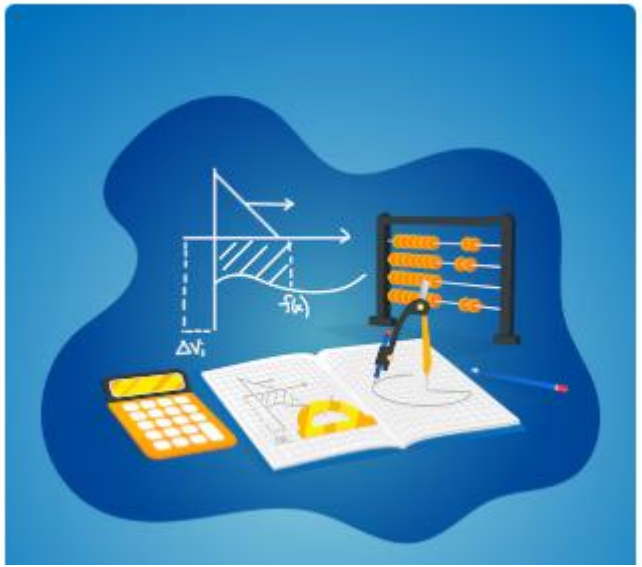
PW SKILLS | Aptitude Training

Quantitative Aptitude:

Number Systems

Remainders

Remainders: Problems Part 3



Q1. What is the remainder when 62^{62} is divided by 5?

A. 2 B. 1 C. 0 D. 4

$\frac{62^{62}}{5}$ R $\Rightarrow \frac{2^{62}}{5} = \frac{(2^2)^{31}}{5} = \frac{4^{31}}{5}$

$(x^a)^b = x^{a \times b}$

R $\Rightarrow (-1)^{31} = -1$

$\Rightarrow$ Rem $= 5 + (-1)$

$= 4$

Q2. What is the remainder when 27^{27} is divided by 5?

A. 2 B. 1 C. 0 ~~D. 3~~ ✓

$\frac{27^{27}}{5}$ R $\Rightarrow$ $\frac{2^{27}}{5} = \frac{2^{26} \times 2^1}{5} = \frac{(2^2)^{13} \times 2}{5}$

$= \frac{4^{13} \times 2}{5}$ R $\Rightarrow (-1)^{13} \times 2 = -1 \times 2$

$= -2$

$\Rightarrow$ Rem $= 5 + (-2)$

$= 3$

$x^a \times x^b = x^{a+b}$

Q3. What is the remainder when 24^{47} is divided by 7?

A. 2 B. 1 C. 5 D. 3

$\frac{24^{47}}{7}$ R $\Rightarrow \frac{3^{47}}{7} = \frac{3^{45} \times 3^2}{7}$

$= \frac{(3^3)^{15} \times 9}{7} = \frac{27^{15} \times 9}{7}$

R $\Rightarrow (-1)^{15} \times 2 = -1 \times 2 = -2$

$\Rightarrow$ Rem $= 7 + (-2) = 5$

$7)27(3$
-21
6

$7)27(4$
-28
-1

Q4. What is the remainder when $(1^1 + 2^2 + 3^3 + \ldots + 100^{100})$ is divided by 4?

A. 2 B. 1 C. 0 D. 3

$$\frac{1^1 + 2^2 + 3^3 + 4^4}{4}$$

$\Rightarrow 1 + 0 + (-1) + 0$

$= 0$

$$\frac{5^5 + 6^6 + 7^7 + 8^8}{4}$$

$$\Rightarrow \frac{1^5 + 2^6 + (-1)^7 + 0}{4}$$

$\Rightarrow 1 + 0 - 1 + 0$

$= 0$

Q5. What is the remainder when $(7^{21} + 7^{22} + 7^{23} + 7^{24})$ is divided by 25?

A. 24 B. 1 C. 0 D. 12

$$\frac{7^{20}\left(7 + 7^2 + 7^3 + 7^4\right)}{25}$$

$$\frac{(7^2)^{10}\left(7 + 7^2 + 7^2 \times 7 + (7^2)^2\right)}{25}$$

R $\rightarrow (-1)^{10}\left[7 + (-1) + (-1) \times 7 + (-1)^2\right]$

$= 1\left[7 - 1 - 7 + 1\right]$

$= 0$

25)49(2
-50
-1

Q6. $P = (1!)^2 + (2!)^2 + (3!)^2 + \ldots + (100!)^2$.

The remainder when 5^{2P} is divided by 13 is:

A. 1 ~~B. 12~~ C. 0 D. 2

$\frac{5^{2P}}{13} = \frac{25^{P}}{13} \xrightarrow{R} (-1)^{P}$

P: even Rem: $(-1)^{\text{even}} = 1$

P: odd Rem: $(-1)^{\text{odd}} = -1 \Rightarrow 12$

$13\overline{)25}\,(2$

-26

-1

$P = (1!)^2 + \underbrace{(2!)^2 + (3!)^2 + \ldots + (100!)^2}_{\text{even}}$ $n \geq 2$; $n!$ = even

odd + even

P = odd

Rem = 13 + (−1) = 12

Q7. Find the remainder:

$$\frac{1234567^{81} \times 369^{135} + 654^{999!}}{5}$$

$\Rightarrow \frac{2^{81} \times (-1)^{135} + (-1)^{999!}}{5}$

$= \frac{2^{80} \times 2^{1} \times (-1) + 1}{5}$

$= \frac{(2^{2})^{40} \times 2 \times (-1) + 1}{5}$

$= \frac{4^{40} \times 2 \times -1 + 1}{5}$ $\Rightarrow (-1)^{40} \times 2 \times -1 + 1$

$= 1 \times 2 \times (-1) + 1$

$= -2 + 1 = -1$

$\Rightarrow$ Rem $= 5 + (-1)$

$= 4$

Thanks!