Long May They Reign: How Majestic Dragons Can Survive In The New Realm
愿它们长久统治：巨龙如何在新的领域生存

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Abstract

The dragon has always been imagined by people in both East and West. If a dragon lives in our world, it will be very meaningful and interesting to study its characteristics and interaction with ecosystem.

First, the basic physical characteristics of the dragon are studied. According to the growth characteristics of creatures, we establish **the Dragon's Weight Logistic Growth Model**. On the basis of this model, it is estimated that the weight of a mature dragon is between 10 tons and 20 tons. Then, by using the mechanics knowledge and analogy method, we analogize the trunk of the dragon to the elastic cylinder and estimate that the length of the trunk of the mature dragon is about 4 meters to 6 meters and that the length of the whole body of the mature dragon is about 20 meters to 35 meters. **Secondly**, we analyze the daily energy intake and expenditures of the dragon. Comparing the dragon with the creatures that have similar characteristics, we estimate the daily energy intake and diet of the dragon, and the rate of energy consumption during its flight. In addition, given the conditions of breathing fire, we establish **the Dragon's Fire Chemical Kinetics Model** so as to analyze the mechanism and energy consumption of breathing fire. **Thirdly**, on the basis of the results above, we consider the minimum requirement of the habitat for maintaining the dragon. In order to analyze the impact of the dragon on the ecosystem, we establish the **Dragon-Sheep-Habitat** Model based on the Logistic model. More importantly, we compare the difference of the resources for maintaining a dragon in arid, warm temperate and arctic regions, and analyze the living status of the dragon in these regions.

What's more, in the sensitivity analysis, we focus on three factors, namely the inherent growth rate of a dragon's weight, environmental factors and environmental temperature, in order to analyze their impacts on the weight of a mature dragon as they change. **Finally**, we write a letter to George R.R. Martin, the author of A Song of Ice and Fire, to provide some advice on maintaining the realistic ecological basis of the story. Although the dragon does not exist in the real world, the models established in this paper can also be applied to solve the problem about the growth of dinosaurs, the invasion of alien species and the design of new flamethrowers.

Note: To avoid copyright infringement, we draw and generate all the figures on our own.

龙一直存在于东西方人的文化之中。如果一条龙生活在当今世界，研究它的特点和它对生态系统的影响将会十分有趣和有意义。

首先，我们研究了龙的基本物理特征。根据生物的生长特点，我们建立了**龙体重的 Logistic 生长模型**。在这个模型的基础上，我们估算了一条成年龙的体重在 10 吨到 20 吨之间。然后，利用力学知识和类比法，我们将龙的躯干简化成弹性圆柱体。由此我们估算出了成年龙的躯干长度约为 4 米至 6 米，而成年龙的身体全长大约为 20 米到 35 米。**其次**，我们分析了龙的日常能量摄入和支出。对比与龙具有相似特征的生物，我们估算了龙的日常能量摄入和饮食，以及其飞行过程中的能量消耗率。此外，根据龙喷出火焰（龙焰）的条件，我们建立了**龙焰的化学动力学模型**，以分析龙焰的机理和能耗。**最后**，根据上述结果，我们考虑了维持龙生存的栖息地的最低要求。为了分析龙对生态系统的影响，我们基于 Logistic 模型建立了**龙-羊-栖息地模型**。而且，我们比较了干旱、温暖和寒冷地区的资源差异对龙的影响，并分析了龙在这些区域的生存状态。

此外，在灵敏度分析中，为了分析参数变化对成年龙体量的影响，我们重点考查了三个因素，即龙体量的固有增长率、环境因素和环境温度。**最后**，我们给《冰与火之歌》的作者乔治 · R · R · 马丁写了一封信，并提供了一些关于故事的现实生态基础的建议。虽然龙在现实世界中并不存在，但本文建立的模型也可能有助于解决恐龙生长、外来物种入侵和新型火焰喷射器设计等难题。

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Contents

1 Introduction | 引言

1.1 Problem Restatement | 问题重述

Dragon, the most legendary behemoths from antiquity appeared in the phenomenal TV series Game of Thrones adapted by George R.R. Martin's best-seller Song of Ice and Fire, which attracted eyeballs around the world. In the story, at the very beginning of their birth, they were small and thin, on the other hand, they were able to grow up at a rapid rate. Now, we assume that the three dragons in the series live in the real world and their growth meets basic ecological principles. We need to build mathematical models to solve the following problems:

1. Analyze the physical characteristics of a dragon and estimate its daily energy expenditures and the minimum of calorie intake.

2. Investigate the impact of the dragon's appearance on the ecological environment in the real world, and figure out the area that could sustain the dragons' life as well as the ecological conditions and the extent of the biomes that satisfy the necessary living conditions of them.

3. Resembling other animals, the dragon would migrate to various places as the local temperature changes. Therefore, we need evaluate the differences in the vital requirement of them living in an arid region, a warm temperate region, and the arctic region.

龙是自古以来最具传奇色彩的巨兽，出现在现象级电视连续剧《权力的游戏》之中，吸引了全世界的眼球。该剧是由乔治 R.R. 马丁的畅销书《冰与火之歌》改编而来。在剧中，刚出生的龙宝宝又小又瘦，但是他们的成长速度很快。在本文中，我们假设剧中的三条龙均生活在现实世界中，它们的生长规律遵循基本生态学原理（basic ecological principle）。我们需要建立数学模型来解决以下问题：

1. 分析龙的物理特征，估算其日常能量消耗和卡路里摄入量的最小值。

2. 研究龙的存在对现实世界中生态环境的影响，确定能够维持龙日常生活的区域大小，以及满足龙生存所需的生态条件和生物群落范围。

3. 随着当地温度的变化，与其他动物类似，龙也会迁徙到不同的地方。因此，我们需要分析他们在干旱、温暖和寒冷地区生存需求的差异。

1.2 Problem Analysis | 问题分析

Since the dragon is a kind of beasts in fiction, for avoiding starting from scratch, we ought to consider them by analogy to animals existing in reality. We apply the related knowledge about ecology and biology etc. and the rule of thumb to help us clarify the ecological characteristics of the dragon. To help us conceive the image of the dragon better, we draw the conceptual graph, cf. Figure 1. n.b. To avoid disputes caused by copyright law, all figures in this paper are drawn, or generated by computer.

由于龙是科幻题材中的一种生物，为了避免从零开始，我们类比现实中存在的动物，以此来分析龙的特征。我们借助生态学、生物学等相关知识及经验法则来帮助我们弄清龙的生态特征。为了帮助我们更好地构思龙的形象，我们绘制了龙的示意图，参见图1。注意：为了避免版权法引起的纠纷，本文中的所有插图都是由我们自己用计算机绘制或生成的。

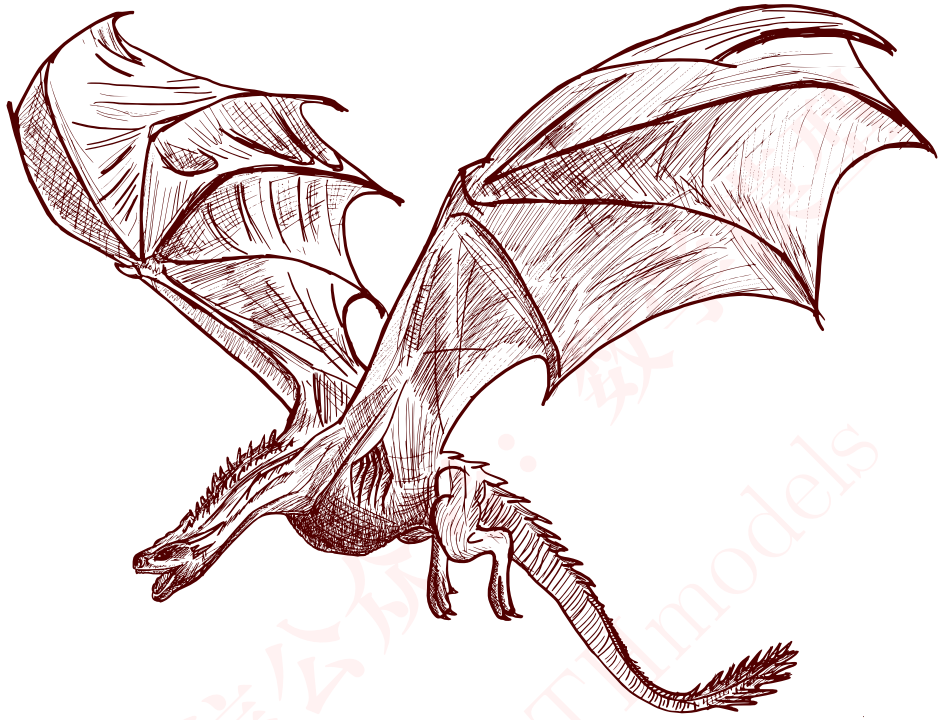

Figure 1: The Hand Drawing Schematic Diagram of Dragon | 龙的手绘示意图

First of all, concerning the physical characteristics of the dragon, we focus on the trend between their weight and length change in accordance of time. Considering the above change is continuous, we set up a logistic growth differential equation, using the dragon's birth weight and the weight next year as two initial values to obtain the weight function changing by time. Consequently, we could estimate the approximate range of the mature dragon's weight.

When analyzing the length change of the dragon, we assume that the dragon's limbs are on the ground when it is staying still, and regard it as an elastic cylinder. Based on mechanics, we can calculate the ratio of weight and length. Hence according to the initial value of weight and length of the dragon's birth, we can fi-

首先，为了考虑龙的物理特征，我们研究了龙的体重和长度随时间变化的关系。考虑到以上的变化是连续的，我们建立了一个 logistic 生长微分方程，并利用龙出生时和次年的体重这两个初始值，得到了体重随时间变化的函数。最终，我们能够估算出成年龙体重的大概范围。

在分析龙的长度变化时，我们假设龙的四肢静止不动地站在地面上，并把它当作一个弹性圆柱体。借助力学知识，我们可以推导出龙的体重与长度的比例关系。因此，根据龙出生时体重和长度的初始值，可以最终确定比例系数以及长

nally confirm the scale coefficient and the functional relationship of length to time. Considering that a dragon needs to maintain primary metabolism and life activities, the more weight it holds, the more energy it consumes. We estimate the daily energy consumption of a mature dragon according to an adult man. To maintain its long-term life activities, the dragon's daily energy intake must be approximately equal to its consumption.

Based on the above analysis, we study the habitat area of the three dragons and the conditions for the ecosystem and community size of the dragon. We assume that the three dragons will not compete with each other so that we only need to deal with the individual dragon entering a balanced ecosystem. Without loss of generality, we consider the dragon as the top predator of the ecosystem, regarding all animals as sheep and the area of the ecosystem as the habitat area of the dragon.

Then we let the number of sheep represent the supply of resources to the dragon in the habitat, and assume that this value is proportional to the ecosystem's area. Considering the factors affecting the number of sheep, we establish the Dragon-Sheep-Habitat Model based on differential equations and obtain a curve of the number of sheep over time.In order to ensure the sustainable supply of the ecosystem to the dragon, the initial number of sheep should have a minimum, otherwise the ecosystem will collapse due to the slow recovery.

To meet the minimum requirements for the dragon's activities, we consider this minimum as the community's capability to support the dragon in the habitat as the minimum standard for measuring the size of the ecosystem. Then, from this minimum value, we can refer the minimum habitat area for maintaining the dragon's survival. Hence, based on the above results, we discuss the impact of dragons on the ecosystem.

