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Formation Control and Obstacle Avoidance for Multi-agent Systems Using Barrier Lyapunov Functions

Jie Lian and Yang Meng
School of Control Science and Engineering
Dalian University of Technology
Dalian 116024, P. R. China
Email: jielian@dlut.edu.cn
my2015@mail.dlut.edu.cn

Li-li Li
College of Marine Electrical Engineering
Dalian Maritime University
Dalian 116024, P. R. China
School of Mathematics
Liaoning Normal University
Dalian 116029, P. R. China
Email: tolilili@gmail.com

***Abstract*—This paper focuses on formation control of multi-agent systems with obstacle avoidance. A control protocol with bounded constraints is proposed to achieve the prescribed formation, where an exponential decay term is introduced. The convergence of the proposed algorithm is presented based on the barrier Lyapunov function method. In addition, the proposed control protocol can keep agents remaining in communication area with neighbour agents while avoiding obstacles settled in their moving area. Simulations are provided to illustrate the effectiveness of the proposed control law.**

***Keywords*—multi-agent systems; Barrier Lyapunov function; formation control**

I. Introduction

Interest in research in multi-agent systems is increasing in recent years, motivated by its practical applications such as exploration and emergency rescue. Some basic concepts and conditions for stability of multi-agent systems are presented in [1], [2]. Multi-agent systems under some reality conditions such as parametric uncertainties, unknown Lipschitz terms are discussed in [3], [4], [5], [6], [7], [8].

A significant problem arising from multi-agent systems is the formation control problem. It's am basical problem in cooperative control. In [9], the authors design a feedback controller for agents with limited number of communication channels. A formation problem of multi-agent systems modeled by Lagrangian dynamics using asymptotic decoupling is discussed in [10]. Two kinds of formation control cases with fixed topology and switching topology are investigate in [11]. Based on adaptive fuzzy control method, the study achieves both formation forming and target tracking in [12]. In real workspace for agents, there are many obstacles to be avoided. Then obstacle avoidance is an inevitable problem for agents working in the limited workspace with a specific formation. In [13], [14], [15], [16], authors discuss the obstacle avoidance problem in formation control. Control laws based on the segment function is presented in [13]. A formation control protocol based on graph theoretic considerations is proposed in [14]. Using the Pontryagins maximum principle, formation problem and optimization problem are researched in [15]. In [16], the author discusses formation problem in time domain.

Coordination and control protocol based on information sharing and barrier function is naturally presented, due to the communication limitation of agents as well as other reasons like external unexpected disturbance and unknown obstacles. Much research work is to employ the barrier Lyapunov function to ensure constraints conditions. A barrier function $f(x)$ is a continuous function whose value tends to be infinity as x approaches the boundary of the feasible region. In the multi-agent systems area, the method mainly has two kinds of applications. One is to construct constraints conditions. By encoding different physical constraints into mathematical expression, agents can move in limited area and complete motions like avoidance, tracking, stopping. In [17], the barrier Lyapunov function is applied into building control laws to achieve formation and flocking behavior. In [18], the barrier Lyapunov function is used to make the system remain in the healthy region and finally force agents to the desired points. A control protocol using the barrier function method is constructed to ensure collision avoidance in [19]. The other is to build a Lyapunov function and combine it with the backstepping method. It is used to ensure state constraints conditions for high-order multi-agent systems. For example, in [20], the barrier Lyapunov function is introduced into the first step of the backstepping method to make the output of agents constrained. In [21], an integral barrier Lyapunov functions is added in each step of the backstepping method to get all the states constrained. A situation that the barrier function is used in double integrator dynamic model of multi-agent systems is discussed in [22].

The main contributions of this paper are twofold: (1) a control law based on the barrier Lyapunov function is proposed to solve formation control problems while forcing agents to avoid obstacles settled in the workspace; (2) the proposed control law allows agents to remain in the communication area with their neighbour agent, considering the communication performance and the perception distance of agents.

