Appendix: Dynamics Equations in the Sagittal Plane of Wheeled Foot Bipedal Robots

$$\tau = \mathrm{M}(q,\theta_0)\ddot{q} + C\left(q,\dot{q},\theta_0,\dot{\theta}_0\right) + G(q,\theta_0)a$$

$$\mathrm{M}(q,\theta_0) = \begin{Bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{Bmatrix}$$

$$C\left(q,\dot{q},\theta_0,\dot{\theta}_0\right) =$$

$$\begin{Bmatrix} m_3 l_3 l_1 s_{23} + m_2 x_{2c} l_1 s_2 + m_3 l_2 l_1 s_2 & m_3 l_3 l_2 s_3 - m_1 x_{1c} l_2 s_2 - m_2 l_1 x_{2c} s_2 & -m_3 l_2 l_3 s_3 - m_3 l_1 l_3 s_{23} \\ m_3 l_3 l_1 s_{23} + m_2 x_{2c} l_1 s_2 + m_3 l_2 l_1 s_2 & m_3 l_3 l_2 s_3 & -m_3 l_3 l_3 s_3 \\ m_3 l_3 l_1 s_{23} & m_3 l_3 l_2 s_3 & 0 \end{Bmatrix} \begin{Bmatrix} \left(\dot{\theta}_0 + \dot{\theta}_1\right)^2 \\ \left(\dot{\theta}_0 + \dot{\theta}_1 + \dot{\theta}_2\right)^2 \\ \left(\dot{\theta}_0 + \dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3\right)^2 \end{Bmatrix}$$

$$m_{11} = I_1 + m_3 l_3 l_1 c_{23} + m_2 x_{2c} l_1 c_2 + m_3 l_2 l_1 c_2 + m_1 {x_{1c}}^2 + m_3 {l_1}^2 + m_2 {l_1}^2 + I_2 + m_3 l_3 l_2 c_3 + m_2 {x_{2c}}^2 + m_3 {l_2}^2 + m_3 l_1 l_2 c_2 + m_2 l_1 x_{2c} c_2 + I_3 + m_3 {l_3}^2 + m_3 l_2 l_3 c_3 + m_3 l_1 l_3 c_{23}$$

$$m_{12} = I_2 + m_3 l_3 l_2 c_3 + m_2 {x_{2c}}^2 + m_3 {l_2}^2 + m_3 l_1 l_2 c_2 + m_2 l_1 x_{2c} c_2 + I_3 + m_3 {l_3}^2 + m_3 l_2 l_3 c_3 + m_3 l_1 l_3 c_{23}$$

$$m_{13} = I_3 + m_3 {l_3}^2 + m_3 l_2 l_3 c_3 + m_3 l_1 l_3 c_{23}$$

$$m_{21} = m_3 x_3 l_1 c_{23} + m_2 x_2 l_1 c_2 + m_3 l_2 l_1 c_2 + I_2 + m_3 l_3 l_2 c_3 + m_2 {x_2}^2 + m_3 {l_2}^2 + I_3 + m_3 {l_3}^2 + m_3 l_2 l_3 c_3$$

$$m_{22} = I_2 + m_3 l_3 l_2 c_3 + m_2 {x_{2c}}^2 + m_3 {l_2}^2 + I_3 + m_3 {l_3}^2 + m_3 l_2 l_3 c_3$$

$$m_{23} = I_3 + m_3 {l_3}^2 + m_3 l_2 l_3 c_3$$

$$m_{31} = m_3 l_3 l_1 c_{23} + m_3 l_3 l_2 c_3 + I_3 + m_3 {l_3}^2$$

$$m_{32} = m_3 l_3 l_2 c_3 + I_3 + m_3 {l_3}^2$$

$$m_{33} = I_3 + m_3 {l_3}^2$$

$$G(q,\theta_0) = \begin{Bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{Bmatrix}$$

$$g_{11} = -(m_3 l_3 c_{0123} + m_2 x_{2c} c_{012} + m_3 l_2 c_{012} + m_1 x_{1c} c_{01} + m_3 l_1 c_{01} + m_2 l_1 c_{01})$$

$$g_{12} = -(m_3 l_3 c_{0123} + m_2 x_{2c} c_{012} + m_3 l_2 c_{012})$$

$$g_{13} = -m_3 l_3 c_{0123}$$

$$g_{21} = -(m_3 l_3 s_{0123} + m_2 x_{2c} s_{012} + m_3 l_2 s_{012} + m_1 x_{1c} s_{01} + m_3 l_1 s_{01} + m_2 l_1 s_{01})$$

$$g_{22} = -(m_3 l_3 s_{0123} + m_2 x_{2c} s_{012} + m_3 l_2 s_{012})$$

$$g_{23} = -m_3 l_3 s_{0123}$$

$$g_{31} = I_1 + m_3 l_3 l_1 c_{23} + m_2 x_{2c} l_1 c_2 + m_3 l_2 l_1 c_2 + m_1 {x_{1c}}^2 + m_3 {l_1}^2 + m_2 {l_1}^2 + I_2 + m_3 l_3 l_2 c_3 + m_2 {x_{2c}}^2 + m_3 {l_2}^2 + m_3 l_1 l_2 c_2 + m_2 l_1 x_{2c} c_2 + I_3 + m_3 {l_3}^2 + m_3 l_2 l_3 c_3 + m_3 l_1 l_3 c_{23}$$

$$g_{32} = I_2 + m_3 l_3 l_2 c_3 + m_2 {x_{2c}}^2 + m_3 {l_2}^2 + m_3 l_1 l_2 c_2 + m_2 l_1 x_{2c} c_2 + I_3 + m_3 {l_3}^2 + m_3 l_2 l_3 c_3 + m_3 l_1 l_3 c_{23}$$

$$g_{33} = I_3 + m_3 {l_3}^2 + m_3 l_2 l_3 c_3 + m_3 l_1 l_3 c_{23}$$