Hardware & Software Verification

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Lecture 4: Dafny
28 October 2024

The need for Dafny

- We need to be able to **reason about** the programs we write, not merely **test** them. There is a large and growing need for this.
- Dafny is a **verification-oriented** programming language. Its compiler will refuse to produce executable code until it has proven the code to be **correct**.

But what does correct mean?

Demo: max of a pair

- named output parameters
- postconditions
- overly weak/strong specifications

Straight-line code

true

```
x:=5;
```

x=5

```
y:=8;
```

x=5 ∧ y=8

```
z:=x+y;
```

x=5 ∧ y=8 ∧ z=x+y

```
x:=x+1;
```

x=6 ∧ y=8 ∧ z=x-1+y

(x+1)=6 ∧ y=8 ∧ z=(x+1)-1+y

```
x:=x+1;
```

x=6 ∧ y=8 ∧ z=x-1+y

P

```
x:=E;
```

P ∧ x=E

✗ (works if E and P don't mention x)

P[E/x]

```
x:=E;
```

P

✓

If-statements

$x=5 \wedge y=8 \wedge z=x+y$

```
if (w > 5) {
```

$x=5 \wedge y=8 \wedge z=x+y \wedge w>5$

```
 w:=5;
```

$x=5 \wedge y=8 \wedge z=x+y \wedge w=5$

```
} else {
```

$x=5 \wedge y=8 \wedge z=x+y \wedge w\leq 5$

```
 x:=10;
```

$x=10 \wedge y=8 \wedge z=x-5+y \wedge w\leq 5$

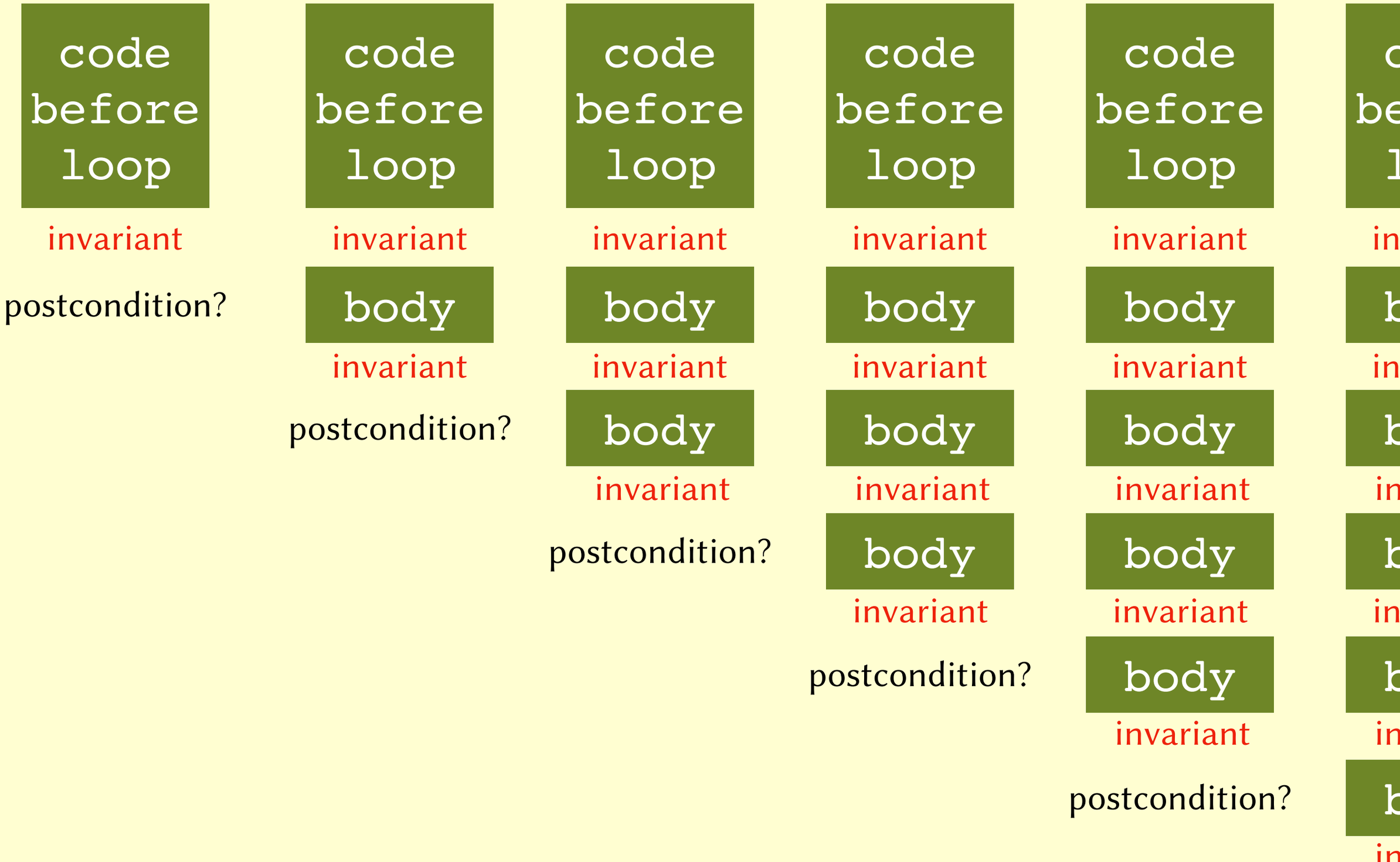
```
}
```

$(x=5 \wedge y=8 \wedge z=x+y \wedge w=5) \vee (x=10 \wedge y=8 \wedge z=x-5+y \wedge w\leq 5)$

$y=8 \wedge ((x=5 \wedge z=x+y \wedge w=5) \vee (x=10 \wedge z=x-5+y \wedge w\leq 5))$

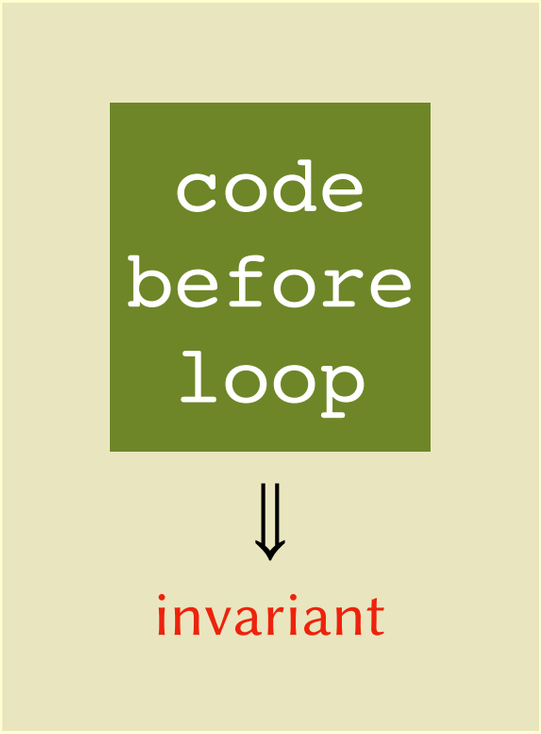
Demo: max of an array

The problem with loops

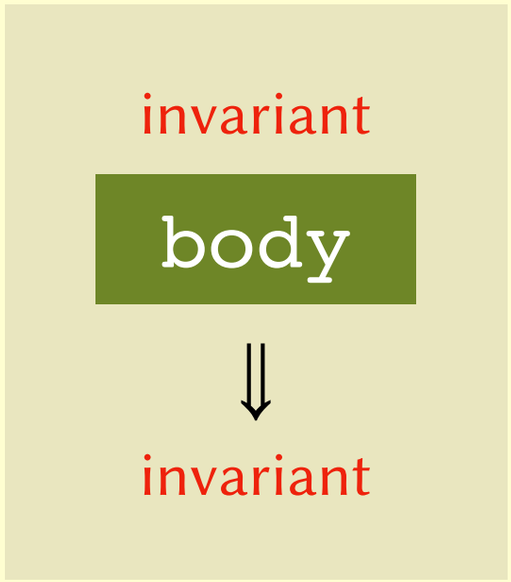


Loop invariants

1.



