

Complex Numbers - I $(\underline{\underline{3-4Q}})$

Sankalp 2022 (NDA I): Complete Course on Maths



COMPLEX NUMBERS-1

$$\underline{x^2 + 1 = 0}$$

✓

$$\underline{x = \sqrt{-1}}$$

$$(i)$$

$$\underline{x^2 = -1}$$

$$\underline{x = \sqrt{-1}} \rightarrow \text{iota}$$

ARPIT CHOUDHARY

Note: $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

only if either of a or b is positive or 0. केवल यदि a और b धनात्मक और शून्य हो

Q 1. If both a & b are negative (यदि a और b ऋणात्मक हो तो) then, $\sqrt{a} \times \sqrt{b} = ?$

- (a) \sqrt{ab}
- ~~(b) $-\sqrt{ab}$~~
- (c) $\pm\sqrt{ab}$
- (d) N.O.T.

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

$$a = -7$$

$$b = -5$$

$$a \rightarrow \ominus$$

$$i = \sqrt{-1}$$

$$b \rightarrow \ominus$$

$$i^2 = -1$$

$$\sqrt{a} \times \sqrt{b}$$

$$\sqrt{-7} \times \sqrt{-5} = \sqrt{-1} \times \sqrt{7} \times \sqrt{-1} \times \sqrt{5}$$

$$= (-1) \times (-1) \times \sqrt{7} \times \sqrt{5}$$

$$= 1 \times \sqrt{35} = \sqrt{35}$$

$$a \rightarrow \oplus$$

$$b \rightarrow \oplus$$

$$a \rightarrow 0$$

$$b \rightarrow 0$$

$$= \sqrt{35}$$

$$i = \sqrt{-1}$$

$$i^2 = -1$$

$$i^3 = -i$$

$$i^4 = 1$$

$$i^2 = -1$$

$$i^{40} = 1$$

$$i^{12} = 1$$

$$i^2 \cdot i = -i$$
$$i^3 = -i$$

$$i^{96} = 1$$

$$i^{100} = 1$$

$$i^3 \cdot i = -i \cdot i$$

$$i^4 = -i^2$$

$$i^{104} = 1$$

$$i^4 = -(-1) = +1$$

$$i^{4k} = 1$$

$$k \in \mathbb{I}$$

$$i^{100} = 1$$

$$i^{99} = i^{(96)+3} = 1 \cdot i^3 = -i$$

$$i^{4k+1} = i$$

$$i^{97} = i^{96+1} = i^{96} \cdot i^1 = 1 \cdot i = i$$

$$i^{4k+2} = -1$$

$$= 1 \cdot i^2 = -1$$

$$i^{4k+3} = -i$$

$$i^{98} = i^{(96)+2} = 1 \cdot i^2 = -1$$

$$= -1 \cdot i^2 = -1$$

$$\lambda \cdot \textcircled{500}$$

$$= \lambda \cdot 10$$

$$\textcircled{501}$$

$$\lambda' = \textcircled{i}$$

$$= \textcircled{1} \checkmark$$

$$\begin{array}{r} \textcircled{125} \\ 4 \overline{) 501} \\ \underline{-500} \\ \textcircled{1} \end{array}$$

$$\frac{1}{i} = -i$$

Note :-

Kalhi Ihi Dm ista

kehre Naikena

$$\frac{1 \times i^3}{i \times i^3} = \frac{i^3}{i^4} = i^3 = -i \quad \checkmark$$

