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Brief paper

Enclosing a target by nonholonomic mobile robots with bearing-only measurements☆

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ABSTRACT

This paper addresses the problem of steering a single or a group of autonomous nonholonomic mobile robots to enclose a target of interest. We develop control schemes which require only local bearing measurements, and deal with encirclement of two types of targets: point target and disk target. When a single robot is used, circumnavigation schemes are proposed to achieve effective encirclement of the target. It is shown that under the proposed control schemes, the robot can circle the target from a prescribed distance without distance measurement. When multiple robots are deployed, the control schemes developed for single robot are modified by introducing a coordination mechanism, which drives the robots to distribute evenly around the target. We further discuss a control scheme that moves the robots to form a static circular formation around the target based on feedback linearization. All the proposed control schemes are validated in experiments on a group of two-wheeled differential drive mobile robots.

1. Introduction

Enclosing a target or an area of interest represents a promising application of mobile robots in many areas such as security and surveillance, search and rescue, orbit maintenance, and source localization. Autonomous mobile robots, either a single robot or a group of robots, can be deployed to perform such tasks (Feddema, Lewis, & Schoenwald, 2002). Enclosing a target by holonomic mobile robots was reported in the literature, such as (Guo, Yan, & Lin, 2010; Kim & Sugie, 2007; Yamaguchi, 1999). Nonholonomic mobile robots, although their control is much more challenging, were also studied in some works, for example Ceccarelli, Di Marco, Garulli, and Giannitrapani (2008), Lan, Yan, and Lin (2010) and Zheng, Lin, Fu, and Sun (2013). Some commercial products, such as the Kestrel autopilot system, also provide target monitoring function for unmanned aerial vehicles.

In most of these existing works, it is commonly assumed that measurements of relative positions amongst the robots are available, which, however, restricts many practical applications. For example, for a mobile robot equipped with single omnidirectional camera, or underwater vehicle equipped with radar and sonar operating in passive listening mode, it is difficult or even impossible to obtain the distance measurement. Motivated by these potential applications, formation control utilizing bearing-only measurements has attracted a lot of attention recently. In bearing-based formation control, it is assumed that the robots can only measure the local bearings of their surrounding objects. Compared with relative position sensors or distance sensors, bearing-only sensors are simpler and cheaper but still span a large spectrum of common sensor types such as monocular camera systems. Some works on single robot target enclosing using bearing measurements have been reported. For example, a circumnavigation strategy was proposed for holonomic robot in Deghat, Shames, Anderson, and Yu (2010) and was extended to nonholonomic robot in Deghat et al. (2012). In Deghat et al. (2012) and Deghat et al. (2010), a nonlinear periodically time-varying algorithm was used to estimate the position of the target. The control algorithm was then built based on the estimated position. However, this method assumes that the robot knows its own position in an inertial coordinate frame.

The problem of enclosing a target with multiple robots using bearing-only measurements has been rarely studied. Guo, Yan, and Lin (2011) proposed a bearing-only control strategy with inter-agent gossip communication to drive a group of robots to achieve

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and maintain a balanced circular formation around a target. Bearing-based control strategies have been studied to solve some other problems in multirobot systems. For example, Moshtagh, Michael, Jadbabaie, and Daniilidis (2009) designed a control law based on the bearing angle, the optical flow and time to collision for a group of constant-speed nonholonomic robots to achieve parallel and circular formations. Bearing-only triangular formation control of three mobile agents was considered in Basiri, Bishop, and Jensfelt (2010), in which each agent used locally measured bearings to establish and maintain a pre-specified angular separation relative to the other two agents. The controller in Basiri et al. (2010) was relaxed in Bishop (2011a). For both papers, the proof of global stability is based on Poincaré-Bendixson theorem and thus the number of agents is limited. Quadrilateral formation control using similar controllers, given in Bishop (2011b), was proved using Lyapunov based method. It would be of great interest to generalize the formulation to arbitrary number of agents, which still remains unsolved. Zhao, Lin, Peng, Chen, and Lee (2013) proved the local finite-time stability of angle-constraint cyclic formation with arbitrary number of holonomic agents under a discontinuous control law. The rendezvous problem of nonholonomic robots using bearing-only control strategies was studied in Zheng and Sun (2013a) and was experimentally tested in Zheng and Sun (2013b).

In addition, direction-constraint formation control was studied by Bishop, Shames, and Anderson (2011). Eren (2012) and Franchi et al. (2012) studied bearing-constraint formation control for nonholonomic robots. It is worth noting that unlike bearing-based formation control, both direction-constraint and bearing-constraint formations only require the desired geometrical relationships to be specified in terms of relative angular constraints between agent-pairs, which implies that the distance measurement may be still needed while implementing the controllers.

In this paper, we investigate the bearing-based target enclosing problem using single robot and multiple robots, respectively. The proposed control schemes are validated by means of real experiments employing a team of mobile robots. Our main contributions in this paper are twofold. First, we develop novel control schemes for nonholonomic mobile robots to enclose a target with relative bearing measurements only. Unlike Deghat et al. (2012, 2010), we assume that global position information is not available and the robots are memoryless so they cannot know their own positions in any inertial coordinate frame. Unlike Guo et al. (2011), we assume that there is no inter-agent communication so that the robots cannot share their bearing measurements amongst each other. Hence, the proposed controller schemes are simple and can be easily implemented on real robots. Second, collision avoidance between the robot and disk-area target is guaranteed with our control schemes. Inter-agent collision avoidance is also considered by exploiting a simple behavior-based algorithm.

The rest of the paper is organized as follows. Section 2 presents our control schemes to achieve circumnavigation using a single robot while Section 3 discusses the distributed circumnavigation and balanced circular formation by using multiple robots. In Section 4, we present experimental results to demonstrate the effectiveness of the proposed control schemes. Finally, we draw conclusions and suggest future research directions in Section 5.