The structure of this paper is as follows: Section II gives some background knowledge and the dynamics description of the multi-agent system. The control objective of multi-agent system is also given in II. Section III shows the construction steps of the barrier Lyapunov function. Section IV gives the control laws containing barrier Lyapunov function and ensures its convergence. Section V shows the effectiveness of control protocol by simulations. Section VI shows conclusion of the paper.

II. Problem Formulation

A. Graph Theory

Here let's recall some basic knowledge of graph theories which will be useful for research on the paper. $G=(V,E)$ is used to describe a directed graph. $V=\{v_1,v_2,...,v_N\}$ is a nonempty set which represents all the nodes. The set of edges is $E\subseteq V\times V$, if $(v_i,v_j)\in E$. It denotes that there is an edge from node i to node j. In other words, node j can transmit its information to node i, but it is not vice versa for a directed graph. $N_i=\{v_j\in V:e_{ij}\in E$ represents the set of adjacency nodes of node i. $A=[a_{ij}]\in R\times R$ is used to represent the topology of undirected graph G, $a_{ij}=1$ if $(v_i,v_j)\in E$, otherwise $a_{ij}=0$. A directed path is a list S of nodes such that $\{S(i),i\in\{1,...r\}:e_{S(i),S(i+1)}\in E\}$. A directed graph is strongly connected if there is a directed path for any two distinct nodes j and i. A directed graph has a directed spanning tree if there exists at least one node called root node which has a directed path to all the other nodes. A graph is said to be a subgraph of the graph G if its nodes set and edges set are the subsets of nodes set and edges set of the graph G.

B. System Statement

Definition 1 [23]: A Barrier Lyapunov Function $V(x)$ is a scalar function, defined with respect to the system $\dot{x}=f(x)$ on an open region $\mathscr{D}$ containing the origin. The scalar function $V(x)$ is continuous, positive definite. It has continuous first-order partial derivatives at every point of $\mathscr{D}$. It has the property $V(x)\to\infty$ as x approaches the boundary of $\mathscr{D}$, satisfies $V(x)\leq c$, $\forall t\geq 0$ along the solution $\dot{x}=f(x)$ for $x(0)\in\mathscr{D}$ and some positive constant c, which has relation to the initial value of x. c is introduced to describe the upper bound of this function.

Consider a limited work space ℓ containing $N(N>1)$ agents and $M(M>0)$ obstacles. Assume that agent $N=1$ as the leader and other agents $j=2,...N$ are the followers. Dynamic model of agent j is

$$\dot{z}_j=\begin{bmatrix}\cos\alpha_j & \sin\alpha_j & 0\end{bmatrix}^T u_j+\begin{bmatrix}0 & 0 & 1\end{bmatrix}^T\omega_j \tag{1}$$

where $z_j=\begin{bmatrix}x_j & y_j & \alpha_j\end{bmatrix}^T$ denotes the configuration vector of agent j, x_j and y_j are coordinate value in X axis and Y axis respectively. $\alpha_j\in[0,\ 2\pi)$ is the orientation of the agent j. u_j and w_j are control input. They respectively represent the linear and angle velocity of agent j. Define $r_j=\begin{bmatrix}x_j & y_j\end{bmatrix}^T$ as the position vector of agent j. Each agent sends the location information to other agents within a maximum communication distance because of the limitation of wireless communication. Assume that the communication area between agents is a circular area. The maximum connective radius is represented as R_0. There is an avoidance radius R_1 for each agent similarly. Assume that the avoidance radius of each obstacles is R_2. $R_3>R_1+R_2$ represents minimum radius for agents avoiding obstacles.