$$\frac{1}{i} = -i$$

$$\frac{1}{i^3} = i$$

$$\frac{1}{i^2} = -1$$

$$\frac{1}{i^4} = 1$$

$$\lambda^{-67} + \lambda^{-61} = ?$$

$$\frac{1}{i^{67}} + \frac{1}{i^{61}}$$

$$\frac{1}{i^3} + \frac{1}{i^1}$$

$$i^{-i} = 0$$

$$i^4 = 1$$

$$\begin{array}{r} 16 \\ 4 \overline{) 67} \end{array}$$

$$\begin{array}{r} -64 \\ \hline 03 \end{array}$$

Properties of IOTA

1. Sum of 4 consecutive power of $i = 0$.

$$i^n + i^{n+1} + i^{n+2} + i^{n+3} = 0$$

Or

$$i^{4n} + i^{4n+1} + i^{4n+2} + i^{4n+3} = 0$$

$$i^1 + i^2 + i^3 + i^4 = 0.$$

$$i^4 + i^5 + i^6 + i^7 = 0$$

$$i^{100} + i^{101} + i^{102} + i^{103} = 0$$

$$\cancel{i^1} + \cancel{i^3} = 0$$

Properties of IOTA

1. Sum of 2 consecutive even or odd power of i = 0.

$$i^n + i^{n+2} = 0$$

Or

$$i^{n+1} + i^{n+3} = 0$$

$$i^{97} + i^{99} = 0$$

$$i^{48} + i^{50} = 0$$

$$i^{47} + i^{49} = 0$$

$$i^{36} + i^{38} = 0$$

$$\cancel{i^1} + \cancel{i^3} = 0$$

$$\cancel{i^1} - \cancel{i^3} = 0$$

$$i^1 + i^3 = 0$$

$$-i + i = 0$$

$$\underline{i^2} + \underline{i^4} = 0$$

$$\text{Q2. } i^{500} + i^{501} + i^{502} + i^{503} =$$

0

$$\text{Q3. } i^{100} + i^{102} + i^{104} + i^{106} =$$

0

$$\text{Q 4. } i^{115} + i^{117} =$$

0

Q. $i^{15} + \frac{1}{i^{19}} + i^{25} + \frac{1}{i^{25}} = ?$

Ans.

$$i^{15} + \frac{1}{i^{19}} + i^{25} + \frac{1}{i^{25}}$$

$$i^3 + \frac{1}{i^3} + i + \frac{1}{i}$$

$$-i + i + i - i = 0$$

$$i^{10.0} + i^{10.1} + i^{10.2} + i^{10.3}$$

$$1 + i + i^2 + i^3 = 0$$

Q. 6 $\sum_{n=1}^{25} (i^n + i^{n+1}) = ?$

- (a) $i - 1$
- (b) $i + 1$
- (c) $1 + i$
- (d) $-1 - i$

let $N = 1$

$$(i^1 + i^{1+1})$$

$$i + i^2$$

$$= \underline{i - 1}$$



Q. $\sum_{n=1}^{30} (i^n + i^{n+1}) = ?$

- (a) 0 (b) 1
(c) -2 (d) $1+i$

$$\sum_{n=1}^{30} i^n (1+i) \Rightarrow (1+i) \sum_{n=1}^{30} i^n$$

$$(1+i)(i^{29} + i^{30})$$

$$(1+i)(i + i^2)$$

$$(1+i)(i-1)$$

$$= i^2 - 1 = -1 - 1 = -2$$

Q. 8 $\sum_{n=4}^{16} (i^n + i^{n+3}) = ?$

(a) $1 - i$

(b) 0

(c) 1

(d) N.O.T

$$\sum_{n=4}^{16} (i^n + i^n \cdot i^3)$$

$$\sum_{n=4}^{16} i^n (1 + i^3)$$

$$\sum_{n=4}^{16} i^n (1 - i)$$

$$(1 - i) \sum_{n=4}^{16} i^n$$

$$(1 - i) (i^4 + i^5 + i^6 + i^7 + i^8 + i^9 + i^{10} + i^{11} + i^{12} + i^{13} + i^{14} + i^{15} + i^{16})$$

$$(1 - i) (i^{16}) = (1 - i) (1) = 1 - i$$

Q 9 $1 + i^2 + i^4 + i^6 + \dots + i^{100} = ?$
 (a) ~~-1~~
 (b) 1
 (c) -1
 (d) N.O.T

$$+ \cancel{i^{102}} = i^2 = -1$$

$$1 + \cancel{i^2} + \cancel{i^4} + \cancel{i^6} + \dots + \cancel{i^{100}}$$

$$n = 50$$

$$1$$

Q 10. $1 + i^2 + i^4 + i^6 + i^8 + \dots$ 2n items =?
 (a) 0
 (b) 1
 (c) -1
 (d) N.O.T

Ans.