2. Enclosing a target by single robot

Consider a nonholonomic, two-wheeled differential drive robot moving in a 2D workspace. The kinematic equation of each robot can be written as

$$\dot{x} = v\cos\theta, \qquad \dot{y} = v\sin\theta, \qquad \dot{\theta} = \omega, \tag{1}$$

where $(x, y) \in \mathbb{R}^2$ is the location of the robot and θ the heading angle with respect to a global coordinate frame ${}^g\Sigma$. Control input $v \in [-v_{\max}, v_{\max}]$ is the forward speed in the direction of θ, and another control input $\omega \in [-\omega_{\max}, \omega_{\max}]$ is the angular speed. Here $v_{\max}$ and $\omega_{\max}$ (both positive) are the bounds for forward and angular speeds, respectively.

Consider a static target $\mathcal{T}$ centered at (x_b, y_b) in the plane, which can be either a point or a disk area. To accomplish the mission, the robot equips an onboard sensor. However, this sensor only allows the robot to detect the bearing information of its surrounding objects, i.e., distance information is not available. It is also assumed that the target can be always detected by the sensor.

When there is only one single robot, a possible way to efficiently surveil or protect the target is to let the robot circle around the target at a stand-off distance. Following works such as Cao, Muse, Casbeer, and Kingston (2013), Shames, Dasgupta, Fidan, and Anderson (2012) and Tang, Shinzaki, Gage, and Clark (2012), we call it *circumnavigation*. To facilitate the analysis, we define

$$\rho := \sqrt{(x - x_b)^2 + (y - y_b)^2}, \qquad \alpha := \arctan2(\tilde{y}_b, \tilde{x}_b), \tag{2}$$

where $\arctan2(y, x)$ represents a two-argument arctangent function retuning the angle of point (x, y) as a numeric value between $-\pi$ and π radians. And $(\tilde{x}_b, \tilde{y}_b)$ is that Cartesian coordinates of the target $\mathcal{T}$ in the robot's local frame ${}^r\Sigma$, i.e.,

$$\begin{bmatrix}\tilde{x}_b \\ \tilde{y}_b\end{bmatrix} = \begin{bmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}x_b - x \\ y_b - y\end{bmatrix}.$$

In our setting, only α is measurable. Because ρ cannot be measured, the relative position $(\tilde{x}_b, \tilde{y}_b)$ is unavailable. After straightforward calculation, the dynamics of ρ and α can be described as

$$\dot{\rho} = -v\cos\alpha, \qquad \dot{\alpha} = -\omega + v\sin\alpha/\rho, \tag{3}$$

where $(\rho, \alpha) \in (0, \infty) \times [-\pi, \pi)$.

2.1. Controller 1: single robot enclosing a point target

We first consider the case of enclosing a point target by a single robot. The following control law is proposed and analyzed:

Controller 1:

$$\begin{cases} v = k_v v_{\max}\cos(\alpha + \varphi), \\ \omega = k_\omega \omega_{\max}\sin(\alpha + \varphi), \end{cases} \tag{4}$$

where $k_v, k_\omega \in (0, 1]$ are control gains for forward speed and angular speed, respectively. The following theorem shows that the desired circling behavior can be achieved by setting

$$\varphi = -\arctan\frac{\rho^* k_\omega \omega_{\max}}{k_v v_{\max}}, \tag{5}$$

where ρ^* is the predefined stand-off distance.

Theorem 1. *Under the controller* 1 *with* (5)*, a robot described by* (1) *will eventually move on a circle centered at* (x_b, y_b) *with radius* ρ^* *counter-clockwise at speed* $k_v v_{\max}\sin(-\varphi)$.

Proof. First of all, it is clear that under controller 1, $v \in [-v_{\max}, v_{\max}]$ and $\omega \in [-\omega_{\max}, \omega_{\max}]$. Consider the following point

$$\begin{bmatrix}x_c \\ y_c\end{bmatrix} := \begin{bmatrix}x \\ y\end{bmatrix} + \rho^*\begin{bmatrix}-\sin\theta \\ \cos\theta\end{bmatrix}, \tag{6}$$

we can see that (x_c, y_c) is the center when the robot performs counter-clock uniform circular motion on a circle of radius ρ^*. We will show that $(x_c, y_c) \to (x_b, y_b)$ as $t \to \infty$.

The derivative of (x_c, y_c) with respect to time can be calculated as

$$\begin{bmatrix}\dot{x}_c \\ \dot{y}_c\end{bmatrix} = \begin{bmatrix}\cos\theta \\ \sin\theta\end{bmatrix}(v - \rho^*\omega).$$

Under (4) and (5), we have

$$v - \rho^*\omega = k_v v_{\max}\cos(\alpha+\varphi) - \rho^* k_\omega \omega_{\max}\sin(\alpha+\varphi)$$
$$= k_1[\cos(-\varphi)\cos(\alpha+\varphi) - \sin(-\varphi)\sin(\alpha+\varphi)] = k_1\cos\alpha,$$

where we define $k_1 := \sqrt{(k_v v_{\max})^2 + (\rho^* k_\omega \omega_{\max})^2}$ for brevity. Furthermore,

$$\cos\alpha = [\cos\theta(x_b - x) + \sin\theta(y_b - y)]/\rho$$
$$\overset{(6)}{=} [\cos\theta(x_b - x_c) + \sin\theta(y_b - y_c)]/\rho. \tag{7}$$

Consider the following Lyapunov function candidate,

$$V = \frac{1}{2}[(x_c - x_b)^2 + (y_c - y_b)^2], \tag{8}$$

whose derivative is

$$\dot V = -\frac{k_1}{\rho}[\cos\theta(x_c - x_b) + \sin\theta(y_c - y_b)]^2 \le 0.$$

It can be seen that $\dot V = 0$ only when

$$\cos\theta(x_c - x_b) + \sin\theta(y_c - y_b) = 0, \tag{9}$$

i.e., $\cos\alpha = 0$ according to (7).

