



Robotics formular sheet

Robotics (Technische Universität München)

Robotik

1 Mathematische Grundlagen

1.1 Taylorentwicklung

$$Tf(x; a) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x)}{n!} (x-a)^n$$

$$Tf(x; x+h) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x)}{n!} (-h)^n$$

$$= f(z) - \frac{h}{1!} f'(x) + \frac{h^2}{2!} f''(x) + \frac{(-h)^k}{k!} f^{(k)}(\xi)$$

$$Tf(x; x-h) = f(x) + \frac{h}{1!} f'(x) + \frac{h^2}{2!} f''(x) + \frac{h^k}{k!} f^{(k)}(\xi)$$

1.2 Trigonometrische Beziehungen

$$\sin \theta = -\sin(-\theta) = -\cos(\theta + 90^\circ) = \cos(\theta - 90^\circ)$$

$$\cos \theta = \cos(-\theta) = \sin(\theta + 90^\circ) = -\sin(\theta - 90^\circ)$$

Identitäten

$$\cos(\theta_1 \pm \theta_2) = c_{12} = c_1 c_2 \mp s_1 s_2$$

$$\sin(\theta_1 \pm \theta_2) = s_{12} = s_1 c_2 \pm c_1 s_2$$

$$s_{2\theta} = 2s_\theta c_\theta$$

$$c_{2\theta} = c_\theta^2 - s_\theta^2 = 2c_\theta^2 - 1 = 1 - 2s_\theta^2$$

$$s_1 c_2 = \frac{1}{2} (s_{1-2} + s_{12})$$

$$s_{12} c_1 = \frac{1}{2} [s_2 + \sin(2\theta_1 + \theta_2)]$$

$$s_1 c_{12} = \frac{1}{2} [s_{-2} + \sin(2\theta_1 + \theta_2)]$$

2 Grundlagen der linearen Algebra

2.1 Adjunkte

$$\text{adj } \mathbf{A} = \text{Cof } \mathbf{A}^T = \tilde{\mathbf{A}}^T = \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \cdots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \cdots & \tilde{a}_{nn} \end{bmatrix}^T$$

$$= \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \cdots & \tilde{a}_{n1} \\ \tilde{a}_{12} & \tilde{a}_{22} & \cdots & \tilde{a}_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{1n} & \tilde{a}_{2n} & \cdots & \tilde{a}_{nn} \end{bmatrix}$$

mit den Kofaktoren

$$\tilde{a}_{ij} = (-1)^{i+j} \cdot M_{ij}$$

$$= (-1)^{i+j} \cdot \det \begin{bmatrix} a_{1,1} & \cdots & a_{1,j-1} & a_{1,j+1} & \cdots & a_{1,n} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ a_{i-1,1} & \cdots & a_{i-1,j-1} & a_{i-1,j+1} & \cdots & a_{i-1,n} \\ a_{i+1,1} & \cdots & a_{i+1,j-1} & a_{i+1,j+1} & \cdots & a_{i+1,n} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & \cdots & a_{n,j-1} & a_{n,j+1} & \cdots & a_{n,n} \end{bmatrix}$$

2.2 Inverse

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow \mathbf{A}^{-1} = \frac{\text{adj } \mathbf{A}}{\det \mathbf{A}} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

2.3 Determinante

$$\mathbf{A}, \mathbf{B} \in \mathbb{R}^{n \times n}$$

$$\det \mathbf{A} = \sum_{(i \vee j)=1}^n (-1)^{i+j} a_{ij} M_{ij}, \quad \text{wobei } i, j = 1, 2, \dots, n$$

Bsp. Entwicklung nach der ersten Zeile

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\det \mathbf{A} = (-1)^2 a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^3 a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^4 a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