C. Control Objective

Control objectives for agents we focus on are as follows:

- Connectivity
  Each follower moves in the connectivity region of its neighbour agents. In other words, $\|r_i-r_j\|<R_0$ always holds where i represents the neighbour agent of follower j.
- Obstacle avoidance
  It is a practical problem that agents face more complicated situations in real working space. To simplify the problem, suppose that obstacles are stationary. There exists a minimal avoidance distance R_3. Thus the condition $R_3\geq R_1+R_2$ can ensure agents move in safety and each agent satisfies $\|r_j-r_l\|>R_3$ where r_l represents the obstacle's position vector and $l=N+1,...N+M$.
- Formation
  Each follower does not only converge to connectivity area, but also forms a specific formation finally. The condition $\|r_i-r_j-c_{ij}\|=0$ is used to force agents to form a desired formation. c_{ij} is a vector to be designed to the distance between agents in the formation.

Assumption 1: The leader's initial position is set out of the collision region.

Assumption 1 means that the condition $\|r_1-r_l\|>R_3$ always holds, where r_l represents the obstacle's position vector. Followers' final target points are also set out of the collision region, or say, $\|r_{jd}-r_l\|>R_3$ always holds where $r_{jd}=\begin{bmatrix}x_{jd} & y_{jd}\end{bmatrix}$ represents the goal position vector of follower j.

Assumption 2: The graph G is connected and the graph G contains a directed spanning tree where the leader is the root node of the graph G.

III. Barrier Lyapunov Function Construction

In this section, a barrier Lyapunov function containing connectivity and obstacle avoidance for each follower is constructed.

A. Barrier Lyapunov Function Construction for Connectivity

In order to ensure connectivity maintenance of communication between followers and their neighbour agent and ensure formation control objective, the following inequality is constructed

$$C_j(r_j,r_i)={R_0}^2-(x_j-x_i)^2-(y_j-y_i)^2>0 \tag{2}$$

For all $j \in [2,3,...N]$, connectivity is maintained as long as conditions hold. An inverse function B_j is defined to encode the connectivity based on the barrier Lyapunov function

$$B_j(r_j,r_i) = 1/C_j(r_j,r_i)$$

where B_j tends to be ∞ when C_j approaches zero. A method called recentered barrier function is given in [24]. This method is used to reprocess above function

$$R_j = B_j(r_j,r_i) - B_j(r_{jd},r_i) - \nabla B_j(r_{jd},r_i)^T \Delta r_j \tag{3}$$

where $\Delta r_j = r_j - r_{jd}$, $\nabla B_j = \left[\begin{array}{cc} \frac{\partial B_j}{\partial x_j} & \frac{\partial B_j}{\partial y_j} \end{array}\right]^T$. The constructed function can be non-zero except reaching the goal position in the connective area. Similarly, the function also tends to be ∞ at the boundary of the constrained region ℓ. A nonnegative function is defined

$$V_{j1} = R_j^2. \tag{4}$$

So the barrier function for connectivity is constructed. It can ensure that agents remain in communication with neighbour agents.

B. Barrier Lyapunov Function Construction For Collision Avoidance

For each follower, there is a minimum avoidance distance R_3. Thus an inequality is obtained

$$C_{jl}(r_j,r_l) = (x_j - x_l)^2 - (y_j - y_l)^2 - {R_3}^2 > 0. \tag{5}$$

As long as the inequality (5) holds, agents can avoid collisions with obstacles. Such function construction is similar to that with connectivity maintenance.

Firstly, an inverse function is constructed as

$$B_{jl}(r_j,r_l) = 1/C_{jl}(r_j,r_l).$$

Based on the recentered barrier method, the following equation is gotten

$$R_{jl} = B_{jl}(r_j,r_l) - B_{jl}(r_{jd},r_l) - \nabla B_{jl}(r_{jd},r_l)\Delta r_j.$$

Lastly, a nonnegative function is constructed as

$$V_{j2} = \sum_{l=2,l\neq j}^{N+M} R_{jl}^2. \tag{6}$$

The barrier function V_{j2} is built for agents to avoid the obstacles.

C. Barrier Lyapunov Function For Each Agent

Define

$$V_j = V_{j1} + \varepsilon V_{j2} \tag{7}$$

where ε is a positive constant to be designed. ε is used to adjust the effect of V_{j2}. V_j combines V_{j1} with V_{j2}. It can make agents remain in communication distance while avoiding obstacles in the process of the formation forming.