We next show that the largest invariance set in $\{(x_c, y_c)|\dot V = 0\}$ is $\{(x_b, y_b)\}$. When $\cos\theta(x_c - x_b) + \sin\theta(y_c - y_b) = 0$, $(\dot x_c, \dot y_c) = (0, 0)$, we then have

$$\frac{d}{dt}[\cos\theta(x_c - x_b) + \sin\theta(y_c - y_b)]$$
$$= \omega[-\sin\theta(x_c - x_b) + \cos\theta(y_c - y_b)].$$

When $\cos\alpha = 0$, $\omega = k_\omega\omega_{\max}\sin(\pm\pi/2 + \varphi) \neq 0$. To satisfy $\cos\theta(x_c - x_b) + \sin\theta(y_c - y_b) \equiv 0$, it requires that

$$-\sin\theta(x_c - x_b) + \cos\theta(y_c - y_b) = 0. \tag{10}$$

Combining both (9) and (10), we get

$$\underbrace{\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}}_{A}\begin{bmatrix}x_c - x_b\\ y_c - y_b\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}.$$

Since matrix A is full rank, the largest invariance set in $\{(x_c, y_c)|\dot V = 0\}$ is $\{(x_b, y_b)\}$. Then according to LaSalle's invariance theorem (Khalil, 2002, Theorem 4.4), the trajectory of (x_c, y_c) converges to (x_b, y_b), i.e., the trajectory of (x, y) converges to a circle centered at (x_b, y_b) with radius ρ^*. The eventual speed of the robot can be calculated accordingly. □

2.2. Controller 2: single robot with fixed forward speed enclosing a point target

For some kinds of mobile vehicles, such as those considered in Deghat et al. (2012) and Matveev, Hoy, and Savkin (2013), the forward speed v is fixed and requires $v \ge v_{\min} > 0$. This requirement motivates us to propose the following control law:

Controller 2:

$$\begin{cases} v = \min(k_v v_{\max}, \eta k_\omega \omega_{\max}\rho^*),\\ \omega = v/\rho^* - (1-\eta)k_\omega\omega_{\max}\cos\alpha, \end{cases} \tag{11}$$

where control gains $k_v, k_\omega \in (0, 1]$ and $\eta \in (0, 1)$ is a design parameter which can be used to adjust the forward speed.

Theorem 2. *Given $\bar v = \min(k_v v_{\max}, \eta k_\omega \omega_{\max}\rho^*) \ge v_{\min}$, under controller 2, a robot described by (1) with constant forward velocity $\bar v$ will eventually move on circle centered at (x_b, y_b) with radius ρ^* counter-clockwise.*

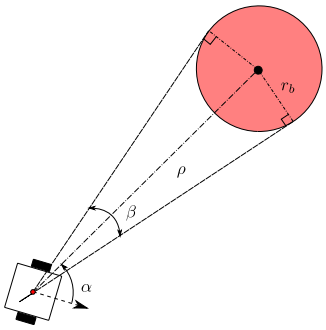


Fig. 1. An illustration of defined variables. The disk represents the target.

Proof. Clearly, $v \in [v_{\min}, v_{\max}]$. For ω, one has $\omega \le \eta k_\omega\omega_{\max} + (1-\eta)k_\omega\omega_{\max} \le \omega_{\max}$ and $\omega \ge v_{\min}/\rho^* - (1-\eta)k_\omega\omega_{\max} \ge -k_\omega\omega_{\max}$, so $\omega \in [-\omega_{\max}, \omega_{\max}]$.

Under controller 2, we obtain the following equation based on (6):

$$\begin{bmatrix}\dot x_c\\ \dot y_c\end{bmatrix} = \begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix}(v - \rho^*\omega) = k_1\begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix}\begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix}^T\begin{bmatrix}x_b - x_c\\ y_b - y_c\end{bmatrix},$$

where we define $k_1 := (1-\eta)k_\omega\omega_{\max}\rho^*/\rho$ for brevity. Consider the Lyapunov function candidate (8), it is obtained that

$$\dot V = -k_1[\cos\theta(x_c - x_b) + \sin\theta(y_c - y_b)]^2 \le 0.$$

The following argumentation is similar to that of Theorem 1 and is omitted here. □

2.3. Controller 3: single robot enclosing a disk target with guaranteed collision avoidance

In this subsection, we consider the case when the target is a disk area (see Fig. 1). Let r_b be the radius of the target disk. Because the onboard sensor can only provide bearing measurement, r_b is unavailable to the robot. However, unlike the case of enclosing a point target, collision avoidance between the robot and the disk target must be taken into account, i.e., it should be guaranteed that $\rho(t) > r_b$ for all $t \ge 0$. To do that, we utilize the measured subtended angle $\beta \in (0, \pi)$ (see Fig. 1).

The proposed control law is given by

Controller 3:

$$\begin{cases} v = \bar v\cos(\alpha+\varphi),\\ \omega = v/\rho^* - (1-\eta)k_w\omega_{\max}\cos\alpha, \end{cases} \tag{12}$$

where $\bar v := \min(k_v v_{\max}, \eta k_\omega\omega_{\max}\rho^*)$, $\eta \in (0, 1)$ is a design parameter. Moreover, the offset angle φ is designed to be

$$\varphi = -\mu\pi - (1-\mu)\pi\exp[k_\beta(\beta - \beta_{\max})] \tag{13}$$

with $\mu \in (0, 1)$, $k_\beta > 0$, and $\beta_{\max} \in (0, \pi)$ is the maximum allowed subtended angle. The value of μ affects the convergence rate significantly. For example, if μ is too small ($\approx$0), then the eventual forward speed around the target may also be too small; If μ is too large ($\approx$1) and it happens that $\cos\alpha \approx 0$, then the robot will approach the target very slowly when it is far away from the target disk.

We first show in the following theorem that under controller 3, a robot can avoid collision with the target disk.

Theorem 3. *Under controller 3 with (13), if $\beta(0) < \beta_{\max}$,[1] then $\rho(t) > r_b$ for all $t \ge 0$.*

[1] That is, $\rho(0) > r_b\csc\frac{\beta_{\max}}{2}$.

Proof. Under controller 3, from (3) one obtains

$$\dot{\rho} = -\bar{v}\cos(\alpha+\varphi)\cos\alpha.$$

As $\rho \to (r_b \csc\frac{\beta_{\max}}{2})^+$, $\beta \to (\beta_{\max})^-$, so $\varphi \to -\pi$ according to (13). As a result, we have $\dot{\rho} \to \bar{v}\cos^2\alpha \geq 0$. Therefore $\rho(t) > r_b \csc\frac{\beta_{\max}}{2} > r_b$ for all $t \geq 0$. □

The following theorem shows that a robot can achieve circumnavigation around the target disk using the proposed control law.