Eigenschaften:

$$\det \mathbf{AB} = \det \mathbf{A} \det \mathbf{B}$$

$$\det \mathbf{A}^k = (\det \mathbf{A})^k$$

$$\det(\lambda \mathbf{A}) = \lambda^n \det \mathbf{A}$$

3 Kinematik

Vorwärts- und Rückwärtskinematik:

$$\underline{x} = f(\underline{\Theta}) \quad \text{"Vorwärtskinematik"}$$

$$\underline{\Theta} = f^{-1}(\underline{x}) \quad \text{"Rückwärtskinematik"}$$

Verallgemeinerte Koordinaten

$$\underline{\tilde{x}} = \begin{bmatrix} \underline{x} \\ \underline{\Theta} \end{bmatrix}, \quad \text{Pose}$$

$$\underline{\nu} = \begin{bmatrix} \underline{v} \\ \underline{\omega} \end{bmatrix} := \begin{bmatrix} {}^i \underline{v}_i \\ {}^i \underline{\omega}_i \end{bmatrix}, \quad \text{verall. Geschwindigkeit}$$

$$\underline{\mathcal{F}} = \begin{bmatrix} \underline{F} \\ \underline{N} \end{bmatrix} := \begin{bmatrix} {}^i \underline{f}_i \\ {}^i \underline{n}_i \end{bmatrix}, \quad \text{verall. Kräfte}$$

3.1 Eigentliche Eulerwinkel

3.2 Rotationsmatrizen

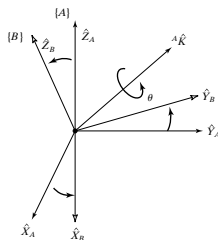
$$\hat{\mathbf{R}}_X = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix}$$

$$\hat{\mathbf{R}}_Y = \begin{bmatrix} \cos \alpha & 0 & \sin \alpha \\ 0 & 1 & 0 \\ -\sin \alpha & 0 & \cos \alpha \end{bmatrix}$$

$$\hat{\mathbf{R}}_Z = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3.3 Achsen-Winkel-Darstellung – Rodrigues Formel

Ausgehend von deckungsgleichen Koordinatensystemen $\{A\}$ und $\{B\}$, rotiere $\{B\}$ entlang dem Vektor ${}^A \hat{\mathbf{K}}$ mit dem Winkel θ entsprechend der Rechten-Hand-Regel.



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Notation:

$$\mathbf{R} := \mathbf{R}_K(\theta)$$

Definitionen:

$${}^A \hat{\mathbf{K}} = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix}, \quad \| {}^A \hat{\mathbf{K}} \| = 1$$

$$\mathbf{K} = \begin{bmatrix} 0 & -k_3 & k_2 \\ k_3 & 0 & -k_1 \\ -k_2 & k_1 & 0 \end{bmatrix}$$

$${}^A_B \mathbf{R} = \mathbf{1} + (1 - \cos \theta) \mathbf{K}^2 + \sin \theta \mathbf{K}$$

$$= \begin{bmatrix} k_1^2 v \theta + c \theta & k_1 k_2 v \theta - k_3 s \theta & k_1 k_3 v \theta + k_2 s \theta \\ k_1 k_2 v \theta + k_3 s \theta & k_2^2 v \theta + c \theta & k_2 k_3 v \theta - k_1 s \theta \\ k_1 k_3 v \theta - k_2 s \theta & k_2 k_3 v \theta + k_1 s \theta & k_3^2 v \theta + c \theta \end{bmatrix}$$

mit $v \theta = 1 - \cos \theta$, $c \theta = \cos \theta$, $s \theta = \sin \theta$

$$\mathbf{R}^{-1}(\theta) = \mathbf{R}^T(\theta) = \mathbf{R}(-\theta)$$

Inverse Problemstellung – \mathbf{R} gegeben, finde ${}^A \hat{\mathbf{K}}$ und θ :

$$\theta = \arccos \left(\frac{\text{spur } \mathbf{R} - 1}{2} \right)$$

$${}^A \hat{\mathbf{K}} = \frac{1}{2 \sin \theta} \begin{bmatrix} r_{32} - r_{23} \\ r_{13} - r_{31} \\ r_{21} - r_{12} \end{bmatrix}$$

3.4 Homogene Transformation

$$\begin{bmatrix} {}^A \underline{P} \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} {}^A_B \hat{\mathbf{R}} & {}^A_B \underline{P}_{\text{BORG}} \\ 0 & 1 \end{bmatrix}}_{:= {}^A_B \mathbf{T}} \begin{bmatrix} {}^B \underline{P} \\ 1 \end{bmatrix}$$

$${}^A \underline{P} = {}^A_B \mathbf{T} {}^B \underline{P}$$

Verknüpfung von Transformationen:

$${}^A_C \mathbf{T} = \begin{bmatrix} {}^A_B \hat{\mathbf{R}} {}^B_C \hat{\mathbf{R}} & {}^A_B \hat{\mathbf{R}} {}^B_C \underline{P}_{\text{CORG}} + {}^A_B \underline{P}_{\text{BORG}} \\ 0 & 1 \end{bmatrix}$$