2.

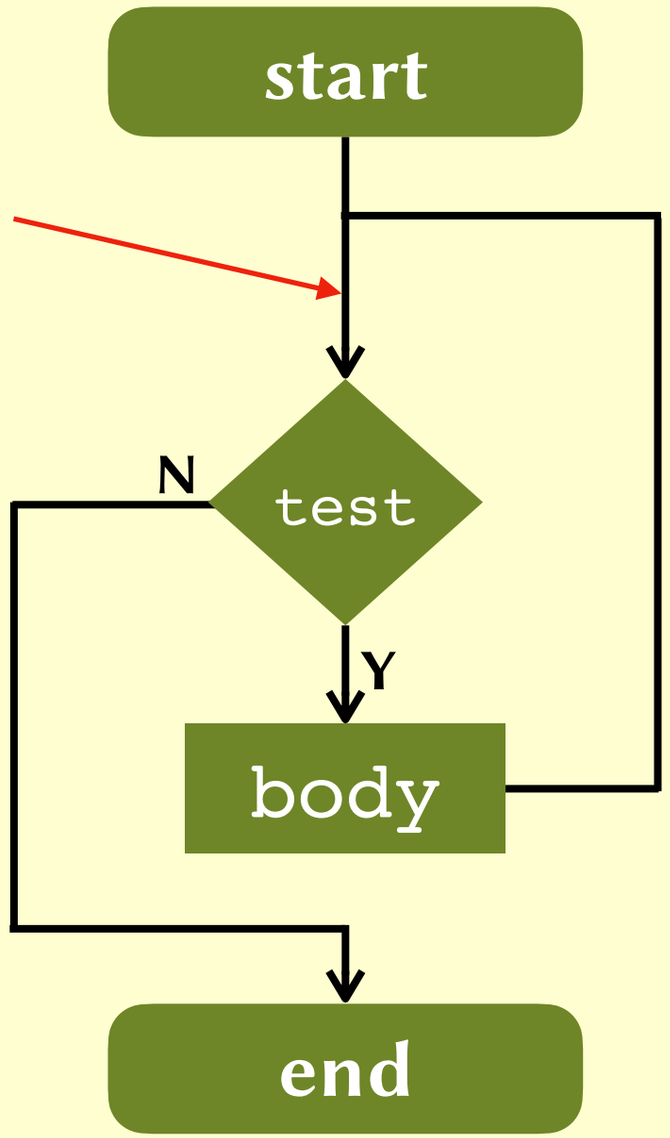


3.

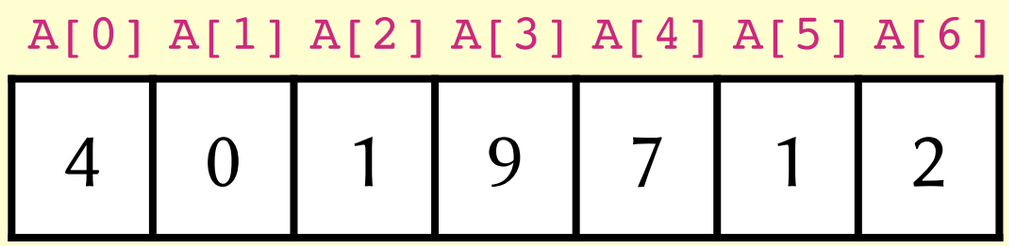
Loop invariants

```
while test
   invariant foo
{
   body
}
```

foo must hold here!