Theorem 4. *Under controller* 3 *with* (13)*, if* $\beta_{\max} > 2\arcsin\frac{r_b}{\rho^*}$ *and* $\rho(0) > r_b\csc(\beta_{\max}/2)$*, then a robot described by* (1) *will eventually move on a circle centered at* (x_b, y_b) *with radius* ρ^* *at speed*

$$\bar{v}\sin\{\mu\pi + (1-\mu)\pi\exp[k_\beta(\beta^* - \beta_{\max})]\}$$

with $\beta^* = 2\arcsin(r_b/\rho^*)$.

Proof. The proof is similar to that of Theorems 1 and 2 and is omitted here. □

Under controllers 1 and 2, it would cause no problem when the initial position of the robot is close to the target. However, the controller 3 requires that the initial distance from the robot to the center of the target disk $\rho(0) \geq r_b\csc\frac{\beta_{\max}}{2}$. Otherwise, collision may happen.

3. Enclosing a target by multiple robots

In this section, we deal with the case that multiple robots enclose a target. Consider a group of n ($\geq$3) mobile robots described by

$$\dot{x}_i = v_i\cos\theta_i, \qquad \dot{y}_i = v_i\sin\theta_i, \qquad \dot{\theta}_i = \omega_i, \tag{14}$$

where $i = 1, 2, \ldots, n$. For multirobot case, we require that the robots not only converge to the desired stand-off circle but also form an optimal configuration. In the target enclosing problem, an optimal configuration often corresponds to an equiangular spaced formation around the target (Bishop, Fidan, Anderson, Doğançay, & Pathirana, 2010). We also require the control schemes to be distributed which allow the team to be robust to insertion and deletion of team members.

3.1. Controller 4: multiple robots enclosing a point target

In this subsection, we modify the control schemes proposed in Section 2 to work in a multirobot scenario.

Let ρ_i and α_i be defined as ρ and α in (2) by replacing (x, y, θ) with (x_i, y_i, θ_i). To be more practical, it is assumed that the robots cannot detect each other until their distance is smaller than a threshold distance r_s. We denote the neighbor set of robot i as $\mathcal{N}_i = \{j | \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2} \leq r_s,\ j \neq i\}$. The bearing angle of robot j in robot i's local frame can be defined formally as $\alpha_j^{\mathrm{i}} = \arctan2(y_j^{\mathrm{i}}, x_j^{\mathrm{i}})$, $j \in \mathcal{N}_i$, where $(x_j^{\mathrm{i}}, y_j^{\mathrm{i}})$ is the relative position of robot j in robot i's local frame, i.e.,

$$\begin{bmatrix} x_j^{\mathrm{i}} \\ y_j^{\mathrm{i}} \end{bmatrix} = \begin{bmatrix} \cos\theta_i & \sin\theta_i \\ -\sin\theta_i & \cos\theta_i \end{bmatrix} \begin{bmatrix} x_j - x_i \\ y_j - y_i \end{bmatrix}.$$

As mentioned before, $(x_j^{\mathrm{i}}, y_j^{\mathrm{i}})$ is unmeasurable and only α_j^{i} is measurable. For convenience of presentation, we also define

$$\sigma_+^{\mathrm{i}} = \begin{cases} \min\limits_{j\in\mathcal{N}_+^{\mathrm{i}}}(\alpha_j^{\mathrm{i}}), & \text{if } \mathcal{N}_+^{\mathrm{i}} \neq \emptyset, \\ \pi/2, & \text{if } \mathcal{N}_+^{\mathrm{i}} = \emptyset, \end{cases} \tag{15}$$

and

$$\sigma_-^{\mathrm{i}} = \begin{cases} \pi - \max\limits_{j\in\mathcal{N}_-^{\mathrm{i}}}(\alpha_j^{\mathrm{i}}), & \text{if } \mathcal{N}_-^{\mathrm{i}} \neq \emptyset, \\ \pi/2, & \text{if } \mathcal{N}_-^{\mathrm{i}} = \emptyset, \end{cases} \tag{16}$$

where $\mathcal{N}_+^{\mathrm{i}} = \{j \in \mathcal{N}_i | \alpha_j^{\mathrm{i}} \in (0, \pi/2)\}$ and $\mathcal{N}_-^{\mathrm{i}} = \{j \in \mathcal{N}_i | \alpha_j^{\mathrm{i}} \in (\pi/2, \pi)\}$.

We then modify the control laws proposed in Section 2 to a multirobot scenario by introducing a new control coefficient and defining new control inputs (v_i', ω_i') as

Controller 4:

$$\begin{cases} v_i' = f(\sigma_+^{\mathrm{i}}/\sigma_-^{\mathrm{i}})\, v_i, \\ \omega_i' = f(\sigma_+^{\mathrm{i}}/\sigma_-^{\mathrm{i}})\, \omega_i. \end{cases} \tag{17}$$

Here (v_i, ω_i) can be any one of controllers 1, 2 and 3. In order to achieve an even distribution, all the robots use same control gains and parameters. Here

$$f : (0, +\infty] \to (0, 1] \tag{18}$$

is any strictly increasing continuous function. For example, we can let $f(x) = 1 - \exp(-x)$.

Define ϕ_i to be the bearing of robot i position with respect to the target position: $\phi_i = \arctan2(y_i - y_b, x_i - x_b) \bmod 2\pi$. For the convenience of analysis, we label the robots at time t according to their positions in a counterclockwise radial order around the target, i.e., $j > k \Leftrightarrow \phi_j \geq \phi_k$. We also define $\delta\phi_i = \phi_{i+1} - \phi_i$ for $i \in \{1, \ldots, n-1\}$ and define $\delta\phi_n = \phi_1 + 2\pi - \phi_n$. Note that $\delta\phi_i \in [0, 2\pi)$ for all i under this labeling.

Theorem 5. *Under controller* 4*, if* $r_s > 2\rho^*\sin(\pi/n)$*, the equilibrium point given by* $\{\rho_i = \rho^*, \alpha_i = \pi/2, \delta\phi_i = 2\pi/n,\ \forall i\}$ *is (locally) asymptotically stable.*

Proof. Based on the analysis in Section 2, because $f(\sigma_+^{\mathrm{i}}/\sigma_-^{\mathrm{i}}) > 0$ for all $t \geq 0$, then for all i, $\rho_i \to \rho^*$ and $\alpha_i \to \pi/2$ as $t \to \infty$.

