Mathematical Modeling

ព្រះរាជាណាចក្រកម្ពុជា

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DEPARTMENT: I3-AMS-A

Project: Battery Discharge

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I. Introduction

This project, we will observe the battery discharge of my phone. We will get data about battery's percentages and time during using. The battery discharge project focuses on studying and modeling the behavior of a battery's energy depletion over time. This involves observing how the battery's percentages and time during using as it discharges under various conditions. The project aims to use mathematical modeling to establish proportional relationships between these parameters and predict battery performance under different scenarios.

II. Objective of Battery Discharge

- **Understand Discharge Characteristics:** Analyze how a battery's percentages and capacity decrease over time during discharge.
- **Develop a Mathematical Model:** Create a model that describes the proportional relationship between key variables such as time during using and battery's percentages.
- **Validate the Model:** Compare the theoretical predictions from the model with experimental data to assess accuracy.

III. General Assumption

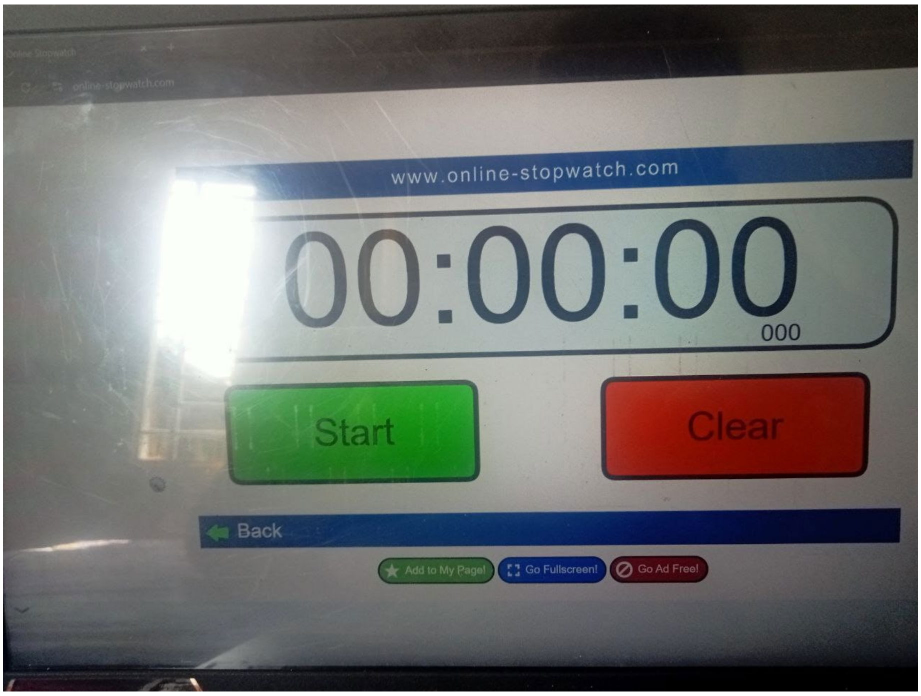
- Battery Percentage Drops Proportionally to Usage: In typical use, the battery percentage decreases in proportion to the intensity and duration of usage. Assume we use phone to only play a game until we finish experiment.
- Higher Screen Brightness and Features Consume More Power: Bright screens, background apps, and features like GPS, Bluetooth, and mobile data increase battery consumption significantly. Assume that we don't use other feature, back ground and open the constant screens bright.
- Estimate of Time Remaining Based on Current Usage: Many devices provide an estimated time remaining based on the current usage pattern. If you're performing heavy tasks, the time estimate will be shorter, whereas for lighter tasks, it will increase. Assume that we use the stop watch in my computer to measures.
- Battery Efficiency Declines Over time: A battery's capacity diminishes due to wear, meaning a fully charged battery will provide less usage time as the device ages. Assume that we use only a phone for our experiment.
- Faster Drain from 100%-50% vs. 50%-0%: On some devices, the battery may appear to drain faster in the higher percentage range (100%-50%) and more slowly as it approaches lower levels, though this depends on the battery calibration and software optimizations. Assume that decreasing of the battery's percentage are almost constants in another scenario.
- Idle Consumption Still Happens: Even when the device is not actively used, background processes and standby features (like notifications) continue to consume a small percentage of battery over time. Assum that we block all notifications during experiment.
- Temperature Impacts Battery Life: Extreme temperatures (both hot and cold) can cause the battery to deplete faster or impact its ability to hold a charge. Assume that temperature of the is constant.