IV. Main Results

Based on the construction of the barrier Lyapunov function, the following control laws are gotten

$$u_j = k_{j1}\mathrm{X}, \tag{8}$$

$$w_j = k_{j3}(\alpha_j - \beta_j) + \dot{\beta}_j \tag{9}$$

where $\mathrm{X} = 1 - e^{k_{j2}\|r_j - r_{jd}\|}$, $k_{j1} > 0$, $k_{j2} < 0$, $k_{j3} < 0$, $\beta_j = \mathrm{atan}(-\frac{\partial V_j}{\partial y_j}, -\frac{\partial V_j}{\partial x_j})$.

Remark 1: In control laws (8) and (9), u_j is used to control the speed of follower j, w_j is used to control the angle of follower j. k_{j1} and k_{j2} ensure that u_j is always a positive constant to let the followers keep moving. When the followers move toward to the leader, the speed value becomes small, finally becomes zero when agents reach the goal position. When the followers move far away from the leader, the value becomes large. Under the effect of angle control, the follower can finally move towards the leader without breaking the constraints.

Remark 2: Compared with [25], k_{j2} in (8) is added to control the speed of agents forming a desired formation. By selecting a sufficiently large k_{j2}, we can get a faster convergence speed. (8) is designed in an exponential term. Due to the characteristic of the control law. There exists a boundary value. When $\|r_j - r_{jd}\|$ goes larger than it, the value of X can be regarded as 1. When $\|r_j - r_{jd}\|$ goes smaller than the boundary value, X starts to converge to 0. By setting a sufficiently large k_{j2}, a small boundary value is gotten.

Theorem 1: Consider a multi-agent system (1) satisfying Assumptions 1 and 2. Under the control laws (8) and (9), each follower $j \in [2,3,...N]$ finally converges to its dynamic target point and forms a formation while remaining in communication area and avoiding obstacles in the region ℓ.

Proof: It is notable that there are two control subsystems according to model (1): position trajectories subsystem and orientation subsystem. In order to construct the control laws and simplify the stability analysis, assume that the orientation trajectories are controlled by a faster speed when compared to the position trajectories. The orientation trajectories are quickly controlled, then the fast dynamics (9) is rewritten as

$$\dot{\alpha}_j = k_{j3}(\alpha_j - \beta_j) + \dot{\beta}_j. \tag{10}$$

Define $\eta_j = \alpha_j - \beta_j$. The following equation is obtained

$$\frac{d\eta_j}{dt} = \frac{d}{dt}(\alpha_j - \beta_j) = k_{j3}(\alpha_j - \beta_j) = k_{j3}\eta_j.$$

The deduction indicates that η_j is thus globally exponentially stable which further means α_j is global exponentially stable to β_j. As we know $\beta_j = \mathrm{atan}(-\frac{\partial V_j}{\partial y_j}, -\frac{\partial V_j}{\partial x_j})$, based on the control law (9), the angle control law can force followers to move forward the safe area. Under the control law (8), u_j is a nonnegative number and u_j is bounded in $(0, k_{j1}]$. It ensures that the follower always moves to follow the direction of the safe gradient vector dictated by the barrier Lyapunov function.

In the next section, we focus on the position trajectories subsystem. Firstly, consider the follower k which communicates with the leader directly. Choosing the candidate Lyapunov function V_k, we can get its derivative:

$$\dot{V}_k = u_k \zeta_k^T \begin{bmatrix} \cos\alpha_k \\ \sin\alpha_k \end{bmatrix} \tag{11}$$

where

$$\zeta_k = \begin{bmatrix} \frac{\partial(V_{k1}+\varepsilon V_{k2})}{\partial x_k} & \frac{\partial(V_{k1}+\varepsilon V_{k2})}{\partial y_k} \end{bmatrix}^T.$$