The condition $r_s \geq 2\rho^*\sin(\pi/n)$ guarantees that neither $\mathcal{N}_+^{\mathrm{i}}$ nor $\mathcal{N}_+^{\mathrm{i}}$ is empty around the equilibrium point. Therefore, when $\rho_i = \rho^*$ and $\alpha_i = \pi/2$ for all i, using simple trigonometry, we have (see Fig. 2)

$$\sigma_+^{\mathrm{i}} = \delta\phi_i/2 \quad \text{and} \quad \sigma_-^{\mathrm{i}} = \delta\phi_{i-1}/2.$$

Around the equilibrium point, it is obtained that

$$\dot{\phi}_i = \omega_i' = f(\delta\phi_i/\delta\phi_{i-1})\,\omega^*,$$

where ω^* is the angular velocity when $\rho_i = \rho^*$ and $\alpha_i = \pi/2$. The above equation leads to

$$\frac{d}{dt}\delta\phi_i = [f(\delta\phi_{i+1}/\delta\phi_i) - f(\delta\phi_i/\delta\phi_{i-1})]\,\omega^*.$$

Define $\overline{M} = \{j | \delta\phi_j = \max\delta\phi_i\}$ and $\underline{M} = \{j | \delta\phi_j = \min\delta\phi_i\}$. When $\overline{M} \neq \underline{M}$, we have

$$\frac{d}{dt}\frac{\sum\limits_{j\in\overline{M}}\delta\phi_j}{|\overline{M}|} < 0 \quad \text{and} \quad \frac{d}{dt}\frac{\sum\limits_{j\in\underline{M}}\delta\phi_j}{|\underline{M}|} > 0.$$

So we conclude that the equilibrium point defined by $\{\rho_i = \rho^*, \alpha_i = \pi/2, \delta\phi_i = 2\pi/n,\ \forall i\}$ is locally asymptotically stable. □

Given the stand-off distance ρ^*, the equilibrium point in Theorem 5 is fixed. When given different ρ^*, the robots can move on different orbits. However, in that case, an even distribution will not be achieved.

3.2. Controller 5: multiple robots enclosing a disk target

By keeping the robots in motion, controllers 1–4 provide solutions to effectively surveil or protect a target when the number of robots is limited. However, if there are enough robots to achieve a tight enclosing, an alternative way is to drive the robots to form a balanced circular formation around the target. In this way, because

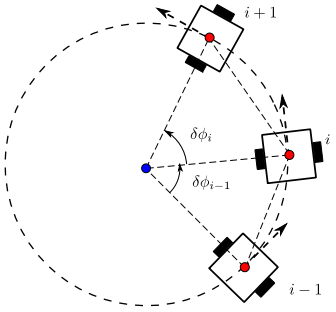


Fig. 2. An illustration of σ_+^{i} and σ_-^{i} when $\rho_i = \rho^*$ and $\alpha_i = \pi/2$ for all i.

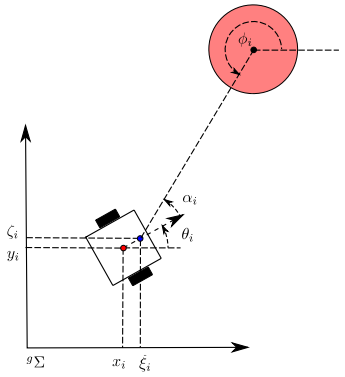


Fig. 3. Pseudo-linearization.

the robots do not need to move all the time, the energy needed for movement can be reduced dramatically.

In this subsection, we consider the case of multiple robots enclosing a disk target. To simplify the design of the control law, assume that the onboard sensors are positioned at the point (ξ_i, ζ_i) (see Fig. 3) defined as

$$\begin{bmatrix}\xi_i\\ \zeta_i\end{bmatrix} := \begin{bmatrix}x_i\\ y_i\end{bmatrix} + L\begin{bmatrix}\cos\theta_i\\ \sin\theta_i\end{bmatrix}, \quad i = 1, 2, \ldots, n.$$

Instead of moving (x_i, y_i) into a formation, we design a control law to move (ξ_i, ζ_i) into a formation. The benefit to introduce (ξ_i, ζ_i) is that its kinematics is holonomic for $L \neq 0$ (Lawton, Beard, & Young, 2003), which is given by

$$\begin{bmatrix}\dot\xi_i\\ \dot\zeta_i\end{bmatrix} = \begin{bmatrix}\cos\theta_i & -\sin\theta_i\\ \sin\theta_i & \cos\theta_i\end{bmatrix}\begin{bmatrix}v_i\\ L\omega_i\end{bmatrix}.$$

Design the control input (v_i, ω_i) as follows:

$$\begin{bmatrix}v_i\\ \omega_i\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1/L\end{bmatrix}\begin{bmatrix}\cos\theta_i & \sin\theta_i\\ -\sin\theta_i & \cos\theta_i\end{bmatrix}\mathbf{u}_i, \tag{19}$$

where $\mathbf{u}_i \in \mathbb{R}^{2\times 1}$. We then obtain that $[\dot\xi_i, \dot\zeta_i]^T = \mathbf{u}_i$.