IV. Experiment

- **Equipment**

1. The phone

2. The stop watch



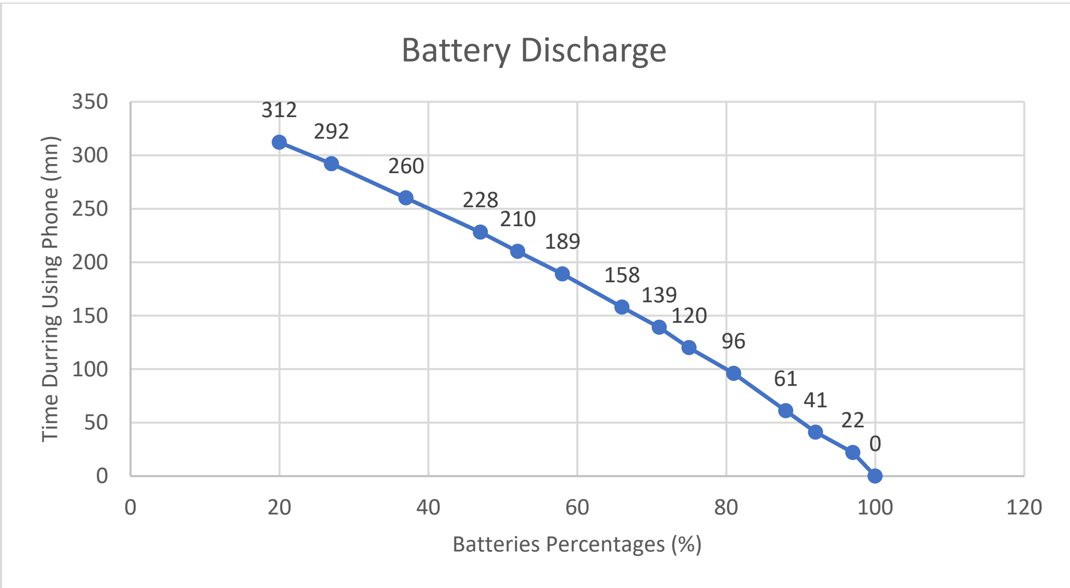
➢ **The Experiment**

- We will charge the phone until it full.
- We use the silent mode to block all notifications.
- We check to verify that another mode is close.
- We start the stop watch.
- And then we spend the time to play only a game until battery's percentage is lower than 20%.
- We collect the data from the battery's percentage and the time during using at stop watch.
- Use the data to plot the graph to know the nature of data.
- We will make the model which the proportional model and fit the model.
- We will verify the model with the data.
- We use the model to predict the time when the battery's percentage die.
- We will make conclusion.

The Data

Time Durring Using Phone	0	22	41	61	96	120	139	158	189	210	228	260	292	312
Batteries Percentages	100	97	92	88	81	75	71	66	58	52	47	37	27	20

Plot line graph

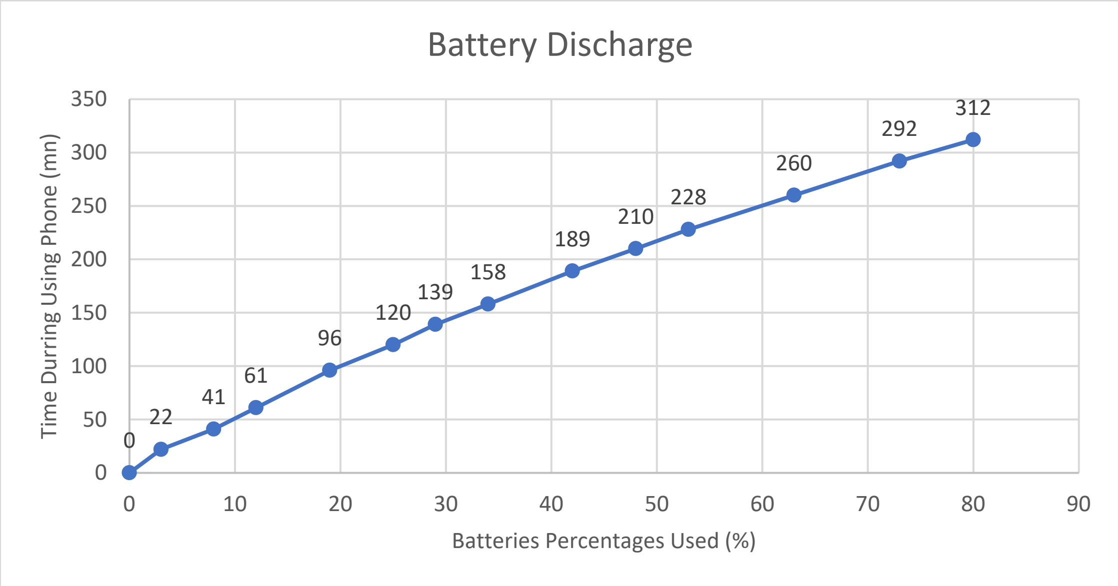


It seems that it maybe not to be proportionality. We will consider with Batteries Percentage Used.