Considering the migration of dragons, the survival pressures of dragons in varied climate regions are different. We focus on the three different climate regions of the arid region, the warm temperate regions and the arctic region, to analyze the effect of different regions have on the survival resources of mature dragons. According to the above analysis of the daily energy intake of the dragon and the Dragon-Sheep-Habitat Model, we discuss the minimum requirement of resources for the dragon's long-term survival based on the different climatic conditions, the dragon's consumption and the ecosystem's supply difference, as well as the impact of the

度与时间的函数关系。考虑到龙需要维持必要的新陈代谢和生命活动，它的体重越大，其消耗的能量就越多。因此，我们以一个成年男子的日常消耗为标准，来估计一条成年龙的每日能量消耗。而且，为了保持其长期稳定的生命活动，龙的日常能量摄入量必须大致等于其消耗量。

基于上述分析，我们研究了三条龙的栖息地面积以及该区域的生态系统和群落规模状况。假设三条龙不会相互竞争，所以我们只需要研究一条龙进入平衡生态系统的情况。不失一般性，我们认为龙是生态系统中的顶级捕食者，并且把其它所有动物均视为羊，生态系统区域视为龙的栖息地。

然后，我们让一定数量的羊代表栖息地中的龙的资源供应，并假设这个值与生态系统的区域大小成正比。考虑到影响羊数量的因素，利用微分方程，我们建立了龙-羊-栖息地的模型，并得到了羊数随时间变化的曲线。为了保证龙的生态系统的可持续供给，羊数量的初值应该大于某个最小值，否则生态系统将因恢复缓慢而崩溃。

为了满足龙生存的最低要求，我们将这个最低要求视为栖息地的群落支撑龙的能力，同时也是衡量生态系统规模的最低标准。然后，根据这个最小值，我们可以得到维持龙生存的最小栖息地面积。因此，基于上述结果，我们讨论了龙对生态系统的影响。

考虑到龙的迁徙，不同气候区域的龙的生存能力是不同的。我们重点从干旱、温暖和寒冷三个不同气候区域入手，分析不同地区的生存资源对成年龙的影响。根据以上对成年龙每日能量摄入和龙-羊-栖息地模型的分析，并从不同的气候条件，龙的能量消耗和生态系统的供给差异，以及龙对当地生态系统的影响，我们讨论了龙长期生存的最低资源要求。

dragon on the local ecosystem.

2 Assumptions and Justifications | 假设和理由

By adequate analysis of the problem, to simplify our model, we make the following well-justified assumptions.

1. The growth of dragons conforms to fundamental biological laws. If the dragon lives in the real world, as a species on the earth, its growth and maturation should be the same as other creatures, to meet the fundamental biological laws.

2. The dragon is a homothermal animal whose body temperature is not affected by the environment.

3. The dragon can fly and withstand huge wounds, and it will not easily die due to physical, chemical and biological attacks. According to the mythology and the TV series, we can consider that the dragon is with a capability to fly, and its flight satisfies the aerodynamics. In the meantime, the dragon is overgrowing, and its matured shape surpasses all the terrestrial creatures, and any species is unable to bear the Dracarys from the dragon.

4. The dragon's daily energy intake at least meets its daily energy consumption for maintaining its primary life activities and growth.

5. We suppose the birth length of a dragon is 30-40 cm. Since the dragon's Physiological information is lack of scientific basis, we acquire the above assumption by inferring from ancient reptile larvae and the TV series.

6. Once the dragon enters an ecosystem, it immediately becomes the top predator. Nevertheless, it will not cause catastrophic destruction to the biosphere. Besides, humankind can efficiently control the dragon.

全面地分析问题后，为了简化我们的模型，我们做出以下合理假设。

1. 龙的生长符合基本生物规律。作为地球上的一个物种，如果龙生活在现实世界中，那么它的生长和发育应该与其他生物一样，符合基本的生物学规律。

2. 龙是一种恒温动物，其体温不受环境影响。

3. 龙可以在空中飞翔，它的护甲能够抵御强烈的攻击，不会轻易死于物理、化学和生物攻击。
根据神话传说和电视连续剧，我们认为龙具有飞行能力，它的飞行符合空气动力学。同时，龙生长发育迅速，其成年的形态巨大，超过了所有陆生生物，任何物种都无法承受龙焰。

4. 龙的日常能量摄入量必需满足其日常能量消耗，以维持龙的基本日常生活和生长发育。

5. 我们假设龙的出生时的长度为 30–40 cm。由于龙的生理信息缺乏科学依据，我们从古代爬行动物幼体和电视剧的信息中得到上述假设。

6. 一旦龙进入生态系统，它便会成为顶级捕食者。尽管如此，龙并不会对生物圈造成灾难性的破坏。此外，我们假设人类能够有效地控制龙的行为。

3 Notations | 符号定义

For convenience, we use the following symbols in our models, cf. Table 1.

Table 1: Symbols and its Description | 符号和描述

Symbols | 符号	Description	描述
t	The time	时间
$W(t)$	The dragon's weight function changing by time	龙的体重随时间变化的函数
$W_{\max}$	The theoretical weight of the mature dragon	成年龙的理论最大体重
$L(t)$	The dragon's length function changing by time	龙的长度随时间变化的函数
E	The daily energy consumption of the dragon	龙每日的能量消耗量
T	The environmental temperature	环境温度
r	The inherent growth rate of the dragon	龙的固有生长速率
σ	The environmental factor	环境因子
A	The area of a balanced ecosystem	平衡生态系统的面积大小
$N(t)$	The number of sheep function changing by time	羊的数量随时间变化的函数
K	The maximum carrying capacity of an ecosystem	生态系统最大的承载力
b	The environmental carrying capacity coefficient	环境承载能力的参数

4 Dragon's Physical Characteristics and Diet | 龙的身体特征与饮食

4.1 Description of Dragon's Weight and Length | 龙的体重和长度的描述

To clearly describe the physiological characteristics of a dragon [1], we consider that the dragon lives in a warm temperate region with a suitable climate, assuming that the temperature of the region is maintaining at 25 °C. Therefore, the dragon can obtain sufficient food from the region during its growth and maturation. Also, based on morphology, the size of creatures cannot be increased indefinitely. Similarly, the dragon should be the same, which guarantees that when the dragon grows to maturation, its final weight and length will be limited. In the meantime, we define the dragon's body length as the straight distance from shoulder to hip.

为了清楚地描述龙的生理特征 [1]，我们认为龙生活在温暖的温地区，气候适宜，气温保持在 25 °C。龙在生长和发育期间可以从该地区获得足够的食物。此外，根据形态学，生物的尺寸不能无止境地生长。同样，龙也应该不能无止境地生长，这保证了当龙成年时，其最终的体重和长度将是有限的。同时，我们把龙的身体长度定义为从肩部到臀部的直线距离。

4.1.1 Dragon's Weight Logistic Growth Model | 龙体重 Logistic 增长模型

Considering when the dragon is born, its weight $W(t)$ changes continuously by time. In the process of its growth to maturation, we regard that it is always healthy and will not die. Based on the above assumption, we denote that the theoretical weight of the mature dragon is $W_{\max}$ kg.

By the Logistic retarded growth principle, we reckon that the effect of the weight growth rate should be embodied in the retardation force among the dragon's weight limitation and the environmental carrying capacity etc.

For now, we consider the most basic case: the growth without any limitation such as the environment and its maximum weight. Under the above situation, the dragon's weight should meet the equation (1).

$$\frac{\mathrm{d}W(t)}{\mathrm{d}t} = rW(t) \tag{1}$$

However, because of the morphological limitation of the dragon per se, the right side of the equation (1) should include the maximum weight retardation factor $(1-W(t))/W_{\max}$. Hence, we correct the equation (1) to the equation (2)

$$\frac{\mathrm{d}W(t)}{\mathrm{d}t} = rW(t)\left(1-\frac{W(t)}{W_{\max}}\right) \tag{2}$$

Besides, we consider the effect of the environment on the dragon's weight. Since for many organisms, their final size will decrease with increasing temperature [2]. In our model, we only consider the effect of temperature on the dragon's weight to substitute the environment effect.

We assume that the effect of temperature on the growth rate of dragons is linear, so we let σT refer to the inherent retardation of temperature on the growth rate. Thus correcting the retardation factor in equation (2) to $(1-W(t)/W_{\max}-\sigma T)$, we finally obtain the weight-time differential equation (3).

$$\frac{\mathrm{d}W(t)}{\mathrm{d}t} = rW(t)\left(1-\frac{W(t)}{W_{\max}}-\sigma T\right) \tag{3}$$

We choose the dragon's birth weight 10 kg as the initial value of the equation (3), i.e., $W(0) = 10$. We can possess the weight-time first-order nonlinear differential equation's Cauchy problem (4)

考虑到龙出生后，它的体重 $W(t)$ 便开始随时间而不断增长。在它生长发育的过程中，我们认为它总是健康的，并且不会死亡。基于上述假设，我们将成年龙的理论体重表示为 $W_{\max}$。

根据 Logistic 阻滞增长原理，我们认为减缓体重增长速率的因素应该体现在龙的最大体重和环境承载能力等因素上。

目前，我们考虑的最基本情况：龙的生长不受任何限制，如环境及其最大体重。在上述情况下，龙的体重应符合下式

$$\frac{\mathrm{d}W(t)}{\mathrm{d}t} = rW(t) \tag{1}$$

然而，由于龙本身的形态学限制，等式 (1) 的右边应该包括最大体重的阻滞因子 $(1-W(t))/W_{\max}$。我们将等式 (1) 修正为

$$\frac{\mathrm{d}W(t)}{\mathrm{d}t} = rW(t)\left(1-\frac{W(t)}{W_{\max}}\right) \tag{2}$$

与此同时，我们还考虑环境对龙的体重的影响。类比其他生物，他们最终的尺寸大小会随着环境温度的上升而减小 [2]。在我们所建立的模型中，我们只考虑温度对龙体重的影响，以此来代替整个环境的作用。

我们假设环境温度对龙生长速率的影响是线性的，并用 σT 来表示温度对生长速率的固有阻滞。由此，将式 (2) 中的阻滞因子修正为 $(1-W(t)/W_{\max}-\sigma T)$，我们最终得到了体重与时间的微分方程：

$$\frac{\mathrm{d}W(t)}{\mathrm{d}t} = rW(t)\left(1-\frac{W(t)}{W_{\max}}-\sigma T\right) \tag{3}$$

我们选择龙出生时的体重 10 kg 作为微分方程 (3) 的初始值，即 $W(0) = 10$。我们可以得到体重随时间变化的一阶非线性微分方程柯西（Cauchy）问题：

$$\begin{cases} \dfrac{\mathrm{d}W(t)}{\mathrm{d}t} = rW(t)\left(1 - \dfrac{W(t)}{W_{\max}} - \sigma T\right) \\ W(0) = 10 \end{cases} \tag{4}$$

To solve the Cauchy problem (4), we denote that $\alpha = 1 - \sigma T$. Eventually, we find $W(t)$, the Logistic weight-time function (5).

为了求解柯西问题，我们令 $\alpha = 1 - \sigma T$。最终，我们得到了 Logistic 体重-时间函数 $W(t)$：

$$W(t) = 10 \cdot \alpha \cdot W_{\max} \cdot \frac{\mathrm{e}^{\alpha r t}}{\alpha W_{\max} + 10\left(\mathrm{e}^{\alpha r t} - 1\right)} \tag{5}$$

Since the dragon's weight range from 30 to 40 kg next year, we assume that the growth rate of the dragon's weight is a constant. Based on the change in the dragon's weight during the year, we calculate the growth rate of the dragon's weight (6).

龙次年的体重在 30-40kg 之间，我们假设体重的增长速率是恒定的。根据体重的改变，我们算出了龙体重的增长速率：

$$r = \frac{\Delta W(t)}{t} \tag{6}$$

We can know that the range of growth rate r is between 20 and 30 kg per year. Without loss of generality, we set the following parameters, cf. Table 2.

我们能够得到增长速率 r 的范围在 20-30 kg 每年。不失一般性，我们设置了以下参数，参见表 2。

Table 2: The Parameter Values | 参数值列表

$W(1)$ (kg)	r (kg/year)	T (°C)	σ
35	20	25	0.04

Hence, we can generate the figure of the equation (5), cf. Figure 2.

由此，我们可以绘制出函数 (5) 的图像，参见图2。

译者注：方程 (4) 考虑温度的方式并不太好，其解 (5) 并不稳定。此外，如果按表2取值 $\sigma = 0.04$，$T = 25$，则方程 (4) 将退化。通过论文末尾所给程序可以判断，实际上 $\sigma = 0.03744$。另外，按照式 (5) 得到的图像也与图2有差别，应该是作者在写程序时出现了一些失误。

4.1.2 Dragon's Length Analysis Based on Analogy | 基于类比的龙长度分析

Consider the mature dragon is lying on the ground, with the limbs landing, staying still. Then we denote L m as the dragon's length, W kg as the dragon's

考虑一条成年的龙四肢着地、静止地站在地面上。我们记该龙的长度为 L m，体重为 W kg。为了简化这个问题，我

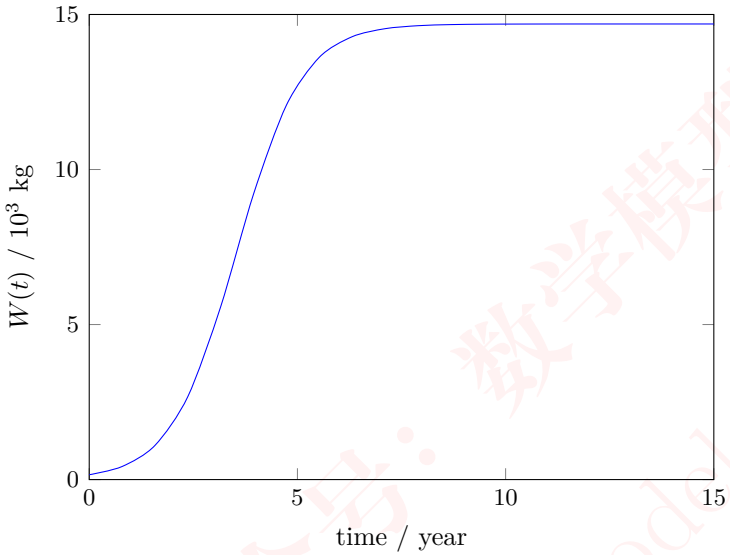

Figure 2: Dragon's Weight-Time Curve | 龙的体量-时间曲线

weight. For simplifying the problem, we regard the dragons body as an elastic cylinder with length L m, diameter D m, base area S m^2 , cf. Figure 3.