With a little abuse of notation, in this subsection, we use the notation ρ_i, ϕ_i, α_i, $\delta\phi_i$ and $\mathcal{N}_i$ as before but using (ξ_i, ζ_i) instead of (x_i, y_i) wherever necessary. The derivative of ρ_i is

$$\dot\rho_i = \frac{\begin{bmatrix}\xi_i - x_b & \zeta_i - y_b\end{bmatrix}}{\rho_i}\begin{bmatrix}\dot\xi_i\\ \dot\zeta_i\end{bmatrix} = \begin{bmatrix}\cos\phi_i & \sin\phi_i\end{bmatrix}\begin{bmatrix}\dot\xi_i\\ \dot\zeta_i\end{bmatrix}.$$

And the derivative of ϕ_i is

$$\dot\phi_i = \frac{\begin{bmatrix}-(\zeta_i - y_b) & \xi_i - x_b\end{bmatrix}}{\rho_i^2}\begin{bmatrix}\dot\xi_i\\ \dot\zeta_i\end{bmatrix} = \frac{\begin{bmatrix}-\sin\phi_i & \cos\phi_i\end{bmatrix}}{\rho_i}\begin{bmatrix}\dot\xi_i\\ \dot\zeta_i\end{bmatrix}.$$

From the above two equations, it is obtained that

$$\begin{bmatrix}\dot\rho_i\\ \rho_i\dot\phi_i\end{bmatrix} = \begin{bmatrix}\cos\phi_i & \sin\phi_i\\ -\sin\phi_i & \cos\phi_i\end{bmatrix}\mathbf{u}_i,$$

that is,

$$\mathbf{u}_i = \begin{bmatrix}\cos(-\phi_i) & \sin(-\phi_i)\\ -\sin(-\phi_i) & \cos(-\phi_i)\end{bmatrix}\begin{bmatrix}\dot\rho_i\\ \rho_i\dot\phi_i\end{bmatrix}.$$

Substituting the above equation into (19), we get

$$\begin{bmatrix}v_i\\ \omega_i\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1/L\end{bmatrix}\begin{bmatrix}\cos(\theta_i - \phi_i) & \sin(\theta_i - \phi_i)\\ -\sin(\theta_i - \phi_i) & \cos(\theta_i - \phi_i)\end{bmatrix}\begin{bmatrix}\dot\rho_i\\ \rho_i\dot\phi_i\end{bmatrix}.$$

Because $\theta_i - \phi_i = -\pi - \alpha_i$ (see Fig. 3), we have

Controller 5:

$$\begin{bmatrix}v_i\\ \omega_i\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1/L\end{bmatrix}\begin{bmatrix}-\cos\alpha_i & \sin\alpha_i\\ -\sin\alpha_i & -\cos\alpha_i\end{bmatrix}\begin{bmatrix}\dot\rho_i\\ \rho_i\dot\phi_i\end{bmatrix}. \tag{20}$$

In order to achieve the balanced circular formation, we just need to design appropriate dynamics for both ρ_i and ϕ_i. Here we choose the dynamics of ρ_i as

$$\dot\rho_i = k_\rho \mathrm{sgn}(\beta_i - \beta^*)\left|(\beta^* - \beta_i)/\pi\right|^a, \tag{21}$$

where $k_\rho > 0$ is the control gain, $\beta^* \in (0, \pi)$ is the desired subtended angle, and $a \in (0, 1)$. To design the dynamics of ϕ_i, we define $\gamma_j^{\mathrm{i}} := \arctan2(\zeta_j^{\mathrm{i}}, \xi_j^{\mathrm{i}})$, where

$$\begin{bmatrix}\xi_j^{\mathrm{i}}\\ \zeta_j^{\mathrm{i}}\end{bmatrix} = \begin{bmatrix}\cos(\phi_i + \pi/2) & \sin(\phi_i + \pi/2)\\ -\sin(\phi_i + \pi/2) & \cos(\phi_i + \pi/2)\end{bmatrix}\begin{bmatrix}\xi_j - \xi_i\\ \zeta_j - \zeta_i\end{bmatrix}.$$

Further, we define

$$\varsigma_+^{\mathrm{i}} = \begin{cases}\min\limits_{j\in N_+^{\mathrm{i}}}(\gamma_j^{\mathrm{i}}), & \text{if } N_+^{\mathrm{i}} \neq \emptyset,\\ \pi/2, & \text{if } N_+^{\mathrm{i}} = \emptyset,\end{cases}$$

and

$$\varsigma_-^{\mathrm{i}} = \begin{cases}\pi - \max\limits_{j\in N_-^{\mathrm{i}}}(\gamma_j^{\mathrm{i}}), & \text{if } N_-^{\mathrm{i}} \neq \emptyset,\\ \pi/2, & \text{if } N_-^{\mathrm{i}} = \emptyset,\end{cases}$$

where $N_+^{\mathrm{i}} = \{j \in \mathcal{N}_i | \gamma_j^{\mathrm{i}} \in (0, \pi/2)\}$ and $N_-^{\mathrm{i}} = \{j \in \mathcal{N}_i | \gamma_j^{\mathrm{i}} \in (\pi/2, \pi)\}$. The dynamics of ϕ_i is then designed to be

$$\dot\phi_i = \frac{k_\phi}{\rho_i} f(\varsigma_+^{\mathrm{i}}/\varsigma_-^{\mathrm{i}}), \tag{22}$$

where $f(\cdot)$ is defined in (18). Notice that although ρ_i appears in the dynamics of ϕ_i in (22), it is not needed for the control inputs (20).

Theorem 6. *Choose k_ρ and k_ϕ such that $\sqrt{k_\rho^2 + k_\phi^2} \leq \min(v_{\max}, \omega_{\max}|L|)$. Then under the controller 5 with (21) and (22), a group of n mobile robots described by (14) achieve a balanced circular formation around the target almost globally provided that $r_s \geq 2r_b \csc(\beta^*/2)\sin(\pi/n)$.*

Proof. Firstly, because $|\dot\rho_i| \leq k_\rho$ and $|\rho_i\dot\phi_i| \leq k_\phi$, it can be proved from (19) that if $\sqrt{k_\rho^2 + k_\phi^2} \leq \min(v_{\max}, \omega_{\max}|L|)$, then $|v_i| \leq \sqrt{k_\rho^2 + k_\phi^2} \leq v_{\max}$ and $|\omega_i| \leq \sqrt{k_\rho^2 + k_\phi^2}/|L| \leq \omega_{\max}$.

Secondly, it can be proved that under the control scheme (21) the point $\{\rho_i = r_b \csc\frac{\beta^*}{2}, \forall i\}$ is finite-time stable (Bhat & Bernstein, 1997), and thus all $\rho_i = r_b \csc\frac{\beta^*}{2}$ after some instant t_0.

Finally, when $t \geq t_0$, we define $\overline{M} = \{j | \delta\phi_j = \max \delta\phi_i\}$ and $\underline{M} = \{j | \delta\phi_j = \min \delta\phi_i\}$.