Finding invariants




```
r := A[0];
var i := 1;
while i < A.Length {
   if r < A[i] {
      r := A[i];
   }
   i := i+1;
}
```

i	r
1	4
2	4
3	4
4	9
5	9
6	9
7	9

Finding invariants

A[0]	A[1]	A[2]	A[3]	A[4]	A[5]	A[6]
4	0	1	9	7	1	2

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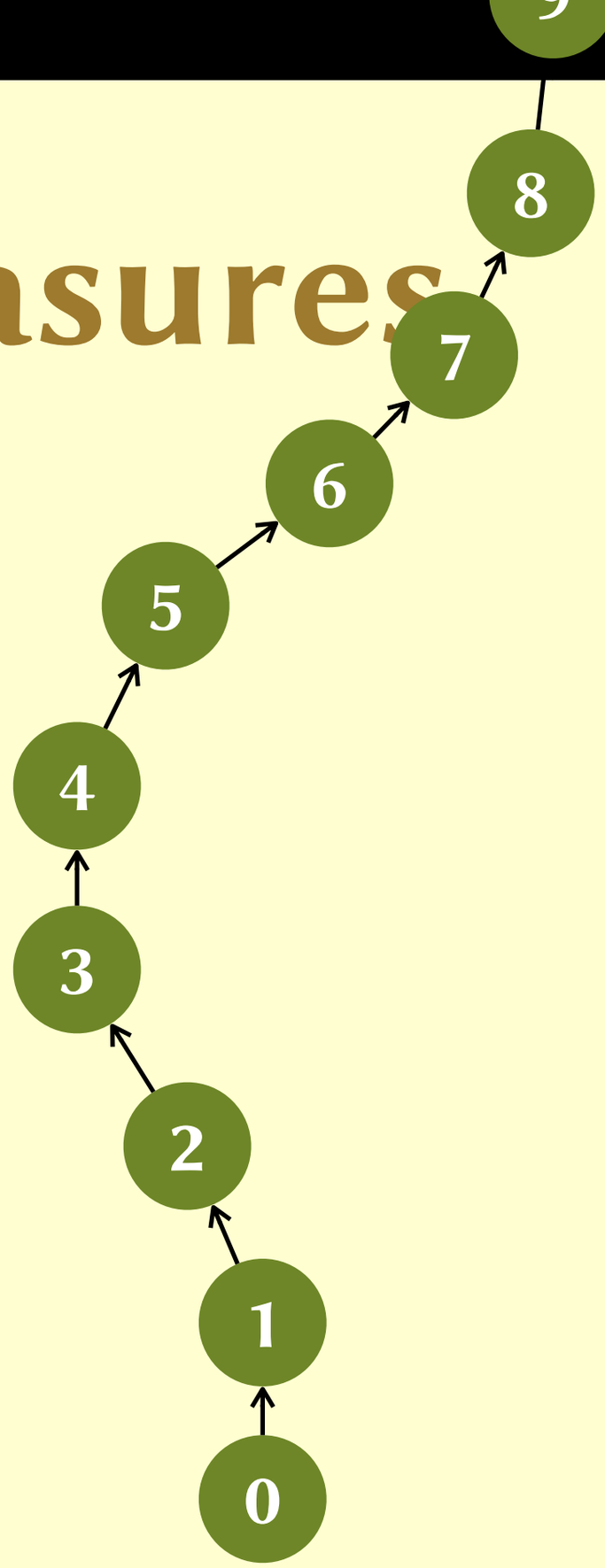
Demo: max of an array

Termination measures

- A *measure* is an expression that evaluates to a non-negative integer.
- The measure must *strictly decrease* every time we go round the loop.
- Hence we can't go round the loop forever!
- E.g.: `A.Length - i`
- "Theory of well-founded relations"