Notice that obstacles and the leader are stationary. Their derivatives are all zero. α_k is global exponentially stable to β_k. (11) is rewritten as

$$\lim_{t\to\infty} \dot{V}_k = u_k \zeta_k^T \begin{bmatrix} \cos\beta_k \\ \sin\beta_k \end{bmatrix}. \tag{12}$$

According to the control law on angle control (9), one has

$$-\frac{\partial V_{k1}}{\partial x_k} - \varepsilon\frac{\partial V_{k2}}{\partial x_k} = \|\zeta_k\|\cos\beta_k,$$
$$-\frac{\partial V_{k1}}{\partial y_k} - \varepsilon\frac{\partial V_{k2}}{\partial y_k} = \|\zeta_k\|\sin\beta_k.$$

(12) can be rewritten as

$$\lim_{t\to\infty} \dot{V}_k = -u_k \|\zeta_k\|.$$

Obviously $\dot{V}_k \le 0$. We can conclude that follower k can finally converge to the target point.

Consider followers which can not get leader's information directly. The neighbour agent should be considered into the derivative of $V_j (j \neq k)$

$$\dot{V}_j = u_j \zeta_j^T \begin{bmatrix} \cos\alpha_j \\ \sin\alpha_j \end{bmatrix} + u_i \zeta_{ji}^T \begin{bmatrix} \cos\alpha_i \\ \sin\alpha_i \end{bmatrix} \tag{13}$$

where

$$\zeta_j = \begin{bmatrix} \frac{\partial(V_{j1}+\varepsilon V_{j2})}{\partial x_j} & \frac{\partial(V_{j1}+\varepsilon V_{j2})}{\partial y_j} \end{bmatrix}^T,$$
$$\zeta_{ji} = \begin{bmatrix} \frac{\partial(V_{j1}+\varepsilon V_{j2})}{\partial x_i} & \frac{\partial(V_{j1}+\varepsilon V_{j2})}{\partial y_i} \end{bmatrix}^T.$$

Due to the characteristic of α_j, (13) is rewritten as

$$\lim_{t\to\infty} \dot{V}_j = u_j \zeta_j^T \begin{bmatrix} \cos\beta_j \\ \sin\beta_j \end{bmatrix} + u_i \zeta_{ji}^T \begin{bmatrix} \cos\beta_i \\ \sin\beta_i \end{bmatrix}.$$

Using similar method the following equation is obtained

$$\lim_{t\to\infty} \dot{V}_j = -u_j \|\zeta_j\| + u_i \zeta_{ji}^T \begin{bmatrix} \cos\beta_i \\ \sin\beta_i \end{bmatrix}. \tag{14}$$

According to vector inequality property, the following inequality is gotten

$$\zeta_{ji}^T \begin{bmatrix} \cos\beta_i, \\ \sin\beta_i \end{bmatrix} \le \|\zeta_{ji}^T\|.$$

The following result is obtained

$$\lim_{t\to\infty} \dot{V}_j \le -u_j \|\zeta_j\| + u_i \|\zeta_{ji}\|. \tag{15}$$

Consider the situation that the follower is out of collision region which means V_{j2} is small enough to be ignored. In this situation, $\|\zeta_j\| = \|\zeta_{ji}\|$, which leads to

$$\lim_{t\to\infty} \dot{V}_j \le -(u_j - k_{i1}) \|\zeta_j\|. \tag{16}$$

When $\|r_j - r_{jd}\|$ is larger than boundary value b, the inequality (16) becomes

$$\lim_{t\to\infty} \dot{V}_j \le -(k_{j1} - k_{i1}) \|\zeta_j\|.$$

When k_{j1} satisfies that $k_{j1} > k_{i1}$, followers can converge to a circular region to the target point.

Remark 3: When $\|r_j - r_{jd}\|$ is smaller than the boundary value, which means that x_j is quite close to x_{jd}, X becomes smaller than 1. Then the invariant set become large, $\|r_j - r_{jd}\|$ may go large because of status unknown in invariant set, then X becomes 1 again when $\|r_j - r_{jd}\|$ becomes larger than boundary value.

