Lecture 1: Motion of Particles in Cartesian Coordinates

Vm240: Introduction to Dynamics and Vibration

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- Introduction to Vm240
- Motion of Particles in Cartesian Coordinates
- Analyzing Straight Line Motion of Particles
- Tutorial examples

Introduction to Vm240

From a viewpoint of Engineering Mechanics

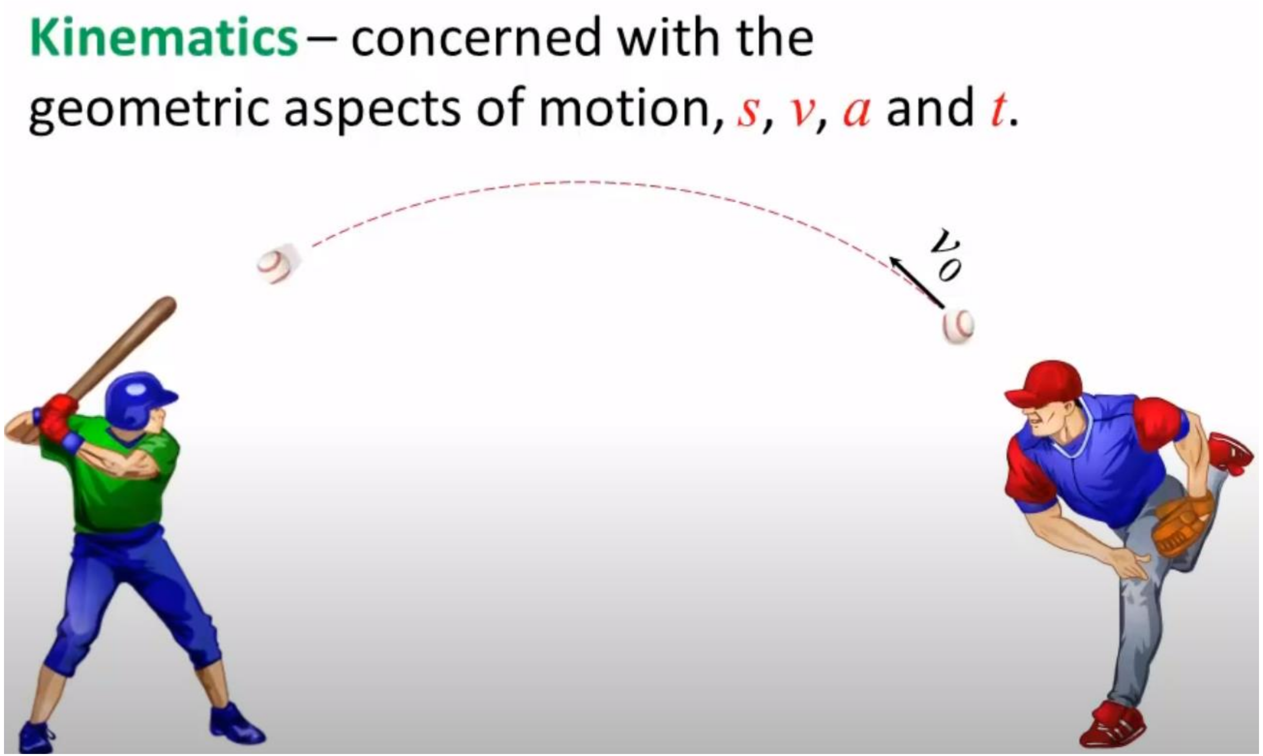
Engineering Mechanics: The study of how rigid bodies react to forces acting on them

Statics: the study of bodies in equilibrium

Dynamics: The study of motion

1. **Kinematics:** concerned with the geometric aspects of motion, s, v, a and t.
2. **Kinetics:** concerned with how forces causing the motion.

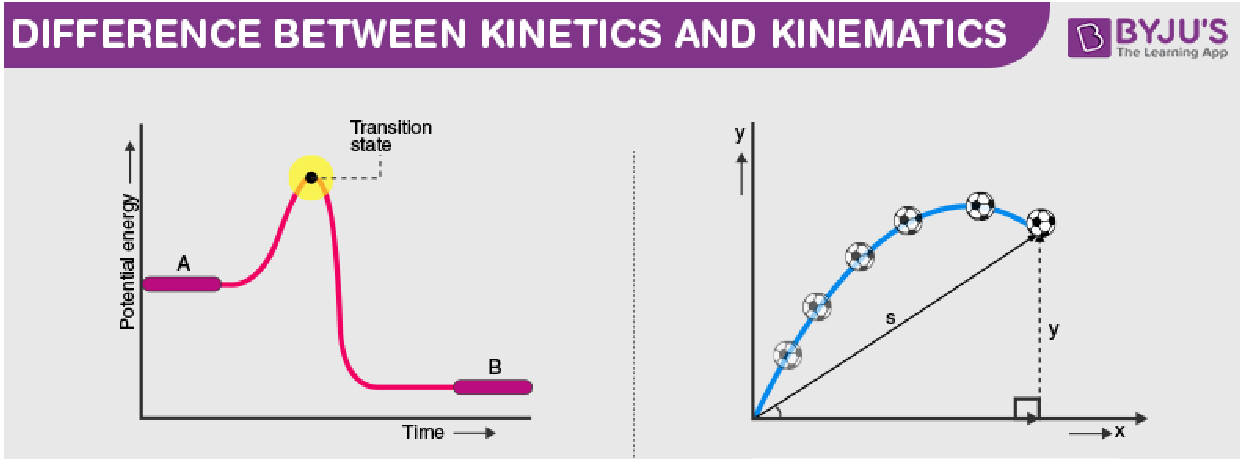
An Example of Kinematics



Credits: Yiheng Wang
DCC EGR245 Online Course

What is Dynamics?