Since the dragon's body sags under the influence of the gravity, we set the bending degree of the elastic cylinder to C m, which we can have the relationship (7) from mechanics [3].

$$\frac{C}{L} \propto \frac{WL^3}{Sd^2} \tag{7}$$

We know that during the evolution, C/L ought to be a constant [4]. Otherwise, creatures are unable to sustain their weight. Meanwhile, according to the relationship between mass and volume, base and diameter of the cylinder, we can obtain the relationship (8).

$$W \propto SL, \qquad S \propto d^2 \tag{8}$$

我们把龙的身体视为一个长 L m，直径为 D m，底面积为 S m^2 的弹性圆柱体。参见图3。

由于在重力的作用下，龙的身体会下弯，我们设下弯的长度为 C m，根据力学知识，有以下关系 [3]

我们知道在生物进化的过程中，C/L 应该是一个常数。否则，生物将无法承受自身的体重。同时，由质量与体积、底面积与直径的关系，我们得到了以下关系

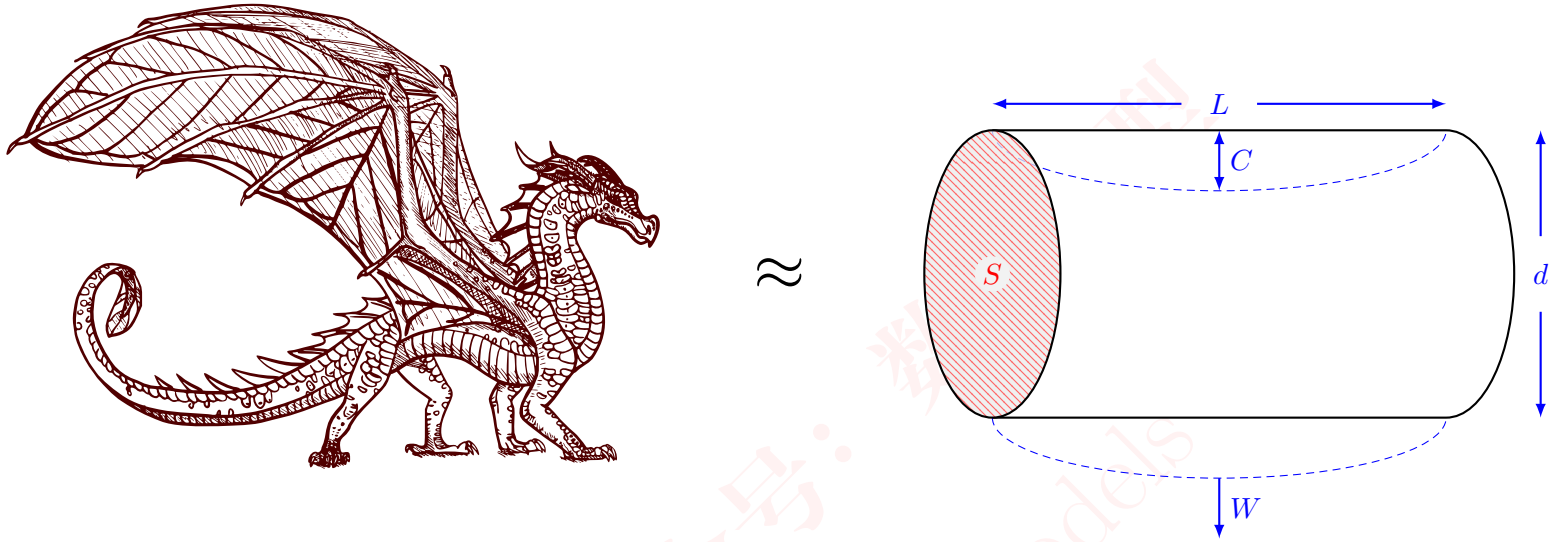

Figure 3: Regarding Dragon as an Elastic Cylinder by Analogy | 把龙比作一个弹性圆柱体

Via combining (7) and (8), we can finally obtain the equation (9), n.b. K is a scale coefficient.

$$W = kL^4 \tag{9}$$

Referring to the TV series, we assume that the infant dragon's length is from 0.7–0.9 m [5]. Combined with the infant dragon's weight, we can calculate the scale coefficient k is around 15.24 kg/m^4–41.65 kg/m^4. Since the mature dragon's weight is about 15 ton, according to the equation (9), we can know the mature dragon's length is around 4.35–5.63 m.

综合式 (7) 和 (8)，我们最终可以得到等式 (9)。注意：K 是一个比例系数。

根据电视剧的信息，我们假设龙出生时的长度在 0.7–0.9 m 之间 [5]。结合龙出生时的体重，我们计算得到比例系数 k 约为 15.24 kg/m^4–41.65 kg/m^4。鉴于成年龙的体重大约为 15 吨，由等式 (9)，我们能得到成年龙的长度约为 4.35–5.63 m。

译者注：作者推导过程和结果都令人费解，通常我们认为 $W \propto L^3$。作者的推导过程和结果应该都是错误的。

We choose 0.8 m as the initial value of the dragon's length. Based on Figure 2, we generate the change graph between length and time, cf. Figure 4.

我们选择 0.8 m 作为龙的长度的初始值。根据图 2，我们绘制了龙的长度随时间变化的关系图。参见图 4。

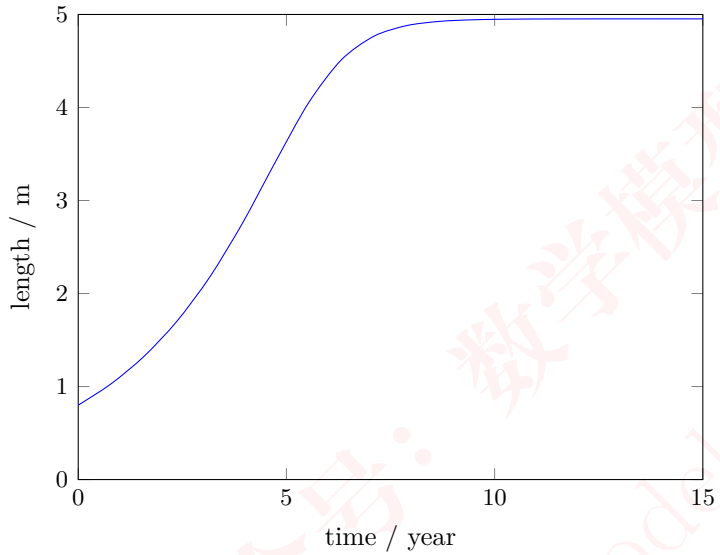

Figure 4: Dragon's Length-Time Curve | 龙的长度-时间曲线

4.2 Dragon's Energy Expenditure and Caloric Intake | 龙的能量消耗与摄入

Inheriting the assumption of section 4, we consider that the dragon is a homeothermic animal [6]. The dragon's daily energy intake must guarantee its primary metabolism. At the same time, as a carnivore, the dragon can be analyzed by its daily meat intake. When examining its energy consumption, we believe that its most crucial energy consumption comes from the flight and fire breathing, thus establishing the primary models [7].

接着 4 的假设，我们认为龙是一种恒温动物 [6]。所以，其龙每日的能量摄入必需满足最基本的新陈代谢。同时，作为食肉动物，龙的每日能量摄入可以用肉的消耗量来分析。在研究龙的能量消耗时，我们认为龙的打斗和使用龙焰是其最主要的能量消耗 [7]。

4.2.1 Dragon's Diet and Energy Consumption during Flight

To simplify the problem, we assume that the dragon is unable to breathe fire in this model. Hence, we can just investigate the energy consumption and intake in flight-only case. We compare the mature dragon with a Northman, a direwolf and a sparrow in different aspects [8].

为了简化问题，我们假设龙在这个模型中不能喷火。因此，我们只分析飞行情况下的能量消耗和摄入量。我们将成熟的龙与北境人、冰原狼和麻雀在不同的方面进行比较 [8]。

Based on the daily energy intake of an adult Northman (c. 2,000 cal/day), we estimate the daily energy intake of the mature dragon by calculating the ratio of the weight of an adult Northman to the mature dragon, and it is about 400,000 calories [9].

Next, based on the daily meat intake of a direwolf (c. 12 kg/day), by calculating the ratio of the direwolf's weight to the mature dragon. We estimate that the mature dragon's daily meat consumption is roughly 900 kg, if the sheep's weight is 100 kg, then the dragon needs to consume about sheep 9 pcs/day.

Finally, based on the fat consumption of a sparrow per hour under normal flight, it is about 0.08 g/hour. By calculating the ratio of the sparrow's weight to the mature dragon, similarly, we estimate that the mature dragon will consume around 80 kg/hour. For a more intuitive comparison among the different creatures and dragon, we draw the schematic diagram cf. Figure 5.

4.2.2 Dragon's Fire Chemical Kinetics Model | 龙火化学动力学模型

Estimation of the Temperature of Dragon's Fire. We can estimate the temperature by the status of burning objects. In the TV series, the Queen was fighting against the enemy troops while she was riding on the dragon. We suppose that the equipment of soldiers is made of iron. After the dragon's fire attack, all material is melt and vapored incompletely. The melting point of iron is 1538 °C, and the boiling point is 2862 °C [10]. As a result, it can be certain that the temperature of the dragon's fire is around 2862 °C.

Chemical Component of Fire's Fuel. According to the approximate temperature of 2862 °C, we list the burning temperature of common organic fuels [11], cf. Table 3.

From Table 3, the burning temperature of ether is most approximate to the dragon's fire. Hence, we could regard ether as the fire's fuel.

Chemical Kinetics Mechanism for Producing Ether. According to the Ockham's Razor Principle [12], the mechanism of chemicals produced by an organism should be the simplest. The simplest preparation method of ether is the dehydration of ethanol to ether under the catalysis of high temperature and concentrated sulfuric acid. We assume that the dragon can produce special catalysts,

根据成年北境人每天的能量摄入（约 2000 大卡/天），我们通过计算成年北境人和成年龙的体重比来估算成年龙每天的能量摄入，其值约为 400,000 大卡 [9]。

接下来，根据冰原狼每天的肉摄入量（约 12 公斤/天），计算冰原狼的体重与成年龙的体重之比。我们估算出成年龙每天的肉消费量大约是 900 公斤。如果一只羊按 100 公斤算，那么龙每天需要消耗大约 9 只羊。

最后，根据一只麻雀正常飞行下每小时的脂肪消耗（大约是每小时 0.08 克），计算麻雀的体重与成年龙的体重之比，我们估算出成年龙每小时大约消耗 80 公斤脂肪。为了更直观地比较不同的生物与龙的关系，我们绘制了示意图，参见图 5。

估计龙焰的温度。我们可以通过燃烧物体的状态来估计温度。在电视剧中，女王骑着龙与敌军作战。我们认为士兵的装备是铁制的。在受到龙焰攻击后，所有的兵器都不同程度地被熔化和汽化了。铁的熔点是 1538 °C，沸点是 2862 °C。由此可以确定，龙焰的温度大约为 2862 °C。

龙焰燃料的化学成分。根据龙焰的温度大约为 2862 °C，我们列了以下常见有机燃料的燃烧温度。参见表 3。

从表 3 可以看出，乙醚的燃烧温度与龙焰的温度最为接近。因此，我们可以把乙醚看作龙焰的燃料。

产生乙醚的化学动力学机理。根据奥卡姆剃刀原理（Ockham's Razor Principle）[12]，生物体产生化学物质的机制应该是最简单的。我们知道，乙醚最简单制备方法是在高温和浓硫酸的催化下将乙醇脱水成醚。我们假设龙能产生特殊的催化剂，可以放宽严格的反应条件，该化学反应的方程式如