According to the proof above, followers communicating with the leader directly can converge to desired points. Then their neighbour agent will also converge to the target points as they remain in the connective area.

Under the condition that V_{j2} is considered, it's obvious that

$$\lim_{t\to\infty} \dot{V}_j \le -u_j \|\zeta_j\| + u_i \|\zeta_{ji}\| = -u_j \|\zeta_{j\alpha} + \zeta_{j\beta}\| + u_i \|\zeta_{ji}\|$$

where

$$\|\zeta_{ji}\| = \|\zeta_{j\alpha}\| = \begin{bmatrix} \frac{\partial V_{j1}}{\partial x} & \frac{\partial V_{j1}}{\partial y} \end{bmatrix}^T,$$
$$\|\zeta_{j\beta}\| = \begin{bmatrix} \frac{\partial V_{j2}}{\partial x} & \frac{\partial V_{j2}}{\partial y} \end{bmatrix}^T.$$

We can always find a constant λ_j and it satisfies that

$$k_{j1} \ge \lambda_j \ge \frac{\|\zeta_{j\alpha}\|}{\|\zeta_{j\alpha} + \zeta_{j\beta}\|} k_{i1}.$$

where $\|\zeta_{j\alpha}\|, \|\zeta_{j\beta}\|$ are bounded. Then the following condition is obtained

$$u_j \ge \frac{\|\zeta_{j\alpha}\|}{\|\zeta_{j\alpha} + \zeta_{j\beta}\|} k_{i1}$$

According to the proof above, followers can avoid obstacles and converge to target point to form a specific formation.

V. Illustrative Examples

The effectiveness of control laws is demonstrated though several simulations.

A. Formation Control with One Static Leader

Consider a multi-agent system with one leader and three followers. The topological graph is shown in Fig. 1. Connective radius is set as $R_0 = 8$. The leader is stationary. Followers need to move to their target points. All followers can get the position information only from the neighbour agent. The leader's initial position is set as follows

$$x_1 = 0, \ y_1 = 0.$$

The c_{ij} are set as $c_{21} = [0, \ -1], c_{32} = [1, \ 0], c_{42} = [-1, \ 0]$.

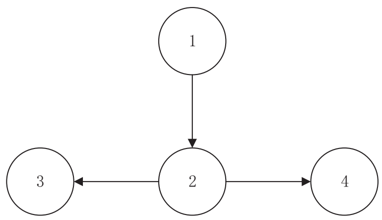

Fig. 1. Topological graph with one static leader

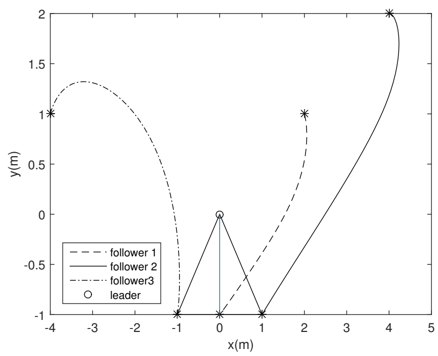

Fig. 2. Formation with one static leader

Then the desired position for agents are

$$x_2 = 0,\ y_2 = -1,$$
$$x_3 = 1,\ y_3 = -1,$$
$$x_4 = -1,\ y_4 = -1.$$

Fig. 2 indicates the effectiveness of the control laws. When $t = 10s$, the agents' positions are

$$x_2 = 0,\ y_2 = -1,$$
$$x_3 = 1,\ y_3 = -0.99,$$
$$x_4 = -0.99,\ y_4 = -0.99.$$

As is shown in Fig. 2, followers reach the goal positions. Followers and the leader can finally form a triangle formation.

Notice that there is an error between the final position and the desired position. That's because a control law $u_j = k_{j1}\mathrm{X}$ is used instead of tradition methods like a *tanh* function. Using X can achieve a faster convergence speed than tradition *tanh* function by setting k_{j2} appropriately large. But it decreases steeply when approaching zero, too. If the agent approaches the desired point really close, the control value tends to be really small. That's the reason of the error between the final position and the desired position.

