状态空间方程

$$x_k = Ax_{k-1} + Bu_{k-1} + w_{k-1}$$ ← process 噪声

$$z_k = Hx_k + v_k$$ ← 测量噪声

$$P(w) \sim (0, \overset{E}{\downarrow}Q)$$ ← 协方差矩阵

$$P(v) \sim (0, R)$$

$$Q = E(w\,w^T)$$

$$x_k: \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad E\left[\begin{bmatrix} w_1 \\ w_2 \end{bmatrix}\begin{bmatrix} w_1 & w_2 \end{bmatrix}\right] = \begin{bmatrix} E_{w_1^2} & E_{w_1w_2} \\ E_{w_2w_1} & E_{w_2^2} \end{bmatrix}$$

$$VAR(X) = E(X^2) - \underbrace{E^2(X)}_{0}$$

先验 → $$\hat{x}_k^- = A\hat{x}_{k-1} + Bu_{k-1}$$

$$z_k = Hx_k \rightarrow \hat{x}_{mea} = H^- z_k$$

后验 $$\hat{x}_k = \hat{x}_k^- + G(H^- z_k - \hat{x}_k^-)$$ (Data fussion)

$$(G \in (0, 1))$$

注：一般误差服从高斯分布

令 $G = K_k H$

卡尔曼滤波 $$\boxed{\hat{x}_k = \hat{x}_k^- + K_k(z_k - H\hat{x}_k^-)}$$ $K_k \in (0, H^-)$

目标：寻找 K_k，使得 $\hat{x}_k \rightarrow x_k$ 实际值.

引入误差 $e_k = x_k - \hat{x}_k$.

$$P(e_k) \sim (0, P)$$

$$P = E(e\,e^T) = \begin{bmatrix} \sigma_{e_1}^2 & \sigma_{e_1}\sigma_{e_2} \\ \sigma_{e_2}\sigma_{e_1} & \sigma_{e_2}^2 \end{bmatrix}$$

期望选取合适的 K，使得 $tr(P)$ min

即方差最小.

$$x_k - \hat{x}_k = x_k - (\hat{x}_k^- + K_k(z_k - H\hat{x}_k^-))$$

$$= x_k - \hat{x}_k^- - K_k z_k + K_k H\hat{x}_k^-$$

$$z_k = Hx_k + v_k$$

$$= x_k - \hat{x}_k^- - K_kHx_k - K_kv_k + K_kH\hat{x}_k^-$$

$$= (I - K_kH)\underbrace{(x_k - \hat{x}_k^-)}_{e_k^-} - K_kv_k$$

$$P = E[e\,e^T]$$

$$= E[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T] \longrightarrow$$

$$= E[((I - K_kH)e_k^- - K_kv_k) \sim^T]$$

$$= E[(I - K_kH)e_k^- e_k^{-T}(I - K_kH)^T - (I - K_kH)\underbrace{e_k^- v_k^T}_{0} K_k^T - K_k\underbrace{v_k e_k^{-T}}_{0}(I - K_kH)^T + K_kv_kv_k^TK_k^T$$