let $n = 1$

$$\cancel{1 + i^2} + \cancel{i^4} + \cancel{i^6} + \cancel{i^8} + \cancel{i^{10}} + \dots = 0$$

$$2(3) = 6$$

$$2(1) = 2 \text{ terms}$$

$$2(2) = 4 \text{ terms}$$

let $n = 2$

$$\cancel{1 + i^2} + \cancel{i^4} + \cancel{i^6} + \dots = 0$$

Q 11. $1 + i^2 + i^4 + i^6 + i^8 + \dots$

(a) 0

(b) 1

(c) -1

(d) N.O.T

Ans.
=

1 +

$$\cancel{1 + i^2} = 0$$

$$i^{2n} = ?$$

$$n = 50$$

$$i^{2(1)} = i^2$$

odd

$$\text{let } n = 1$$

$$i^{2(2)} = i^4$$

$$\text{let } n = 2 \text{ even}$$

$$\cancel{1 + i^2 + i^4}$$

$$= 1$$

$$n = 51$$

Some important values

1. $(1+i)^2 = 2i$

2. $(1-i)^2 = -2i$

3. $\sqrt{i} = \pm \left(\frac{1+i}{\sqrt{2}} \right)$

4. $\sqrt{-i} = \pm \left(\frac{1-i}{\sqrt{2}} \right)$

$$(1+i)^2 = 2i$$

$$1 + 2i + i^2 = 2i$$

$$(1-i)^2 = -2i$$

$$\sqrt{i} = \pm \left(\frac{1+i}{\sqrt{2}} \right)$$

$$\sqrt{-i} = \pm \left(\frac{1-i}{\sqrt{2}} \right)$$

$$\boxed{\sqrt{i} + \sqrt{-i} = ?} \quad \checkmark$$

$$\frac{1 + \cancel{i}}{\sqrt{2}} + \frac{\cancel{1} - i}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \textcircled{\sqrt{2}}$$

$$5. i^{\frac{1}{3}} = \frac{\sqrt{3}+i}{2} \Rightarrow \sqrt{3}+i = 2i^{\frac{1}{3}}$$

$$6. i^{-\frac{1}{3}} = \frac{\sqrt{3}-i}{2} \Rightarrow \sqrt{3}-i = 2i^{-\frac{1}{3}}$$

$$7. \frac{1+i}{1-i} = i \text{ \& } \frac{1-i}{1+i} = -i$$

$$8. \frac{\sqrt{3}+i}{\sqrt{3}-i} = i^{\frac{2}{3}}, \frac{\sqrt{3}-i}{\sqrt{3}+i} = i^{-\frac{2}{3}}$$

$$i^{\frac{1}{3}} = \frac{\sqrt{3}+i}{2} ; \sqrt{3}+i = 2i^{\frac{1}{3}}$$

$$i^{-\frac{1}{3}} = \frac{\sqrt{3}-i}{2} ; \sqrt{3}-i = 2i^{-\frac{1}{3}}$$

$$\frac{1+i}{1-i} = i$$

$$\frac{1-i}{1+i} = -i$$

$$\frac{(1+i)(1+i)}{(1-i)(1-i)} = \frac{(1+i)^2}{1^2-i^2}$$

$$= \frac{2i}{2} \cdot i$$

$$i^{2/3} = \frac{\sqrt{3} + i}{\sqrt{3} - i}$$

$$i^{-2/3} = \frac{\sqrt{3} - i}{\sqrt{3} + i}$$

$$i^{1/3 + 1/3} = \frac{i^{1/3}}{i^{-1/3}} = \frac{i^{1/3}}{\frac{1}{i^{1/3}}} = \frac{i^{1/3} \cdot i^{1/3}}{1} = i^{2/3} = \frac{\sqrt{3} + i}{\sqrt{3} - i}$$