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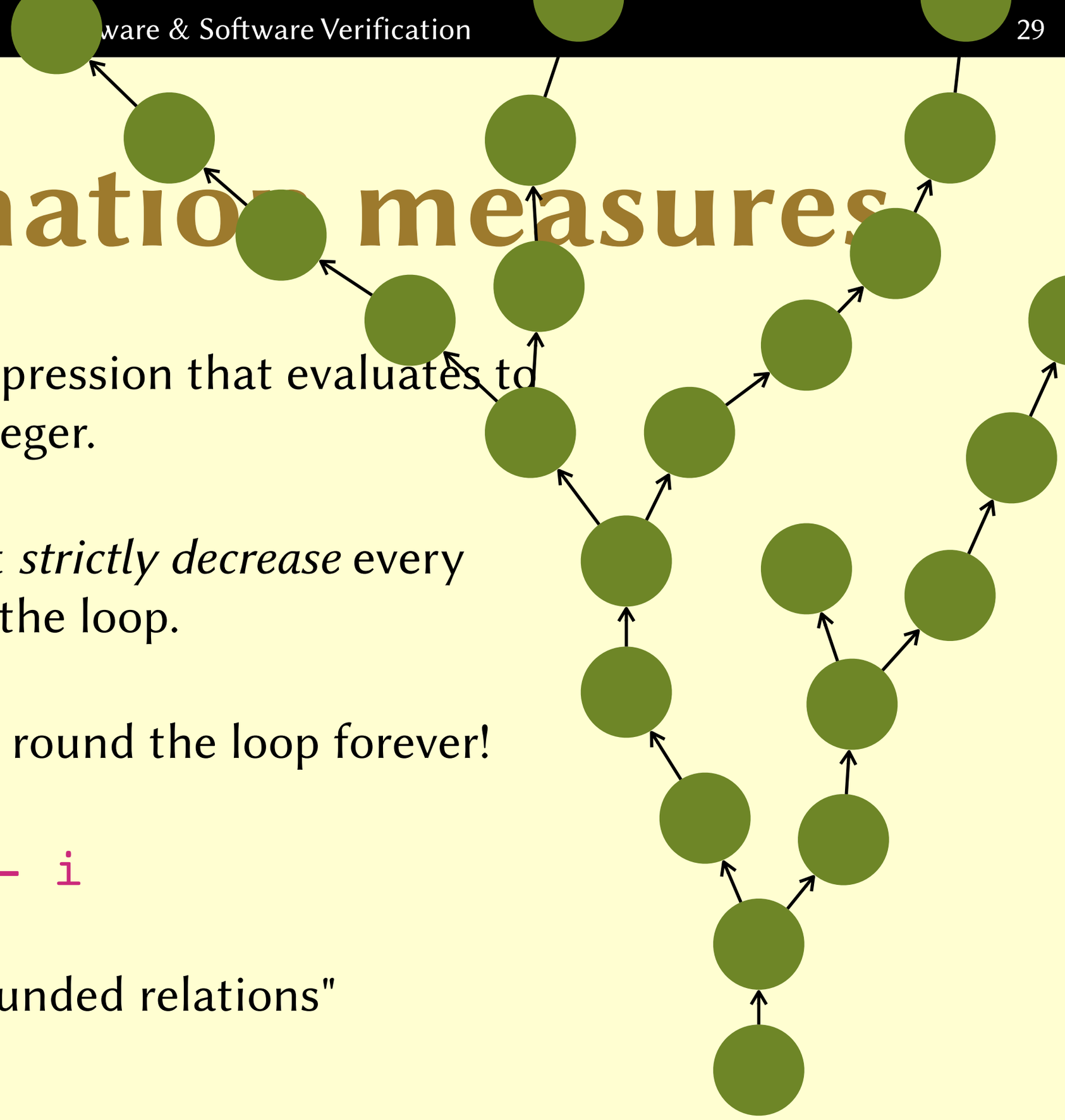

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6	9	✔	✔
7	9	✔	✔

Demo: max of an array

- syntax for variables (`var`) and arrays (`array<...>`)
- preconditions (`requires`)
- termination measures (`decreases`)
- universal (`forall`) and existential (`exists`) quantification
- loop invariants (`invariant`)
- predicates (`predicate`)