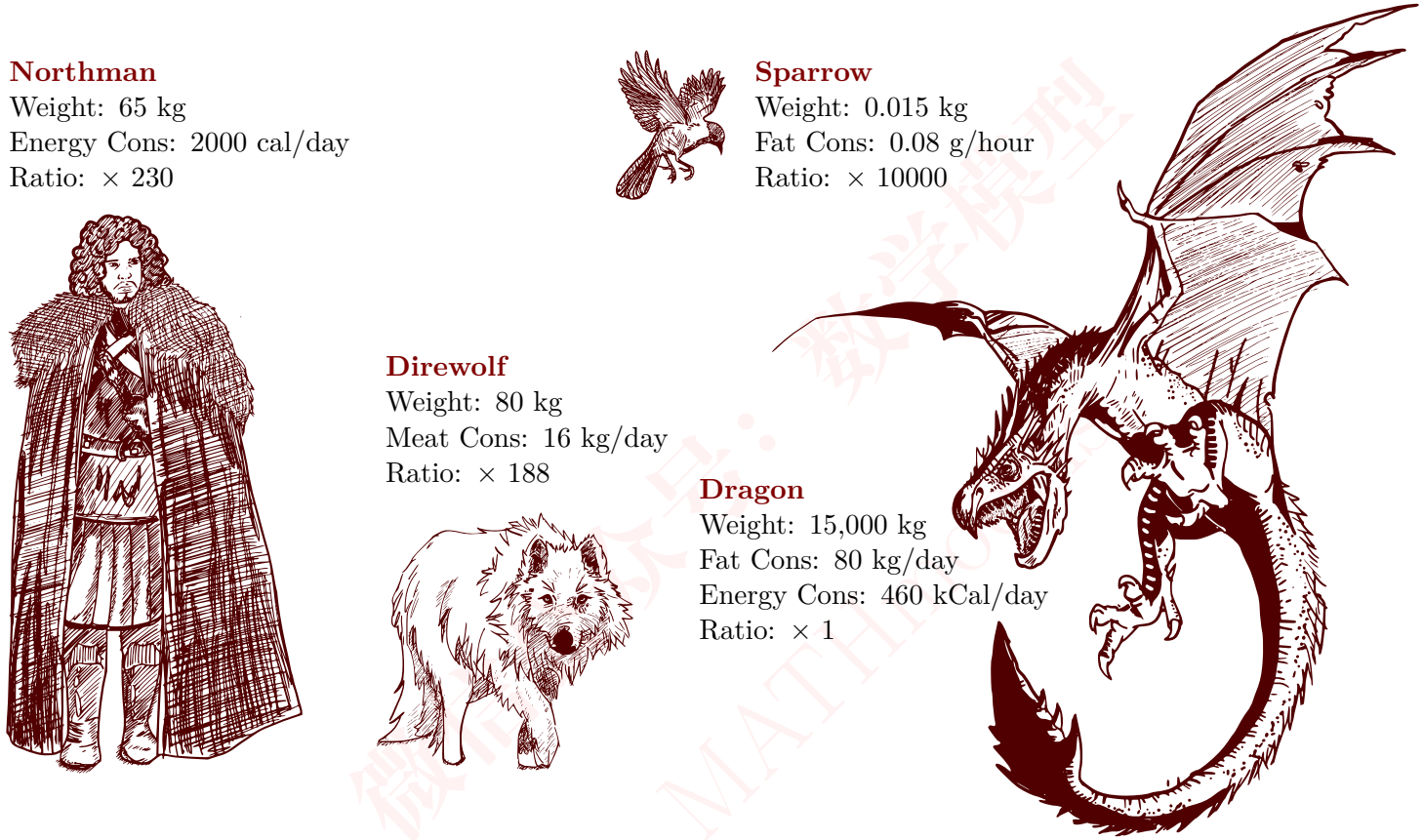

Figure 5: Consumption and Intake of Dragon vs. Other Creatures | 龙的消耗和摄取与其他生物的对比

which can liberalize the strict reaction condition, as shown by the following chemical equation.

下：

$$2CH_3CH_2OH(C_2H_6O) \xrightarrow[\text{Concentrated } H_2SO_4]{\text{special Catalyst}} CH_3CH_2OCH_2CH_3(C_4H_{10}O)\uparrow + H_2O\uparrow$$

Thus, we can construct the structure of the dragon's fire-breathing organ con-

由此，我们可以构建出龙焰产生器官的结构，该器官主

Table 3: Burning Temperature List | 燃烧温度表

燃料（Fuel）	温度（Temp.）(°C)	燃料（Fuel）	温度（Temp.）(°C)	燃料（Fuel）	温度（Temp.）(°C)
甲醇（Methanol）	1100	原油（Crude）	1100	天然气（Gas）	2020
乙醇（Ethanol）	1180	汽油（Gasoline）	1200	液化石油气（LPG）	2120
丙酮（Acetone）	1000	丙烯（Acetylene）	2127	甲烷（Methane）	1800
乙醚（**Ether**）	2861	氢气（Hydrogen）	700	乙烷（Ethane）	1895

sisting of gland A providing concentrated sulfuric acid, gland B providing anhydrous ethanol, and gland C as a reaction vessel. The high pressure inside the gland C and the catalyst can promote the reaction and keep the ether produced in a liquid state, which is easy to store and enhance the ignition point.

The sphincter is a valve controlling the excretion of ether. When the sphincter is tightened, the ether is locked in the gland C. When the sphincter relaxes, with the air entering, the pressure of the gland C rapidly decreases. Finally, the ether evaporates and ejects from the valve. The gaseous ether is mixed with the special catalyst, the ignition point is lowered to below the environmental temperature when reacted with oxygen in the air to generate carbon dioxide and water. Then a large amount of heat is released. The schematic diagram of the gland structure of the dragon and the chemical equation of the ether combustion is shown in Figure 6.

The sulfuric acid in the gland is produced by the stomach acid and sulfur-containing amino acids (methionine, cystine, cysteine) under the action of a catalyst, wherein the methionine needs to be obtained from meat. It also explains why the dragon needs to consume a large amount of meat for maintaining fuel production. Ethanol is an intermediate product in the decomposition of carbohydrates.

Estimation of Energy Consumption by Breathing Fire. Through combustion heat of ether and the storage volume of gland C, we can estimate the energy consumption when the dragon is breathing fire. According to the TV series, we assume that the storage volume of gland C is roughly equivalent to the human's full stomach, denoting the gland storage volume V , the burning heat of ether Q at 25 °C, 100 kPa. Suppose the pressure inside the gland C is the critical pressure of ether 3637.6 kPa. Based on that, we define the density of liquid ether as ρ, the

要包含三个腺体，腺体 A 提供浓硫酸，腺体 B 提供无水乙醇，腺体 C 作为反应容器。腺体 C 内部的高压和催化剂可以促进反应，并使生成的乙醚保持液态，易于储存，提高燃点。

括约肌是一个控制乙醚释放的阀门。当括约肌收紧时，乙醚被锁在腺体 C 中。当括约肌舒张时，空气进入，腺体 C 的压力迅速下降。最后，乙醚从阀门中汽化并喷出。气态的乙醚与特殊催化剂混合后，使混合物的着火点降低至比环境温度更低的值。当混合物与空气中的氧气反应时生成二氧化碳和水。然后大量的热量被释放出来。龙的腺体结构示意图和乙醚燃烧的化学方程如图 6 所示。

腺体中的硫酸是由胃酸和含硫氨基酸（蛋氨酸、胱氨酸、半胱氨酸）在催化剂作用下产生的，其中蛋氨酸需要从肉类中获得。这也解释了为什么龙需要消耗大量的肉类，原因是龙需要维持燃料的生产。乙醇是碳水化合物分解的中间产物。

估计龙焰的能量消耗。通过乙醚的燃烧热和腺体 C 的储存体积，可以估算出龙在喷火时的能量消耗。根据电视剧，我们假设腺体的储存体积大致相当于人的整个胃，腺体的储存体积表示为 V，乙醚在 100kPa 气压 25 °C 的环境下的燃烧热为 Q 。假设腺体 C 内的压强为乙醚的临界压力 3637.6 kPa。在此基础上，我们定义乙醚液体的密度为 ρ，乙醚的摩尔质量为 M，物质的量为 N 和质量为 m。假设龙一天只消

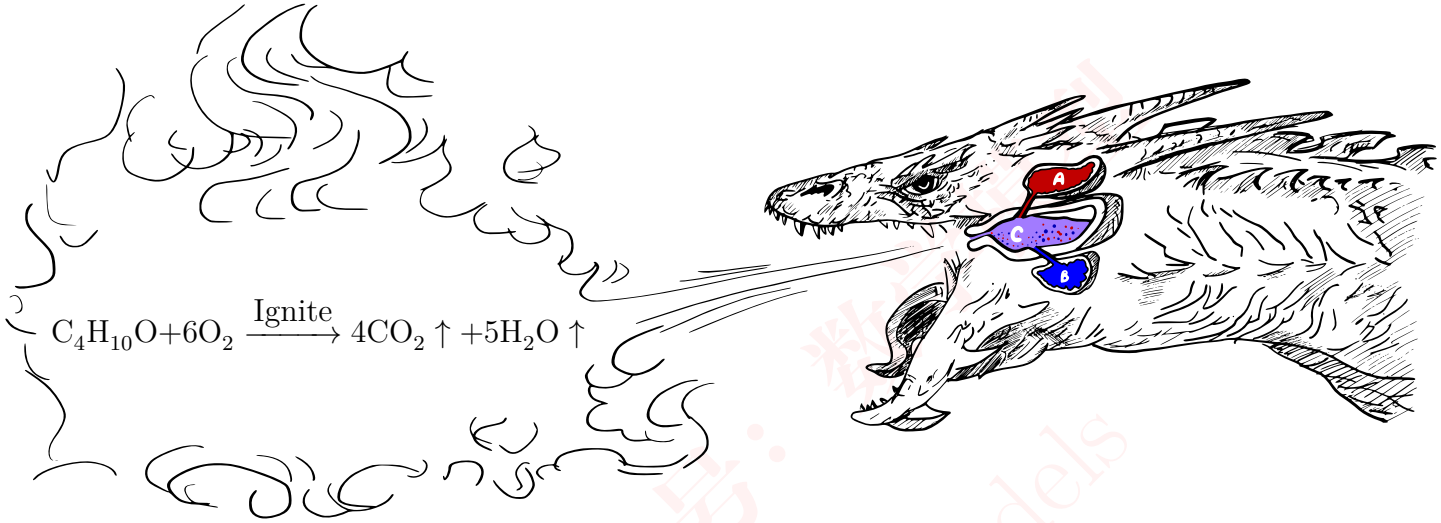

Figure 6: The Schematic Diagram of the Gland Structure | 腺体结构示意图

molar mass of ether as M , the amount of substance as N, and the mass as m. Suppose the dragon consumes one gland of ether in a day, we finally define the energy consumption as Σ, cf. Table 4.

耗一个腺体体积的乙醚，则我们定义能耗为 Σ，参见表 4。

Table 4: Parameters and Their Values | 参数值表

V (m^3)	Q (kJ/mol)	ρ (kg/m^3)	m (kg)	M (kg/mol)	N (k)
0.002	2752.9	265	0.74	$\rho V = 0.53$	$m/M = 7.16$

We can calculate the energy consumption: $\Sigma = Q\rho V/M = 19716.72$kJ. As a result, provided the dragon breathes fire, the energy consumption in a day will increase by 19716.72 kJ.

n.b. Since there are no fire-breathing creatures in the real world, in order to ensure the stcapability and capability of our models, in the subsequent analysis, we will not consider the consumption of energy by the dragon's fire.

我们能够计算龙焰能量消耗 $\Sigma = Q\rho V/M = 19716.72$ kJ。所以，由于喷火，龙每日的能量消耗会增加 19716.72 kJ。

注意：由于地球上根本不存在会喷火的生物，为了确保模型的普适性，在后续的分析中，我们不会考虑龙焰的能量消耗。

5 Dragon-Sheep-Habitat Model | 龙-羊栖息地模型

5.1 Analysis of Dragon's Habitat Demand | 龙的栖息地需求分析

We consider the ecosystem that has reached an ecological balance, with an area of S. At some point, the mature dragon enters the ecosystem, denoting this moment as the initial time. We assume that the dragon can prey the rest of the animals. Without loss of generality, we can regard all the animals as sheep, the current ecosystem as a grassland ecosystem.

