Chapter 2 (TD2)

The Modeling Process centering Proportionality, and Geometric Similarity

Exercise in Class

1). Show graphically the meaning of the proportionality $y \propto \dfrac{u}{v}$

2). The height of a tree is $h(m)$, is in direct proportion on with its age, y-years. A 10 year old tree is $16m$ tall.

a). Set up a formula to model this variation.

b).what is the height of a tree 14 years old?

3). The Temperature (T) of water cools in direct proportion to time in minutes (m). Over 6 minutes there is drop of $18°C$.

a). State the equation connecting T and M.

b). Calculate the change in temperature for 10 minutes.

4). At an hourly wage job, you work for 5 hours, and get paid \$83.75.

a). How much money will you earn if you work 7 hours? (Assume that you asre not making overtime pay, or any other sort of special pay rate.)

b). what is the constant of proportionality in this situation and what does it mean in context?

5). A road map has scale $1inch = 8miles$. You measure the distantce from home to the ski resort you plan to go visit as $11.75inches$ How many miles will you be traveling? What assumption are you making? 6). Determine whether the following data support a proportionality argument for $y \propto z^{1/2}$. If so, estimate the slope.

y	3.5	5	6	7	8
z	3	6	9	12	15

7). Determine whether the data set supports the stated proportionality model.

a). Force $\propto$ Stretch

Force	10	20	30	40	50	60	70	80	90
Stretch	19	57	94	134	173	216	256	297	343

b). $y \propto x^3$

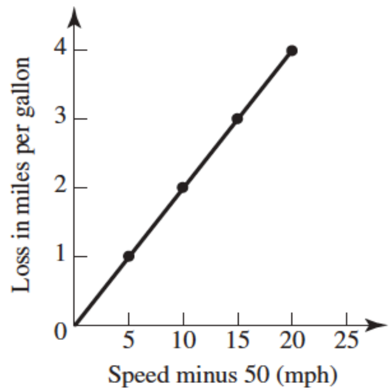
y	19	25	32	51	57	71	113	141	123	187	192	205	252	259	294
x	17	19	20	22	23	25	28	31	32	33	36	37	38	39	41

8). Consider a $20-lb$ pink flamingo that stands 3 ft in height and has legs that are 2 ft inlength. Model the height and leg length of a $100-lb$ flamingo. What assumptions are necessary? Are they reasonable assumptions?

9). An object is sliding down a ramp inclined at an angle of θ radians and attains a terminal velocity before reaching the bottom. Assume that the drag force caused by the air is proportional to Sv^2 , where $S$is the cross-sectional area perpendicular to the direction of motion and v is the speed. Further assume that the sliding friction between the object and the ramp is proportional to the normal weight of the object. Determine the relationship between the terminal velocity and the mass of the object. If two different boxes, weighing 600 and $800lb$, are pushed down the ramp, find the relationship between their terminal velocities.

10). In the automobile gasoline mileage example, support yoy plot miles per gallon againt speed as a graphical repersentaion of the general rule (insteasd of the graph depicted in **Figure 2.7**). explain why it would be difficult to deduce a proportionality relationship from that graoh.

Figure 2.27
For every 5 mph over 50 mph, there is a loss of 1 mile to the gallon.



Assignmet

Deadline: 08-November, 2025.

1). A randrop starts falling from a cloud at a considerable height above the surface of the earth. During the fall of the raindrop experiences retardation due to air resistance. The retardation is directly proportional to the instantaneous speed of the drop. Find an expression for distance traveled.

2). Consider an automobile suspension system. Build a model that relates the stretch (or compression) of the spring to the mass it supports. If possible, obtain a car spring and collect data by measuring the change in spring size to the mass supported by the spring. Graphically test your proportionality argument. If it is reasonable, find the constant of proportionality.

3). Consider the models $W \propto l^2G$ and $W \propto g^3$. Interpret each of these models geometrically. Explain how these two models differ from Models (2.11) and (2.13), respectively. In what circumstances, if any, would the four models coincide? Which model do you think would do the best job of predicting W ? Why? In Chapter 3 you will be asked to compare the four models analytically.

a). Let $A(x)$ denote a typical cross-sectional area of a bass, $0 \leq x \leq l$, where l denotes the length of the fish. Use the mean value theorem from calculus to show that the volume V of the fish is given by

$$V = l.\overline{A}$$

where A is the average value of A(x).

b). Assuming that A is proportional to the square of the girth g and that weight density for the bass is constant, establish that

$$W \propto lg^2$$

4). Heart Rate of Mammals—The following data relate the weights of some mammals to their heart rate in beats per minute. Based on the discussion relating blood flow through the heart to body weight, as presented in Project 2, construct a model that relates heart rate to body weight. Discuss the assumptions of your model. Use the data to check your model.

Mammal	Body weight (g)	Pulse rate (beats/min)
Vespergo pipistrellas	4	660
Mouse	25	670
Rat	200	420
Guinea pig	300	300
Rabbit	2000	205
Little dog	5000	120
Big dog	30000	85
Sheep	50000	70
Man	70000	72
Horse	450000	38
Ox	500000	40
Elephant	3000000	48

5). In the automobile gasoline mileage example, assume the drage forces are proportinal to Sv, where S is the cross-sectional area perpendicular to the direction of the moving car and v is its speed. what conclusions can you draw? Discuss the factors that might influence the choice of Sv^2 over Sv for the drage forces submodel. How could you test the sumodel?

6). Discuss several factors that were completely ignored in our analysis of the gasoline mileage problem.

7). Describe in detai the data you would like to obtain to test the various submodels suporting Model (2.21) (in Text book page 102). How would you go about collection the data?

8). Tests exsit to measure the percentage of body fat. Assume that such tests are accurate and that a great many carefully collected data are available. You may specify any other statistics, such as waist size and height, that you would like collected. Explain how the data could be arranged to check the assumptions underlying the submodels in this section. For example, suppose the data for males between ages 17 and 21 with constant body fat and height are examined. Explain how the assumption of constant density of the inner core could be checked.

9). Lumber Cutters—Lumber cutters wish to use readily available measurements to estimate the number of board feet of lumber in a tree. Assume they measure the diameter of the tree in inches at waist height. Develop a model that predicts board feet as a function of diameter in inches. Use the following data for your test:

x	17	19	20	23	25	28	32	38	39	41
y	19	25	32	57	71	113	123	252	259	294

The variable x is the diameter of a ponderosa pine in inches, and y is the number of board feet divided by 10.

(a.) Consider two separate assumptions, allowing each to lead to a model. Completely analyze each model.

(i.) Assume that all trees are right-circular cylinders and are approximately the same height.

(ii.) Assume that all trees are right-circular cylinders and that the height of the tree is proportional to the diameter.

(b.) Which model appears to be better? Why? Justify your conclusions.

10). A popular measure of physical condition and personal appearance is the pinch test. To administer this test, you measure the thickness of the outer core at selected locations on the body by pinching. Where and how should the pinch be made? What thickness of pinch should be allowed? Should the pinch thickness be allowed to vary with height?

11). Consider an endurance test that measures only aerobic fitness. This teat could be a swimming test, running test, or bike test. Assume that we want all competitors to do an equal amount of work. Build a mathematical model that relates work done by the competitor to some measurable characteristic, such as height or weight. Next consider a refinement using kinetic energy in your model. Collect some data for one of these aerobic tests and determine the reasonableness of these models.