For $j \in \overline{M}$, we have $\varsigma_+^{\mathrm{j}+1} \leq \varsigma_-^{\mathrm{j}+1} = \varsigma_+^{\mathrm{j}} \geq \varsigma_-^{\mathrm{j}}$, therefore

$$\frac{d}{dt}\frac{\sum\limits_{j\in\overline{M}}\delta\phi_j}{|\overline{M}|} = \frac{1}{|\overline{M}|}\sum_{j\in\overline{M}}\left[f(\varsigma_+^{\mathrm{j}+1}/\varsigma_-^{\mathrm{j}+1}) - f(\varsigma_+^{\mathrm{j}}/\varsigma_-^{\mathrm{j}})\right] \leq 0.$$

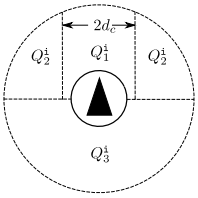


Fig. 4. The partition of Q_1^i, Q_2^i, and Q_3^i when $v_i > 0$.

Similarly, for $j \in \underline{M}$, we have $\varsigma_+^{j+1} \geq \varsigma_-^{j+1} = \varsigma_+^j \leq \varsigma_-^j$. Moreover, when $\underline{M} \neq \overline{M}$, the condition $r_s \geq 2r_b \csc(\beta^*/2)\sin(\pi/n)$ guarantees that there exists at least one $j \in \underline{M}$ with $\varsigma_+^j < \varsigma_-^j$, so

$$\frac{d}{dt}\frac{\sum_{j\in\underline{M}} \delta\phi_i}{|\underline{M}|} = \frac{1}{|\underline{M}|}\sum_{j\in\underline{M}}\left[f(\varsigma_+^{j+1}/\varsigma_-^{j+1}) - f(\varsigma_+^j/\varsigma_-^j)\right] = \frac{1}{|\underline{M}|}\sum_{j\in\underline{M}}\left[f(\varsigma_+^{j+1}/\varsigma_+^j) - f(\varsigma_+^j/\varsigma_-^j)\right] > 0.$$

As a result, $\delta\phi_i \to 2\pi/n$ for all i as $t \to \infty$.

The circular formation is "almost globally" stable because there is a measure-zero set characterized by those configurations that two or more robots have same ϕ_i. However, these equilibria are unstable. □

4. Experiments

An experimental platform that contains a team of e-puck robot is developed in our laboratory for demonstrating the effectiveness of the proposed control schemes. Because the sensing capability of the e-puck robot is very limited, in the current implementation, bearing measurements are provided by a central computer through a calibrated overhead camera. Color triangular marks are placed on the top of each robot. These marks are identified by the camera for tracking robot pose. However, only relative bearing information are utilized in order to mimic local and distributed implementation of the proposed control schemes. Although this is not a real distributed implementation, it still provides us a platform to test the performance of the proposed controllers under many practical issues, such as unmodeled dynamics and errors, noise, delays caused by sensing and information processing, and also the switching of controllers to avoid collision with others robots and boundaries which is not accounted in the theoretical analysis.

4.1. Collision avoidance

In the theoretical analysis, the robots are treated as moving points. However, a real robot is never a point, and therefore collision avoidance cannot be negligible when several robots work together in a region. Collision avoidance itself, especially for nonholonomic mobile robots, is a challenging problem (see Kan, Dani, Shea, & Dixon, 2012, Mastellone, Stipanović, Graunke, Intlekofer, & Spong, 2008 and references therein). Designing a totally distributed and mathematically provable bearing-only collision avoidance algorithm remains an unsolved problem and we are still working on it. In our experiments, a behavior-based algorithm is adopted, which is straightforward and computationally effective.

It should be pointed out that the current collision avoidance scheme uses not only bearing information. We need to assume that within a certain sensing zone $Q^i := \{j | \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2} \leq d_s,\ j \neq i\}$, a robot can detect both the bearing and range information about other robots (see Fig. 4). This can be done, for example, by using short-range infra-red proximity sensors mounted on the body of the mobile robot. The forward speeds v_i are multiplied by a variable m_i and become $\tilde{v}_i = m_i v_i$, where

$$m_i = \begin{cases} 1, & \text{if } Q^i = \emptyset, \\ 0, & \text{if } Q_1^i \neq \emptyset, \\ 0, & \text{if } Q_2^i \neq \emptyset \text{ and } |v_i| < |v_j|,\ \exists j \in Q_2^i, \\ 2, & \text{if } Q_1^i = \emptyset, Q_2^i \neq \emptyset \text{ and } |v_i| > |v_j|,\ \forall j \in Q_2^i, \\ 2, & \text{if } Q_1^i = \emptyset, Q_2^i = \emptyset, \text{ and } Q_3^i \neq \emptyset. \end{cases}$$

The partitions Q_1^i, Q_2^i, and Q_3^i are formally defined as

$$Q_1^i := \left\{j | x_j^i \cdot v_i \geq 0 \wedge |y_j^i| \leq d_c\right\} \cap Q^i,$$
$$Q_2^i := \left\{j | x_j^i \cdot v_i \geq 0 \wedge |y_j^i| > d_c\right\} \cap Q^i,$$
$$Q_3^i := \left\{j | x_j^i \cdot v_i < 0\right\} \cap Q^i.$$

Notice that $\tilde{v}_i$ may not be achieved due to saturation of the actuator. This collision avoidance algorithm was found to work well in the experiment. One of the reasons is that the proposed controllers themselves tend to separate the robots from each other.

Since the working area is bounded, the robots should also avoid collision with the boundaries and keep inside the working area. To do that, on detecting the boundaries, a robot stops moving forward and rotates at the maximal angular speed for a short time span until its forward direction (the direction of v_i) points into the bounded area again.

4.2. Experimental results

Due to space limitation, only the experimental results for the multirobot case are shown here. We performed tests of enclosing a target with five e-puck robots. The parameters used in the experiments are given in Table 1.

Table 1
Parameters used in experiments.