- The study of the motions of bodies and the forces that accompany or cause those motions.
  - Kinematics: The branch of dynamics that deals with only space and time
  - Kinetics: The branch of dynamics that deals with the relationships between forces and motions.
- Governing laws: Newton's Laws of Motion



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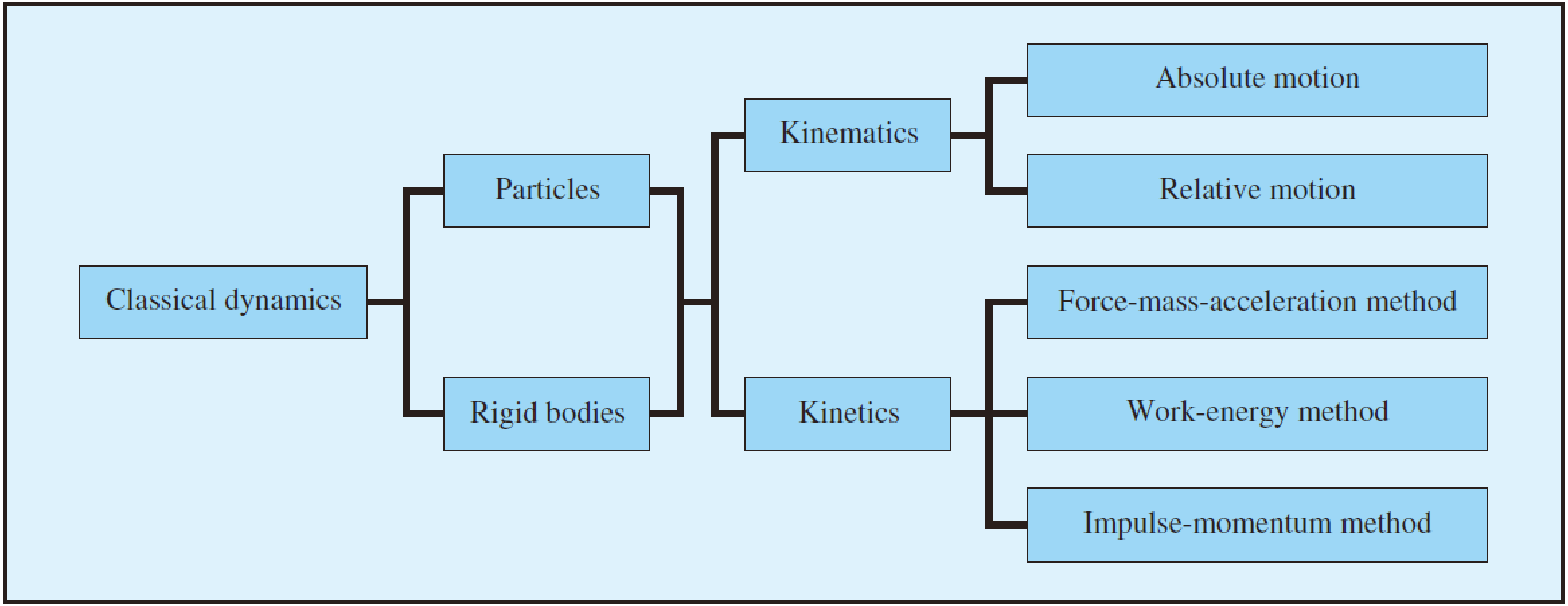
Subdivisions of Classical Dynamics



kinetics 畅通词汇

英 [kɪ'netɪks] 美 [kɪ'netɪks]

n. [物]动力学

kinematics

英 [ˌkɪnə'mætɪks] 美 [ˌkɪnə'mætɪks]

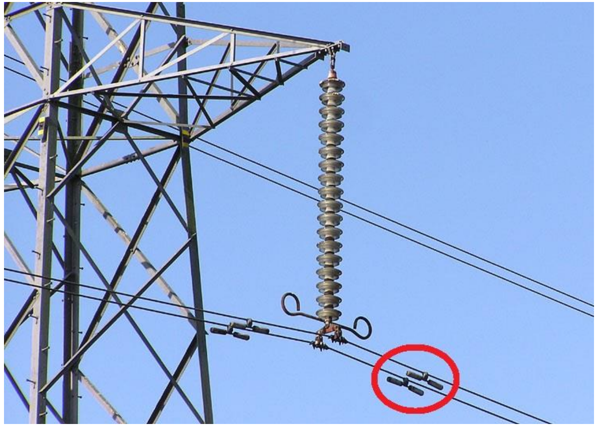
n. 运动学；动力学

A Brief History

- Newton: The 3 laws of motion, law of universal attraction
- 18th Century
  - Bernoulli: Principle of Virtual Work
  - D'Alambert: D'Alambert's principle
  - Euler: Rigid-body dynamics
- 19th. Century
  - Lagrange, Poisson, Hamilton and Jacobi: Analytical mechanics (rational mechanics) – a much richer mathematical structure of Newtonian mechanics
- 20th. Century
  - Limits of Newtonian mechanics found: Systems moving at speeds comparable with the speed of light, or systems of dimensions comparable to the size of the atom.