We reckon that the number of sheep $N(t)$ can be regarded as the supply of resources to the dragon at the time t. We regard the area of the ecosystem as the habitat area of the dragon. The sheep in the ecosystem embodies an indicator to measure the capability of the community to support the dragon in the habitat, and it can also be regarded as an indicator of the size of the community.

Denote the maximum carrying capacity of the ecosystem to the sheep is K, which is proportional to the area S. Since the ecosystem is balanced before the initial time ($t = 0$), we can obtain the initial condition (10) that the ecosystem should satisfy.

$$N(0) = K \tag{10}$$

Consider the limitation of environmental resources on the number of sheep and the growth of sheep per se. According to the retardation growth principle, in the absence of dragons, we can obtain the differential equation (11) that the number of sheep should satisfy [13]. n.b. The parameter p is the inherent growth rate of the sheep.

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = pN(t)\left(1 - \frac{N(t)}{K}\right) \tag{11}$$

Since the dragon's entry will influent the ecosystem and affect the number of sheep, we modify the equation (11) to the following form (12). n.b. The parameter a is the theoretical predation of a mature dragon within a day, and its unit is pcs/day.

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = pN(t)\left(1 - \frac{N(t)}{K}\right) - a \tag{12}$$

We combine the equation (10) and (12) to obtain the number of sheep change

我们假设生态系统已经达到了生态平衡，其面积为 S。在某一时刻，成年的龙进入了生态系统，记此时为初始时刻。我们认为龙可以捕食其它所有动物。不失一般性，我们可以把其它所有动物都看作是羊，把当前的生态系统看作是草地生态系统。

我们认为，羊的数量 $N(t)$ 可以看作是 t 时刻向龙的资源供给。我们把该生态系统视为龙的栖息地。将生态系统中的羊作为一个指标，衡量群落的规模，以及支持栖息地中龙生存的群落能力。

设生态系统中羊的最大承载能力为 K，K 与面积 S 成正比。由于生态系统在初始时刻（$t = 0$）之前就已经平衡，可以得到生态系统应满足的初始条件为：

首先我们仅考虑环境资源对羊群的限制和羊自身的增长。根据阻滞增长原理，在不考虑龙的情况下，羊的数量应该满足式 (11) 所示的微分方程 [13]。注意：参数 p 是羊群的固有增长速率。

由于龙的进入会影响生态系统和羊群的数量，我们将等式 (11) 修正为等式 (12)。注意：参数 a 是成年龙理论的捕猎数量，其单位是只 / 天。

我们联立式 (10) 和 (12)，得到了当龙进入生态系统后，

one-order non-linear equation's Cauchy problem (13) when the dragon enter the ecosystem.

$$\begin{cases} \dfrac{\mathrm{d}N(t)}{\mathrm{d}t} = pN(t)\left(1 - \dfrac{N(t)}{K}\right) - a \\ \\ N(0) = K \end{cases} \tag{13}$$

Through our analysis of the daily energy intake of the mature dragon, we can know that the mature dragon needs to consume about 900 kg meat per day (sheep's weight is about 100 kg), that is about 9 sheep. Without loss of generality, we can set that $a = 9$, $p = 0.01$. Furthermore, we select the different values of the ecosystem carrying capacity of sheep, and then we can solve the equation (13) to the graph about the number of sheep changing by time. e.g. Taking $K = 5000$, we obtain the graph as shown in Figure 7.

羊群数量变化的一阶非线性方程的柯西问题。

通过对成年龙一天的能量摄入分析，我们知道成年龙每日需要 900 kg 的肉类（一只羊的重量是 100 kg），也就是九只羊的量。不失一般性，我们设 $a = 9$，$p = 0.01$。接着，我们选择不同值的羊群的生态承载能力，然后我们可以重新求解微分方程 (13)，可得到羊的数量随时间变化的图像。例如 $K = 5000$，我们得到如图 7 所示的结果。

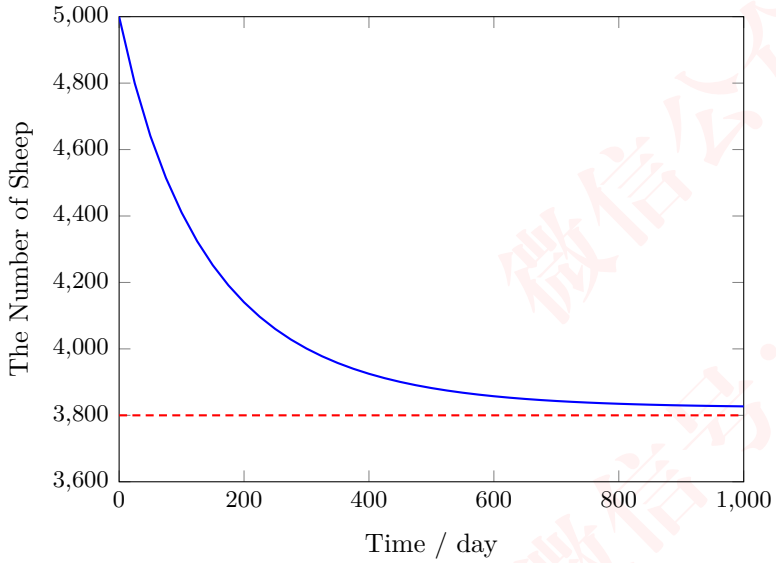

Figure 7: The Number of Sheep Changing by Time when $K = 5000$
$K = 5000$ 时羊数量随时间的改变

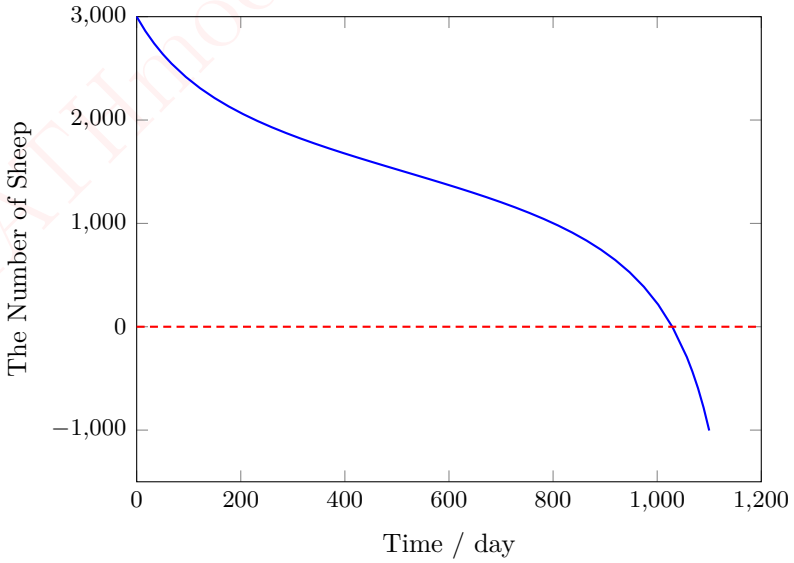

Figure 8: The Number of Sheep Changing by Time when $K = 3000$
$K = 3000$ 时羊数量随时间的改变

It can be seen from Figure 7, as the dragon enters the ecosystem, the number of sheep continues to decline, but the rate of decline gradually slows down. At about $t = 700$ (around the second year), the ecosystem is almost balanced. In fact, the value of K cannot be arbitrary, in order to satisfy the long-term survival of the mature dragon, when the K value is too small, the rate at the dragon preying the sheep is far greater than the recovery rate of the number of sheep. The number of sheep will soon reach zero, which will cause the ecosystem to collapse. e.g. Taking $K = 3000$, we obtain the graph as shown in Figure 8.

Figure 8 shows that $N(t)$ reaches zero at about $t = 1000$ (around the third year). Therefore, we conclude that in order to meet the basic living needs of the dragon, and for maintaining the ecosystem's sustainable development, the value of K should have critical value. We choose different K values and finally obtain the forms of the corresponding ecosystem, cf. Table 5.

从图 7 可以看出，龙进入生态系统之后，羊的数量持续下降，但下降的速度逐渐放缓。在 $t = 700$（大约在第二年）的时候，这个生态系统基本上稳定平衡了。事实上，K 的取值不能是任意的，为了满足成年龙的长期生存，当 K 值很小的时候，被猎杀的羊的数量远远大于羊自身繁衍能力。因此，羊群很快就会消失，这将导致生态系统崩溃。例：取 $K = 3000$，得到如图 8 所示的图像。

图 8 表明，$N(t)$ 在 $t = 1000$ 时（大约第三年）达到零。因此，我们得出结论，为了满足龙的基本生活需求，并保证生态系统的可持续发展，K 值应该具有最小临界值。选择不同的 K 值，我们最终可以得到对应生态系统的结果，如表 5 所示。

Table 5: K Value and Corresponding Ecosystem | 不同的 K 值和对应的生态系统

K Value	Ecosystem Development Trend	生态系统发展趋势
100	Ecosystem Collapses in 10 Days	10 天内生态系统崩溃
500	Ecosystem Collapses in 2 Months	2 个月内生态系统崩溃
1000	Ecosystem Collapses in 5 Months	5 个月内生态系统崩溃
2000	Ecosystem Collapses in 1 Year	1 年内生态系统崩溃
3000	Ecosystem Collapses in 3 Years	3 年内生态系统崩溃
3500	Ecosystem Collapses in 10 Years	10 年内生态系统崩溃
4000	Ecosystem Maintains Long-Term Balance with 2600 Sheep	生态系统保持长期平衡，平衡时羊数量为 2600 只
5000	Ecosystem Maintains Long-Term Balance with 3800 Sheep	生态系统保持长期平衡，平衡时羊数量为 3800 只

From Table 5, the critical value of K is about 4000. We assume that the ecosystem's area A is proportional to its maximum environmental carrying capacity. Then we have the relationship (14). n.b. The parameter b is the coefficient of the environmental carrying capacity, indicating the number of sheep that can be carried per square meter of the ecosystem's area.

$$K = bA \tag{14}$$

In the case that the pasture will not degrade, about 30 mu of grassland can raise

由表 5 可知，K 的临界值约为 4000。我们假设生态系统的面积 A 与其最大环境承载力成正比，即关系式 (14)。注意：参数 b 为环境承载能力系数，表示生态系统每平方米可承载的羊的数量。

在草场不退化的情况下，30 亩左右的草地可以养一只羊，

one sheep, taking into account the active area of the sheep, and about 150mu of land carries one sheep. From this ratio, if you want to carry 4,000 sheep, about 600,000 mu of land (i.e., 400 km^2). Therefore, we estimate that the habitat of dragons can be at least 400 km^2 . Considering that the area of Yellowstone National Park is 900,000 hectares [14], so the Park can feed about 22 dragons.

考虑到羊的活动面积，假设 150 亩左右的土地养一只羊。由这个比例，如果想承载 4000 只羊，那么大约需要 60 万亩的土地（即 400 平方公里）。因此，我们估算出龙的栖息地至少有 400 平方公里。举个例子，美国黄石国家公园的面积是 90 万公顷，其面积大约可以养活 22 条龙。

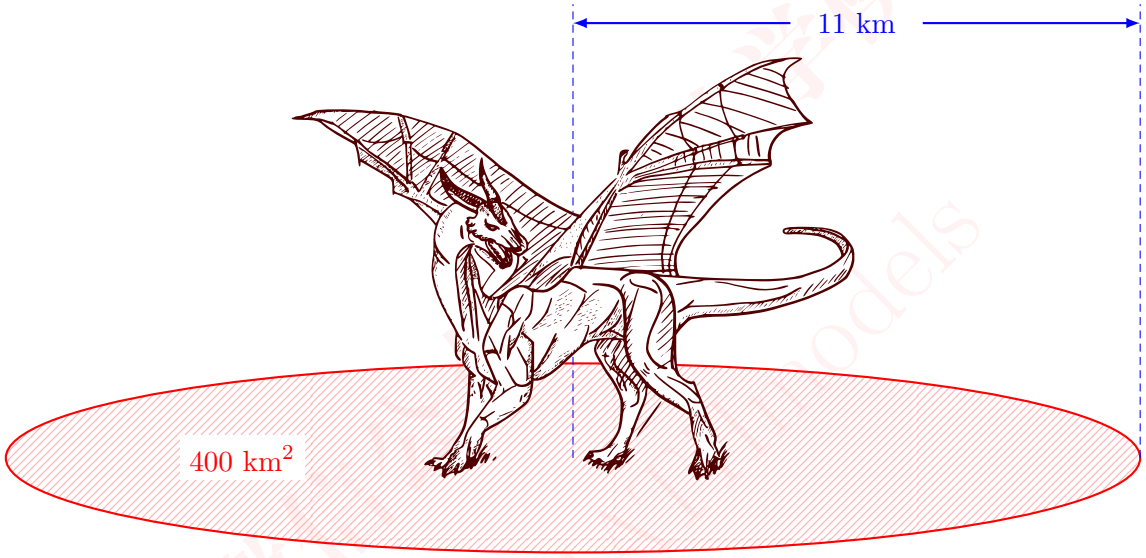

Figure 9: The Minimum Area of Dragon's Habitat | 龙栖息地的最小面积

Based on the above analysis, we can obtain the minimum requirements for the dragon's habitat and the minimum standards for the community's capability to support the dragon in the habitat. Here are our conclusions:

- The habitat for an individual dragon is at least 400 km^2, around 120 Central Parks in New York [15], and we draw a schematic graph cf. Figure 9.
- At least 4,000 sheep in the habitat could guarantee the long-term support for the dragon. i.e., The community of the ecosystem needs at least 4,000 herbivores and carnivores. From biology, the energy transfer efficiency between food chains is about 20%, so the number of herbivores in the community is

根据以上分析，我们可以得出龙对栖息地面积的最小需求以及群落供给能力的最低标准。以下是我们的结论：

- 单独一条龙至少需要 400 平方千米的栖息地，这大约是 120 个纽约中央公园的面积，我们绘制了示意图 9。
- 初始时至少有 4,000 只羊的栖息地可以保证对龙的长期供给。即整个生态系统至少需要 4,000 只食草动物和食肉动物。从生物学上看，食物链之间的能量传递效率约为 20%，所以群落中食草动物的数量约为 3000

about 3,000, and carnivores is about 1,000. We draw the trophic structure of the community which dragons participated, cf. Figure 10.