Inversion:

$${}^B_A \mathbf{T} = \begin{bmatrix} {}^A_B \hat{\mathbf{R}}^T & -{}^A_B \hat{\mathbf{R}}^T {}^A_B \underline{P}_{\text{BORG}} \\ 0 & 1 \end{bmatrix}$$

$${}^B_A \mathbf{T} = {}^A_B \mathbf{T}^{-1}$$

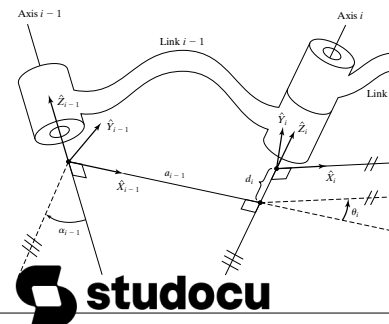
Operatoren

$$\mathbf{D}_Q(q) := \begin{bmatrix} 1 & 0 & 0 & q_x \\ 0 & 1 & 0 & q_y \\ 0 & 0 & 1 & q_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{R}_i(\alpha) := \begin{bmatrix} \hat{\mathbf{R}}_i & 0 \\ 0 & 1 \end{bmatrix}$$

3.5 Denavit-Hartenberg-Transformation

Verwendung der *distalen* Beschreibung, d.h. Transformation von dem Koordinatensystem $i-1$ nach dem System i .



Regeln – Distal:

- α_{i-1} = Winkel zwischen \hat{z}_{i-1} und \hat{z}_i um \hat{x}_{i-1}
- a_{i-1} = Abstand von \hat{z}_{i-1} nach \hat{z}_i gemessen entlang \hat{x}_{i-1}
- θ_i = Winkel zwischen \hat{x}_{i-1} und \hat{x}_i um \hat{z}_i
- d_i = Abstand von \hat{x}_{i-1} nach \hat{x}_i gemessen entlang \hat{z}_i

i	a_{i-1}	α_{i-1}	d_i	θ_i
1	a_0	α_0	d_1	θ_1
2	a_1	α_1	d_2	θ_2
\vdots	\vdots	\vdots	\vdots	\vdots

Proximale Transformationsmatrix:

$${}^{i-1}_i \mathbf{T} = \mathbf{R}_X(\alpha_{i-1}) \mathbf{D}_X(a_{i-1}) \mathbf{R}_Z(\theta_i) \mathbf{D}_Z(d_i)$$

$$= \begin{bmatrix} c \theta_i & -s \theta_i & 0 & a_{i-1} \\ s \theta_i c \alpha_{i-1} & c \theta_i c \alpha_{i-1} & -s \alpha_{i-1} & -s \alpha_{i-1} d_i \\ s \theta_i s \alpha_{i-1} & c \theta_i s \alpha_{i-1} & c \alpha_{i-1} & c \alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Beispiele? Sonderfälle?

Arbeitsraumattribute

- Design Effizienz (möglichst gering)

$$L = \sum_{i=1}^n (a_{i-1} + d_i)$$

$$Q_L = \frac{L}{\sqrt[3]{W}}$$

mit W Arbeitsraumvolumen.

- Manipulator Maß – Yoshikawa (möglichst hoch)

$$w = \sqrt{\det \mathbf{J} \mathbf{J}^T}$$

$$= |\det \mathbf{J}| \quad (\text{nicht-redundant})$$

3.6 Jacobimatrix

Definition:

$$\mathbf{J}_f = \frac{\partial \underline{f}_i}{\partial \underline{x}_j} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \frac{\partial f_m}{\partial x_2} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$