What is vibration?

- The branch of dynamics that deals with periodic or oscillatory motion. Examples:
  - Response of civil engineering structures to dynamic loading, ambient conditions and earthquakes
  - Vibration of unbalanced rotating machines
  - Vibration of power lines due to wind excitation
  - Aircraft wings



Aeolean Vibration Dampers for Power Lines

Credits: https://electronics.stackexchange.com/

oscillatory

英 ['ɒsɪleɪˌtərɪ] 美 ['ɒsələˌtourɪ]

adj. 振动的；动摇的；变动的

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Example: Treadmill Vibration Isolation System

Because running in the International Space Station might cause unwanted **vibrations**, they have installed a Treadmill Vibration Isolation System.

Credits: McGraw-Hill Companies, Inc.

Example: Spring Vibration Isolators

Kinetics Spring Vibration Isolators are used to reduce the transmission of noise, shock, and vibration produced by mechanical, industrial, or process equipment into or within a building structure.

Contents of this Course

- Dynamics
  - Particle kinematics
  - Particle kinetics
  - Rigid body kinematics
  - Rigid body kinetics
- Vibrations
  - Free vibration of a single degree of freedom
  - Damped vibration of a single degree of freedom
  - Forced vibration of a single degree of freedom
  - Vibration of systems with two degrees of freedom

Grading Policy (To be Updated)

- Homework/Quizzes/Attendance (25%)
- Midterm exam (30%)
- Final Exam (45%)

About the Instructor(s)

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- Main research field:
  - Intelligent Transportation Optimization and Control
- Office hours:
  - Wed. 2:00-4:00
  - Fri. 2:00-4:00

Problem-Solving Tips

- Most effective ways: to solve problem
- Two key tools: Newton's Laws of Motion and Vector Algebra
- Present the work in an orderly and logic manner:
  - Read problem carefully
  - Draw necessary diagrams and tabulate the problem data
  - Establish a coordinate system and apply the relevant principles, generally in mathematical form
  - Solve the necessary equations algebraically as far as practical
  - Use technical judgement and common sense
  - Review the problem

Motion of Particles in Cartesian Coordinates

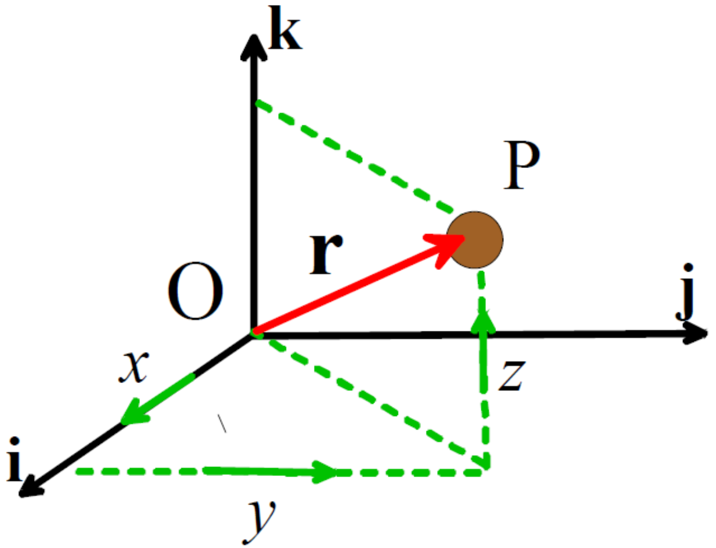
Definition of Particles

- Definition: A particle is a point mass at some position in space
- Properties:
  - Mass
  - Position (Velocity, Acceleration)
  - No shape or orientation
  - Example: Satellite in space

Position-Velocity-Acceleration formula (Cartesian Coordinates)

- Position
- $\{\mathbf{i}, \mathbf{j}, \mathbf{k}\}$ is a Cartesian basis
- $r = x(t)\mathbf{i} + \mathrm{y}(\mathrm{t})\mathbf{j} + \mathrm{z}(t)\mathbf{k}$

- The "Inertial Frame"
  - To use Newton:
    - O must not accelerate
    - $\{\mathbf{i}, \mathbf{j}, \mathbf{k}\}$ must not rotate
  - This is an approximation.