The Data

Time Durring Using Phone	0	22	41	61	96	120	139	158	189	210	228	260	292	312
Batteries Percentages Used	0	3	8	12	19	25	29	34	42	48	53	63	73	80

Plot line graph



Good, it seems that it can be proportionality.We will use proportional model.

- **Proportional Model**

$$f(x) = kx$$

- **Find k by model fitting**

We should use At Least Square Error Method to find k because distance of each our data is far.

- **At Least Square Error Method**

$$min(SSE = \sum_{i=1}^{n}(Y_i - f(x_i))^2)$$

SSE is Sum Square Error

Y is real data

f(x) is model prediction

Then

$$SSE = \sum_{i=1}^{14}(Y_i - kx_i)^2$$

$$\frac{d_{SSE}}{d_k} = -2\sum_{i=1}^{14}(Y_i - kx_i)\, x_i = 0$$

$$\sum_{i=1}^{14}(Y_i - kx_i)\, x_i = 0$$

$$\sum_{i=1}^{14}(Y_i x_i - k{x_i}^2) = 0$$

$$\sum_{i=1}^{14}(Y_i x_i) - k\sum_{i=1}^{14}({x_i}^2) = 0$$

$$k = \frac{\sum_{i=1}^{14}(Y_i x_i)}{\sum_{i=1}^{14}({x_i}^2)}$$

$Battery's\ Percentages\ Used$	0	3	8	12	19	25	29	34	42	48	53	63	73	80
$Y_i x_i$	0	66	328	732	1824	3000	4031	5372	7938	10080	12084	16380	21316	24960
${x_i}^2$	0	9	64	144	361	625	841	1156	1764	2304	2809	3969	5329	6400

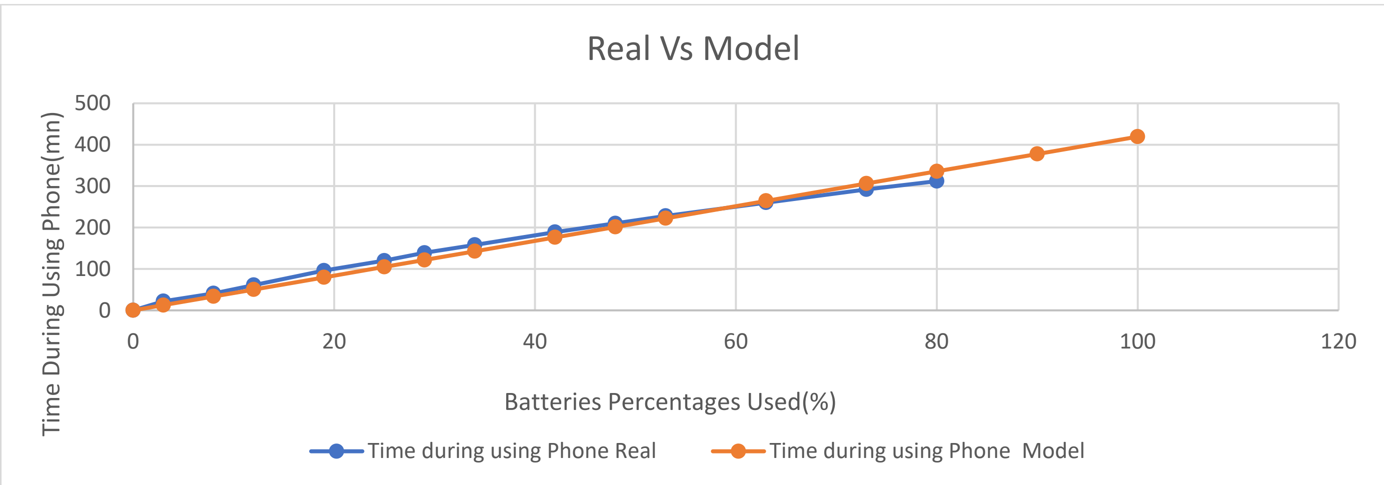
$$k = \frac{\sum_{i=1}^{14}(Y_i x_i)}{\sum_{i=1}^{14}({x_i}^2)} = 4.19441$$