Anwendung:

$$\mathbf{B}_{\underline{v}} := \begin{bmatrix} \mathbf{B}_{\underline{v}} \\ \mathbf{B}_{\underline{\omega}} \end{bmatrix} = \mathbf{B} \mathbf{J}_{\underline{f}}(\underline{\Theta}) \dot{\underline{\Theta}} = \begin{bmatrix} {}^B \mathbf{J}_{\underline{v}}(\underline{\Theta}) \\ {}^B \mathbf{J}_{\underline{\omega}}(\underline{\Theta}) \end{bmatrix} \dot{\underline{\Theta}}$$

wobei gilt:

$${}^A \mathbf{J}_{\underline{f}}(\underline{\Theta}) = \begin{bmatrix} {}^A_B \hat{\mathbf{R}} & 0 \\ 0 & {}^A_B \hat{\mathbf{R}} \end{bmatrix} {}^B \mathbf{J}_{\underline{f}}(\underline{\Theta})$$

Singuläre, isotrope Positionen:

$$\det \mathbf{J}_{\underline{f}}(\underline{\Theta}) \stackrel{!}{=} 0$$

Bestimmung:

1. Geometrische Methode – Vorwärtskinematik

$${}^0\mathbf{p} = {}^0\mathbf{T}_2^1 \mathbf{T} \dots {}^n\mathbf{T} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$${}^0\mathbf{J}_{\mathbf{p}} = \begin{bmatrix} \frac{\partial {}^0\mathbf{p}_i}{\partial \Theta_j} \\ {}^0\dot{\mathbf{z}}_j \end{bmatrix}$$

$${}^n\mathbf{J}_{\mathbf{p}} = \begin{bmatrix} {}^n\mathbf{R} & \mathbf{0} \\ \mathbf{0} & {}^n\mathbf{R} \end{bmatrix} {}^0\mathbf{J}_{\mathbf{p}}$$

2. Winkel- und Lineargeschwindigkeiten

$${}^n\mathbf{v} = {}^n\dot{\mathbf{x}} = \begin{bmatrix} {}^n\mathbf{v}_n \\ {}^n\omega_n \end{bmatrix} \stackrel{\text{Prop.}}{=} \underbrace{\begin{bmatrix} [\dots] \\ [\dots] \end{bmatrix}}_{{}^n\mathbf{J}} \dot{\Theta}$$

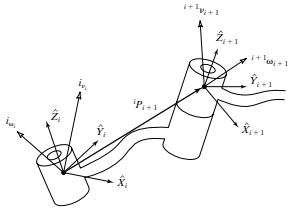
3. Kräfte und Momente

$$\boldsymbol{\tau} = \begin{bmatrix} \tau_1 = \begin{cases} {}^1\mathbf{n}_1^T {}^1\dot{\mathbf{z}}_1, & \text{rot.} \\ {}^1\mathbf{f}_1^T {}^1\dot{\mathbf{z}}_1, & \text{prism.} \end{cases} \\ \tau_2 = \begin{cases} {}^2\mathbf{n}_2^T {}^2\dot{\mathbf{z}}_2, & \text{rot.} \\ {}^2\mathbf{f}_2^T {}^2\dot{\mathbf{z}}_2, & \text{prism.} \end{cases} \\ \vdots \\ \tau_i = \begin{cases} {}^i\mathbf{n}_i^T {}^i\dot{\mathbf{z}}_i, & \text{rot.} \\ {}^i\mathbf{f}_i^T {}^i\dot{\mathbf{z}}_i, & \text{prism.} \end{cases} \end{bmatrix} \stackrel{\text{Prop.}}{=} \underbrace{\begin{bmatrix} [\dots] & [\dots] \end{bmatrix}}_{{}^n\mathbf{J}^T} \begin{bmatrix} F \\ N \end{bmatrix}$$

Pseudoinverse – nicht quadratische Jacobimatrix

$$\mathbf{J}^{-*} = (\mathbf{J}^T \mathbf{J})^{-1} \mathbf{J}^T$$

3.7 Winkel- und Lineargeschwindigkeiten



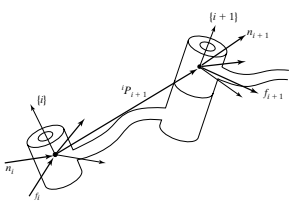
Propagation

$${}^{i+1}\omega_{i+1} = {}^{i+1}\hat{\mathbf{R}}^i \omega_i + \begin{cases} \dot{\Theta}_{i+1} {}^{i+1}\hat{\mathbf{z}}_{i+1}, & \text{rot.} \\ 0 & \text{prism.} \end{cases}$$

$${}^{i+1}\mathbf{v}_{i+1} = {}^{i+1}\hat{\mathbf{R}}^i \left({}^i\mathbf{v}_i + {}^i\omega_i \times {}^i\mathbf{p}_{i+1} \right)$$

$$+ \begin{cases} 0, & \text{rot} \\ {}^i\dot{\mathbf{d}}_{i+1} {}^{i+1}\hat{\mathbf{z}}_{i+1}, & \text{prism.} \end{cases}$$