Cartesian 畅通词汇

英 [kɑː'tiːziən] 美 [kɑːr'tiːziən]

Position-Velocity-Acceleration formula (Cartesian Coordinates)

- Velocity vector
  - Velocity is the derivative of the position vector with respect to time
  - $v = \lim_{\delta t \to 0} \frac{r(t+\delta t)-r(t)}{\delta t} = \dot{r}$
  - Expressed in terms of its cartesian components
  - $\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k}$
    - $v_x = \frac{dx}{dt},\ v_y = \frac{dy}{dt},\ v_z = \frac{dz}{dt}$

Position-Velocity-Acceleration formula (Cartesian Coordinates)

- Acceleration vector
  - $\mathbf{a} = \lim_{\delta t \to 0} \frac{\mathrm{v}(t+\delta t) - \mathrm{v}(t)}{\delta t}$
  - Cartesian representation:$\mathbf{a} = a_x \mathbf{i} + a_y \mathbf{j} + a_z \mathrm{k}$
    - $v_x = \frac{dv_x}{dt} = \frac{d^2 x}{dt^2},\ v_y = \frac{dv_y}{dt} = \frac{d^2 y}{dt^2},\ v_z = \frac{dv_z}{dt} = \frac{d^2 z}{dt^2}$

Analyzing Straight Line Motion of Particles

Particle motion problems

- Given (x,y,z), in terms of time
  - Find (v_x, v_y, v_z) and (a_x, a_y, a_z)
- Formula for a in terms of x
  - Suppose we know v as a function of x, e.g., $v = x^2$

Exercise

- v as a function of x: $v = x^2$
- Calculate a using chain rule:

$$\mathrm{a} = \frac{\mathrm{dv}}{\mathrm{dx}}\frac{\mathrm{dx}}{\mathrm{dt}} = \mathrm{v}\frac{\mathrm{dv}}{\mathrm{dx}}$$

- For $v = x^2$, $\mathrm{a} = 2\mathrm{x} \cdot \mathrm{v} = 2\mathrm{x}^3$

Particle Motion Problems

- Given $\left(a_x, a_y, a_z\right)$ in terms of t, find $\left(v_x, v_y, v_z\right)$ or (x, y, z)
- To do this, we will need to solve “differential equations”, e.g.

$$\frac{d^2x}{dt^2} + x = 0$$

Background Knowledge

- Common dynamics problem, given:
  i. Acceleration $a = f(x, v, t)$
  ii. Speed v and position x at time $t = 0$
- Find: speed v and distance traveled x for $t > 0$
- Approach: solve the differential equations

$$\frac{dx}{dt} = v \qquad \frac{dv}{dt} = \frac{vdv}{dx} = \frac{d^2x}{dt^2} = f(x, v, t)$$

- Some equations can be solved by using separation of variables.

Three Common Cases

1) Acceleration is a known function of time $a = f(t)$

Example: Rocket in space, with thrust that decreases with time (for 0<t<T)

$$a(t) = F_0(1 - t/T)/m$$

2) Acceleration depends on speed (and time) $a = f(v)g(t)$

Example: Dust particle dropping vertically with air resistance

$$a(v) = g - cv/m$$

3) Acceleration depends on position (and speed) $a = f(x)g(v)$

Example: Mass on a spring $a(x) = -kx/m$

Case 1

- Acceleration is a known function of time $a = f(t)$

$\frac{dv}{dt} = f(t), \frac{dx}{dt} = v$ Initial Condition: $x(t=0) = x_0, v(t=0) = v_0$

- <u>Calculating v</u>
  - Step 1: "Separate variables" $dv = f(t)dt$
  - Step 2: Integrate both sides:

$$\int_{v_0}^{v} dv = \int_0^t f(t)dt$$

Case 1: calculating v

- Acceleration:

$$a = \frac{F_0\left(1 - \frac{t}{T}\right)}{m} \quad 0 < t < T$$

- Step 1: $dv = \left\{\frac{F_0\left(1 - \frac{t}{T}\right)}{m}\right\} dt$
- Step 2:

This means substitute the limits

$$\int_{v_0}^{v} dv = \int_0^t \left\{\frac{F_0\left(1 - \frac{t}{T}\right)}{m}\right\} dt$$

$$[v]_{v_0}^{v} = \left[\frac{F_0\left(t - \frac{t^2}{2T}\right)}{m}\right]_0^t \Rightarrow v = v_0 + \frac{F_0\left\{t - \frac{t^2}{2T}\right\}}{m}$$

Case 1: calculating x

- Acceleration is a known function of time $a = f(t)$

$\frac{dv}{dt} = f(t), \frac{dx}{dt} = v$ Initial Condition: $x(t=0) = x_0, v(t=0) = v_0$

- <u>Calculating x</u>
  - Step 1: Separate variables: $dx = v(t)dt$
  - Step 2: Integrate both sides:

$$\int_{x_0}^{x} dx = \int_{0}^{t} v(t)dt$$

Case 1: calculating x

$$a = \frac{F_0\left(1 - \frac{t}{T}\right)}{m}$$

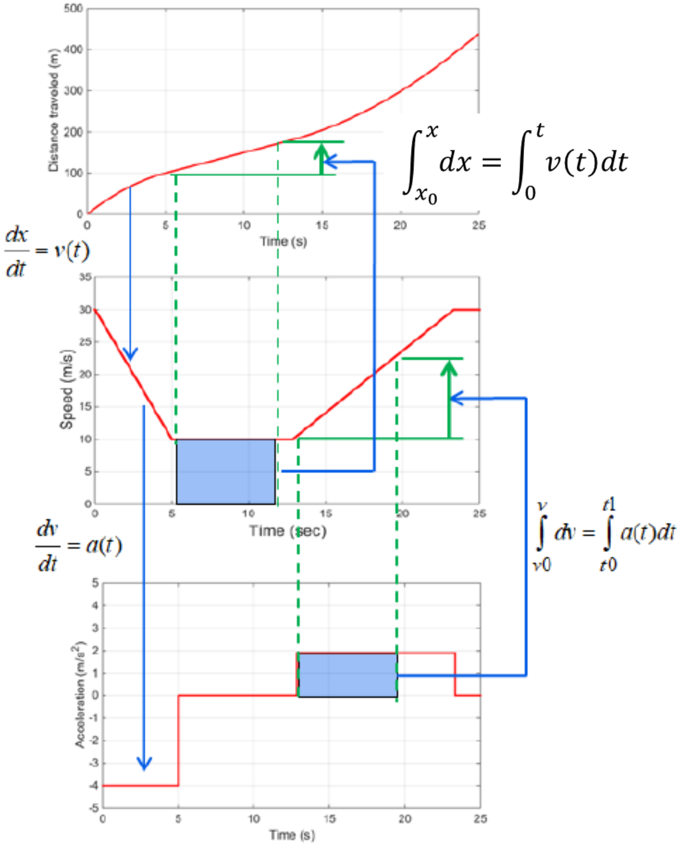
$$v = v_0 + \frac{F_0\left(t - \frac{t^2}{2T}\right)}{m}, 0 < t < T$$

- Step 1: $dx = \left(v_0 + \frac{F_0\left(t - \frac{t^2}{2T}\right)}{m}\right)dt$
- Step 2: $\int_0^x dx = \int_0^t \left(v_0 + \frac{F_0\left(t - \frac{t^2}{2T}\right)}{m}\right)dt \Rightarrow$

$$[x]_{x_0}^{x} = \left[v_0 + \frac{F_0\left(t - \frac{t^2}{2T}\right)}{m}\right]_0^t \Rightarrow x = v_0 t + \frac{F_0\left(\frac{t^2}{2} - \frac{t^3}{6T}\right)}{m} = v_0 t + \frac{F_0 t^2}{2m} - \frac{F_0 t^3}{6mT}$$

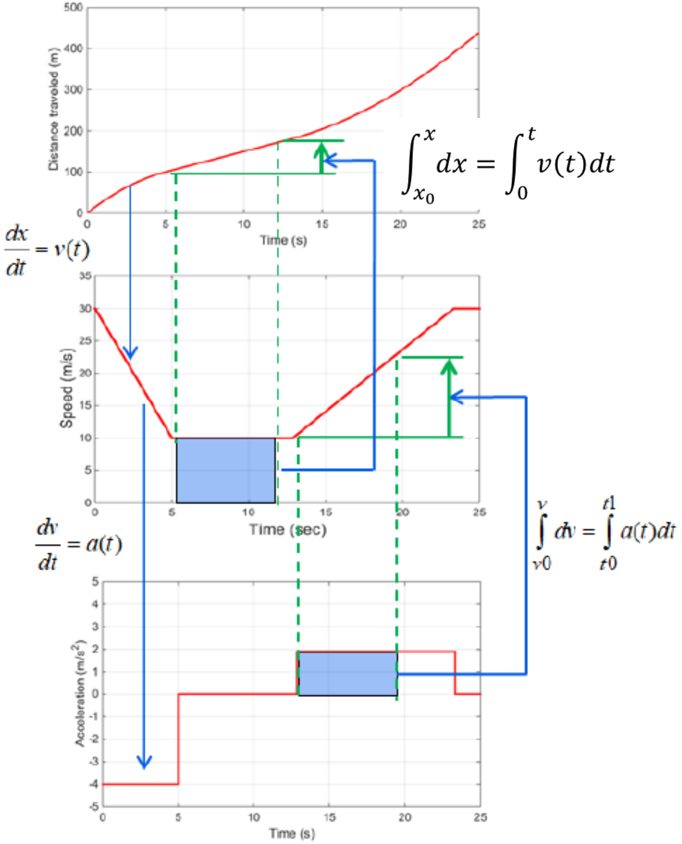
Separation of variables: Case 1

- Graphical Method for a, v, x all functions of time
  - Speed is the slope of the distance-time curve
  - Distance is the area under the speed-time curve



Separation of variables: Case 1

- Graphical Method for a, v, x all functions of time
  - Acceleration is the slope of the speed-time curve
  - Speed is the area under the acceleration-time curve



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Separation of variables: Case 2

- Acceleration depends on speed (and time) $a = f(v)g(t)$

$$\frac{dv}{dt} = f(v)g(t) \quad \frac{dx}{dt} = v$$

- Initial condition: $x(t = 0) = x_0, v(t = 0) = v_0$
- Calculating v:
  - Step 1: separate variables:

$$\frac{dv}{f(v)} = g(t)dt$$

  - Step 2: integrate both sides:

$$\int_{v_0}^{v} \frac{dv}{f(v)} = \int_0^t g(t)dt$$

Case 2: Calculating v

- $a(v) = g - \frac{cv}{m} \leftarrow f(v) = g - \frac{cv}{m}, g(t) = 1$
- Calculating v:
  - Step 1: separate variables:
$$\frac{dv}{g - \frac{cv}{m}} = dt$$
  - Step 2: integrate both sides:
$$\int_{v_0}^{v} \frac{dv}{g - \frac{cv}{m}} = \int_0^t dt \Rightarrow \left[-\frac{m}{c}\log\left(g - \frac{cv}{m}\right)\right]_{v_0}^{v} = t$$
$$\Rightarrow -\frac{m}{c}\log\left(\frac{g - \frac{cv}{m}}{g - \frac{cv_0}{m}}\right) = t \Rightarrow v = \frac{mg}{c} - \left(\frac{mg}{c} - v_0\right)\exp\left(-\frac{ct}{m}\right)$$