MDAP.Y.Q

Q 12. $z = \frac{1+2i}{1-(1-i)^2}$

then, तो $z = ?$

- (a) 0
- ☒ (b) 1
- (c) $1+i$
- (d) $1-i$

$$z = \frac{1+2i}{1-(1-i)^2}$$

$$1 - (1-i)^2$$

$$z = \frac{1+2i}{1-(-2i)}$$

$$\frac{1+2i}{1-(-2i)} = \frac{1+2i}{1+2i} = \textcircled{1}$$

$$= \frac{1+2i}{1+2i} = \textcircled{1}$$

Q 13. $(1+i)^6 + (1-i)^6 = ?$

(a) $16i$

(b) $-16i$

(c) 0

(d) N.O.T

$$\left((1+i)^2\right)^3 + \left((1-i)^2\right)^3$$

$$(2i)^3 + (-2i)^3$$

$$\cancel{8i^3} - \cancel{8i^3} = 0$$

$$\left((1+i)^2\right)^3 + \left((1-i)^2\right)^3$$

$$(2i)^3 + (-2i)^3$$

$$\cancel{8i^3} - \cancel{8i^3}$$

Q 14. $\frac{(1+i)^{4n+2}}{(1-i)^{4n+1}} = ?$

(a) $1 + i$

(b) $1 - i$

(c) $-1 + i$

(d) $-1 - i$

$$\frac{(1+i)^2}{(1-i)^1} \quad \boxed{n=0}$$

$$= \frac{2i(1+i)}{(1-i)(1+i)}$$

$$= \frac{2i(1+i)}{i^2 - i^2} = \frac{2i(1+i)}{1 - (-1)}$$

$$= \frac{2i(1+i)}{2}$$

$$= i + i^2 = i - 1 \quad \text{or} \quad \boxed{-1 + i}$$

Q 15. $z = \left(\frac{1+i}{1-i}\right)^n$, for what minimum value of n , z is purely real n के किस न्यूनतम मान के लिए z वास्तविक होगा?

(a) 1

(b) 2

(c) 4

(d) 8

$$z = \left(\frac{1+i}{1-i}\right)^n$$

$$\frac{1+i}{1-i} = i$$

$$z = i^n$$

$$i^2 = -1$$

$$z = -1$$

NDA 2017

Q 16. $\left(\frac{\sqrt{3}+i}{2}\right)^{24} + \left(\frac{\sqrt{3}-i}{2}\right)^{24} = ?$

- (a) -1
- (b) 2
- (c) 1
- (d) -2

$\left(i^{1/3}\right)^{24} + \left(i^{-1/3}\right)^{24}$

$i^8 + i^8$

De-Moivre's

$1 + \frac{1}{1} = 1 + 1 = 2$

Q 16. $(1+i)^7 + (1-i)^7 = ?$

- (a) 16
- (b) -16
- (c) 0
- (d) N.O.T

$$(1+i)^{\textcircled{7}} + (1-i)^7$$

$$\left((1+i)^2 \right)^{\textcircled{3}} (1+i) + i \left((1-i)^2 \right)^{\textcircled{3}} (1-i)$$

$$8i^3(1+i) + (-8i^3)(1-i)$$

$$-8i(1+i) + 8i(1-i)$$

$$8i(-1-i+1-i)$$

$$8i(-2i) = -16i^2 = -16(-1) = \textcircled{16}$$

Q 17. $(\sqrt{3} + i)^{12} + (\sqrt{3} - i)^{12} = ?$

(a) 2^{13}

(b) $i2^{13}$

(c) 2^{12}

(d) 12^{12}

$$\frac{\sqrt{3} + i}{2} = e^{i\pi/6}$$

$$\sqrt{3} + i = 2e^{i\pi/6}$$

$$\frac{\sqrt{3} - i}{2} = e^{-i\pi/6}$$

$$\sqrt{3} - i = 2e^{-i\pi/6}$$

$$(2e^{i\pi/6})^{12} + (2e^{-i\pi/6})^{12}$$

$$2^{12} \cdot 1 + 2^{12} \cdot 1$$

$$2^{12} (1 + 1) = 2^{12} \cdot 2$$

$$\frac{1}{i^4} = \frac{1}{1} = 1$$

$$= 2^{13}$$

Q 18. $\left(\frac{\sqrt{3}+i}{\sqrt{3}-i}\right)^6 = ?$

- (a) 2
- ~~(b) 1~~
- (c) 64
- (d) N.O.T

NDA P. Y. O
 $(i^2/3)^{36} = i^4 = 1$

Q 19. $(\sqrt{3} + i)^7 + (\sqrt{3} - i)^7 = ?$

(a) $128\sqrt{3}$

(b) $-128\sqrt{3}$

(c) $128\sqrt{3}i$

(d) $-128\sqrt{3}i$

H. W

Repeat

H. W