So $f(x) = 4.19441x$

Real Vs Model

Time during using Phone Real	0	22	41	61	96	120	139	158	189	210	228	260	292	312	Prediction	
Time during using Phone Model	0	12.58	33.56	50.33	79.69	104.86	121.64	142.61	176.17	201.33	222.30	264.25	306.19	335.55	377.50	419.44
Batteries Percentages Used	0	3	8	12	19	25	29	34	42	48	53	63	73	80	90	100

Plot Multiple line Graph



The Model is very similar to the real data. We can say the proportional model is good for battery discharge data. By the way we can predict the time when the phone will battery die. If we want to use 100% of battery, we maybe need spend 419.44mn or 6h 59mn 26 s for playing only a game.

Verification

- **Mean Absolute Error**

$$\text{Mean Absolute Error} = \frac{\sum_{i=1}^{n} |Y_i - f(x_i)|}{n}$$

$$\text{Mean Absolute Error} = \frac{\sum_{i=1}^{14} |Y_i - kx_i|}{14}$$

$Battery's\ Percentages\ Used$	0	3	8	12	19	25	29	34	42	48	53	63	73	80
$\lvert Y_i - kx_i \rvert$	0	9.42	7.44	10.67	16.31	15.14	17.36	15.39	12.83	8.67	5.70	4.25	14.19	23.55

So, $\text{MAE} = \frac{\sum_{i=1}^{14} |Y_i - kx_i|}{14} = 11.49$

- **Mean Square Error**

$$\text{Mean Square Error} = \frac{\sum_{i=1}^{n} (Y_i - f(x_i))^2}{n}$$

$$\text{Mean Absolute Error} = \frac{\sum_{i=1}^{14} (Y_i - k(x_i))^2}{14}$$

$Battery's\ Percentages\ Used$	0	3	8	12	19	25	29	34	42	48	53	63	73	80
$(Y_i - k(x_i))^2$	0	88.68	55.42	113.79	265.89	229.21	301.44	236.85	164.73	75.14	32.45	18.04	201.41	554.73

So $\text{MSE} = \frac{\sum_{i=1}^{14} (Y_i - k(x_i))^2}{14} = 166.98$

- **R-Squared**

$$R^2 = 1 - \frac{SSE}{\sum_{i=1}^{n} (y_i - \bar{y})^2}$$

$\bar{y}\ is\ mean$

$$R^2 = 1 - \frac{SSE}{\sum_{i=1}^{14} (y_i - \bar{y})^2}$$

Time Durring Using Phone	0	22	41	61	96	120	139	158	189	210	228	260	292	312
Batteries Percentages Used	0	3	8	12	19	25	29	34	42	48	53	63	73	80

$$\bar{y} = \frac{1}{n} \sum_{i=1}^{n} (y_i)$$

$$\bar{y} = \frac{1}{14}\sum_{i=1}^{14}(y_i) = 152$$

$Battery's\ Percentages\ Used$	0	3	8	12	19	25	29	34	42	48	53	63	73	80
$(y_i - \bar{y})^2$	16900	12321	8281	3136	1024	169	36	1369	3364	5776	11664	19600	25600	23104

$\sum_{i=1}^{14}(y_i - \bar{y})^2$ =132344

$$SSE = \sum_{i=1}^{14}(Y_i - kx_i)^2$$

$Battery's\ Percentages\ Used$	0	3	8	12	19	25	29	34	42	48	53	63	73	80
$(Y_i - k(x_i))^2$	0	88.68	55.42	113.79	265.89	229.21	301.44	236.85	164.73	75.14	32.45	18.04	201.41	554.73

$SSE = \sum_{i=1}^{14}(Y_i - kx_i)^2 = 2337.80$

So $R^2 = 1 - \frac{2337.80}{132344}$ =0.9823 or 98.23%

By these results, we can say proportional model is very good for battery discharge data prediction. It means that the battery percentage is proportional with the times during using phone.

V. Conclusion

The proportional model accurately predicts the battery discharge rate under controlled conditions, indicating that the battery percentage decreases in proportion to the time the phone is in use. This model can be used to estimate the remaining battery life during tasks that consume power at a consistent rate, such as gaming, provided that external factors (such as background apps or temperature changes) are controlled.

In summary, the proportional model is a useful tool for estimating battery life and can assist users in managing their device usage based on predictable battery consumption

VI. References

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