Case 2: calculating x

- Acceleration depends on speed (and time) $a = f(v)g(t)$

$$\frac{dv}{dt} = f(v)g(t) \quad \frac{dx}{dt} = v$$

- Initial condition: $x(t = 0) = x_0, v(t = 0) = v_0$
- Calculating v:
  - Step 1: separate variables:

$$dx = v(t)dt$$

  - Step 2: integrate both sides:

$$\int_{x_0}^{x} dx = \int_0^t v(t)dt$$

Case 2: Calculating x

$$a(v) = g - \frac{cv}{m}, v = \frac{mg}{c} - \left(\frac{mg}{c} - v_0\right)\exp\left(-\frac{ct}{m}\right)$$

- Calculating v:
  - Step 1: separate variables:

$$dx = \left(\frac{mg}{c} - \left(\frac{mg}{c} - v_0\right)\exp\left(-\frac{ct}{m}\right)\right)dt$$

  - Step 2: integrate both sides:

$$\int_{x_0}^{x} dx = \int_0^t \left(\frac{mg}{c} - \left(\frac{mg}{c} - v_0\right)\exp\left(-\frac{ct}{m}\right)\right)dt \Rightarrow [x]_{x_0}^{x} = \left[\frac{mg}{c} - \left(\frac{mg}{c} - v_0\right)\exp\left(-\frac{ct}{m}\right)\right]_0^t$$

$$\Rightarrow x = x_0 + \frac{mgt}{c} + \left(\frac{m}{c}\right)\left(\frac{mg}{c} - v_0\right)\left(\exp\left(-\frac{ct}{m}\right) - 1\right)$$

Separation of variables: Case 3

- Acceleration depends on position $a = f(x)g(v)$

$$\frac{dv}{dt} = f(x)g(v) \quad \frac{dx}{dt} = v$$

- Initial condition: $x(t = 0) = x_0, v(t = 0) = v_0$
- Calculating v:
  - Step 1: Rewrite acceleration in terms of x:

$$\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = \frac{dv}{dx}v = f(x)g(v)$$

  - Step 2: Separate variables:

$$v\frac{dv}{g(v)} = f(x)dx$$

  - Step 3: Integrate both sides:

$$\int_{v_0}^{v} \frac{vdv}{g(v)} = \int_{0}^{x} f(x)dx$$

Case 3: Calculating v

$$a(x) = -\frac{kx}{m}$$

- Calculating v:
  - Step 1&2:

$$vdv = \left(-\frac{kx}{m}\right)dx$$

  - Step 3:

$$\int_{v_0}^{v} vdv = \int_{x_0}^{x}\left(-\frac{kx}{m}\right)t \Rightarrow \left[\frac{v^2}{2}\right]_{v_0}^{v} = \left[-\frac{kx^2}{2m}\right]_{x_0}^{x}$$

$$\Rightarrow v = \sqrt{v_0^2 + \frac{kx_0^2}{m} - \frac{kx^2}{m}}$$

Case 3: Calculating x

$$a(x) = -\frac{kx}{m}, v = \sqrt{v_0^2 + \frac{kx_0^2}{m} - \frac{kx^2}{m}}$$

- Calculating x:
  - Step 1: separate variables:

$$dx = \sqrt{v_0^2 + \frac{kx_0^2}{m} - \frac{kx^2}{m}}\,dt$$

  - Step 2: integrate both sides:

$$\int_{x_0}^{x} \frac{dx}{\sqrt{v_0^2 + \frac{kx_0^2}{m} - \frac{kx^2}{m}}} = \int_0^t dt \Rightarrow \frac{1}{\sqrt{\frac{k}{m}}}\left[\sin^{-1}\left(\frac{x}{\sqrt{\frac{mv_0^2}{k} + x_0^2}}\right)\right]_{x_0}^{x} = t$$

$$\Rightarrow x = \left(\sqrt{\frac{mv_0^2}{k} + x_0^2}\right)\sin\left(\sqrt{\frac{k}{m}}t + \sin^{-1}\left(\frac{x}{\sqrt{\frac{mv_0^2}{k} + x_0^2}}\right)\right)$$

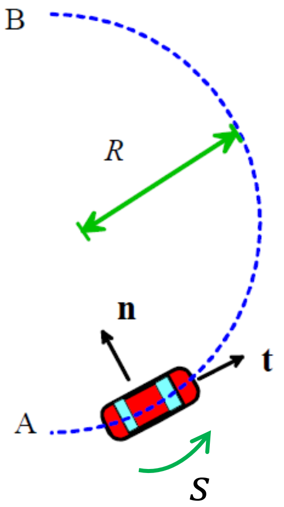
Final note

- We use the same calculus in many other applications
- Example 1: Motion along a curved path
  - Differential equations