3.8 Kräfte und Momente



Propagation

$${}^i\mathbf{f}_i = {}^{i+1}\hat{\mathbf{R}}^{i+1} \mathbf{f}_{i+1}$$

$${}^i\mathbf{n}_i = {}^{i+1}\hat{\mathbf{R}}^{i+1} \mathbf{n}_{i+1} + {}^i\mathbf{p}_{i+1} \times {}^i\mathbf{f}_i$$

Rotations-Gelenk:

$$\tau_i = {}^i\mathbf{n}_i^T {}^i\mathbf{z}_i = {}^i\mathbf{n}_i^T \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Prismatisches-Gelenk:

$$\tau_i = {}^i\mathbf{f}_i^T {}^i\mathbf{z}_i = {}^i\mathbf{f}_i^T \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Prinzip der virtuellen Arbeit

$$\mathcal{F} \cdot \delta \mathbf{X} = \boldsymbol{\tau} \cdot \delta \Theta$$

mit

$$\delta \mathbf{X} = \mathbf{J} \delta \Theta$$

gilt

$$\begin{bmatrix} \tau_1 \\ \tau_2 \\ \vdots \\ \tau_n \end{bmatrix} = \boldsymbol{\tau} = \mathbf{A} \mathbf{J}^T \mathbf{A} \boldsymbol{\varepsilon} = \mathbf{A} \mathbf{J}^T \begin{bmatrix} A f \\ A n \end{bmatrix}$$

4 Kinetik

Newton- und Eulergleichung

$$\mathbf{F} = m \dot{\mathbf{v}}_C$$

$$\mathbf{N} = {}^C\mathbf{I} \dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times {}^C\mathbf{I} \boldsymbol{\omega}$$

4.1 Trägheitstensor

$$\mathbf{I} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{bmatrix}$$

$$I_{xx} = \int_K (y^2 + z^2) dm \quad I_{xy} = - \int_K xy dm$$

$$I_{yy} = \int_K (x^2 + z^2) dm \quad I_{xz} = - \int_K xz dm$$

$$I_{zz} = \int_K (x^2 + y^2) dm \quad I_{yz} = - \int_K yz dm$$

Verschiebung Massenmittelpunkt: $\mathbf{A} \mathbf{d} = (a, b, c)^T$

$$\mathbf{A} \mathbf{I} = {}^C\mathbf{I} + m \left[\mathbf{A} \mathbf{d} \cdot \mathbf{A} \mathbf{d}^T - \mathbf{A} \mathbf{d} \otimes \mathbf{A} \mathbf{d} \right]$$
$$= \begin{bmatrix} I_{xx}^C & I_{xy}^C & I_{xz}^C \\ I_{xy}^C & I_{yy}^C & I_{yz}^C \\ I_{xz}^C & I_{yz}^C & I_{zz}^C \end{bmatrix} + m \begin{bmatrix} b^2 + c^2 & -ab & -ac \\ -ab & a^2 + c^2 & -bc \\ -ac & -bc & a^2 + b^2 \end{bmatrix}$$

$$I_{xx}^A = I_{xx}^C + (b^2 + c^2)m \quad I_{xy}^A = I_{xy}^C - abm$$

$$I_{yy}^A = I_{yy}^C + (a^2 + c^2)m \quad I_{xz}^A = I_{xz}^C - acm$$

$$I_{zz}^A = I_{zz}^C + (a^2 + b^2)m \quad I_{yz}^A = I_{yz}^C - bcm$$

4.2 Newton-Euler-Methode

Notation:

$$\star = \begin{cases} [\dots], & i+1 = \text{rot.} \\ [\dots], & i+1 = \text{prism.} \end{cases}$$