In Fig. 3, an obstacle is set in $[0.5,\ 0]$ in workspace to show the effectiveness of the control law in the obstacle avoidance.

Connective radius R_0 and avoidance radius R_3 are set as $R_0 = 8,\ R_3 = 0.5$.

The followers' initial positions are set as follows

$$x_2 = 1.2,\ y_2 = 1,$$
$$x_3 = 1.5,\ y_3 = 0,$$
$$x_4 = -1,\ y_4 = 0.$$

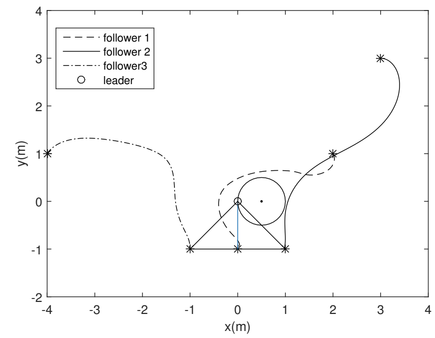

Fig. 3. Collision avoidance with one static leader

As is shown in Fig. 3, followers 1 and 2 can avoid the circular area shown in Fig. 3. When $t = 10s$

$$x_1 = 0,\ y_1 = -1,$$
$$x_2 = -0.98,\ y_2 = -0.99,$$
$$x_3 = 0.99,\ y_3 = -1.$$

After avoiding the obstacle, agents reach their goal position.

B. Formation with One Moving Leader

Consider a multi-agent system with one leader and two followers. The leader moves with the constant angel and speed. Followers need to move to their target points. All followers know the position of the obstacle and can get position information from the leader directly. One obstacle is set in $[-1, 6]$. When $t = 0$, the leader's initial positions are set as follows:

$$x_1 = 1,\ y_1 = 0,$$
$$\dot{x}_1 = 0.3,\ \dot{y}_1 = 0.4.$$

The followers' initial positions are set as follows

$$x_2 = -5,\ y_2 = -1,$$
$$x_3 = -3,\ y_3 = -2.$$

The followers' goal positions are set as follows

$$c_{21} = [-3, 3],$$
$$c_{31} = [-3, -3].$$

Connective radius R_0 and avoidance radius R_3 are set as

$$R_0 = 10,\ R_3 = 1.$$

Fig. 4 indicates the effectiveness of the control law. Followers can all converge their dynamic goal position. Agents constitute an isoceles triangle finally. Follower 2 can fast converge to its goal position. The path of follower 1 shows the effect of the control laws on obstacle avoidance. From the figure we can clearly see that follower 1 avoids the region

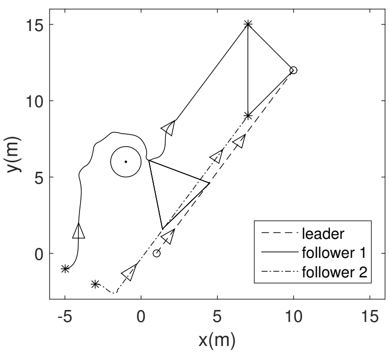

Fig. 4. Target tracking with one moving leader

circular region of center $[-1,\ 6]$ with radius r, then converges to goal position. When $t = 30s$, the agents' position are

$$\begin{aligned} x_1 &= 10.00,\ y_1 = 12.00, \\ x_2 &= \ 6.98,\ y_2 = 14.98, \\ x_3 &= \ 6.98,\ y_3 = \ 8.98, \end{aligned}$$

which means followers can converge to the target point approximately.

VI. Conclusion

The formation control problem in multi-agent systems has been investigated. The obstacle avoidance have been considered as well. The convergence of the proposed control protocol has been analyzed based on the barrier Lyapunov function method. Simulations have demonstrated that the control protocol can make agents remain in communication area avoiding collisions and obstacles to form a desired formation. Future work will focus on considering time-varying constraints conditions.

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