Replaces x

$$\frac{dV}{dt} = \frac{VdV}{ds} = a_t(s, V, t) \quad \frac{ds}{dt} = V$$

Replaces v



- Example 2: Rotation about a fixed axis
  - Differential equations

Replaces x

$$\frac{d\omega}{dt} = \omega\frac{d\omega}{d\theta} = \alpha(\theta, \omega, t) \quad \frac{d\theta}{dt} = \omega$$

Replaces a

Replaces v

Tutorial Examples

Example: Straight line motion with constant acceleration

- A particle has constant acceleration $\mathbf{a} = a\mathbf{i}$, at time t=0, it has velocity $\mathbf{v} = v_0\mathbf{i}$ and position $\mathbf{r} = x_0\mathbf{i}$
- Find $\mathbf{v}(t)$, $\mathbf{v}(x)$ and $\mathbf{r}(t)$.

Example: Straight line motion with constant acceleration

- A particle has constant acceleration $\mathbf{a} = a\mathbf{i}$, at time t=0, it has velocity $\mathbf{v} = v_0\mathbf{i}$ and position $\mathbf{r} = x_0\mathbf{i}$
- Find $\mathbf{v}(t)$, $\mathbf{v}(x)$ and $\mathbf{r}(t)$.
- Solution:
  - Use calculus formulas:

$$\frac{dv}{dt} = a \Rightarrow \int_{v_0}^{v} dv = \int_0^t a dt \Rightarrow v - v_0 = at \Rightarrow v(t) = v_0 + at$$

$$\frac{vdv}{dx} = a \Rightarrow \int_{v_0}^{v} vdv = \int_{x_0}^{x} adx \Rightarrow \left[\frac{1}{2}v^2\right]_{v_0}^{v} = a(x - x_0)$$

$$\Rightarrow \frac{1}{2}v^2 - \frac{1}{2}v_0^2 = a(x - x_0) \Rightarrow v(x) = \sqrt{v_0^2 + 2a(x - x_0)}$$

Example: Straight line motion with constant acceleration

- A particle has constant acceleration $\mathbf{a} = a\mathbf{i}$, at time t=0, it has velocity $\mathbf{v} = v_0\mathbf{i}$ and position $\mathbf{r} = x_0\mathbf{i}$
- Find $\mathbf{v}(t)$, $\mathbf{v}(x)$ and $\mathbf{r}(t)$.
- Solution:

$$\frac{dx}{dt} = v = v_0 + at \Rightarrow \int_{x_0}^{x} dx = \int_0^t (v_0 + at)\,dt$$

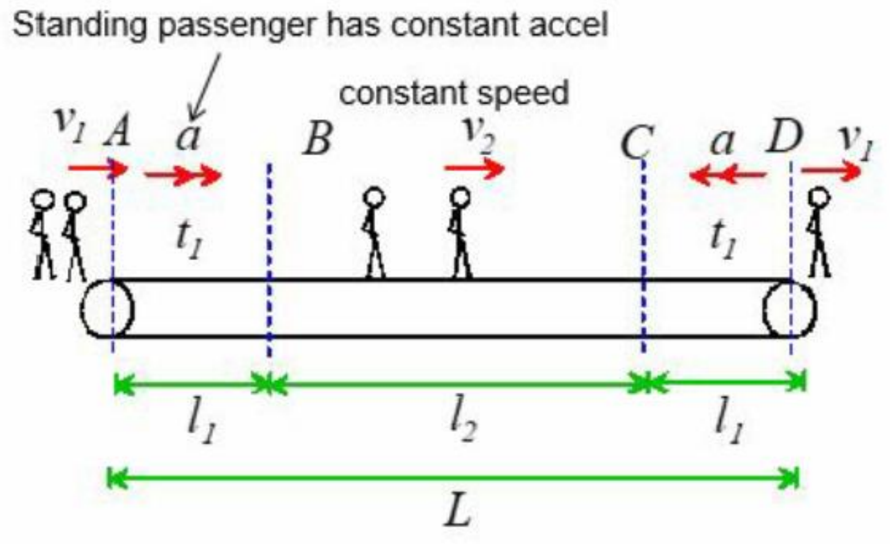
$$\Rightarrow x - x_0 = v_0 + \frac{1}{2}at^2 \Rightarrow x = x_0 + v_0 t + \frac{1}{2}at^2$$

Note: “Constant Acceleration formulas”

- Use these only if a is constant, otherwise separate variables or use MATLAB.

Example: High-speed Walkway

- Given information:
  - Total length L=912 ft
  - $v_1 = 125 \text{ ft/min}$
  - $v_2 = 400 \text{ ft/min}$
  - Travel time $A \rightarrow B = t_1$, approx. 10 sec.
  - Travel time $C \rightarrow D = t_1$, approx. 10 sec.
- Calculate:
  - Acceleration a
  - Time of travel T from $A \rightarrow D$



