact { File + Dir = FSObject }
```

Alloy – facts

- The general syntax for a fact is

  `fact` *`name`* { *`formulas`* }

- *formulas* are Boolean expressions, and by putting them in a fact, we're constraining them to be true.

Alloy – abstract signatures

- (An alternative way to say that all FSObjects must be Dirs or Files would be to declare FSObject **`abstract`**)

Alloy – abstract signatures

- (An alternative way to say that all FSObjects must be Dirs or Files would be to declare FSObject **`abstract`**)
- (This is similar to the use of the **`abstract`** keyword in Java; it means there are no objects that are *directly* of type FSObject; they must be members of some subtype, instead.)

Alloy – operators

Operators are available to construct Boolean expressions.

- subset: **`in`**
  - *`set1`* **`in`** *`set2`* — *`set1`* is a subset of *`set2`*
  - informally: "some *`set2`* are *`set1`*", or "a *`set2`* may be *`set1`*"; but the set-theoretic meaning is more precise.
- set equality: **`=`**
  - *`set1`* **`=`** *`set2`* — *`set1`* equals *`set2`*
- scalar equality: **`=`**
  - `scalar` **`=`** *`value`* — *`scalar`* equals *`value`*

Alloy – subsets

- We saw that subtypes are disjoint.
- We can also declare sub*sets*:

  sig *signame* **in** *supername* { ... }

- Subsets are *not* necessarilly disjoint, and may have multiple parents

Alloy – subsets

```
sig Animal {}
sig Cat extends Animal {}
sig Dog extends Animal {}
sig FurryPet in Cat + Dog {}
```

- "FurryPet" is a subset of the union of Cat and Dog.
- Some dogs and cats may not be furry (hairless breeds).
- We could *make* them all furry as follows:

```
fact { Cat + Dog = FurryPet }
```

- Are there animals other than cats and dogs?
  Can they be furry?

More operators

- We can use Boolean connectives **`and`**, **`or`**, **`implies`**, **`iff`**, **`not`** to join Boolean expressions.
- e.g.

```
fact { A + B = C and X + Y = Z }
```

Relations

- In our file-system example, we also saw things in the *body* of signatures (i.e., between the braces).

- ```
  // A file system object in the file system
  sig FSObject { parent: lone Dir }

  // A directory in the file system
  sig Dir extends FSObject { contents: set FSObject }

  // A file in the file system
  sig File extends FSObject { }
  ```
  ```

Relations

- ```
  // A file system object in the file system
  sig FSObject { parent: lone Dir }

  // A directory in the file system
  sig Dir extends FSObject { contents: set FSObject }

  // A file in the file system
  sig File extends FSObject { }
  ```
- To a first approximation, we can think of relations as behaving like *fields* in an OO language.
- `sig FSObject { parent: lone Dir }` can be read as "Things of type `FSObject` *have a parent*, which is of type `Dir`".
- **`lone`** means "at most one" – i.e., you can have zero or one parents. (We need this because the root directory has no parent.)
  ```

Relations

- ```
  // A file system object in the file system
  sig FSObject { parent: lone Dir }

  // A directory in the file system
  sig Dir extends FSObject { contents: set FSObject }

  // A file in the file system
  sig File extends FSObject { }
  ```
- More precisely, `parent` is a relation between `FSObject` and `Dir`.
  ```

Relations – multiplicities

- `lone` is a type of multiplicity – it says how many of something there are.
- Other multiplicities:
  - **one** - one
  - **some** - at least one; one or more
  - **set** - zero or more
  - **no** - zero
- The default multiplicity is one.

Relations – multiplicities

- In set theory terms . . .
- **`one`** means the relation is a total function –
  `sig Student { name : one String }` –
  for every Student, we can map to a string which is their name.
- **`lone`** means the relation is a *partial* function –
  `sig Student { driverLicenseNum : lone String }` – \
  for every Student, we *may* be able to map to a diver's license number.
  (Here, it's assumed you can't have more than one license.)

Relations

- So, signature declarations will look like:

```
sig SomeName {
  field1 : FieldType,
  field2a, field2b : OtherFieldType
}
```

- The order of declarations doesn't matter – `SomeName`, `FieldType` and `OtherFieldType` could be declared in any order in a file.

Relations

- ```
  // A directory in the file system
  sig Dir extends FSObject { contents: set FSObject }
  ```
- Here, we say that a `Dir` has a field `contents`, which is a *set* of `FSObjects`.
- The could contain one item, many items, or no items.
  ```

Examples

- "A car has one engine"

  `sig Car { engine: one Engine }`, or
  `sig Car { engine: Engine }`

- "People have zero or more hobbies"

  `sig Person { hobbies: set Activity }`

Exercises

- Classes have at least one lecturer, and zero or more students.

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Exercises

- Classes have at least one lecturer, and zero or more students.
- Animals have zero or more legs
- Some animals are carnivores
- Textbooks have one or more pages

Back to the file system example

```
sig FSObject { parent: lone Dir }

sig Dir extends FSObject { contents: set FSObject }

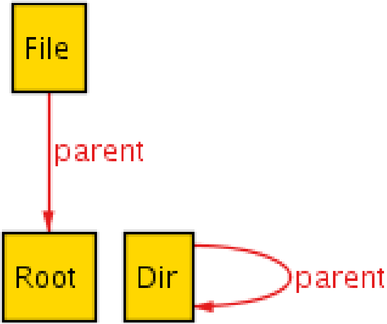
sig File extends FSObject { }

// There exists a root
one sig Root extends Dir { } { no parent }
```

- FSObjects have parents, and directories have contents, and we have constrained the multiplicities ...
- but there's currently no connection between them.

File system

- So we could have this situation:



File system

- We will need to constrain things more, so we'll use a *fact*.

  ```
  // A directory is the parent of its contents
  fact { all d: Dir, o: d.contents | o.parent = d }
  ```

- This says: "for any thing (let's call it *d* for the moment) of type `Dir`, and for any thing (let's call it *o* for the moment) which is in the set `d.contents`:
  o's parent is *d*.
- It uses a *quantifier* ("all") – we'll look at these more later.