1. Phase – Äußere Iteration: $0 \rightarrow n$

Kinematik:

$${}^{i+1}\omega_{i+1} = {}^{i+1}\hat{\mathbf{R}}^i \omega_i + \begin{cases} \dot{\Theta}_{i+1} {}^{i+1}\hat{\mathbf{z}}_{i+1} \\ 0 \end{cases}$$

$${}^{i+1}\dot{\omega}_{i+1} = {}^{i+1}\hat{\mathbf{R}}^i \dot{\omega}_i + \begin{cases} {}^{i+1}\hat{\mathbf{R}}^i \omega_i \times \dot{\Theta}_{i+1} {}^{i+1}\hat{\mathbf{z}}_{i+1} \\ + \dot{\Theta}_{i+1} {}^{i+1}\hat{\mathbf{z}}_{i+1} \\ 0 \end{cases}$$

$${}^{i+1}\dot{\mathbf{v}}_{i+1} = {}^{i+1}\hat{\mathbf{R}}^i \left({}^i\dot{\omega}_i \times {}^i\mathbf{p}_{i+1} + {}^i\omega_i \times ({}^i\omega_i \times {}^i\mathbf{p}_{i+1}) + {}^i\dot{\mathbf{v}}_i \right)$$

$$+ \begin{cases} 0 \\ 2 \cdot {}^{i+1}\omega_{i+1} \times {}^i\dot{\mathbf{d}}_{i+1} {}^{i+1}\hat{\mathbf{z}}_{i+1} + {}^i\ddot{\mathbf{d}}_{i+1} {}^{i+1}\hat{\mathbf{z}}_{i+1} \end{cases}$$

$$(1) \quad {}^{i+1}\dot{\mathbf{v}}_{C_{i+1}} = {}^{i+1}\dot{\omega}_{i+1} \times {}^{i+1}\mathbf{p}_{C_{i+1}} + {}^{i+1}\omega_{i+1} \times ({}^{i+1}\omega_{i+1} \times {}^{i+1}\mathbf{p}_{C_{i+1}}) + {}^{i+1}\dot{\mathbf{v}}_{i+1}$$

Kinetik:

$${}^{i+1}\mathbf{F}_{i+1} = m_{i+1} {}^{i+1}\dot{\mathbf{v}}_{C_{i+1}}$$

$${}^{i+1}\mathbf{N}_{i+1} = {}^{C_{i+1}}\mathbf{I}_{i+1} {}^{i+1}\dot{\omega}_{i+1} + {}^{i+1}\omega_{i+1} \times {}^{C_{i+1}}\mathbf{I}_{i+1} {}^{i+1}\omega_{i+1}$$

2. Phase – Innere Iteration: $n \rightarrow 1$

$${}^i\mathbf{f}_i = {}^{i+1}\hat{\mathbf{R}}^{i+1} \mathbf{f}_{i+1} + {}^i\mathbf{F}_i$$

$${}^i\mathbf{n}_i = {}^{i+1}\hat{\mathbf{R}}^{i+1} \mathbf{n}_{i+1} + {}^i\mathbf{N}_i + {}^i\mathbf{p}_{i+1} \times {}^{i+1}\hat{\mathbf{R}}^{i+1} \mathbf{f}_{i+1} + {}^i\mathbf{p}_{C_i} \times {}^i\mathbf{F}_i$$

Gravitation:

$$\underline{G} \stackrel{\text{ObdA}}{=} \begin{bmatrix} 0 \\ -g \\ 0 \end{bmatrix} \quad {}^0\dot{\mathbf{z}}_0 = -\underline{G}$$

M-V-G und M-B-C-G Form

$$\boldsymbol{\tau} = \tau_i = \begin{cases} {}^i\mathbf{n}_i^T {}^i\hat{\mathbf{z}}_i, & \text{rot.} \\ {}^i\mathbf{f}_i^T {}^i\hat{\mathbf{z}}_i, & \text{prism.} \end{cases}$$

$$= \mathbf{M}(\Theta) \ddot{\Theta} + \mathbf{V}(\Theta, \dot{\Theta}) + \mathbf{G}(\Theta)$$