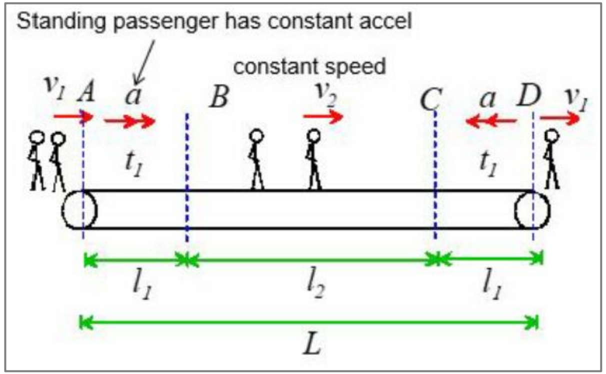
Example: Toronto High-speed Walkway

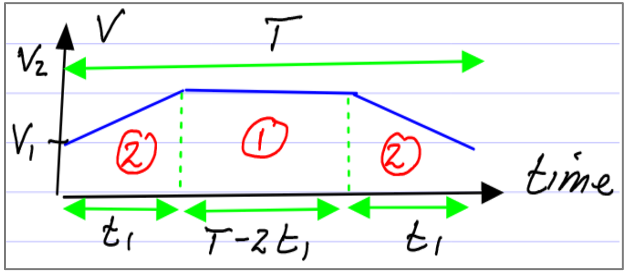
Given information:

- Total length L=912 ft
- $v_1 = 125$ ft/min
- $v_2 = 400$ ft/min
- Travel time $A \to B = t_1$, approx. 10 sec.
- Travel time $C \to D = t_1$, approx. 10 sec.

Calculate:

- Acceleration a
- Time of travel T from $A \to D$





Solution:

Use constant accel formulas between A and B

$$v_2 = v_1 + at_1 \Rightarrow a = \frac{v_2 - v_1}{t_1} = \frac{400 - 125}{60 \times 10}$$

$$\Rightarrow a = 0.46 \frac{\text{ft}}{\text{s}^2}$$

Approx. 0.015g, where g=32ft/s^2

Solution:

Distance = Area under the Blue Line

$$L = v_2(T - 2t_1) + 2\left\{\frac{v_1 + v_2}{2} t_1\right\}$$

Solve for T: $T = \frac{L}{v_2} + 2t_1 - \frac{v_1+v_2}{v_2} t_1$

Substitute numbers: $T = 142.9$ s

Example: Straight Line Motion with Variable Acceleration

Aircraft starts from rest.

Acceleration $a = \dfrac{F_0}{m}\left(1 - \dfrac{v}{v_0}\right)$

Must reach speed v_{TO} to take off

1. Find a formula for speed as a function of time.
2. Find a formula for distance traveled as a function of time
3. Find a formula for the minimum length of runway required to takeoff

Example: Straight Line Motion with Variable Acceleration

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Must reach speed v_{TO} to take off

1. Find a formula for speed as a function of time.
2. Find a formula for distance traveled as a function of time
3. Find a formula for the minimum length of runway required to takeoff

<u>Solution:</u>

1. Use separation of variables:

$$\frac{dv}{dt} = a = \frac{F_0}{m}\left(1 - \frac{v}{v_0}\right) \Rightarrow \int_0^v \frac{dv}{1 - \frac{v}{v_0}} = \int_0^t \frac{F_0}{m} dt$$

$$\Rightarrow \left[-v_0 \log\left(1 - \frac{v}{v_0}\right)\right]_0^v = \frac{F_0}{m} t$$

$$\Rightarrow v = v_0\left(1 - \exp\left(-\frac{F_0}{m v_0} t\right)\right)$$

2.

$$\frac{dx}{dt} = v = v_0\left(1 - \exp\left(-\frac{F_0}{m v_0} t\right)\right)$$

$$\Rightarrow \int_0^x dx = \int_0^t v_0\left(1 - \exp\left(-\frac{F_0}{m v_0} t\right)\right)$$

$$\Rightarrow x = \left[v_0 t + \frac{m v_0^2}{F_0} \exp\left(-\frac{F_0}{m v_0} t\right)\right]_0^t$$

$$\Rightarrow x = v_0 t + \frac{m v_0^2}{F_0}\left(\exp\left(-\frac{F_0}{m v_0} t\right) - 1\right)$$

Example: Straight Line Motion with Variable Acceleration

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Must reach speed v_{TO} to take off

1. Find a formula for speed as a function of time.
2. Find a formula for distance traveled as a function of time
3. Find a formula for the minimum length of runway required to takeoff

Notes:

a. Aircraft must reach v_{TO} before end of runway.

b. We can find time to reach v_{TO} from the solution of question 1; then find distance travel from the solution of question 2.

$$v_{TO} = v_0\left(1 - \exp\left(-\frac{F_0}{mv_0}t\right)\right)$$

Note:

$$-\frac{mv_0 v_{TO}}{F_0} = \frac{mv_0^2}{F_0}\left(1 - \exp\left(-\frac{F_0}{mv_0}t\right)\right)$$

Now solve for t,

$$t = -\frac{mv_0}{F_0}\log\left(1 - \frac{v_{T0}}{v_0}\right)$$

Finally,

$$x = -\frac{mv_0^2}{F_0}\log\left(1 - \frac{v_{To}}{v_0}\right) - \frac{mv_0 v_{TO}}{F_0}$$

Note: $v_{TO} < v_0$ and $\log\beta < 0$ for $\beta < 1$

First term is positive.

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Summary

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- Motion of Particles in Cartesian Coordinates
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  - Acceleration depends on position (and speed)
- Tutorial Examples