只，食肉动物约为 1000 只。我们绘制了包含龙的群落食物链结构，见图 10。

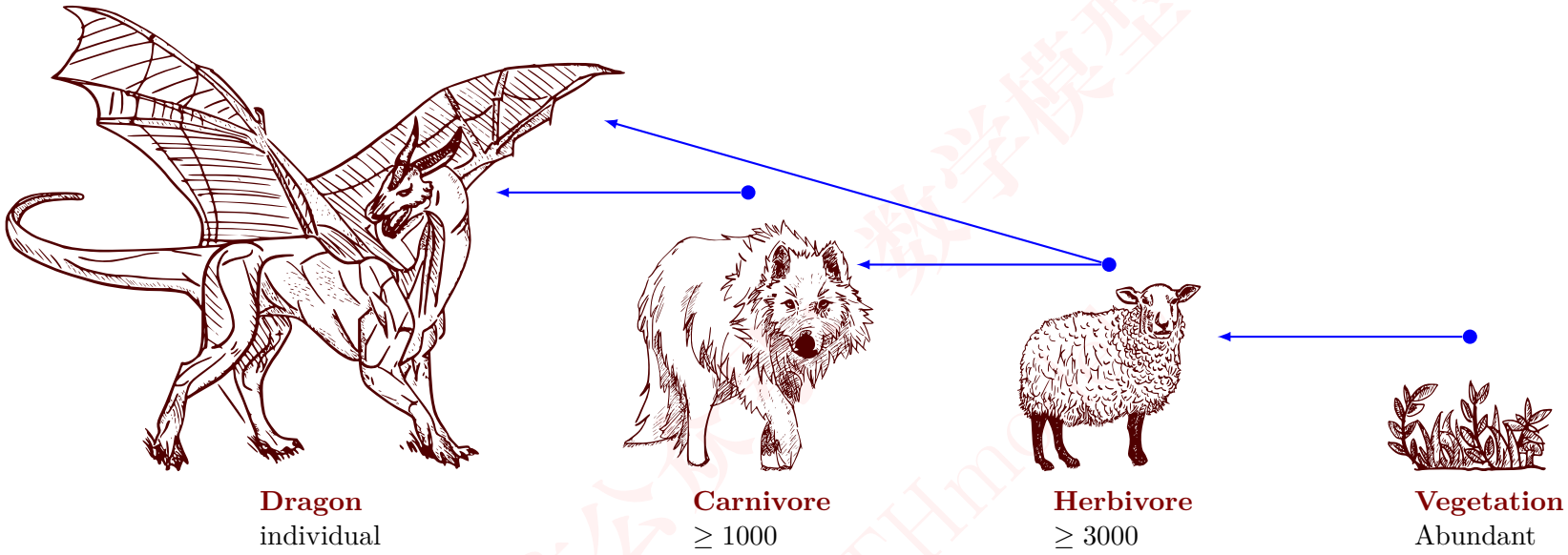

Figure 10: Food Chain with Dragon's Participation | 包含龙的食物链结构

5.2 Dragon's Impact on The Ecosystem: Balance or Collapse? | 龙对生态系统的影响：平衡还是崩溃？

We still consider the ecosystem reached an ecological balance, which can sustain itself in the long run without the dragon. Assuming that the dragon can prey all creatures in the ecosystem, we regard the food chain of the ecosystem as a dragon-sheep-grass model. Based on the above analysis of the habitat and K value of the dragon, we can draw the following conclusions:

- When the K value is very small (e.g. less than 1000), the ecosystem will be destructed in a short term due to the dragon's entry. i.e., the sheep in the food chain are extinct, then the dragon has no food source and will eventually become extinct. i.e., the ecosystem cannot self-recover in time in

即使没有龙，我们也认为这个生态系统可以达到生态平衡，并长期维持下去。假设龙能够猎杀该生态系统中所有的生物，那么我们便可以把生态系统的食物链看作一个龙-羊-草模型。基于以上对龙的栖息地和 K 值的分析，我们可以得出以下结论：

- 当 K 值非常小的时候（例如小于 1000），生态系统会因为龙的进入而在短期内被破坏。即如果食物链中的羊灭绝了，那么龙就没有食物来源，最终龙也会灭绝。在这种情况下，生态系统无法及时自我恢复。

this situation.

- Since the K value has the critical value, when the K value is greater than or equal to the critical value, the number of sheep in the food chain tends to be stable. The ecosystem can provide long-term survival needs for the dragon, i.e., the entire ecosystem will return to balance.

- 由于 K 值具有最小临界值，当 K 值大于或等于该临界值时，食物链中的羊的数量会随时间趋于稳定。生态系统可以为龙提供长期的生存需求，并保持自身平衡。

6 Impact of Three Regions on Dragon's Survival | 三种区域对龙生存的影响

6.1 Difference in Daily Energy Intake of Dragons in Three Regions | 三种区域龙的日能量摄入差异

We still assume that the dragon is a homothermal animal, i.e., it needs to maintain its body temperature relatively stable in the changes of environmental temperature. Consider three different regions in climate: an arid region, a warm temperate region, and the arctic region. The three regions have different temperatures, especially in the arctic region, where the dragon consumes more energy per day to maintain its activities and metabolism.

Combined with the metabolic rate of the human body at different temperatures, we speculate that the dragon's daily energy intake in the arid region and the arctic region should be 0.9 times and 1.3 times, respectively, in the warm temperate region. According to our analysis of the dragon's daily energy intake, we can know that the dragon needs to consume about 460,000 calories per day (i.e., 9 sheep) in the warm temperate region, thus we calculate the energy consumption in the arid region is 414,000 calories and 598,000 calories in the arctic region, which is equivalent to consuming 8 and 12 sheep per day.

我们假设龙是恒温动物，也就是说，当环境温度发生变化的时候，龙的体温能够保持相对稳定。考虑三种不同气候的地区：干旱地区、温暖地区和寒冷地区。这三种地区具有不同的环境温度，尤其是在寒冷地区，在那里龙每天将会消耗更多的能量来维持它的日常活动和新陈代谢。

根据不同温度下人体的代谢率，我们推测，龙在干旱地区和寒冷地区每日的能量摄入量应是温暖地区的 0.9 和 1.3 倍。根据我们对龙每天能量摄入的分析，可知龙每天需要消耗大约 460,000 卡路里的热量（相当于 9 只羊）。按照比率，我们可以算得干旱地区要每天要消耗 414,000 卡路里，相当于每天 8 只羊；寒冷地区每天要消耗 598,000 卡路里，相当于每天 12 只羊。

6.2 Three Regions' Capability about Self-Recovery and Supporting Dragons | 三种区域的自我恢复与对龙的供给能力

There are huge differences in vegetation and animal resources in the three regions. The more the number and variety of animals and plants, the higher the species richness of the ecosystem and the self-recovery capability in the regions (i.e., return to the original state after destructed). It is clear that the species richness in the warm temperate region is much greater than the arid and arctic regions. Therefore the self-recovery capability of the warm temperate region is far greater than that in the arid and arctic regions.

这三种地区的植被和动物资源差异很大。生态系统的动物和植物的数量和种类越多，物种丰富度越高，区域的自我恢复能力越强（破坏后恢复原状的能力）。很明显，温暖地区的生态系统要比干旱和寒冷地区丰富得多。因此，温暖地区的自我恢复能力远远大于干旱和寒冷地区。

Based on the Dragon-Sheep-Habitat Model, we use the parameter p in the model as an indicator of the self-recovery capability of an ecosystem. We assume that in the arid region and the arctic region $p = 0.005$, and in the warm temperate region $p = 0.01$. Furthermore, under different climatic conditions, the distribution density of creatures in an ecosystem is different. In the warm temperate region, the distribution density of creatures is much greater than that in the arid and arctic regions. We set the parameter b in the model as an indicator of the distribution density of the area. We assume that b is 10 pcs/hm^2 in the warm temperate region and 13 pcs/hm^2 in the arid and arctic regions.

基于龙-羊栖息地模型，我们使用模型中的参数 p 作为生态系统自我恢复能力的指标。我们假设干旱区和寒冷地区 $p = 0.005$，温暖地区 $p = 0.01$。此外，在不同的气候条件下，生物在生态系统中的分布密度是不同的。温暖地区生物的分布密度远远大于干旱和寒冷地区。我们用参数 b 来表示该地区的分布密度。假设 b 在温暖地区为 10 只 / 公顷，在干旱和寒冷地区为 13 只 / 公顷。

6.3 Minimum Resource and Habitat Demand for Dragon's Survival in Three Regions | 三种区域龙生存的最低资源和栖息地需求

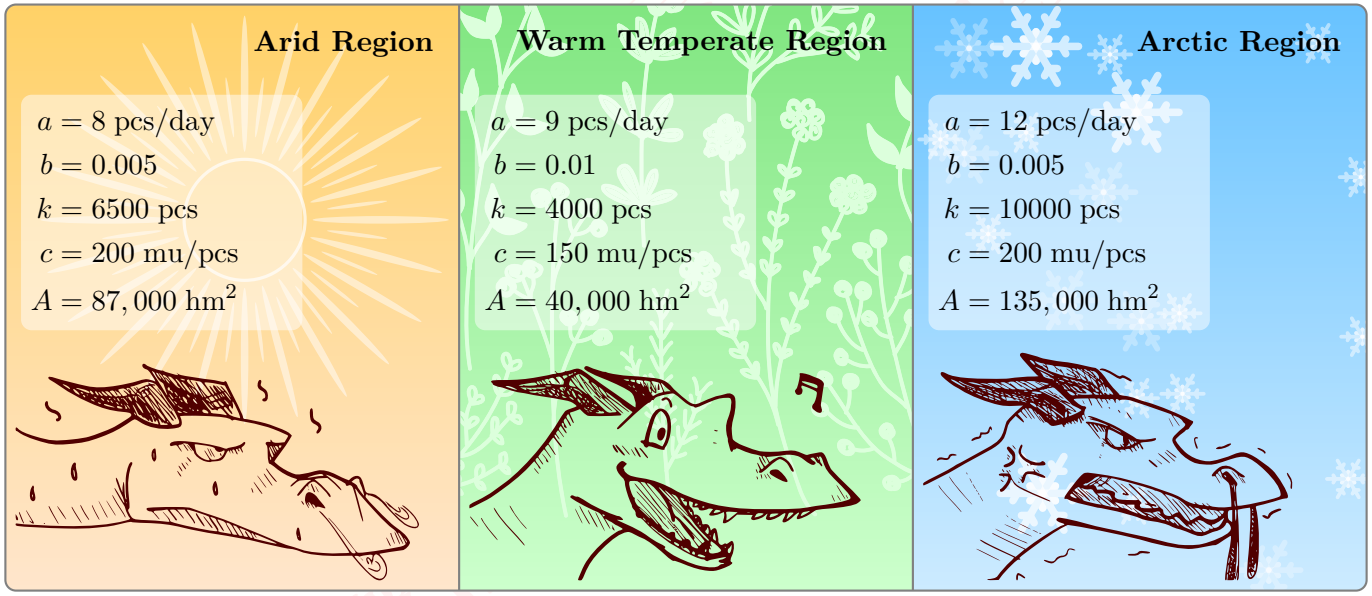

Figure 11: Minimum Demand of Dragon's Survival in Three Regions | 龙在三种地区的生存最低要求

Combining the analysis of 6.1 and 6.2, we substitute the parameters p (i.e.,

结合 6.1 和 6.2, 的分析，我们将不同的 p 值（羊群的固

the inherent growth rate of the number of sheep) and b (i.e., the environmental carrying capacity coefficient) into the equation (13) to calculate the critical value of K (i.e., maximum carrying capacity in three regions). cf. (15).