$$= \mathbf{M}(\Theta) \ddot{\Theta} + \mathbf{B}(\Theta) [\dot{\Theta} \dot{\Theta}] + \mathbf{C}(\Theta) [\dot{\Theta}^2] + \mathbf{G}(\Theta)$$

mit

$$[\dot{\Theta} \dot{\Theta}] = \begin{bmatrix} \dot{\Theta}_1 \dot{\Theta}_2 \\ \dot{\Theta}_1 \dot{\Theta}_3 \\ \vdots \\ \dot{\Theta}_{n-1} \dot{\Theta}_n \end{bmatrix}$$

$$[\dot{\Theta}^2] = \begin{bmatrix} \dot{\Theta}_1^2 \\ \dot{\Theta}_2^2 \\ \vdots \\ \dot{\Theta}_n^2 \end{bmatrix}$$

Kartesische Zustandsraumgleichung

$$\mathcal{F} = \mathbf{M}_x(\Theta) \ddot{\mathbf{x}} + \mathbf{V}_x(\Theta, \dot{\Theta}) + \mathbf{G}_x(\Theta)$$

mit Gleichung (1)

$$\mathbf{M}_x(\Theta) = \mathbf{J}^{-T} \mathbf{M}(\Theta) \mathbf{J}^{-1}$$

$$\mathbf{V}_x(\Theta) = \mathbf{J}^{-T} \left(\mathbf{V}(\Theta, \dot{\Theta}) - \mathbf{M}(\Theta) \mathbf{J}^{-1} \dot{\mathbf{J}} \dot{\Theta} \right)$$

$$\mathbf{G}_x(\Theta) = \mathbf{J}^{-T} \mathbf{G}(\Theta)$$

mit elementweiser Zeitdifferentiation

$$\dot{\mathbf{J}} = \frac{\partial}{\partial t} \left(\frac{\partial f_i}{\partial x_j} \right)$$

4.3 Lagrange-Formalismus

$$k_i = \frac{1}{2} m_i \mathbf{v}_{C_i}^T \mathbf{v}_{C_i} + \frac{1}{2} {}^i\omega_i^T \mathbf{C}_i \mathbf{I}_i {}^i\omega_i$$

$$k = \begin{cases} \sum_{i=1}^n k_i \\ \frac{1}{2} \dot{\Theta}^T \mathbf{M}(\Theta) \dot{\Theta} \end{cases}$$

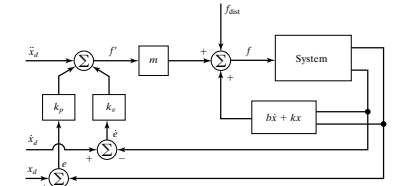
$$u_i = -m_i {}^0\mathbf{g}^T {}^0\mathbf{p}_{C_i} + u_{\text{ref}}$$

Lagrange-Gleichung 2. Art

$$\boldsymbol{\tau} = \frac{d}{dt} \frac{\partial k}{\partial \dot{\Theta}} - \frac{\partial k}{\partial \Theta} + \frac{\partial u}{\partial \Theta}$$

$$\tau_i = \frac{d}{dt} \frac{\partial k}{\partial \dot{\Theta}_i} - \frac{\partial k}{\partial \Theta_i} + \frac{\partial u}{\partial \Theta_i}$$

5 Lineare Regelung



Modellproblem

$$m\ddot{x} + b\dot{x} + kx = 0$$

Fallunterscheidung

$$\text{Verhalten} = \begin{cases} b^2 > 4mk & \text{Überdämpfung} \\ b^2 < 4mk & \text{Unterdämpfung – Oscillation} \\ b^2 = 4mk & \text{Aperiodischer Grenzfall} \end{cases}$$

Dämpfungsverhältnis und Resonanzfrequenz

$$\xi = \frac{b}{2\sqrt{km}}, \quad \omega_n = \sqrt{k/m}$$

5.1 Kontrollgesetz Ansätze

Bewegungsgleichung

$$m\ddot{x} + b\dot{x} + kx = f$$

Position-Regulierung

$$f = -k_p x - k_v \dot{x}$$

Regelungsparameter

$$b' = b + k_v \stackrel{!}{=} 2\sqrt{mk'}, \quad k' = k + k_p$$

Closed-Loop Dynamik

$$m\ddot{x} + b'\dot{x} + k'x = 0$$