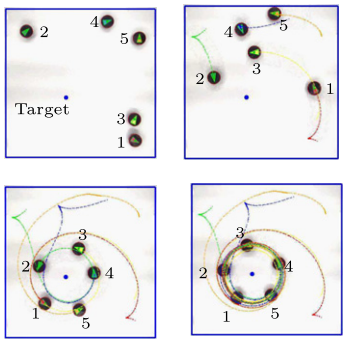
$v_{\max}$	$\omega_{\max}$	ρ^*	r_s	d_s	d_c
6 cm/s	2.43 rad/s	24 cm	40 cm	15 cm	9 cm

We firstly applied the controller 4. Fig. 5(a) illustrates top-view snapshots from the actual trial, which are captured by the overhead camera. We set the coordination function to be $f(\sigma_+^i/\sigma_-^i) = 1 - \exp(-\sigma_+^i/\sigma_-^i)$. To test the performance of controller 4, the robots were placed at different locations with different orientations initially within the working area (see Fig. 5(a)). Figs. 5(b) and (c) illustrate the changes of $\rho_i(t) - \rho^*$ and $\sin[\phi_i + 2(i-1)\pi/n]$, respectively. One can observe that the five robots converged to the desired circle and achieved even distribution over the circle as expected. It is also pointed out here that the collision avoidance scheme does not affect the locally asymptotical stability result in Theorem 5, because at the equilibrium point all robots are in the same operational state, i.e., $m_i = m_j$ for any i and j, where $i \neq j$.

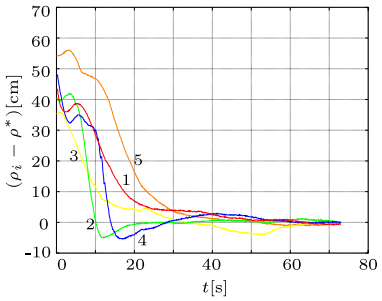
We then tested the performance of controller 5. We set $L = 3$ cm, $\beta^* = 0.8$ rad, $a = 1/2$ and let the function $f(\varsigma_+^i/\varsigma_-^i)$ in (22) be $1 - \exp(-\varsigma_+^i/\varsigma_-^i)$. To meet the requirement $\sqrt{k_\rho^2 + k_\phi^2} \leq \min(v_{\max}, \omega_{\max}|L|)$, k_ρ and k_ϕ should be small. However, small k_ρ and k_ϕ will lead to slow convergence. In the actual experiment, we set both k_ρ and k_ϕ to be 10. Satisfying results were obtained as shown in Figs. 6(a)–(c).

5. Conclusions

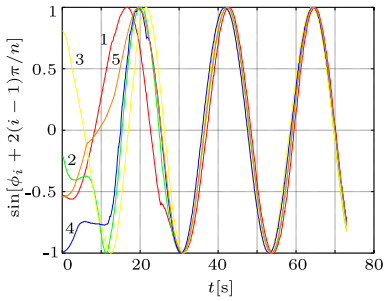
In this paper, the problem of enclosing a target by a single robot and multiple cooperative robots is investigated. We develop novel control schemes based on the local bearing measurements only, which are commonly available by using monocular vision systems. No communication is needed amongst the robots when using the proposed control schemes. We develop controller for single robot circumnavigation first, and then extend them to deal with the multirobot case. Using Lyapunov stability criteria, it has



(a) Trajectories of the robots with $t = 0$ s, 10 s, 30 s, 80 s.

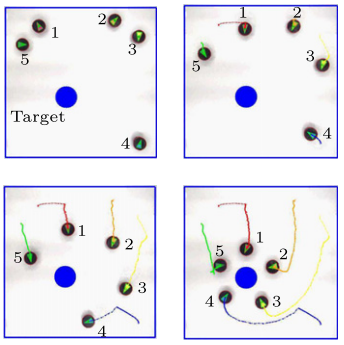


(b) Evolution of $\rho_i(t) - \rho^*$.

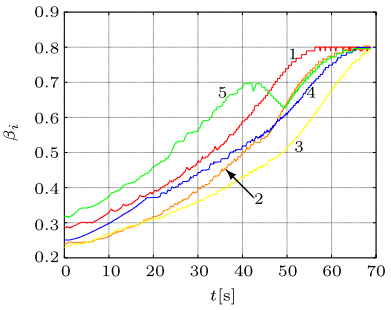


(c) Evolution of $\sin[\phi_i + 2(i-1)\pi/n]$, which indicates the robots achieve even distribution around the target.

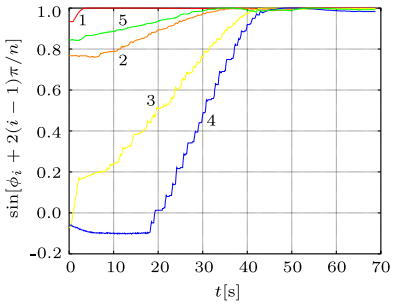
Fig. 5. Experimental result of controller 4.



(a) Trajectories of the robot with $t = 0$ s, 10 s, 30 s, 70 s.



(b) Evolution of $\beta_i(t)$.



(c) Evolution of $\sin[\phi_i + 2(i-1)\pi/n]$, which indicates the robots achieve even distribution around the target.

Fig. 6. Experimental result of controller 5.

been shown that the proposed control schemes can guarantee asymptotic convergence to the desired stand-off orbit despite the lack of direct metric information. In addition, a static circular formation control based on feedback linearization is studied. The experiment on a group of two-wheeled differential drive robots supports the effectiveness of the proposed control design.

As part of our future work, we plan to implement the proposed control schemes in more dynamic environments and multi-target scenarios. In this paper, the robots use the measured subtended angle to avoid collision with disk targets. As a future research topic, we will consider inter-robot collision avoidance by using the subtended angle measurement without range measurement. This approach allows us to treat target enclosing and collision avoidance at the same time, which is very useful when the dynamics of the mobile robots is considered and the smoothness of the controllers becomes important. We also intend to implement the proposed control schemes on mobile robots which have the capability to directly measure the relative bearing information, such as those used in Chen, Sun, Yang, and Chen (2009), Li, Sun, and Yang (2013) and Sun, Wang, Shang, and Feng (2009).

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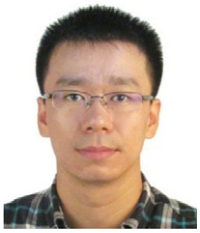

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