有增长速率）和 b 值（环境承载能力的参数）分别代入方程 (13) 求解，并得到了 K 值（生态系统最大的承载力）的临界值。

$$K_{\text{arid}} = 6500, \quad K_{\text{warm}} = 4000, \quad K_{\text{arctic}} = 1000 \tag{15}$$

According to the environmental carrying capacity coefficient b in different regions, we can calculate the minimum habitat area under three regions. cf. (16).

由不同地区中的环境承载能力参数 b，我们得到了三个地区龙最小的栖息地面积，如式 (16) 所示。

$$A_{\text{arid}} = 187,000 \text{ hm}^2, \quad A_{\text{warm}} = 40,000 \text{ hm}^2, \quad K_{\text{arctic}} = 133,000 \text{ hm}^2 \tag{16}$$

In addition, we draw a diagram of the minimum demand of dragon's survival in the three regions as follows. cf. Figure 11.

另外，我们绘制了龙在三种不同地区生存的最小需求的示意图，参见图 11。

7 Sensitivity Analysis | 灵敏度分析

In the Dragon's Weight Logistic Growth Model, the dragon's weight after one year, the inherent growth rate of the dragon's weight, the environmental factor, and the environmental temperature all have impacts on the model's result. Since the dragon's weight after one year is closely related to the inherent growth rate, we only consider the effects of changes in inherent growth rate, environmental factor and environmental temperature on the model.

在龙体重的 Logistic 增长模型中，龙出生一年后的体重、龙的固有体重增长率、环境因素和环境温度都会对模型的结果产生影响。由于一年后龙的体重与固有增长率密切相关，因此我们只考虑固有增长率、环境因子和环境温度的变化对模型的影响。

7.1 Impact of Inherent Growth Rate r on Dragon's Weight | 固有增长率 r 对龙体重的影响

According to the equation (6), we can calculate the inherent growth rate r of the dragon's weight from 20 kg/year to 30 kg/year. We fix the environmental temperature T as 25 °C, the environmental factor σ as 0.04. Then we take a linear increase of r by step 2, and plot the function $W(t)$ as shown in Figure 12.

As we can see in Figure 12, the weight $W(t)$ of the mature dragon varies from 1.2 tons to 4.5 tons at a rate of 2 kg/year, which is weakly relative to its total weight. Therefore $W(t)$ is not sensitive to the change of r, which also shows that the model per se is non-sensitive to r.

根据式 (6)，我们可以算出龙的体量固有增长率 r 在 20 kg/年到 30 kg/年之间。我们控制环境温度 T 恒定为 25 摄氏度，环境因子 σ 为 0.04。然后以步长为 2 的幅度线性增加 r ，绘制出 $W(t)$ 函数的图像，如图 12 所示。

从图12中我们可以看到，当 r 增加 2 公斤 / 年时，成年龙的体重 $W(t)$ 的改变在 1.2 吨到 4.5 吨之间，这表明 r 与龙的总体重相关性不大。因此，$W(t)$ 对 r 的变化不敏感，这也说明模型本身对 r 不敏感。

7.2 Impact of Environmental factor σ on Dragon's Weight | 环境因子 σ 对龙体重的影响

In our model, the environmental factor measures the retardation effect of the

在我们的模型中，环境因子衡量着环境对龙体重的阻滞

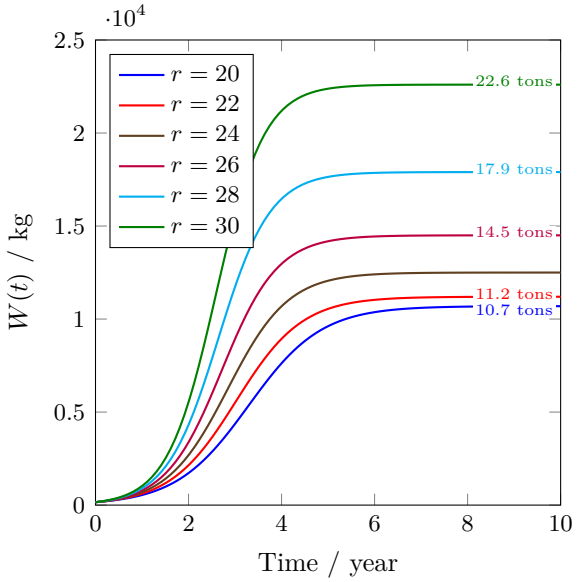

Figure 12: Sensitivity Analysis on Inherent Growth Rate r

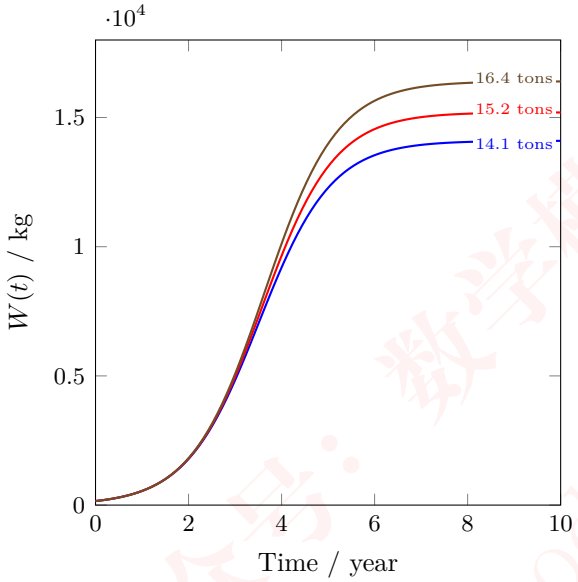

Figure 13: Sensitivity Analysis on Environmental Factor σ

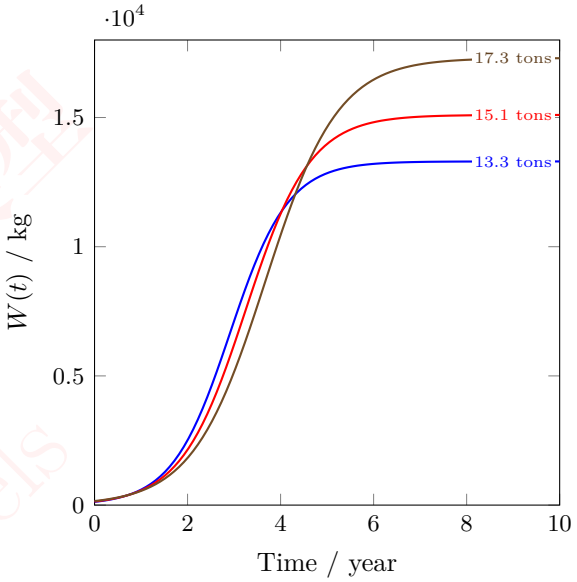

Figure 14: Sensitivity Analysis on environmental Temperature T

environment on the dragon's weight. We fix the environmental temperature T as 25 °C, the inherent growth rate r of the dragon's weight as 20 kg/year. We select different σ values, and plot the function $W(t)$ as shown in Figure 13.

It can be seen from Figure 13 that as the σ increases, the stable value of $W(t)$ is decreasing. It indicates that the environment has effects on the dragon's weight while the temperature remains constant.

效应。控制环境温度 T 为 25 摄氏度，龙体重的固有增长率 r 为 20kg/年，我们选择了不同的 σ 值，绘制出 $W(t)$ 函数的图像，如图 13 所示。

从图 13 可以看出，随着 σ 值的增加，$W(t)$ 的稳定值减小。这表明当环境温度恒定时，环境因子仍然对成年龙的体重有影响。

7.3 Impact of Environmental Temperature T on Dragon's Weight | 环境温度 T 对龙体重的影响

In the end, we consider the effect of environmental temperature on the dragon's weight. We fix r as 20 kg/year and σ as 0.04. Select different T values and plot the function $W(t)$ as shown in Figure 14.

From Figure 14, we can see the difference in temperature has effects on the

最后，我们考虑了环境温度对龙体重的影响。控制龙的体重的固有增长率 r 为 20kg/年，σ 值为 0.04，我们选择了不同的 T 值，绘制出 $W(t)$ 函数的图像，如图 14 所示。

从图 14 中，我们看到环境温度的变化对龙体重 $W(t)$ 有

weight $W(t)$. Due to the correlation between the dragon's length and weight, these parameters reflect the size of the dragon. i.e., the larger their values, the larger the dragon's size. Therefore, according to Figure 14, we can obtain that the size of the mature dragon is smaller in hot regions, on the contrary, the size is larger in cold regions, which satisfies the ecological laws. And also it illustrates the rationality of our model.

影响。由于龙的体重与长度存在连系，这些参数将会影响到龙的大小。具体来说就是：这些参数的值越大，龙的体型就越大。因此，由图 14，我们可以知道在较热的地带，龙的体型会小一些，反之，较冷的地带会大一些。这与自然规律“不谋而合”。同时也说明了我们模型的合理性。

8 Model Evaluation and Further Discussion | 模型评价和展望

8.1 Evaluation of Models | 模型评价

Like the creature in fiction, the dragon is mysterious and enchanting. In order to explore the characteristics of dragons, we combine the characteristics of the real creatures, physics, chemistry, biology, ecology, and other knowledge to establish series of models, the Dragon's Weight Logistic Growth Model, the Dragon's Fire Chemical Kinetics Model, and the Dragon-Sheep-Habitat Model, etc. We answer the following questions precisely by analogy, such as the analysis of the dragon's basic characteristics, the dragon's daily energy intake and consumption, the minimum requirement for the habitat to support the dragons, the minimum area of the habitat, and the impact on local ecosystems with the dragon's entry.

Throughout our modeling, we set reasonable assumptions to provide convenience to build and solve models. We also take the sensitivity analysis which verified the stability of the models.

We innovatively analyze the dragon's fire-breathing mechanism and explore the possibilities and operations of its implementation. Under the assumption that the dragon can breathe fire, we established the Chemical Kinetics Model of the dragon's fire to investigate the physiological and energy characteristics of the fire-breathing mechanism.

However, our models still have the following weaknesses:

- Our model contains many parameters. The values of these parameters largely depend on biological knowledge. Due to the diversity and complexity of the creatures, the values will be very different, which will affect the results of the models.

与小说中的人物一样，龙也是神秘而迷人的。为了探索龙的特点，我们根据真实生物的特点，以及物理、化学、生物学、生态学和其他的知识建立了一系列的模型：龙体重的 Logistic 增长模型，龙焰的化学动力学模型和龙-羊栖息地模型等。通过类比，我们解决了很多问题，如龙的基本特征的分析、龙的每日能量摄入和消耗、龙对栖息地的最低要求、栖息地的最小面积和龙对生态系统的影响。

在整个建模过程中，我们设置了合理的假设，这为构建和解决模型提供了便利。最后，通过灵敏度分析，我们检验了模型的稳定性。

我们创新性地分析了龙焰的机理，并探索其实施的可能性和可操作性。在龙可以喷火的假设下，我们建立了龙焰化学动力学模型，以研究喷火机制的生理学和能量特性。

但是，我们的模型仍然存在以下不足：

- 我们的模型包含了很多参数。这些参数的值在很大程度上与生物学有关。由于生物的多样性和复杂性，参数的值会有很大的不同，这将影响模型的结果。

- To simplify the model, our Dragon-Sheep-Habitat Model is aimed at the mature dragon, which does not account changes in energy intake during the dragon's growth, hence the model has limitations to some extent.

- 为了简化模型，我们的龙-羊栖息地模型针对的是成年的龙，它没有考虑龙在生长过程中能量摄入的变化，因此模型在一定程度上存在局限性。

8.2 Further Discussion | 进一步讨论

In our Dragon-Sheep-Habitat Model, we explore the problems about the mature dragons. De facto, if we add the related variables and functions, based on the original model, we can also investigate the dragon from infancy to maturation, such as its patterns of the energy intake changes, its impact on the ecosystem, and its habitat requirements etc.

Our models are combined with the relevant knowledge, generally satisfy mathematical principles and natural laws. Although the dragon does not exist in the real world, the Dragon-Sheep-Habitat Model can also try to solve the invasion problem of huge alien species, the Dragon's Weight Logistic Growth Model can try to solve the dinosaurs' growth problem, the Dragon's Fire Chemical Kinetics Model can assist the design of a new type flamethrower.

在我们的龙-羊栖息地模型中，我们探讨了关于成年龙的问题。事实上，如果我们添加相关的变量和函数，我们也可以在原始模型的基础上研究龙从婴儿期到成熟期的变化过程，如能量摄入的变化模式、对生态系统的影响、对栖息地需求的变化等。

我们的模型与相关知识相结合，满足一般的数学原理和自然规律。虽然龙不存在于现实世界中，但是龙-羊栖息地模型也或许可以解决大型外来物种入侵问题，龙体重的 Logistic 增长模型可以帮助解决恐龙的生长问题，龙焰化学动力学模型可以帮助设计一种新型的火焰喷射器。

9 A Letter to Martin | 一封给 Martin 的信

(See next two pages)

（见下两页）

Mr. George R.R. Martin
The Creator of
Song of Ice and Fire

Team # 1910246
MCM/ICM Contest
Problem A: Game of Ecology
January 29, 2019

Dear Martin,

We hope everything is fine with you.

Thank you very much for creating the epic series and the outstanding novels A Song of Ice and Fire, which we are immensely addicted, as well as the TV series Game of Thrones adapted from your books. The dragons appearing in the TV series are fantastic and enchanting. The figure of the dragon has struck a chord with us, which aroused our curiosity and interest in researching dragons.

According to your descriptions of dragons in the novel, we start to consider that if a dragon lives in the real world, how can its essential physical characteristics be? What is its living habits like? How many resources should we provide for it to maintain its survival and growth? What impacts would it make on the local ecosystem? Based on these problems, we combine mathematics, physics, chemistry, biology, ecology and other related subjects to try to establish various models to analyze the characteristics of dragons and to make scientific answers to these questions.

From the model we established and the analyze the dragon's daily caloric intake and energy expenditure. We can know that from its infancy to maturation, the food intake of the dragon is increasing. The growth rate of the food intake is about one sheep per year. When the dragon becomes mature, it should at least consume nine sheep per day to maintain its primary activities. Thus, this food intake is quite large, that is to say, the dragon may have a significant impact on the local ecosystem.

To ensure the scientificalness and rationality of the dragons in the books, we have paid attention to the impacts of the dragon on the real ecosystem, especially the migration of dragons. Therefore, we have considered that the difference of the living conditions of the dragons and the impacts they make on the ecosystem when the dragon lives in an arid region, a warm temperate region or the Arctic region. In consequence, we suggest that to maintain the realistic ecological basis to support your story, and you may well pay attention to the following points:

- The best living area for dragons is the warm temperate region, where the climate is humid, with high species richness. Therefore, the capacity for

愿您一切都好。

非常感谢您创作了史诗系列小说《冰与火之歌》，我们很喜欢这个小说以及根据这个小说改编的电视剧《权力的游戏》。电视剧中出现的龙更是令人觉得神奇而迷人。龙的形象引起了我们的共鸣，引起了我们研究龙的好奇心和兴趣。

根据您在小说中对龙的描述，我们开始思考，如果一条龙生活在现实世界中，那么它的基本物理特征是什么？它的生活习惯会是怎么样？我们应该为它提供多少资源来维持它的生存和生长？它会对当地的生态系统产生什么影响？基于这些问题，我们结合数学、物理、化学、生物、生态学等相关学科，尝试建立多个模型来分析龙的特征，并对这些问题做出科学的回答。

通过建立模型，我们分析了龙每日的能量摄入和消耗。由此可以知道，龙从婴儿期到成年期，其每日食物摄取量一直在增加。食物摄取量的增长率大约是每年一只羊。当龙成年时，它每天至少要吃 9 只羊来维持日常活动的能量消耗。这些食物的摄入量是相当大的。因此，龙可能对当地的生态系统有重大的影响。

为了保证论文中龙的科学性和合理性，我们关注了龙对实际的生态系统的影响，尤其考虑了龙迁徙的可能。由此，我们分析了当龙生活在干旱地区、温暖地区及寒冷地区时，龙的生存条件的差异及其对生态系统的影响。为了使故事更显真实性，我们建议了以下几点来维持现实的生态基础：

- 最适宜龙生存的是温暖的环境。温暖的环境下，气候湿润，生态系统复杂程度高。因而有足够大的能力维

maintaining the dragons is enormous enough.

- The dragon should live far away from the arid region and the Arctic region. The dragon itself will not like these kinds of regions with lousy living conditions and climates.
- We try to analyze the mechanism of the dragon breathing fire. We have found that the energy expenditure for breathing fire are very huge. Therefore, we hope that the times for the dragon to breath fire should not be too many per day, because this behavior costs a lot for its body to burden.
- According to our models' conclusions, the weight of the mature dragon may range from 10 ton to 20 ton. Therefore, the energy consumption is enormous when it is flying over a long distance. Be careful about the limitation of the flight distance and time.
- In the foraging activities, a dragon should seek for food in a wide range of areas. Considering the compatibility of the dragon with the ecosystem, to maintain the balance of it, you ought to let the dragons avoid causing catastrophic damage to the ecosystem.

We hope that our suggestions can help you with your subsequent creations, and it will be greatly honored for us if our recommendations are adopted! A Song of Ice and Fire really makes us enchanted and satisfied. We admire your imagination, and your capacity is shown in arranging such a large story setting. By the way, your mania fan in our team draw a picture as a gift for you.

持龙的生存。

- 龙应该居住在远离干旱和寒冷的地方。龙这个物种不太喜欢糟糕的生存条件和生存环境。

- 我们尝试分析了龙焰的机理。通过计算，我们发现，龙使用龙焰时会消耗自身的大量能量。因此，龙每日使用龙焰的次数不能太多，否则将需要捕猎更多的食物，喷火对它的能量消耗来说是一种负担。
- 根据的模型结论，成年的龙的体重大约在 10 至 20 吨之间。因此，龙长距离的飞行将会消耗大量的能量。要注意龙飞行距离和飞行时间的极限。

- 在觅食活动中，龙应该在一个足够大的范围内寻找食物。为了维持生态系统的平衡，考虑到龙与生态系统的兼容性，应该避免让龙对生态系统造成灾难性的破坏。

我们希望我们的建议能对您以后的创作有所帮助，如果我们的建议被采纳，我们将不胜荣幸！《冰与火之歌》真的让我们着迷和喜爱。您在宏大的故事背景布局中表现出的想像力和能力，令我们非常钦佩。我们团队里的骨灰级粉丝画了一幅画作为礼物送给您。

We wish you every success in the coming year. If you want more information, please feel free to write to us. Looking forward to your new books again!

Yours Sincerely,

Team # 1910246

我们祝您在新的一年里万事顺利。如果您需要更多的信息，请随时写信给我们。再次期待您的新书!

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Appendices | 附录

A Plots | 补充图

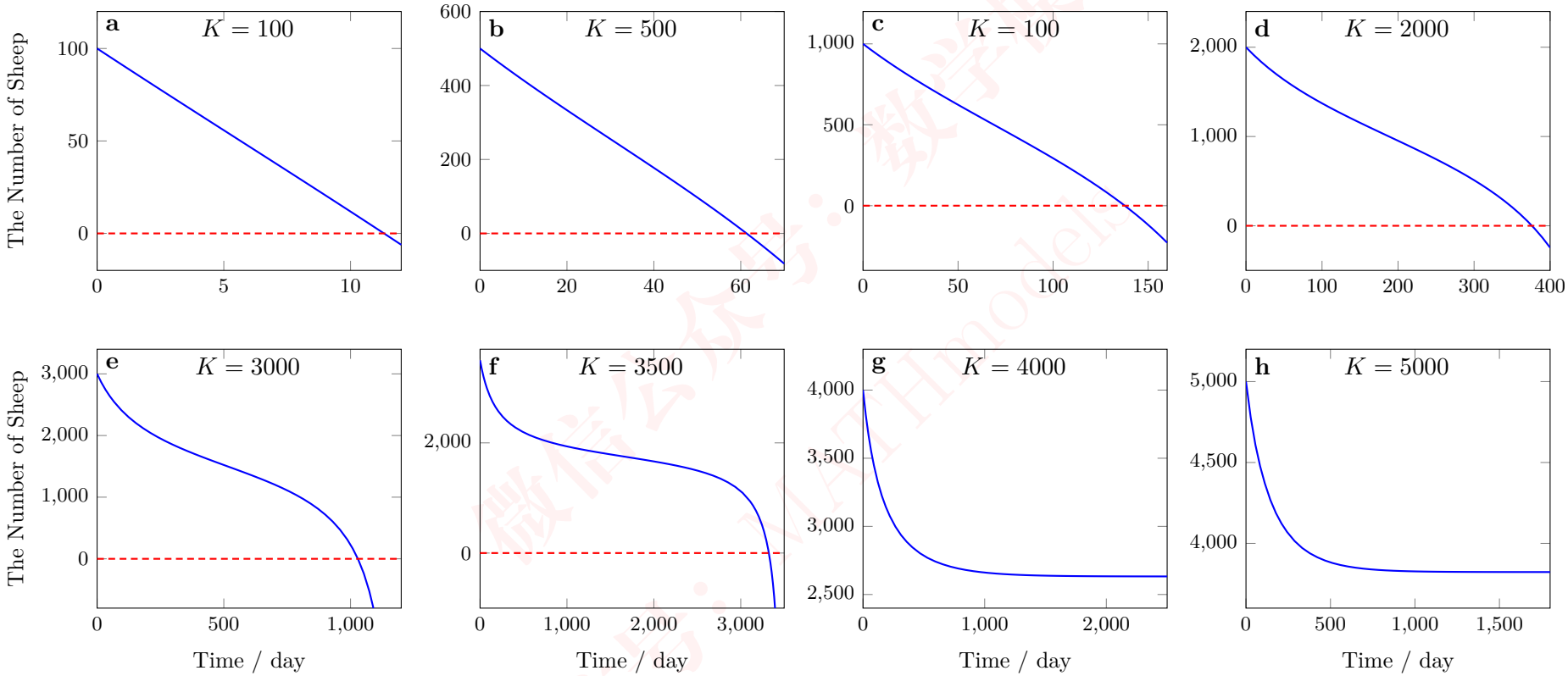

Figure 15: The curve of the function $N(t)$, on the condition of $a = 9$, $p = 0.01$ with different K values. For $K = 100$, 500, 1000, 2000, 3000 and 3500, the ecosystem collapses in 10 days (a), 2 Months (b), 5 Months (c), 1 year (d), 3 years (e) and 10 years (f) respectively. For $K = 4000$ and 5000, Ecosystem Maintains Long-Term Balance (g & h). | 当 $a = 9$, $p = 0.01$ 时，不同 K 值对应的 $N(t)$ 曲线。当 $K = 100$、500、1000、2000、3000 和 3500 时，生态系统分别在 10 天、2 月、5 月、1 年和 3 年崩溃。当 $K = 4000$ 和 5000 时，生态系统能维持长期平衡。

B Codes | 程序

Listing 1: Plotting the Curve of the Function $W(t)$

```
%% weight - time curve

p0 = 10;                % Initial weight value
k = 10694;              % Theoretical upper limit
r = 20;                 % Growth rate
a = 0.064;              % Environmental impact factor
t = 0:0.1:15';          % timeline
p_u = a*k*p0* exp(a*r*t);     % weight fraction molecule
p_l = a*k+p0*(exp(a*r*t)-1); % Weight fraction denominator
p = p_u./p_l/a;         % Weight
% Draw a curve of body weight over time
plot(t,p)
xlabel('time/year')
ylabel('w(t)/kg')
title('weight-time')
```

Listing 2: Plotting the Curve of the Function $L(t)$

```
%% Draw the image of body length changes over time

l0 = 0.8;               % Initial value of body length
l = 10*(p/10).^(1/4); % Relationship between body length and body weight
% Draw an image of body length over time
plot(t,l)
hold on
xlabel('t/year')
ylabel('length/m')
title('length-t')
```

Listing 3: Dragon-Sheep-Habitat Model

```
function [t,x] = function3(a,r,k, tspan)
% Dragon Sheep Habitat Model Code
```

```
x0 = k;
s = @(t,x) r*x.*(1-x/k) - a;
% Numerical Solution[t x] = ode45(s, tspan, x0);
% Draw horizontal line y=0
plot(t,x)
y=zeros(1,length(x));
hold on
plot(t,y)
hold off
```